Large spin bootstrap in Mellin space

Parijat Dey

CHEP, Indian Institute of Science,

Bangalore, India

Based on

arXiv: 1709.06110 with Kausik Ghosh and Aninda Sinha

arXiv: 1711.01173 with Apratim Kaviraj

Flashing the results

Anomalous dimension of large spin double trace operators:
All orders in inverse spin J

$$\gamma_\ell \sim \sum_{i=0}^{\infty} \frac{\gamma^i}{J^{2i}}$$

$$\gamma^i = -\frac{C_m}{J^{\tau_m}} \sum_{q=0}^{i} \sum_{n=0}^{i-q} \sum_{k_1=0}^{i-q-n} (-1)^n \, 2^{1+\ell_m} \, \mathfrak{b}_{k_1}(\Delta_\phi) \, \mathfrak{b}_{i-k_1-n-q}(\frac{\tau_m}{2} - \Delta_\phi + q)$$
$$\times \frac{(\tau_m + 2\ell_m - 1)\, \Gamma(-h + \ell_m + \tau_m + 1)\, \Gamma^2(2\ell_m + \tau_m - 1)}{n!\, q!\, \Gamma(1 - h + q + \ell_m + \tau_m)\, \Gamma^4\left(\ell_m + \frac{\tau_m}{2}\right)\, \Gamma(\ell_m + \tau_m - 1)}$$
$$\times \frac{\Gamma\left(q + \frac{\tau_m}{2}\right) \Gamma\left(n + q + \frac{\tau_m}{2}\right) \Gamma^2(\Delta_\phi)}{\Gamma^2\left(-n - q + \Delta_\phi - \frac{\tau_m}{2}\right)} P(q + \tau_m/2, 0)$$

Flashing the results

Wilson-Fisher point:

Anomalous dimension of higher spin operators in large spin limit

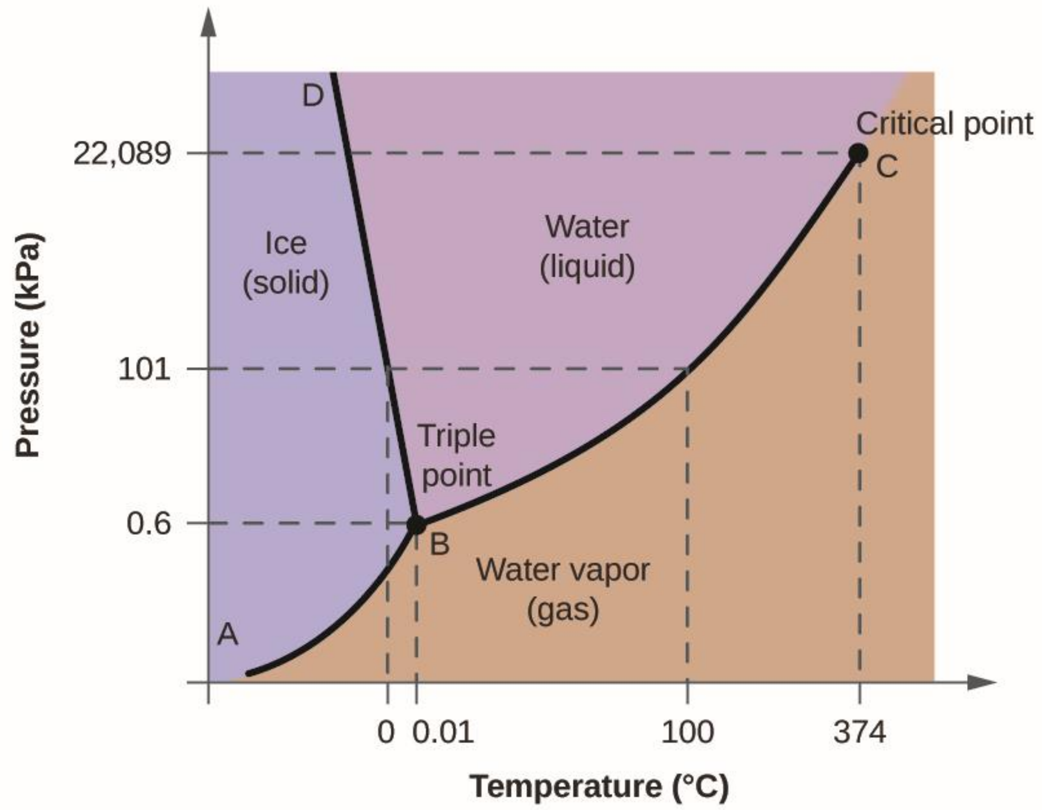
$$\Delta_\ell = 2 + \ell - \epsilon + \frac{1}{54}\epsilon^2 \left(1 - \frac{6}{\ell\left(\ell+1\right)}\right) + \delta_\ell^{(3)}\,\epsilon^3 + \delta_\ell^{(4)}\,\epsilon^4 + \delta_\ell^{(5)}\,\epsilon^5 + \cdots$$

$$\begin{aligned}\delta_\ell^{(5)} \sim \frac{\epsilon^5}{7085880\ell^2}\Big(& 2430\zeta(3)(144\log(\ell) + 144\gamma_E - 59) - 2332800\zeta(5) \\ & - 135\log(\ell)\left(24\log(\ell)(12\log(\ell) + 36\gamma_E - 41) + 48\gamma_E(18\gamma_E - 41)\right. \\ & \left. +162\pi^2 - 31\right) + 27\left(5\gamma_E(24\gamma_E(41 - 12\gamma_E) + 31) + 216\pi^4 - 810\gamma_E\pi^2\right) \\ & + 20385\pi^2 + 33770\Big)\end{aligned}$$

Plan of talk

- Review of conformal bootstrap
- Large spin bootstrap
- Analytic results for double trace operators
- Mellin bootstrap
- Comparing the two bootstraps
- Epsilon expansion at large spin
- Summary

Critical phenomena



- Critical behaviour is universal

- Governed by conformal symmetry

Critical exponents via Epsilon expansion

O(N) model

$$\int d^{4-\epsilon}x\left[(\partial_\mu\phi^i)^2+\lambda(\phi_i\phi^i)^2\right]$$

Wilson-Fisher fixed point

A large class of systems

- 3d Ising model $N=1,\epsilon=1$
- 2d Ising model $N=1,\epsilon=2$
- Superfluid He $N=2,\epsilon=1$

Critical exponents via Epsilon expansion...

- Wilson's renormalization group approach is based on Feynman diagrams

- Perturbative approach

- Locate fixed point for which the beta function vanishes

- Use Callan-Symanzik equation to determine the anomalous dimension

$$\Delta = \Delta_{free} + \gamma$$

Critical exponents via Epsilon expansion...

$$\Delta_\phi = \frac{d-2}{2} + \frac{N+2}{4\,(N+8)^2}\epsilon^2 + \frac{N+2}{4\,(N+8)^2}\left[6\frac{3N+14}{(N+8)^2} - \frac{1}{4}\right]\epsilon^3$$

$$\Delta_{\phi^2} = d - 2 + \frac{N+2}{N+8}\epsilon + \frac{N+2}{2\,(N+8)^2}(13N+44)\epsilon^2$$

Known up to epsilon^5

Kleinert et al

Ising

$$d = 3, N = 1$$

anm. dim.	critical exponent	$\epsilon = 1$	numerics	experiment
Δ_ϕ	$\eta = 2\Delta_\phi - d + 2$	0.519	0.518	0.521
Δ_{ϕ^2}	$\alpha = 2 - \frac{d}{d-\Delta_{\phi^2}}$	1.45	1.41	1.41

Critical exponents via Epsilon expansion...

- Computing anomalous dimension in Wilson-Fisher fixed point
- involves many diagrams

- OPE coefficients are even more difficult to compute, even at
- one loop: involves three point functions

- Does not make use of conformal symmetry at the fixed point

- Conformal bootstrap can be used to gain insight

Rattazzi-Rychkov-Tonni-Vichi (2008)

Ways to find the unknowns

Feynman diagrams **vs** Conformal bootstrap

Feynman diagrams		Conformal bootstrap
→ analytical		→ Mostly numerical
→ Tedious		→ Uses conformal symmetry

Another approach: uses conformal symmetry efficiently and gives an analytic handle: **Mellin bootstrap**

Conventional bootstrap:

Gives an analytic handle in the large spin limit in position space

Mellin bootstrap:

Works efficiently in Mellin space

What is the relation between the two approaches?

To answer this , we need to set up the conventional bootstrap in large spin limit in Mellin space

Bootstrap philosophy

- Focus on the CFT itself and not a specific microscopic system
  - → No Lagrangian
  - → No Feynman diagram
- Only consequences of conformal symmetry
  - → OPE associativity
  - → crossing symmetry

Part I: Conventional Bootstrap

Conventional bootstrap: Quick Review

Operator Product Expansion

$$\phi(x_1)\phi(x_2) = \sum_k \lambda_{12k}\, C(x_{12}, \partial_2)\, \phi(x_2)$$

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_{\Delta,\ell} \overbrace{C_{\Delta,\ell}}^{\text{OPE coefficient}}\, \underbrace{g_{\Delta,\ell}(u,v)}_{\text{Conformal block}}$$

cross-ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Δ_ϕ : Dimension of ϕ

s-channel

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_{\Delta,\ell} C_{\Delta,\ell}\, g_{\Delta,\ell}(u,v)$$

t-channel

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{(x_{14}^2 x_{23}^2)^{\Delta_\phi}} \sum_{\Delta,\ell} C_{\Delta,\ell}\, g_{\Delta,\ell}(v,u)$$

If OPE is associative then these two expansion must give the same result

Obtain all $\Delta, \quad C_{\Delta,\ell}$ You know everything..!!

This is the conventional bootstrap program

Bootstrap equation

s-channel = t-channel

$$\sum_{\Delta,\ell} C_{\Delta,\ell} g_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \sum_{\Delta,\ell} C_{\Delta,\ell} g_{\Delta,\ell}(v,u)$$

Difficult to solve this equation analytically and get information for general operators

Simplifies in a certain limit...large spin limit and for double trace operators

$$\ell \gg 1$$

$$v \ll u \ll 1$$

$$\sum_{\Delta,\ell} C_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \left(1 + \sum_{\Delta,\ell} C_{\Delta,\ell}\, v^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v,u)\right)$$

Large spin bootstrap

$$v \ll u \ll 1$$

Leading term

$$\sum_{\Delta,\ell} C_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\phi}$$

The r.h.s. diverges as v ~ 0.

Each term on the l.h.s. goes as $f_{\Delta,\ell}(u,v) \sim \log v$

Need to sum over infinite large spin double trace operators
on the left to reproduce the divergence on the r.h.s.

Fitzpatrick-Kaplan-Poland-Simmons-Duffin,
Komargodski-Zhiboedov, 2012

Large spin bootstrap

Double trace operators

$$O \sim \phi \partial_{\mu_1} \partial_{\mu_2} \cdots \partial_{\mu_\ell} \partial^{2n} \phi$$

The Unknowns

$$\Delta = 2\Delta_\phi + 2n + \ell + \gamma_{n,\ell}$$

Anomalous dimension

$\ell \gg 1$

$$C_{\Delta,\ell} = C_{n,\ell}(1 + \delta C_{n,\ell})$$

OPE coefficient

$n = 0$ for simplicity

Large spin bootstrap

Subleading term of the bootstrap equation

$$\sum_{\Delta,\ell} C_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \left(\sum_{\Delta,\ell} C_{\Delta,\ell}\, v^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v,u)\right)$$

Small v behavior:power of v is controlled by the twist $\tau = \Delta - \ell$

We focus on the minimal twist operators on the rhs

twist τ_m spin ℓ_m OPE coeff C_m

Expanding the lhs in small u we get $u^{\Delta_\phi} \log u \sum_\ell C_\ell \frac{\gamma_\ell}{2} f_\ell^0(v)$

Subleading term of the bootstrap equation

Coefficient of Log u term on the lhs involves the anomalous dimension

Assume the following expansion of the anomalous dimension in large spin limit

$$\gamma_\ell = \frac{1}{\ell^{\tau_m}}\left(\gamma_0 + \frac{\gamma_1}{\ell} + \cdots\right)$$

Known Mean field OPE coefficient $$C_\ell = \frac{2\,\Gamma^2(\Delta_\phi+\ell)\,\Gamma(2\Delta_\phi+\ell-1)}{\ell!\,\Gamma^2(\Delta_\phi)\,\Gamma(2\Delta_\phi+2\ell-1)}$$

Large spin sum on the lhs can be done at the leading order

Need to match this with the log u term from the rhs

Subleading term of the bootstrap equation

$$u^{\Delta_\phi}\ \log u \sum_\ell C_\ell\, \frac{\gamma_\ell}{2}\, f^0_\ell(v)$$

Summing over $\ell \gg 1$

lhs $$u^{\Delta_\phi} \log u\, v^{\tau_m/2-\Delta_\phi}\, \gamma_0\, (\cdots)$$

We have from the rhs $$\left(\frac{u}{v}\right)^{\Delta_\phi}\left(C_m\, v^{\frac{\tau_m}{2}}\ f_{\tau_m,\ell_m}(v,u)\right)$$

Compare the log u term

$$\gamma_0 = -\frac{2\Gamma^2\left(\Delta_\phi\right)\Gamma\left(2\ell_m+\tau_m\right)}{\Gamma^2\left(\ell_m+\frac{\tau_m}{2}\right)\Gamma^2\left(\Delta_\phi-\frac{\tau_m}{2}\right)}\left(\frac{1}{\ell}\right)^{\tau_m} C_m,$$

Large spin bootstrap

Subleading order corrections can be done following   Alday-Zhiboedov (2015)

Change of variable $\ell \to J = (\ell + \Delta_\phi)(\ell + \Delta_\phi - 1)$

Anomalous dimension: asymptotic expansion in inverse J

$$\gamma_\ell \sim \frac{1}{J^{\tau_m}}\left(\gamma_0 + \frac{\gamma_1}{J^2} + \cdots\right)$$

Involves two recursion relations that one needs to solve

A bit more involved...

Can we do better?

Simplifies in Mellin space...gives an all order expression easily!

Mellin transform

$$\tilde{f}(s) = \int_0^\infty dx\, x^{s-1}\, f(x)$$

Inverse Mellin transform

$$f(x) = \frac{1}{2\,\pi} \int_{-i\infty}^{i\infty} ds\, x^{-s}\, \tilde{f}(s)$$

Mellin transform is relevant for a power law decomposition of a function...like the fourier transform disaplays the harmonic decomposition of a function.

For $\quad \tilde{f}(s) = \dfrac{1}{s-\Delta}$

Scaling behavior of CFT correlation function

$$f(x) = \frac{1}{x^\Delta}$$

Mellin representation captures conformal symmetry automatically

The Mellin amplitude

Mack (2009), Penedones, Costa-Goncalves- Penedones ,
Fitzpatrick-Kaplan- Penedones-Raju, van Rees ....

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \mathcal{A}(u,v)$$

Mellin transform of the 4-pt function

$$\mathcal{A}(u,v) = \int ds\,dt\,u^s\,v^t\,\Gamma(-t)^2\,\Gamma(s+t)^2\,\Gamma(\Delta_\phi - s)^2\,\underbrace{\mathcal{M}(s,t)}_{\textbf{Mellin amplitude}}$$

Mellin transform of s channel conformal block

$$G(u,v)=\sum_\ell \int \frac{ds}{2\pi i}\frac{dt}{2\pi i} u^s\, v^t\, \Gamma^2(s+t)\,\Gamma^2(-t)\,\Gamma^2(\Delta_\phi - s)\, q_\ell^{(s)}(s)\, Q_{\ell,0}^{2s+\ell}(t)$$

Continuous Hahn polynomial

$$Q_{\ell,0}^{2s+\ell}(t)=\frac{2^\ell\,((s)_\ell)^2}{(2s+\ell-1)_\ell}\, {}_3F_2\left[\begin{matrix}-\ell,\ 2s+\ell-1,\ s+t \\ s\ \ ,\qquad s\end{matrix};1\right]$$

These are orthogonal polynomials

$$\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dt\ \Gamma^2(s+t)\Gamma^2(-t)Q_{\ell,0}^{2s+\ell}(t)Q_{\ell',0}^{2s+\ell'}(t) \sim \text{func}(s)\ \delta_{\ell,\ell'}$$

Play a key role in repackaging the equations in Mellin space

Strategy in Mellin space

Double pole at $s = \Delta_\phi$ implies log u and non-log term

Idea now is to look at the Mellin transform of the t channel conformal block and expand the t-dependence in terms of the continuous Hahn polynomials

Boils down to simple equations in Mellin space

Log term gives the anomalous dimension whereas the non log term gives the OPE coefficient of the large spin operators

Bootstrap equation in Mellin space

$$\sum_\ell u^{\Delta_\phi} \log u \int dt\, v^t\, \Gamma^2(\Delta_\phi + t)\Gamma^2(-t)\, (\mathfrak{q}^s_\ell + 2\,\mathfrak{q}^t_\ell)\, Q^{2\Delta_\phi+\ell}_\ell(t) = 0$$

$$\boxed{\mathfrak{q}^s_\ell + 2\,\mathfrak{q}^t_\ell = 0}$$

$$\mathfrak{q}^s_\ell = \beta_\ell(\Delta_\phi)\, \gamma_\ell$$

$$\mathfrak{q}^t_\ell = \sum_{r=0}^{\infty} \beta_\ell(\Delta_\phi)\, C_m\, \frac{(-1)^r}{r!} \Gamma^2(\tau_m/2 + r)\Gamma(h - \Delta_m - r)$$
$$\times\, P(\tau_m/2 + r, 0)\, {}_3F_2\left[\begin{matrix} -\ell,\ 2\Delta_\phi + \ell - 1,\ \tau_m/2 + r \\ \Delta_\phi\ ,\qquad \Delta_\phi \end{matrix}; 1\right]$$

Need the large spin behavior of continuous Hahn polynoimal.

In Mellin space approach the large spin behavior of continuous Hahn polynomial is the key ingredient

These polynomials can be derived from Wilson polynomials in a particular limit

Wilson in 1991 worked out the large argument asymptotics of the Wilson polynomial

Wilson 1991

Use these results to derive the large spin behavior of continuous Hahn polynomial

Asymptotics of continuous Hahn polynomial

PD, K. Ghosh, A. Sinha- 1709.06110

$$
{}_3F_2\left[\begin{matrix}-\ell,\ 2s+\ell-1,\ s+t\\ s\quad,\qquad s\end{matrix};1\right] \sim \sum_{n,k_1,k_2=0}^{\infty} \frac{(-1)^n\,\Gamma^2(s)\,(s+t)_n}{n!\Gamma^2(-t-n)}\,\mathfrak{b}_{k_1}(s)\,\mathfrak{b}_{k_2}(t)J^{-\alpha}
$$

$$
J^2 = (\ell+s)(\ell+s-1)
$$

$$
\alpha = 2k_1 + 2k_2 + 2n + 2s + 2t
$$

Generalised Bernoulli polynomial

$$
\mathfrak{b}_{k_1}(s) \sim \sum_{j=0}^{k_1} \frac{\Gamma(2s-2+2j)}{(2j)!\Gamma(2s-2)}\left(-\frac{1}{2}\right)^{2k_1-2j}\mathcal{B}_{2j}^{3-2s}\left(\frac{3-2s}{2}\right)
$$

$$
\mathfrak{b}_{k_2}(t) \sim \sum_{j=0}^{k_1} \frac{\Gamma(2n+2t+2+2j)}{(2j)!\Gamma(2n+2t+2)}\left(-\frac{1}{2}\right)^{2k_2-2j}\mathcal{B}_{2j}^{-1-2n-2t}\left(\frac{-1-2n-2t}{2}\right)
$$

Only even powers of J will appear

In the large spin limit $$\gamma_\ell \sim \sum_{i=0}^{\infty} \frac{\gamma^i}{J^{2i}}$$

Large spin expansion of anomalous dimension involves only even powers of J. This becomes transparent in Mellin space.

Comparing powers of J from both sides of the bootstrap equation will fix the anomalous dimension at all orders in inverse J

Result: Anomalous dimension

$$\gamma^i = -\frac{C_m}{J^{\tau_m}} \sum_{q=0}^{i} \sum_{n=0}^{i-q} \sum_{k_1=0}^{i-q-n} (-1)^n \, 2^{1+\ell_m} \, \mathfrak{b}_{k_1}(\Delta_\phi) \, \mathfrak{b}_{i-k_1-n-q}(\frac{\tau_m}{2} - \Delta_\phi + q)$$

$$\times \frac{(\tau_m + 2\ell_m - 1) \, \Gamma(-h + \ell_m + \tau_m + 1) \, \Gamma^2(2\ell_m + \tau_m - 1)}{n! \, q! \, \Gamma(1 - h + q + \ell_m + \tau_m) \, \Gamma^4\left(\ell_m + \frac{\tau_m}{2}\right) \Gamma(\ell_m + \tau_m - 1)}$$

$$\times \frac{\Gamma\left(q + \frac{\tau_m}{2}\right) \Gamma\left(n + q + \frac{\tau_m}{2}\right) \Gamma^2(\Delta_\phi)}{\Gamma^2\left(-n - q + \Delta_\phi - \frac{\tau_m}{2}\right)} P(q + \tau_m/2, 0)$$

Expression for anomalous dimension to all orders in 1/J

e.g. i = 0

$$\gamma_0 = -\frac{2\Gamma^2(\Delta_\phi) \, \Gamma(2\ell_m + \tau_m)}{\Gamma^2\left(\ell_m + \frac{\tau_m}{2}\right) \Gamma^2\left(\Delta_\phi - \frac{\tau_m}{2}\right)} \left(\frac{1}{J}\right)^{\tau_m} C_m,$$

Result: OPE coefficient

Similar expression for OPE coefficient comes from the non log term

$$\delta C_{0,\ell} = \sum_{i=0}^{\infty} \frac{\delta C_{0,\ell}^{(i)}}{J^{2i}}$$

$$\delta C_{0,\ell}^{(i)} = \frac{\text{function}(\tau_{\rm m}, \Delta_\phi, \ell_{\rm m})}{J^{\tau_m}}\, C_m$$

Given in terms of generalised Bernoulli polynomial and Mack polynomial

Part II: Mellin Bootstrap

Mellin bootstrap

Polyakov 1974

Gopakumar-Kaviraj-Sen-Sinha, 2017

- A different way of attacking the problem

- Expand the four point function in terms
- of **crossing-symmetric** blocks

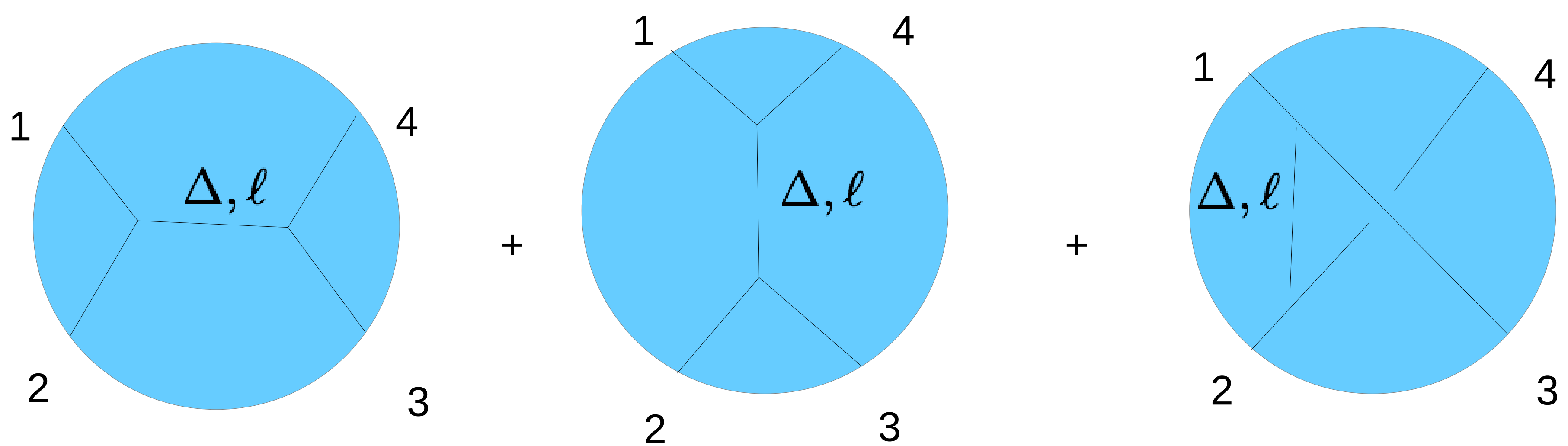
- This description should be consistent with the OPE .

- **These consistency conditions gave nontrivial information**
- **about the CFT .**

New bootstrap

Expand the four point function in terms of **Witten diagrams, instead of conformal blocks**

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \sum_{\Delta,\ell} c_{\Delta,\ell}(W^{(s)}_{\Delta,\ell}(u,v) + W^{(t)}_{\Delta,\ell}(u,v) + W^{(u)}_{\Delta,\ell}(u,v))$$



- Easier to deal in Mellin space...more elegant

Mellin transform of Witten diagram

$$\mathcal{A}(u,v) = \int ds\, dt\, u^s\, v^t\, \Gamma(s+t)^2\, \Gamma(-t)^2\, \Gamma(\Delta_\phi - s)^2\, \mathcal{M}(s,t)$$

$$M^{(s)}(s,t) = \sum_{\Delta,\ell} \frac{C_{\Delta,\ell}}{2s - \Delta + \ell - 2m} \times Q^{\Delta}_{\ell,m}(t)$$

The Mellin amplitude has poles in s (simple poles, Gamma functions)

The Mellin integral can be evaluated using residue theorem

Evaluate residues at the poles

Physical poles

$$\mathcal{A}(u,v) = \int ds\, dt\, u^s\, v^t\, \Gamma(s+t)^2\, \Gamma(-t)^2\, \Gamma(\Delta_\phi - s)^2\, \mathcal{M}(s,t)$$

$$M^{(s)}(s,t) = \sum_{\Delta,\ell} \frac{C_{\Delta,\ell}}{2s-\Delta+\ell-2m} \times Q^{\Delta}_{\ell,m}(t)$$

$$u^{\frac{\Delta-\ell}{2}} \longleftarrow s = \frac{\Delta-\ell}{2}$$

s-channel conformal block expansion

$$\langle\phi\phi\phi\phi\rangle = \sum_{\Delta,\ell} C_{\Delta,\ell}\, g_{\Delta,\ell} = \sum_{\Delta,\ell} C_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}} (1-v)^\ell + \text{higher powers}$$

The s-channel OPE is exactly reproduced..!!

Unphysical poles

$$\mathcal{A}(u,v) = \int ds\, dt\, u^s\, v^t\, \Gamma(s+t)^2\, \Gamma(-t)^2\, \Gamma(\Delta_\phi - s)^2\, \mathcal{M}(s,t)$$

Double pole

Residue at $s = \Delta_\phi + n$

$\downarrow$

(schematically) $u^{\Delta_\phi + n} \log u(\cdots) + u^{\Delta_\phi + n}(\cdots)$ **not present in the OPE**

Must cancel

.Physical poles: Residue expected from the OPE

.Unphysical poles: Residues that are impossible to appear in the OPE

The unphysical terms

The unphysical terms from the s, t and u-channels can also be expanded in the Q polynomials

$$\mathcal{A}(u,v) = \int ds\, dt\, u^s\, v^t\, \Gamma(s+t)^2\, \Gamma(-t)^2\, \Gamma(\Delta_\phi - s)^2\, \mathcal{M}(s,t)$$

$$\sum_\ell \overset{\overset{0}{\|}}{\boxed{q_\ell^s(\Delta_\phi)}}\, Q_\ell^{\Delta_\phi}(t)\, u^{\Delta_\phi}\, \log u + \overset{\overset{0}{\|}}{\boxed{q_\ell^{s\,\prime}(\Delta_\phi)}}\, Q_\ell^{\Delta_\phi}(t)\, u^{\Delta_\phi}$$ from the s-channel

$$\sum_\ell \boxed{q_\ell^t(\Delta_\phi)}\, Q_\ell^{\Delta_\phi}(t)\, u^{\Delta_\phi}\, \log u + \boxed{q_\ell^{t\,\prime}(\Delta_\phi)}\, Q_\ell^{\Delta_\phi}(t)\, u^{\Delta_\phi}$$ from the t-channel

$$\sum_\ell \boxed{q_\ell^u(\Delta_\phi)}\, Q_\ell^{\Delta_\phi}(t)\, u^{\Delta_\phi}\, \log u + \boxed{q_\ell^{u\,\prime}(\Delta_\phi)}\, Q_\ell^{\Delta_\phi}(t)\, u^{\Delta_\phi}$$ from the u-channel

Mellin bootstrap equations

$$q_\ell^s(\Delta_\phi) + q_\ell^t(\Delta_\phi) + q_\ell^u(\Delta_\phi) = 0$$

$$q_\ell^{s\prime}(\Delta_\phi) + q_\ell^{t\prime}(\Delta_\phi) + q_\ell^{u\prime}(\Delta_\phi) + 2q_\ell^{identity}(\Delta_\phi) = 0$$

Only one spin appears in the s-channel unphysical term

But all spins appear in the t and u-channel unphysical terms

Conventional bootstrap vs Mellin bootstrap

$$q^s_\ell(\Delta_\phi) - \mathfrak{q}^s_\ell(\Delta_\phi) \sim O(\gamma_\ell^2)$$

$$q^t_\ell(\Delta_\phi) + q^u_\ell(\Delta_\phi) = 2\mathfrak{q}^t_\ell(\Delta_\phi) \qquad \ell \gg 1$$

The two bootstraps are equivalent to leading order in anomalous dimension, but to all orders in inverse spin J

Difference between the two approaches in the large spin limit $O(\gamma_\ell^2)$

Reflecting the polynomial ambiguities in Mellin amplitudes

Epsilon expansion in the large spin limit

$$S = \int d^{4-\epsilon}x \left[\frac{1}{2}(\partial\phi)^2 + \lambda\phi^4\right]$$

$$\Delta_\phi = 1 - \frac{1}{2}\epsilon + \frac{1}{108}\,\epsilon^2 + \frac{109}{11664}\,\epsilon^3 + \delta_\phi^{(4)}\,\epsilon^4 + \delta_\phi^{(5)}\,\epsilon^5 + \cdots$$

$$\Delta_{\phi^2} = 2 - \frac{2}{3}\,\epsilon + \frac{19}{162}\,\epsilon^2 + \delta_0^{(3)}\,\epsilon^3 + \delta_0^{(4)}\,\epsilon^4 + \delta_0^{(5)}\,\epsilon^5 + \cdots$$

Feynman diagram computations

Known up to epsilon^5

Kleinert et al

Epsilon expansion in the large spin limit

Higher spin operators $\phi \partial_{\mu_1} \partial_{\mu_2} \cdots \partial_{\mu_\ell} \phi$

$$\Delta_\ell = 2 + \ell - \epsilon + \frac{1}{54}\epsilon^2 \left(1 - \frac{6}{\ell(\ell+1)}\right) + \delta_\ell^{(3)}\,\epsilon^3 + +\delta_\ell^{(4)}\,\epsilon^4 + \cdots$$

Known upto epsilon^4 Derkachov-Gracey-Manashov

Using known results from Feynman diagrams and OPE results from Mellin bootstrap one can compute

$$(\cdots)\frac{\epsilon^5}{\ell^2}, \quad (\cdots)\frac{\epsilon^5}{\ell^3}$$

Sampling of new result: Large spin anm. dimension

PD, A. Kaviraj
arXiv:1711.01173

$$\delta_\ell^{(5)} \sim \frac{\epsilon^5}{7085880\ell^2}\Big(2430\zeta(3)(144\log(\ell)+144\gamma_E-59)-2332800\zeta(5)$$
$$-135\log(\ell)\left(24\log(\ell)(12\log(\ell)+36\gamma_E-41)+48\gamma_E(18\gamma_E-41)\right.$$
$$\left.+162\pi^2-31\right)+27\left(5\gamma_E(24\gamma_E(41-12\gamma_E)+31)+216\pi^4-810\gamma_E\pi^2\right)$$
$$+20385\pi^2+33770\Big)$$

One can also compute the OPE coefficients

Summary

Mellin space techniques can be used to obtain the
CFT data for large spin double trace operators to all orders in
inverse conformal spin

Relation between the usual bootstrap and Mellin bootstrap

Can be extended for higher twist operators

External operators with spin

Epsilon expansion using conventional bootstrap

Thank you