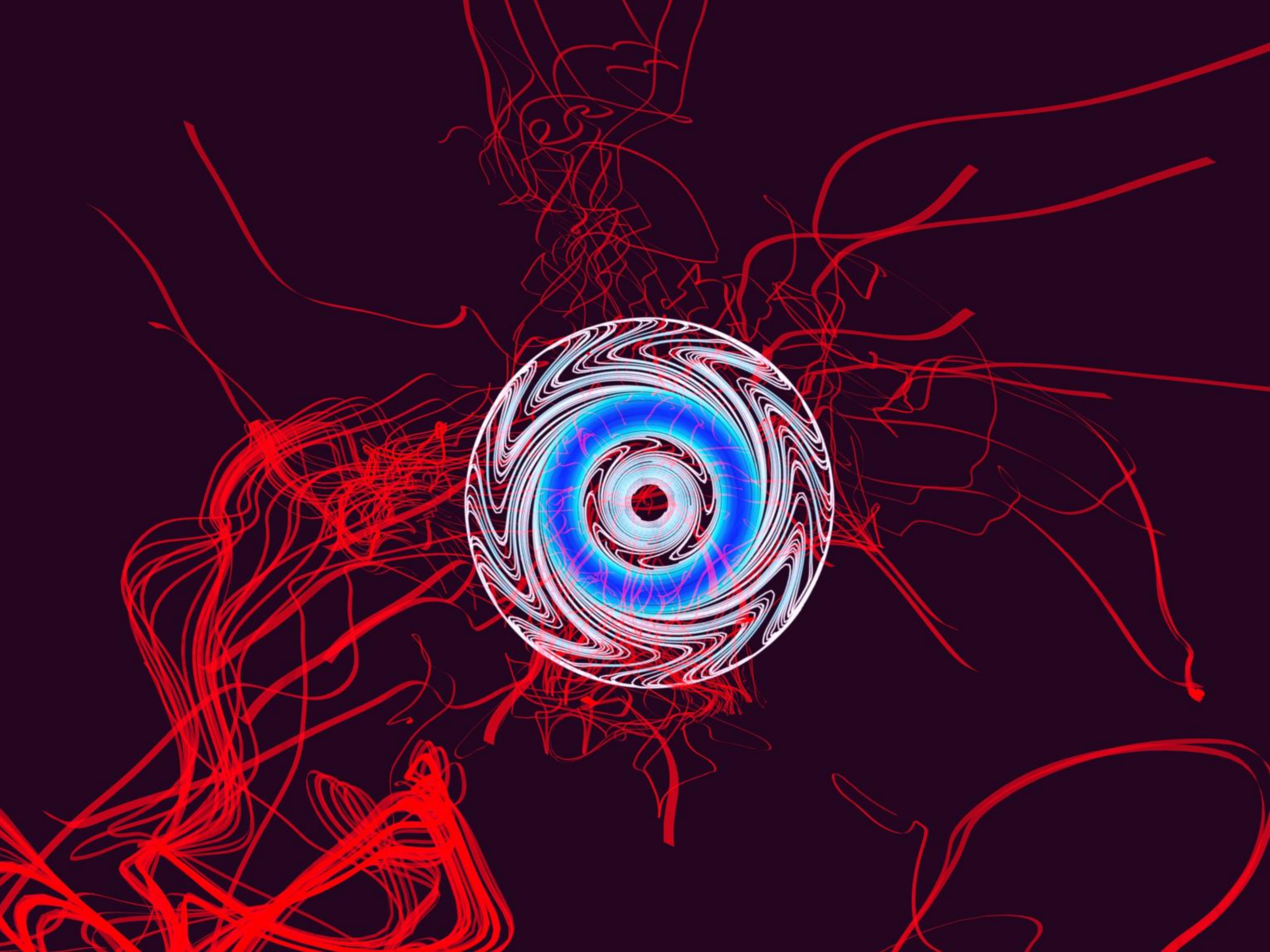
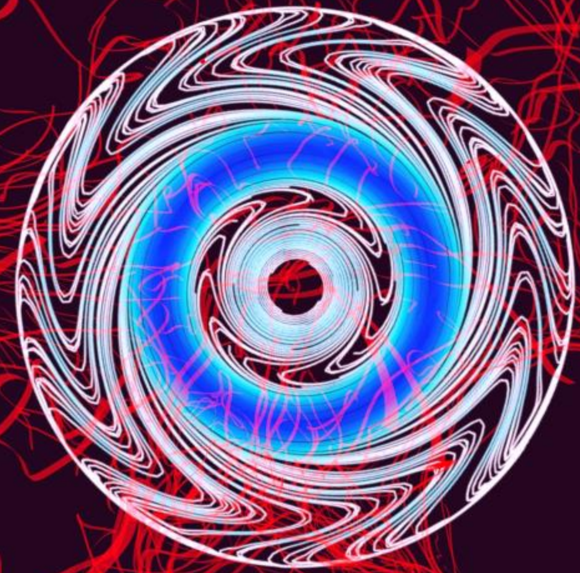


Statistical Mechanics of Turbulence

Bangalore School on Statistical Physics XI



Friction: P. Chakraborty, G. Gioia, H. Kellay, T. Tran, W. Goldburg
Transition: Hong-Yan Shih, Tsung-Lin Hsieh

Partially supported by NSF

Syllabus

1. Philosophy: turbulence, renormalization group and statistical mechanics
2. What is the BIG QUESTION we want to answer?
3. Why is turbulence REALLY HARD?

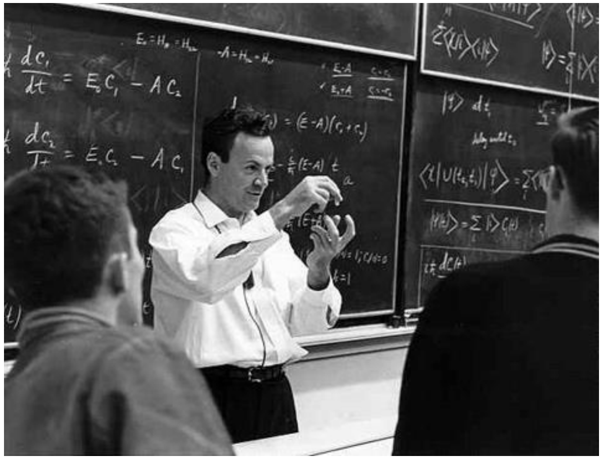
4. Lecture 1: Transition to Turbulence
5. Lecture 2: Fluctuations, dissipation and turbulence

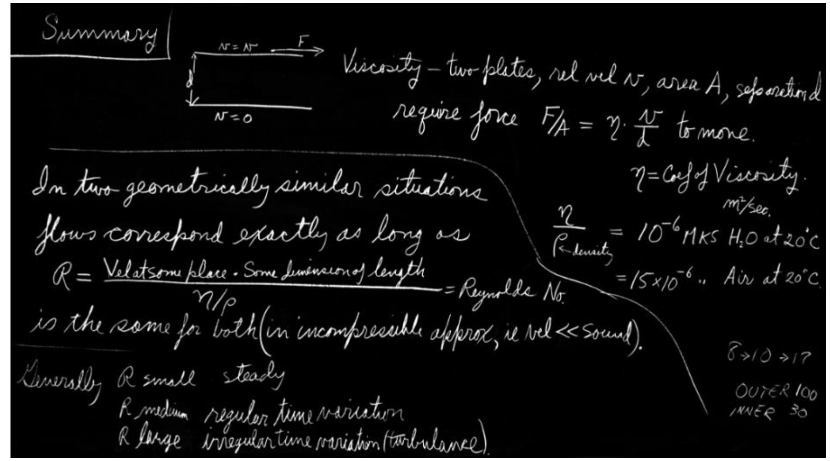
Extra things you will learn!

- How to think of turbulence in a new way
- Advanced similarity methods
- How to use computer simulations to make discoveries and drive analytical work
- Some taste of trans-disciplinary research

Propaganda

Feynman's vision: RG & Turbulence

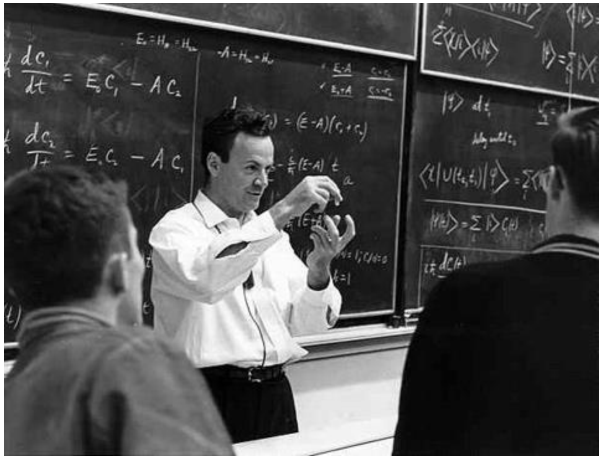


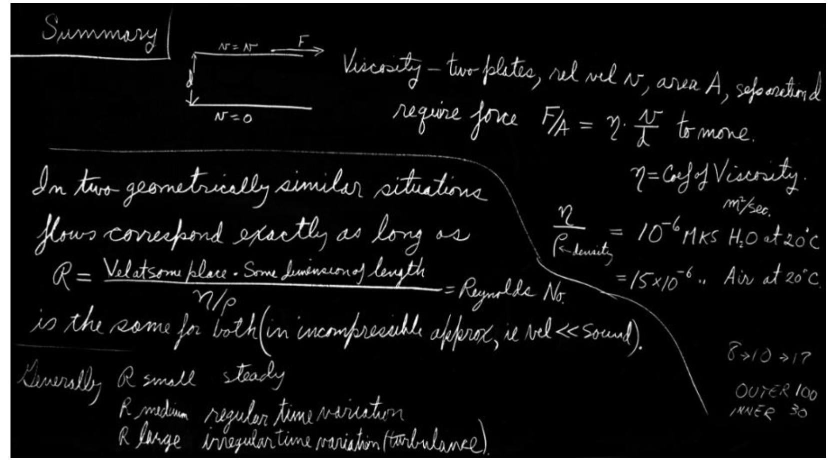


We have written the equations of water flow. From experiment, we find a set of concepts and approximations to use to discuss the solution—vortex streets, turbulent wakes, boundary layers. When we have similar equations in a less familiar situation, and one for which we cannot yet experiment, we try to solve the equations in a primitive, halting, and confused way to try to determine what new qualitative features may come out, or what new qualitative forms are a consequence of the equations.

http://www.feynmanlectures.caltech.edu/II_41.html Scientific American

Feynman's vision: RG & Turbulence



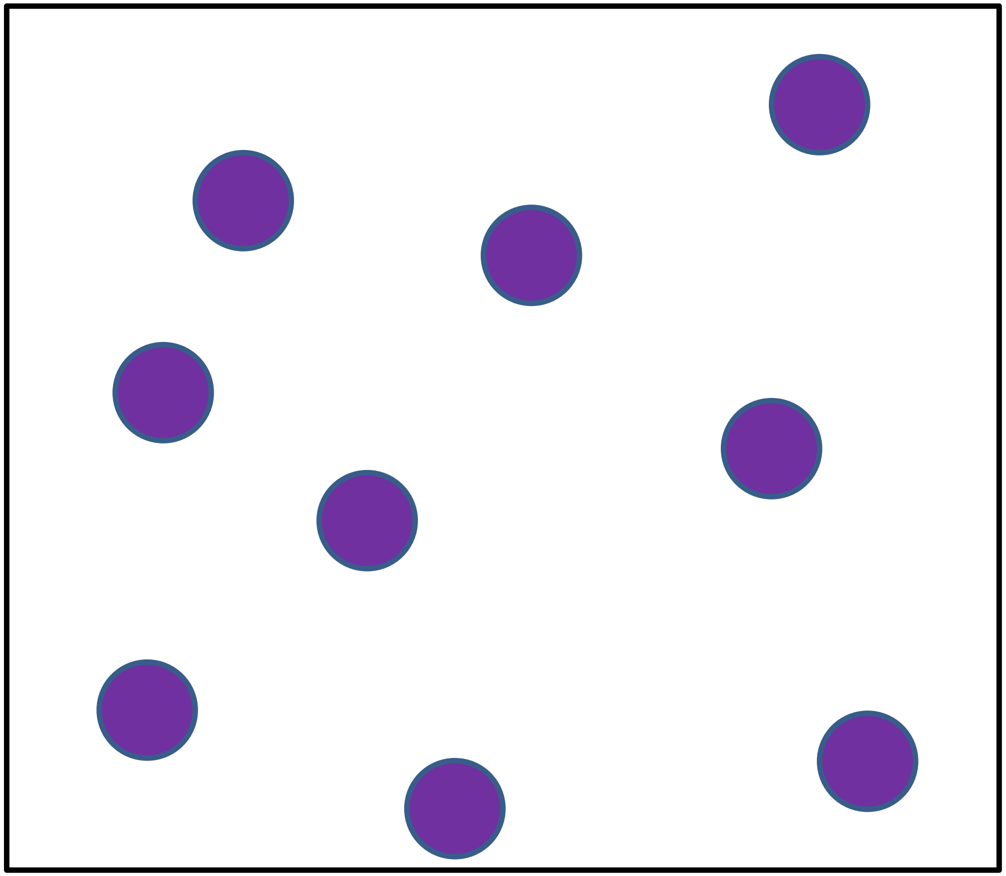


The next great era of awakening of human intellect may well produce a method of understanding the *qualitative* content of equations. Today we cannot. Today we cannot see that the water flow equations contain such things as the barber pole structure of turbulence that one sees between rotating cylinders. Today we cannot see whether Schrödinger's equation contains frogs, musical composers, or morality—or whether it does not.

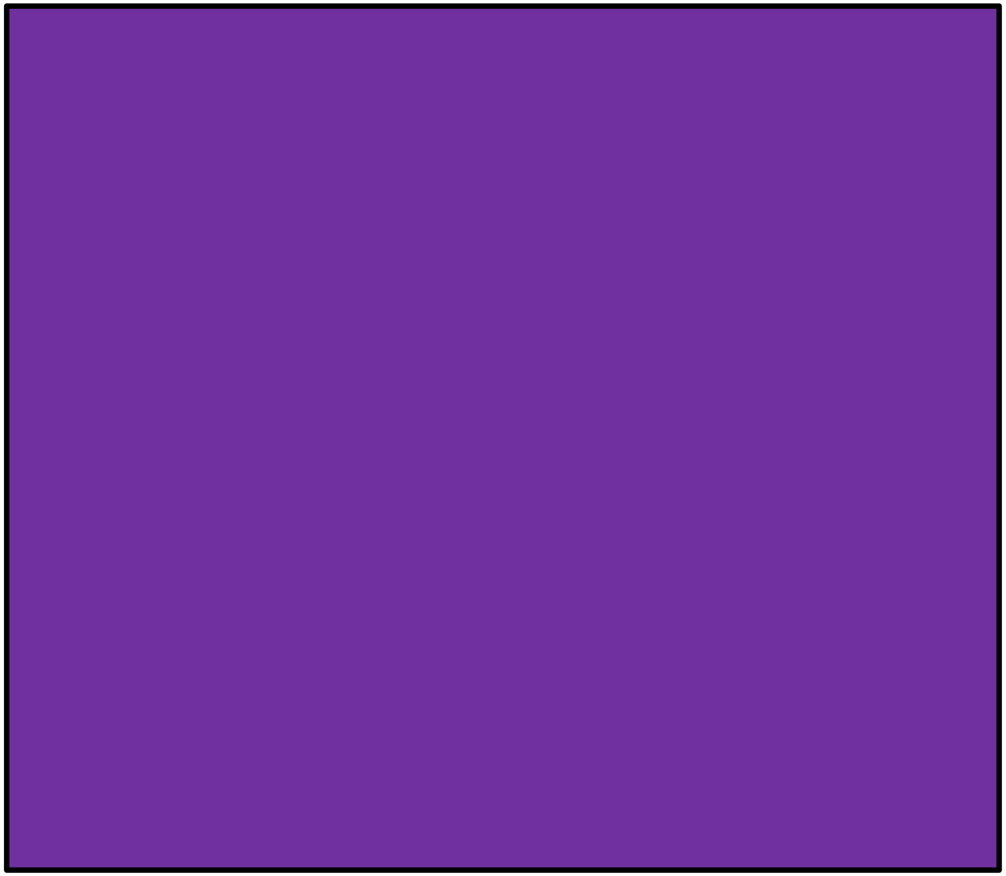
http://www.feynmanlectures.caltech.edu/II_41.html Scientific American

Goal

- The qualitative behavior of matter is the domain of statistical mechanics
- **Our goal: expose the qualitative content of the equations of fluid mechanics, by understanding the phase diagram, universality and scaling laws of turbulence.**
- A start: novel predictions and perspectives based on statistical mechanics.
  - Transitional flows
  - Fluctuation-dissipation relations



Deterministic classical mechanics of many particles in a box ➔ statistical mechanics



Deterministic classical mechanics of infinite number of particles in a box

= Navier-Stokes equations for a fluid

➔ statistical mechanics

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

Deterministic classical mechanics of infinite number of particles in a box

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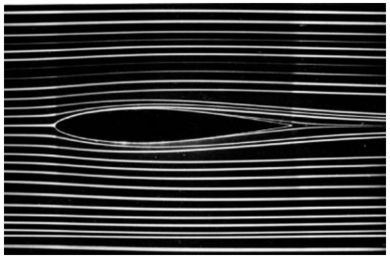
What is turbulence?



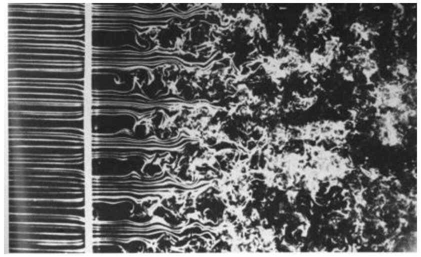
laminar flow

steady

predictable

laminar-turbulent transition

critical behavior



fully-developed turbulence

fluctuating

unpredictable

Re < 1600

Re ~ 2000

Re > 10^5

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

$$\mathrm{Re} = \frac{UL}{\nu}$$

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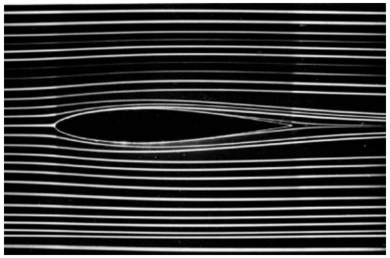

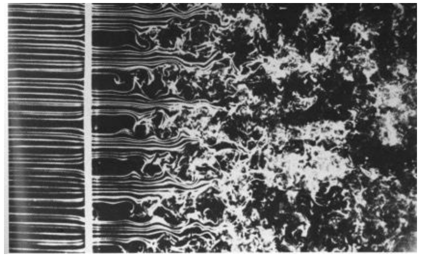

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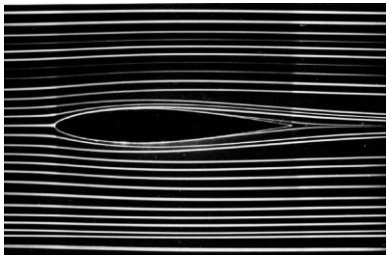
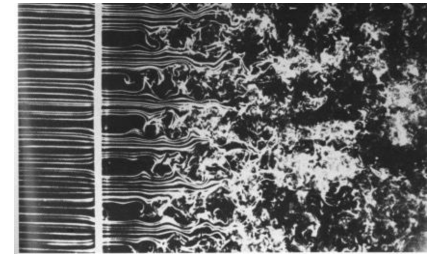
$$\mathrm{Re} = \frac{UL}{\nu}$$

$$\upsilon = \mu/\rho$$

What is turbulence?





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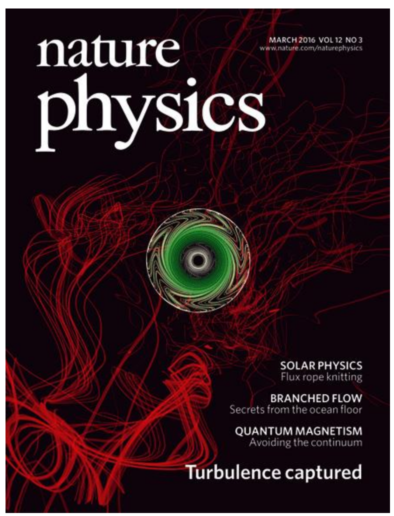
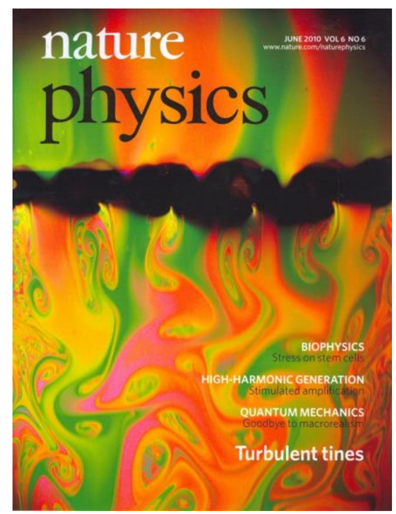
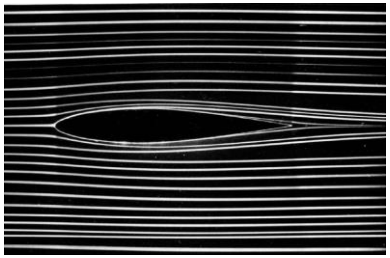


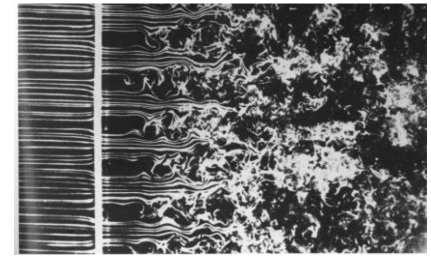


Death

Life

What is turbulence?





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Transitional pipe flow

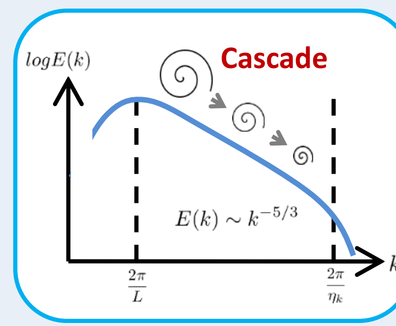
Puff decays

Splitting puffs

laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

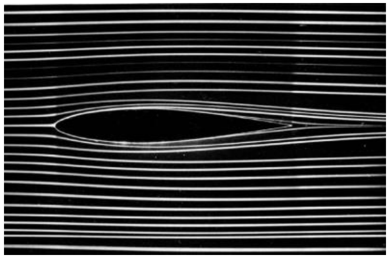
Re — 1775 — 2100 — 2500

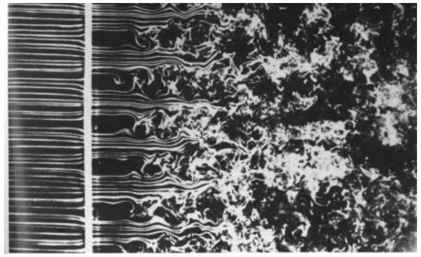
Turbulence & mean flows



$$\epsilon = \nu |\nabla u|^2 \neq 0 \text{ as } \nu \to 0$$

Take-home: 2 types of universality in turbulence





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Critical points with their own scaling laws, crossovers and universality classes

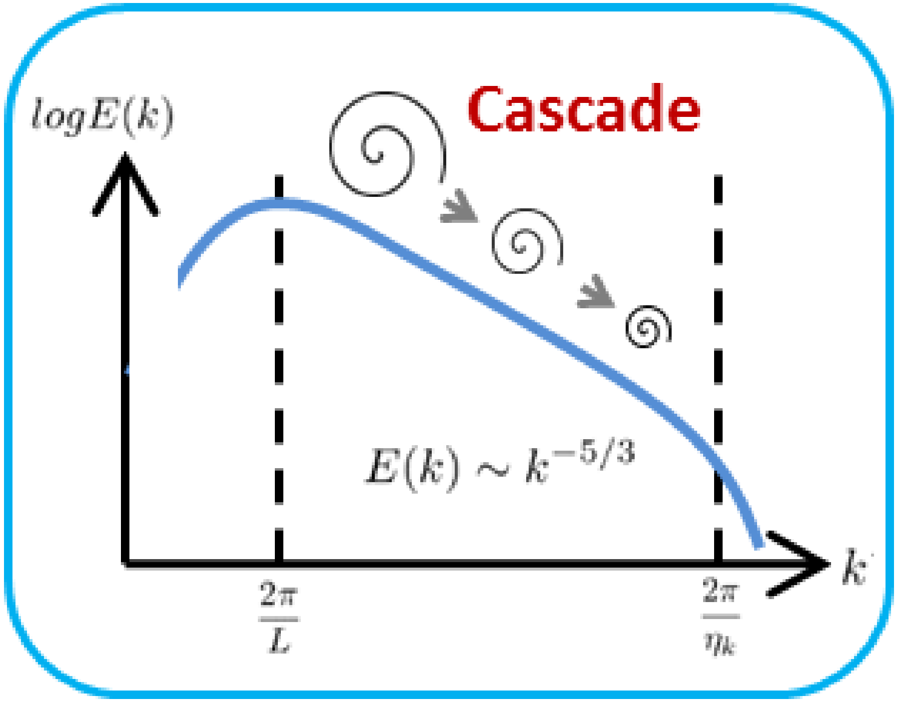
Unifying concepts of fluctuation-dissipation, rare events, mean-flow interactions

What does it mean: “solve turbulence”?

Solve turbulence? Predict the fluctuations at small scales

This is a typical answer (by physicists)

Energy cascade



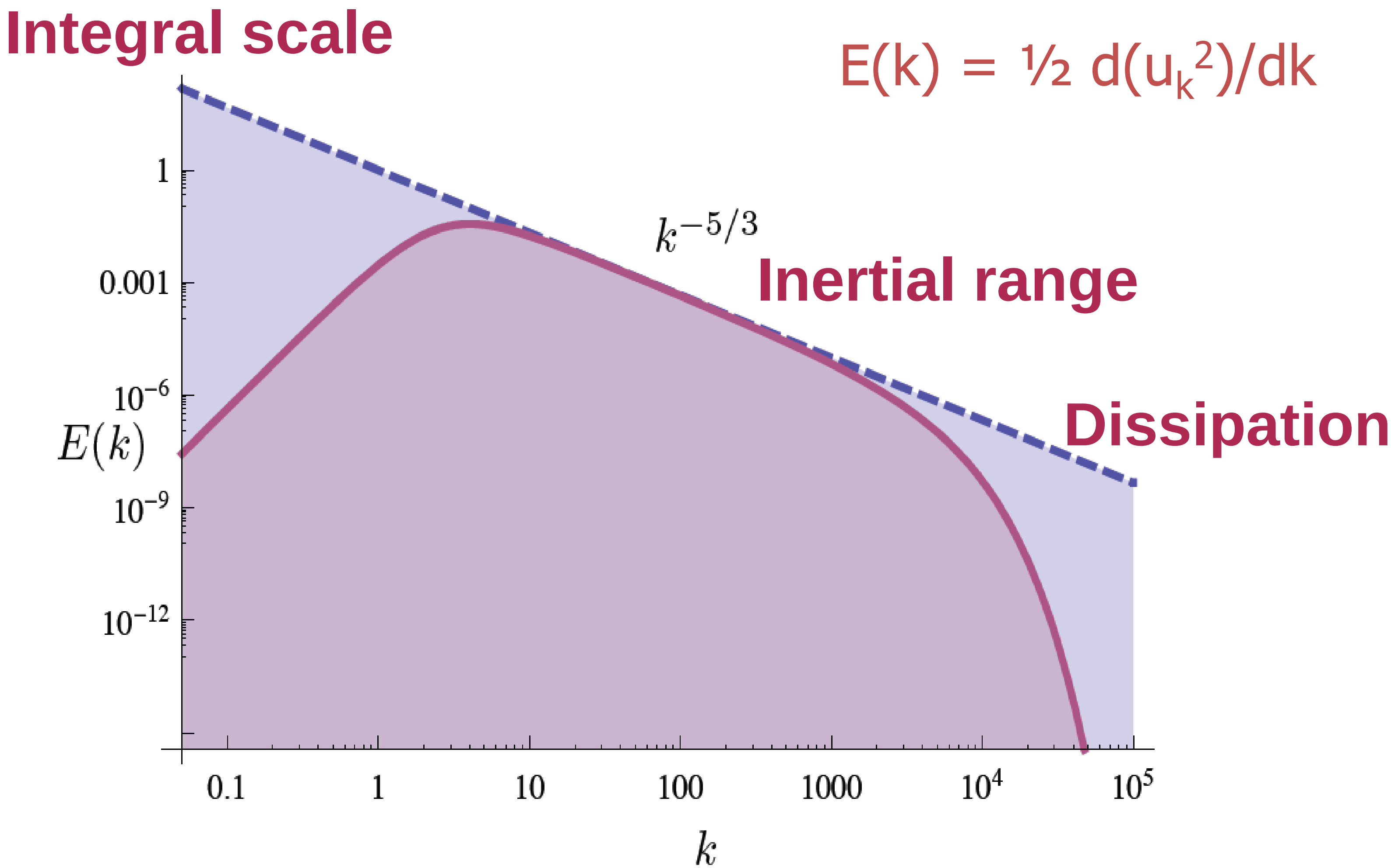
- Eddies spin off other eddies in a Hamiltonian process.
  - Does not involve friction!
  - Hypothesis due to Richardson, Kolmogorov, …
- Implication: viscosity will not enter into the equations

Kolmogorov's similarity hypotheses

- Dimensional analysis
- $E(k) = E(k, \varepsilon, \nu, L)$
  - Kolmogorov length $\eta_K = (\nu^3/\varepsilon)^{1/4}$
- $E(k) = (\nu^2/\eta_K)\, F(k\eta_K, kL)$
  - Complete similarity as $kL \rightarrow \infty$
- $E(k) = (\nu^2/\eta_K)\, F(k\eta_K, \infty)$
  - Complete similarity as $k\eta_K \rightarrow 0$
- $E(k) = F(0, \infty)\, \varepsilon^{2/3}\, k^{-5/3}$

The energy spectrum

$E(k) = \frac{1}{2}\, d(u_k^2)/dk$



Kolmogorov's similarity hypotheses

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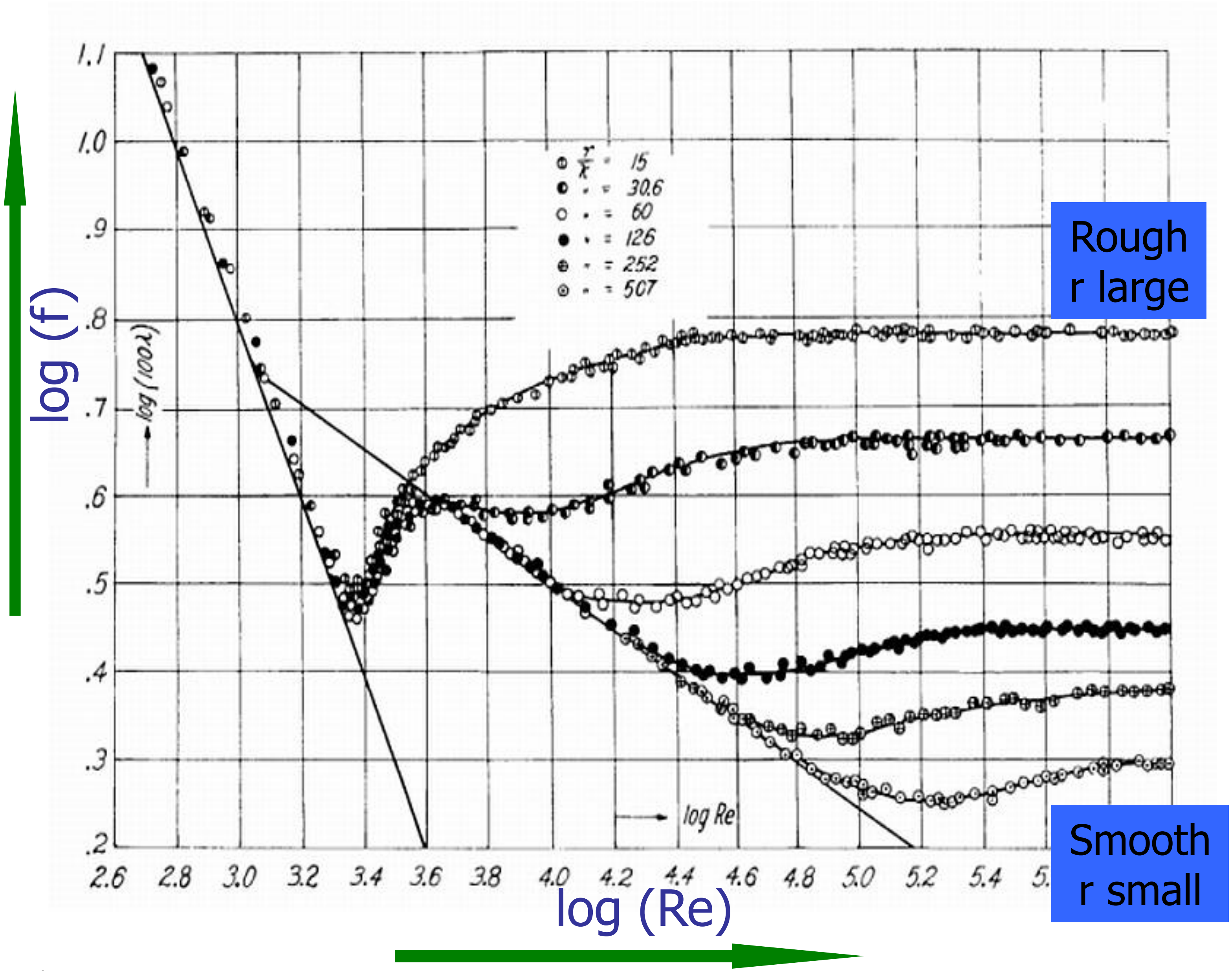
- $E(k) = E(k, \varepsilon, \nu, L)$
  - Kolmogorov length $\eta_K = (\nu^3/\varepsilon)^{1/4}$
- $E(k) = (\nu^2/\eta_K)\, F(k\eta_K, kL)$
  - Incomplete similarity as $kL \to \infty$
- $E(k) = (\nu^2/\eta_K)\, F_1(k\eta_K)\, (kL)^\eta$
  - Complete similarity as $k\eta_K \to 0$
- $E(k) = F_1(0)\, \varepsilon^{2/3}\, k^{-5/3}\, (kL)^\eta$

Intermittency exponent

Solve turbulence? Predict the dissipation experienced at large scales …

This is a typical answer (by engineers)

Friction factor in turbulent rough pipes

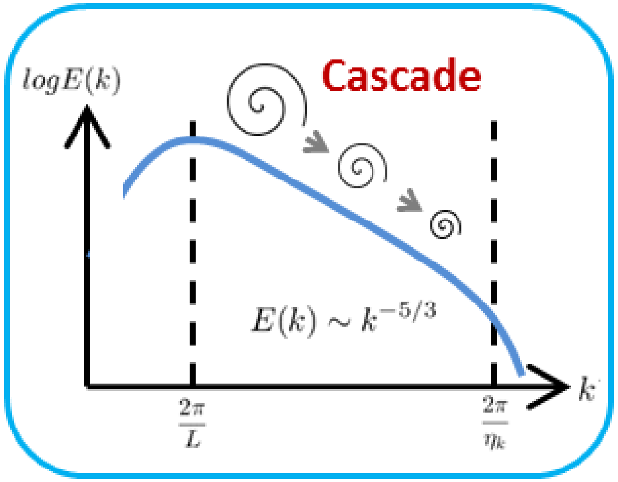


Nikuradse (1933)

Solve turbulence? Connect the scales …

Predict the dissipation experienced at large scales from the fluctuations at small scales

Fluctuations and Dissipation



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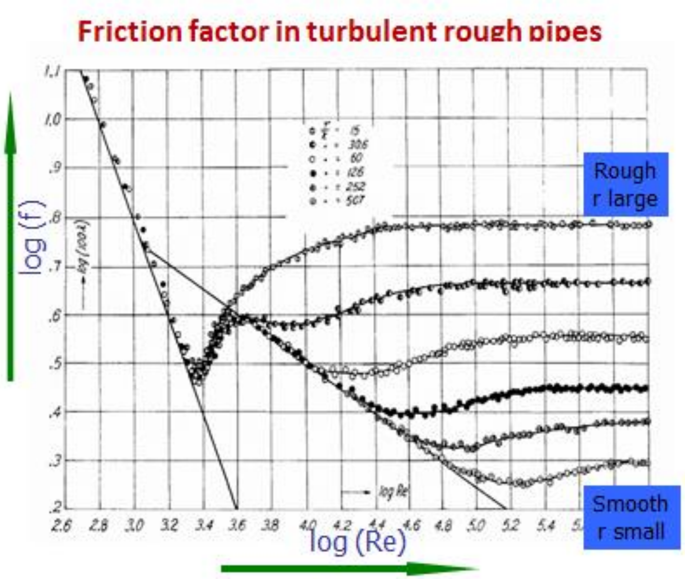
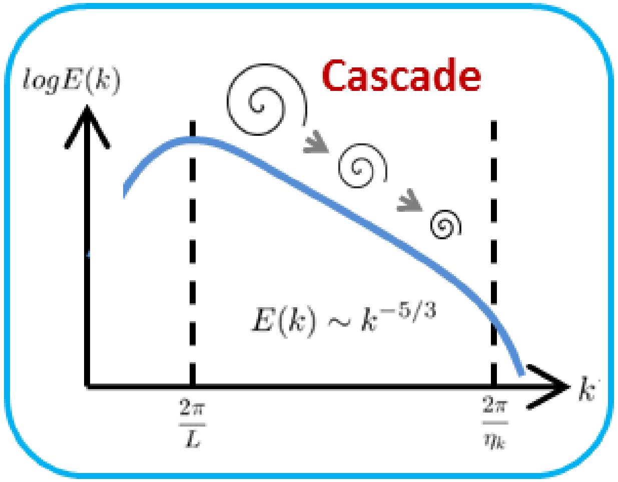


- Small-scale velocity fluctuations determine dissipation experienced at system-wide scales
  - reminiscent of the **fluctuation-dissipation** theorem in equilibrium statistical mechanics
- But this is not applicable here: the system is driven and far from equilibrium

Fluctuations and Dissipation



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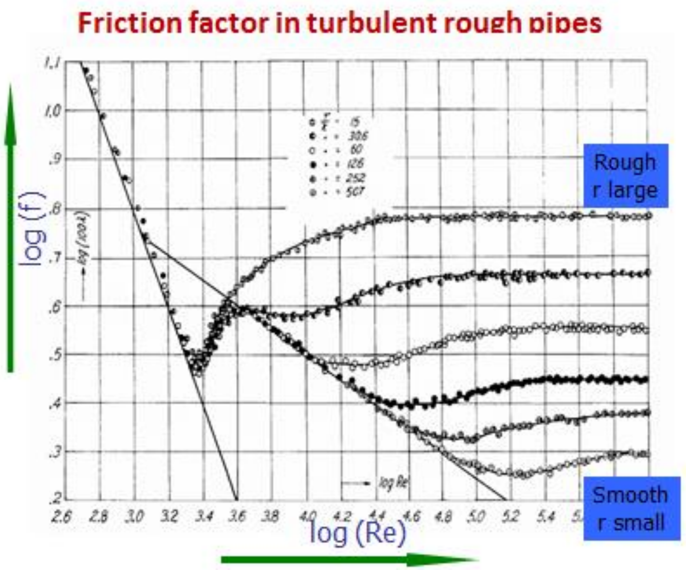


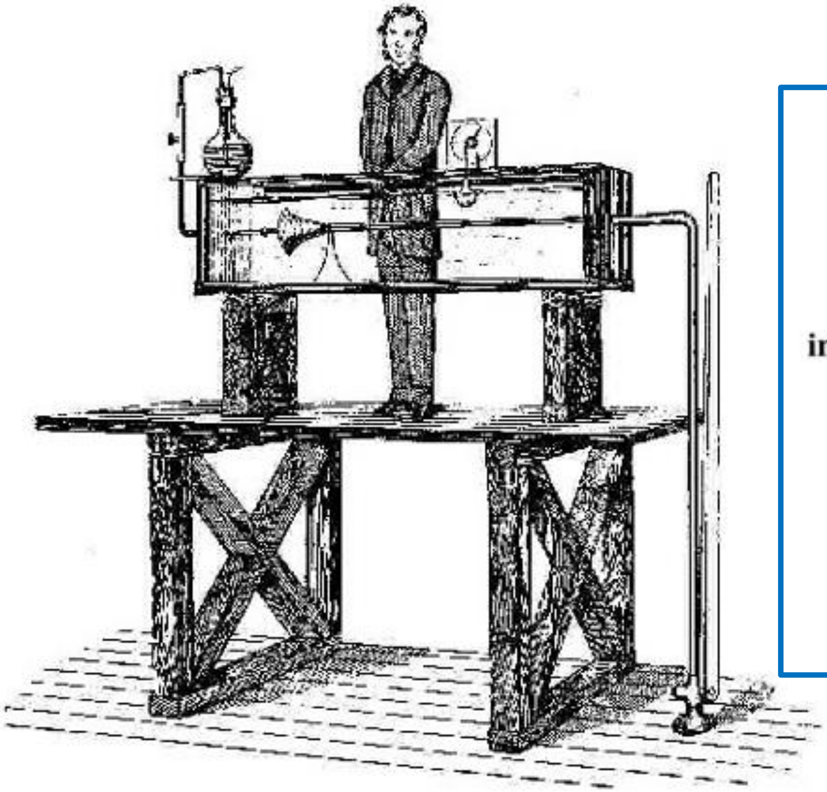
How does the drag experienced at large scales reflect the very nature of the turbulent state at smaller scales?

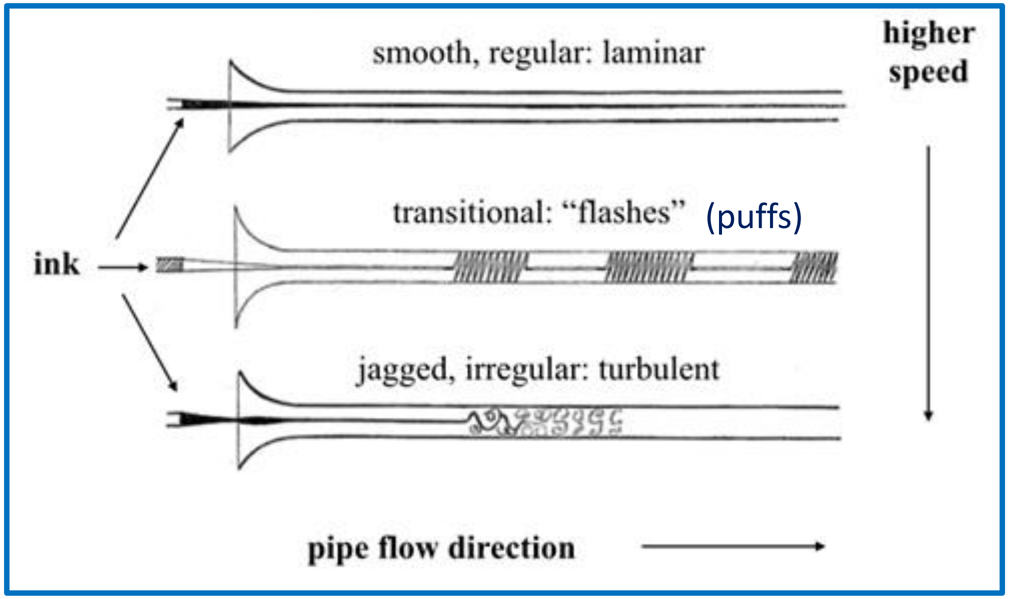
Solve turbulence? Connect the scales …

Predict how fluids become turbulent

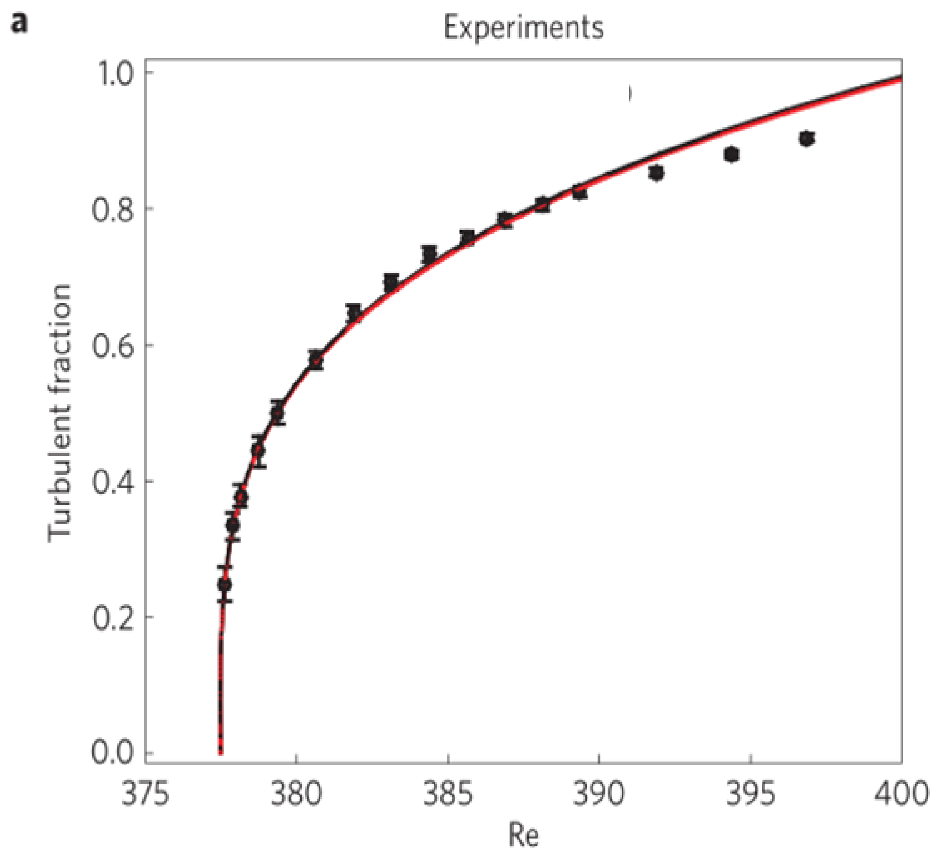
Transitional turbulence in pipe flow: puffs

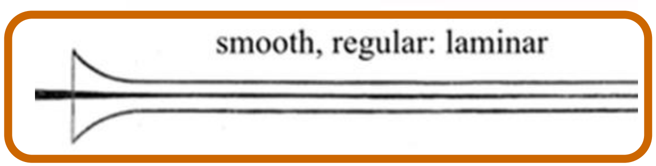
- Reynolds' original pipe turbulence (1883) reports on the transition
- Defined Reynolds' number $\mathrm{Re} = \frac{UL}{\nu}$

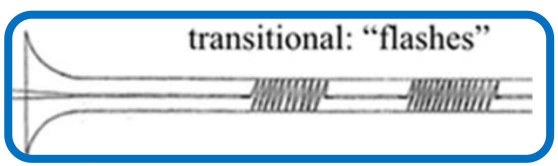


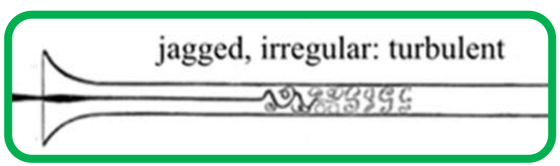


How much turbulence is in the pipe?









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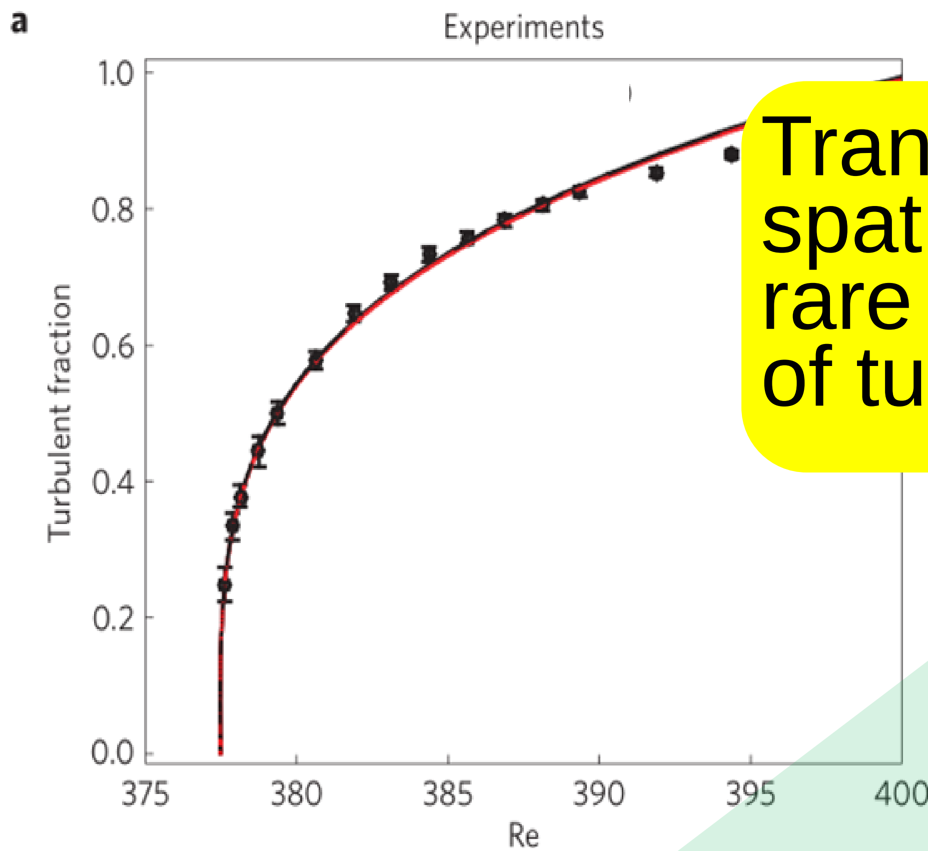
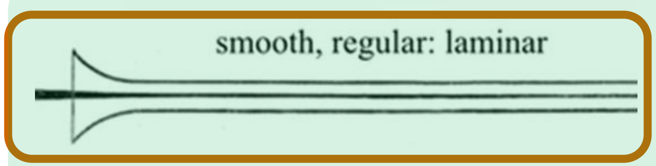
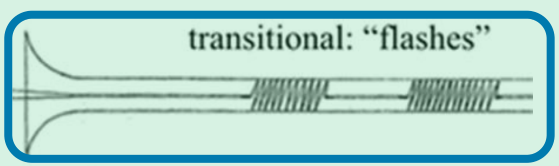
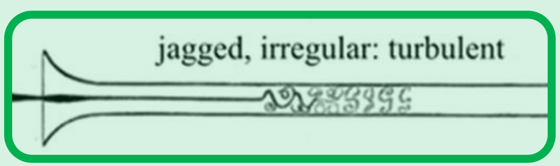


Transition not spatially uniform: rare sharp bursts of turbulence







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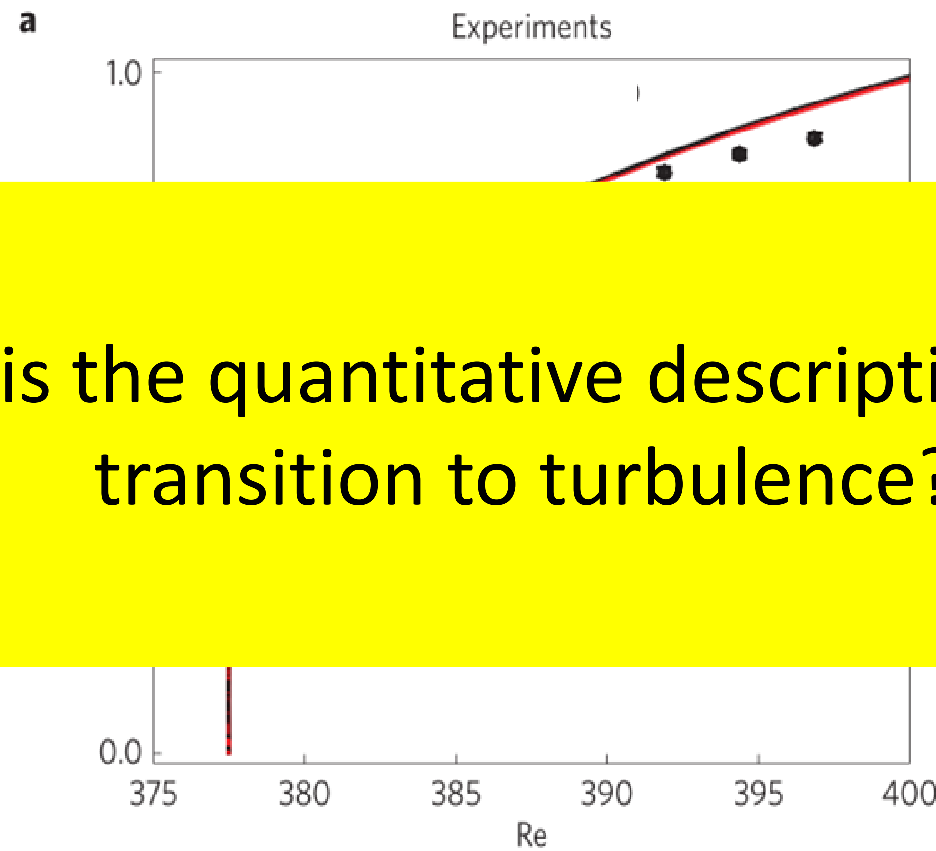
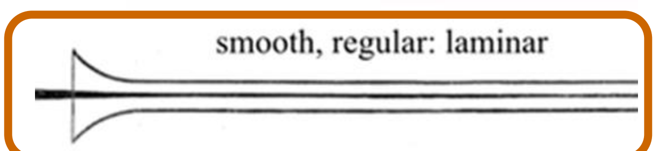
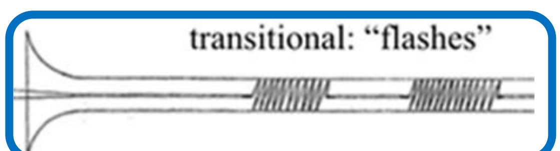
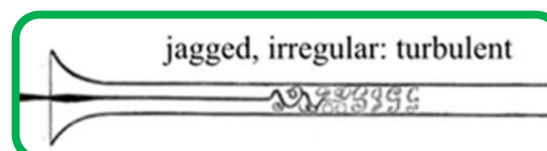


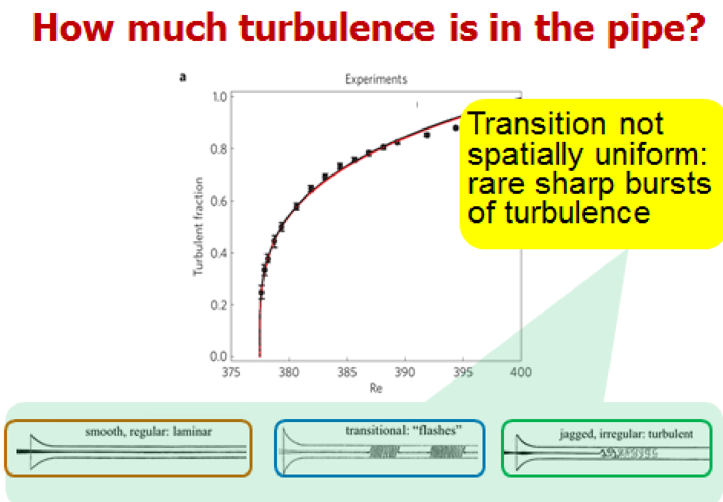
What is the quantitative description of the transition to turbulence?

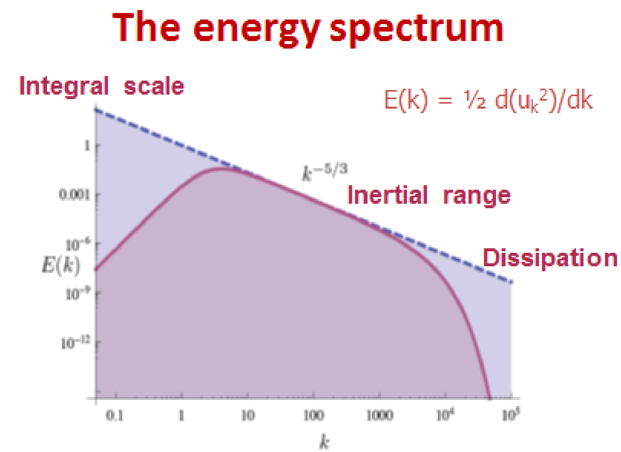






Turbulence & Phase Transitions





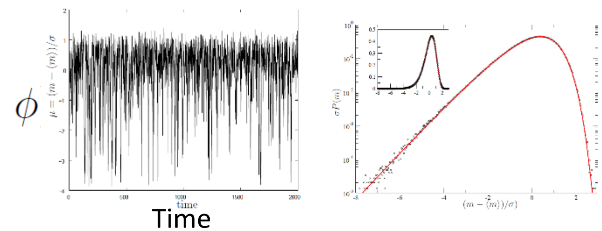
- Turbulent fraction as a function of Re is reminiscent of a continuous phase transition
- Strong power-law correlated fluctuations is reminiscent of a continuous phase transition

Why is fully-developed turbulence hard?

Why is turbulence unsolved?

Why are phase transitions hard?

- **Strong interactions + fluctuations** in the order parameter ϕ :
  - Very non-Gaussian
  - Intermittency



- **No usable small parameter!**

Landau free energy

$$\mathcal{H} = \int d^d\mathbf{r} \left[\frac{1}{2}\gamma\left(\nabla\phi\right)^2 + \frac{1}{2}r_0\phi^2 + \frac{1}{4}u_0\phi^4 \right]$$

$$t = (T - T_c)/T_c \qquad r_0 \sim t$$

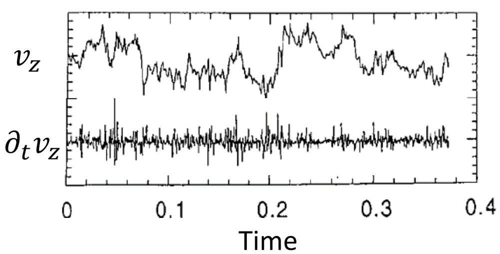
rescaling $\overline{u}_0 \sim u_0 t^{(d-4)/2} \begin{cases} d < 4: & \overline{u}_0 \to \infty \\ d > 4: & \overline{u}_0 \to 0 \end{cases}$

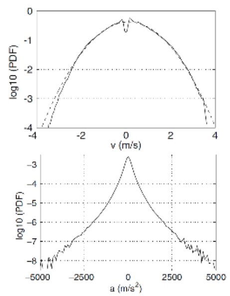
The coefficient of the **interaction** becomes relatively large at **phase transition** $t \to 0$

Why is turbulence unsolved?

Why is turbulence hard?

- **Strong interactions + fluctuations** in the velocity derivatives $\partial_i v_j$:
  - Very non-Gaussian
  - Intermittency: intervals of weak fluctuations interspersed with bursts of strong fluctuations





- **No usable small parameter!**

The Navier-Stokes equation

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{f}$$

Turbulence: $\mathrm{Re} \to \infty,\ \nu \to 0$

The coefficient of the **nonlinearity** becomes relatively large as the **viscosity** $\nu \to 0$

Why are phase transitions hard?

- **Strong interactions + fluctuations** in the order parameter ϕ :
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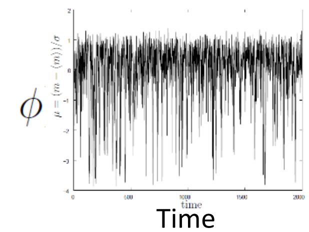
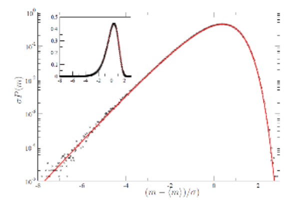
  - Intermittency





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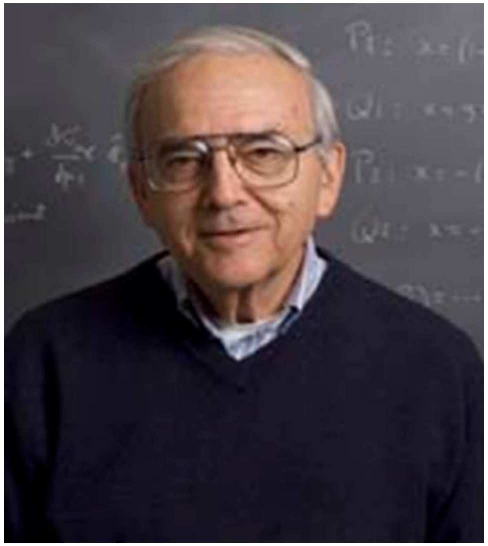
The coefficient of the **interaction** becomes relatively large at **phase transition** $t \to 0$

How was critical phenomena solved?



Ben Widom discovered "data collapse" (1965)

Leo Kadanoff explained data collapse, with scaling concepts (1966)

Ken Wilson developed the RG based on Kadanoff's scaling ideas (1970)

- **Common features**
  - Strong fluctuations
  - Power law correlations

- **Can we solve turbulence by following critical phenomena?**
- **Does turbulence exhibit critical phenomena at its onset?**

Transition to turbulence

Stability of laminar flow

1924. № 15.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 74.

1. *Über Stabilität und Turbulenz von Flüssigkeitsströmen; von Werner Heisenberg.*

Einleitung.

Das Turbulenzproblem, das ganz allgemein den Gegenstand der folgenden Untersuchungen bilden soll, ist im Laufe der Zeit in so vielen Arbeiten von so vielen verschiedenen Gesichtspunkten aus behandelt worden, daß es nicht unsere Absicht sein kann, einleitend über die bisherigen Resultate eine Übersicht zu geben. Wir verweisen zu diesem Zweck auf eine Arbeit von Noether[1]) über den heutigen Stand des Turbulenzproblems, in welcher auch die meisten Literaturangaben zu finden sind.

Für unseren Zweck genügt es, den gegenwärtigen Stand des Turbulenzproblems in ganz groben Umrissen zu skizzieren: die bisherigen Untersuchungen zerfallen in zwei Teile: die einen von ihnen befassen sich mit der Stabilitätsuntersuchung irgendwelcher laminaren Bewegung, die anderen mit der turbulenten Bewegung selbst.

Die ersteren führten anfangs zu dem negativen Resultat, daß alle untersuchten Laminarbewegungen stabil seien. v. Mises[2]) und Hopf[3]) bewiesen auf Grund eines Ansatzes von Sommerfeld[4]) die Stabilität des der Couetteschen Anordnung entsprechenden linearen Geschwindigkeitsprofils, Blumenthal[5]) gelangte bei einem von Noether zur Diskussion gestellten Profile 3. Grades zu demselben Ergebnis. Dagegen gelang es später Noether[6]), ein labiles Profil anzugeben — allerdings

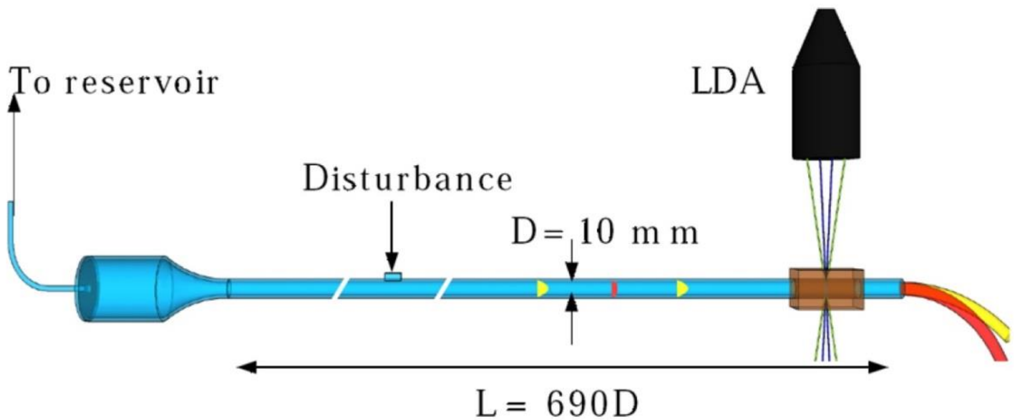
1) F. Noether, Zeitschr. f. angew. Math. u. Mech. 1. S. 125, 1921.
2) R. v. Mises, Beitrag z. Oszillationsprobl.: Heinr. Weber-Festschrift. 1912. S. 252.
3) L. Hopf, Ann. d. Phys. 44. S. 1. 1914.
4) A. Sommerfeld, Atti d. IV. congr. int. dei Mathem. Rom 1909.
5) O. Blumenthal, Sitzungsber. d. bayr. Akad. d. Wiss. S. 563. 1913.
6) F. Noether, Nachr. d. Ges. d. Wiss. Göttingen 1917.

Annalen der Physik. IV. Folge. 74. 39

Werner Heisenberg

Precision measurement of turbulent transition

Q: will a turbulent puff survive to the end of the pipe?



Many repetitions ➔ **Survival probability = P(Re, t)**

Hof et al., *PRL* **101**, 214501 (2008)

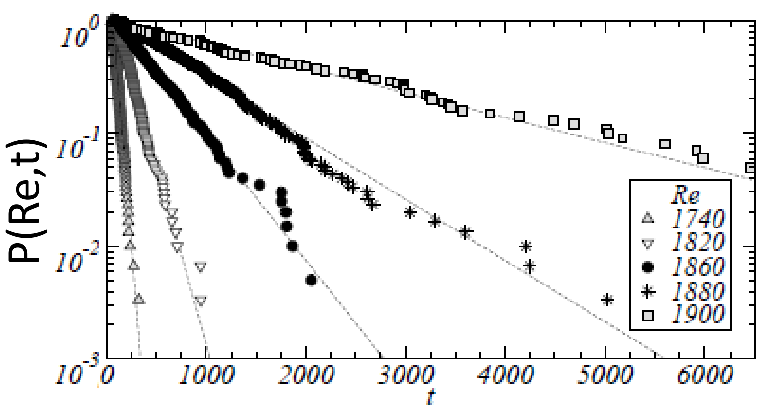
Pipe flow turbulence

Decaying single puff

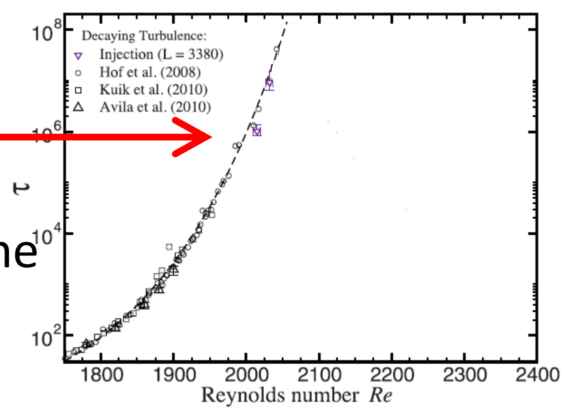
laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

Re: 1775 — 2050 — 2500

Survival probability $P(\mathrm{Re},t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

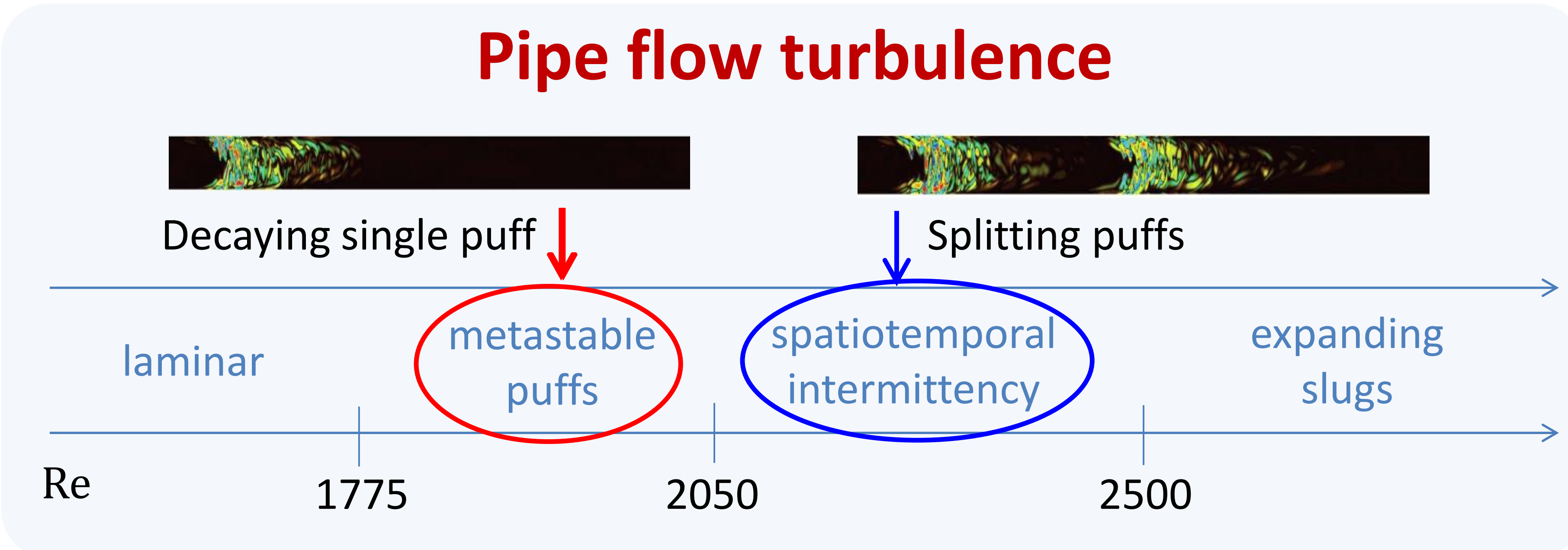


Puff lifetime

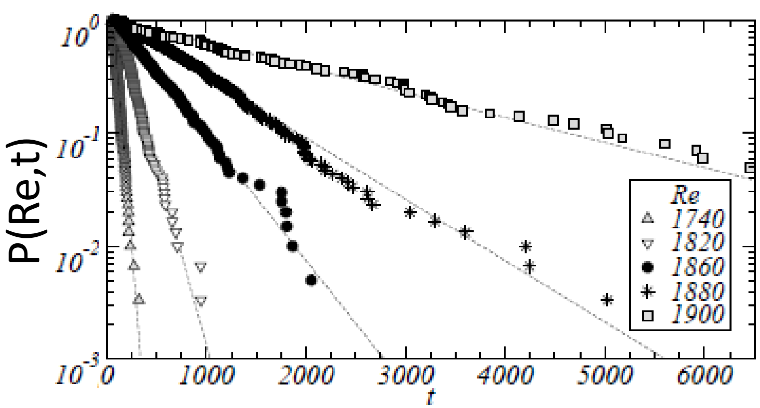


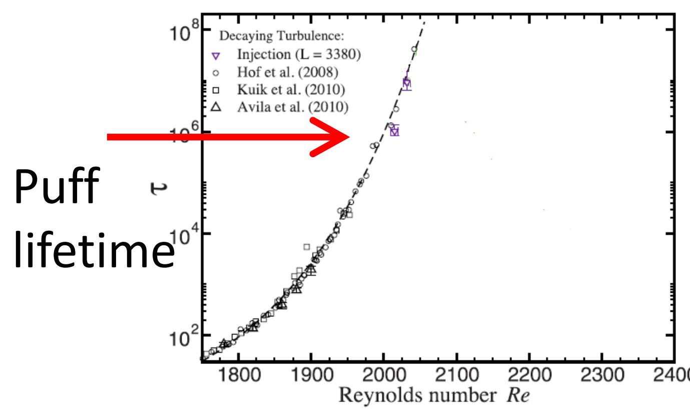
Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



Survival probability $P(\mathrm{Re},t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

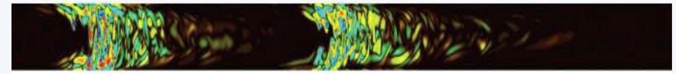




Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence

Decaying single puff

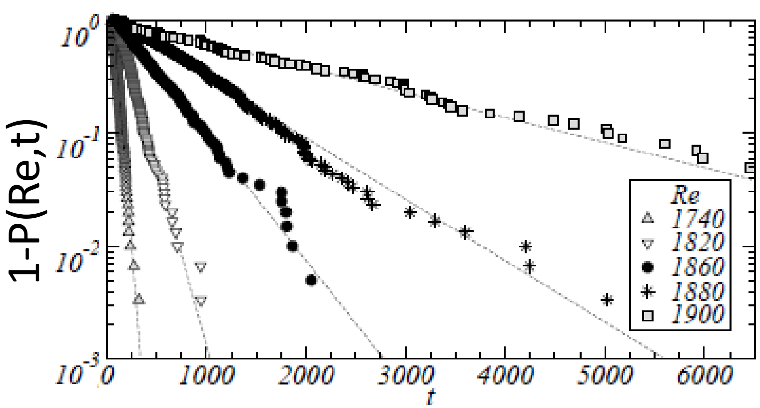


Splitting puffs

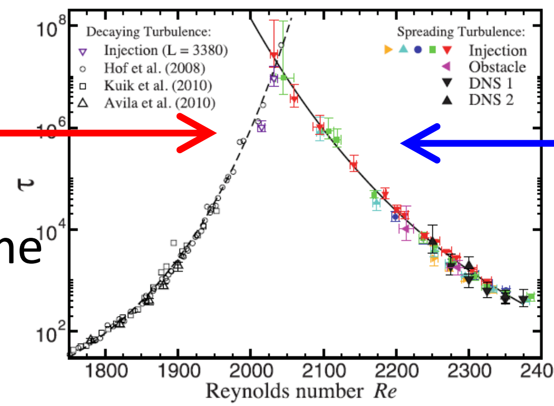
laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

Re: 1775 — 2050 — 2500

Splitting probability $1 - P(\mathrm{Re}, t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$



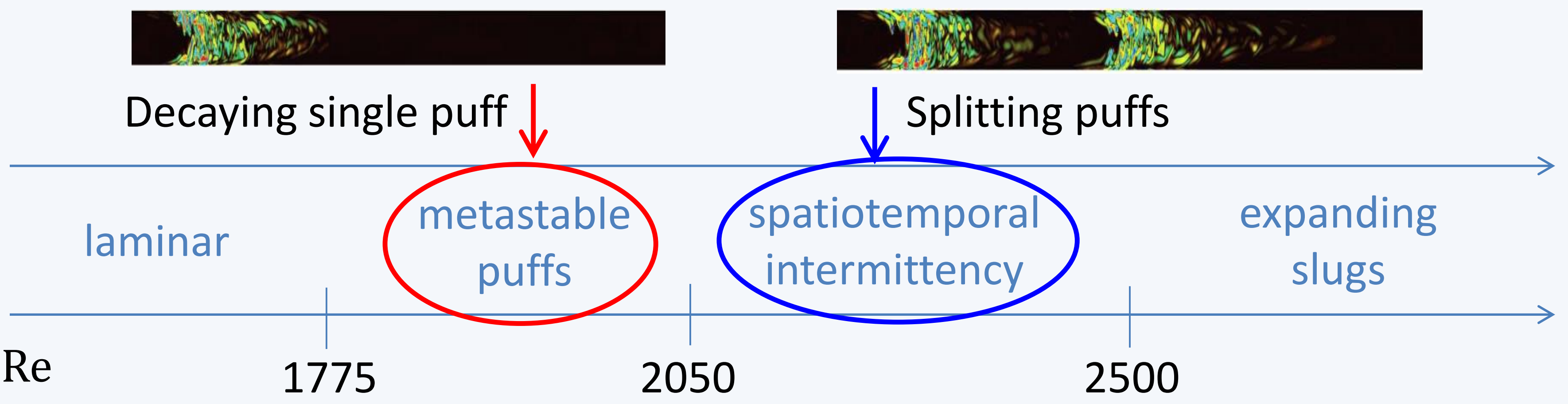
Puff lifetime



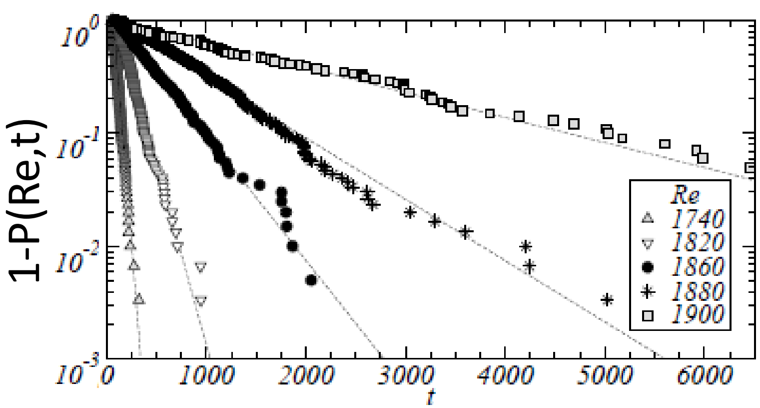
Mean time between split events

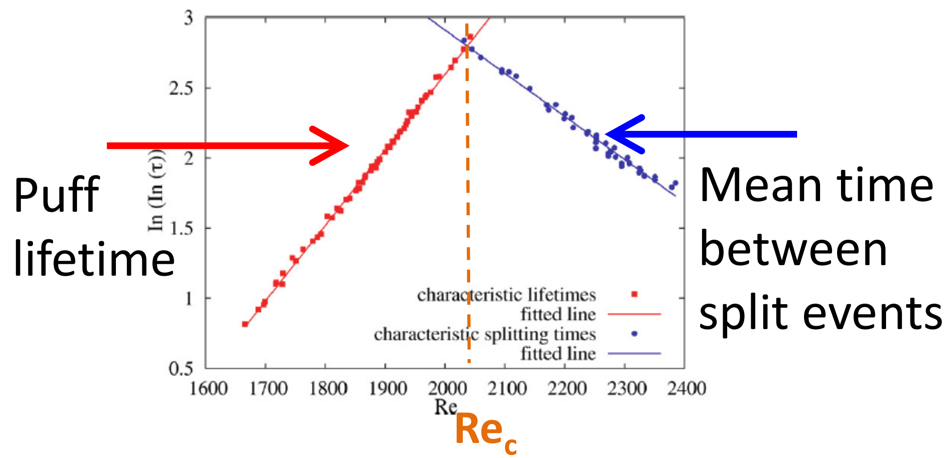
Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



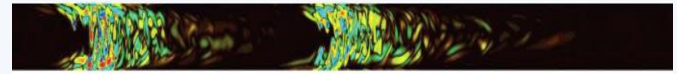
Splitting probability $1 - P(\mathrm{Re}, t) = e^{-\frac{t - t_0}{\tau(\mathrm{Re})}}$





Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



Decaying single puff

Splitting puffs

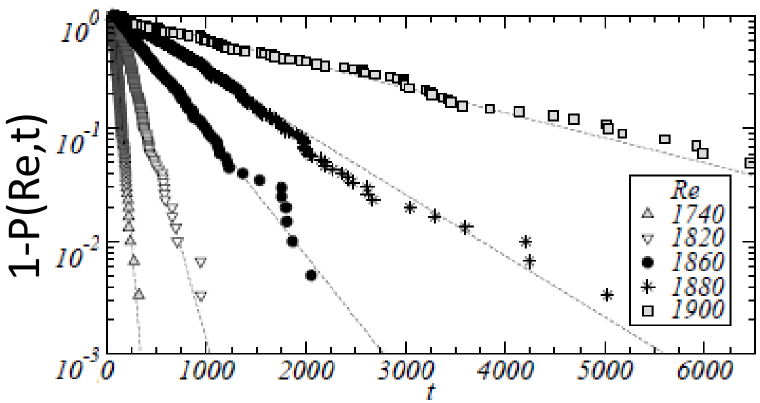
laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re

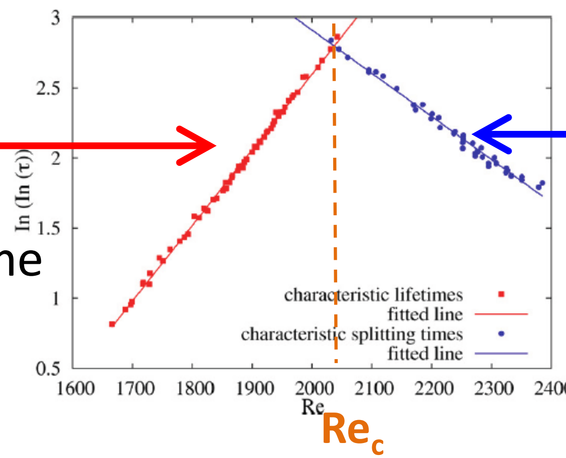
Super-exponential scaling:

$$\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{Re})$$

Splitting p



Puff lifetime



Mean time between split events

Avila *et al.*, (2009)   Avila *et al.*, *Science* **333**, 192 (2011)   Hof *et al.*, *PRL* **101**, 214501 (2008)

Theory for the laminar-turbulent transition in pipe flow

Logic of modeling phase transitions

Magnets

Electronic structure

↓

Ising model

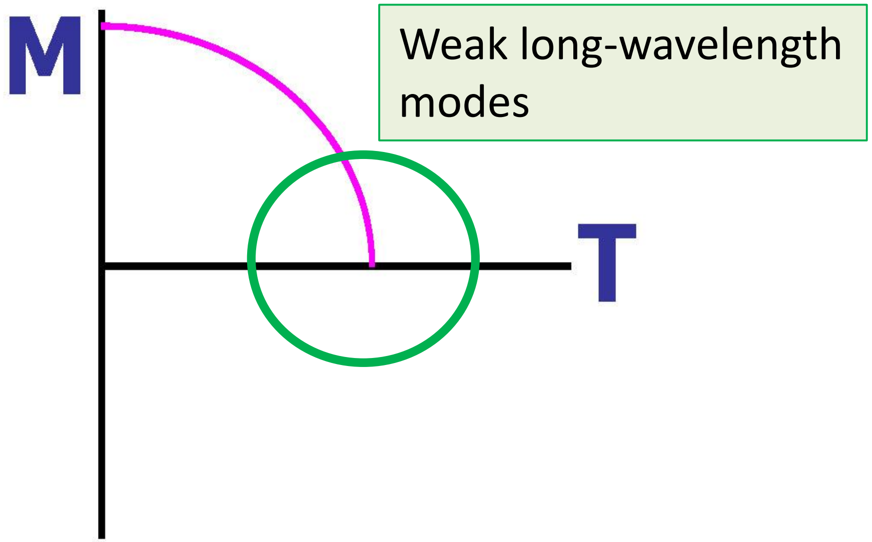
↓

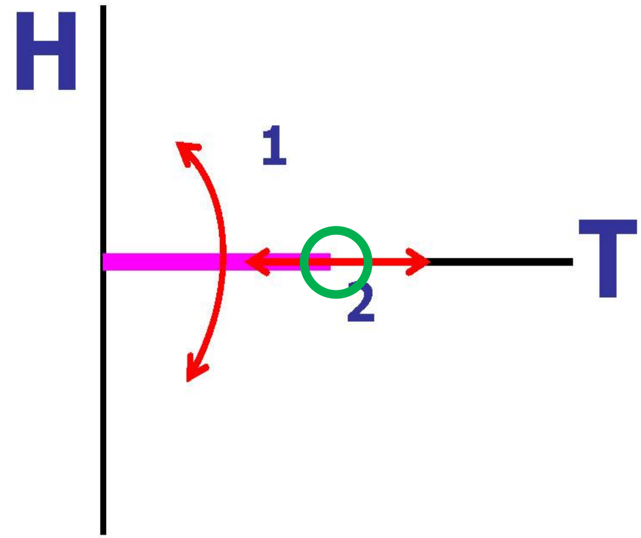
Landau theory

↓

Renormalization group
universality class
(Ising universality class)

Critical phenomena in magnets





$M \sim M_0[|T - T_c|/T_c]^\beta$ for $H = 0$ as $T \to T_c$

Critical isotherm: $M \sim H^{1/\delta}$ for $T = T_c$

- Widom (1963) pointed out that both these results followed from a *similarity formula*:

$$M(t,h) = |t|^\beta f_M(h/t^\Delta)$$

where $t \equiv (T - T_c)/T_c$ for some choice of exponent Δ and scaling function $f_M(x)$

Logic of modeling phase transitions

Magnets

Electronic structure

↓

Ising model

↓

Landau theory

↓

Renormalization group
universality class
(Ising universality class)

Turbulence

Kinetic theory

↓

Navier-Stokes eqn

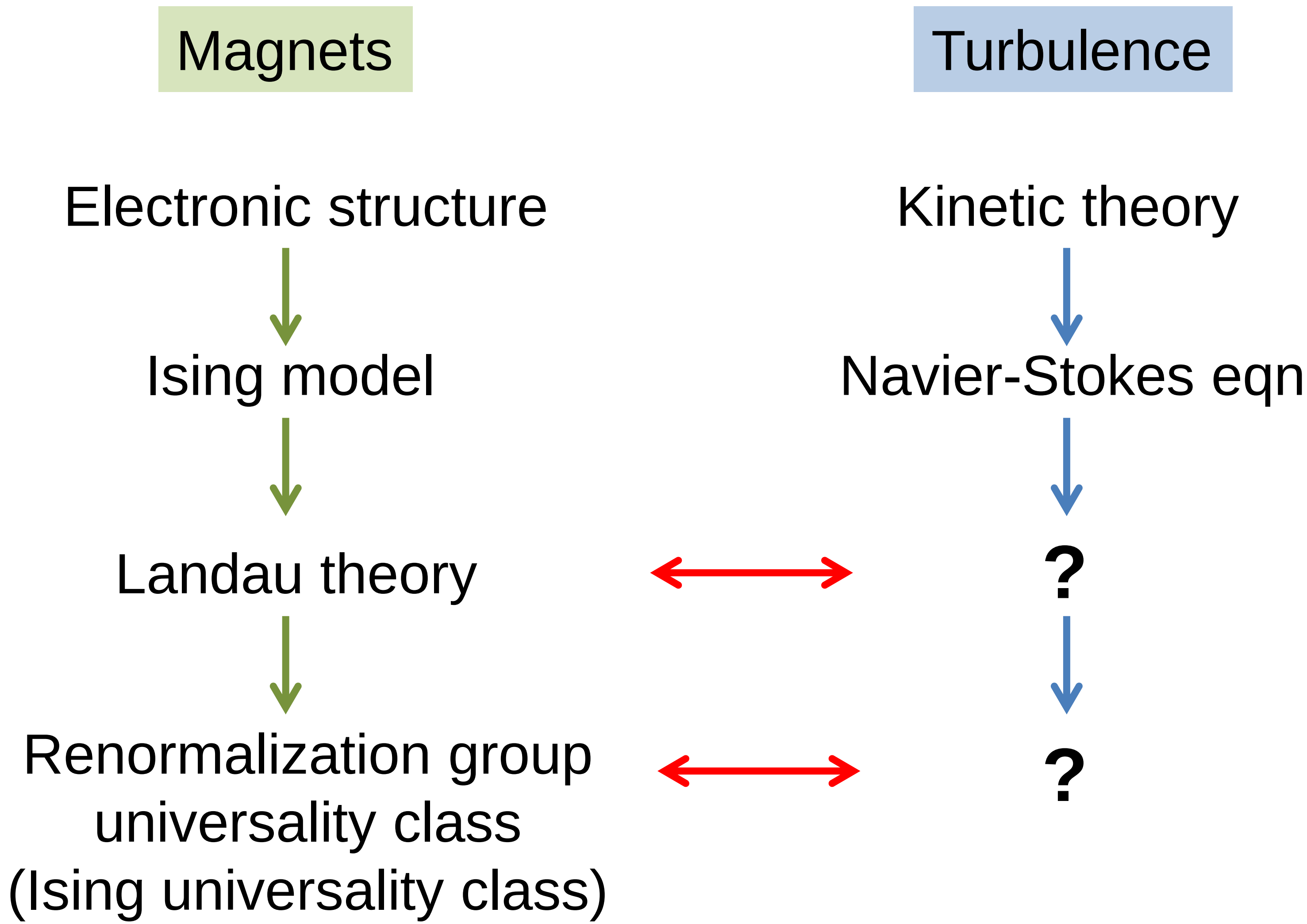
↓

?

↓

?

Logic of modeling phase transitions



Identification of long-wavelength collective modes at the laminar-turbulent transition

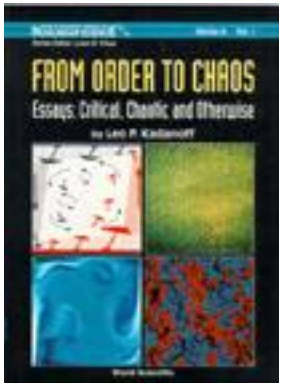
To avoid uncontrolled approximations,
we use **direct numerical simulation** of
the **Navier-Stokes equations**

Digression: how we should use computer simulation as a tool to make discoveries

Computer Simulation & Excessive Realism

- At that time there was a great national push toward understanding the dynamics of urban development. Jay Forrester of MIT had developed a computer model of urban change, which took a very simplified view of a urban society … and then used the output of that model to prescribe social policy. I did not like the policy prescribed.
- Consequently, I set out to use the modeling tools that Forrester had developed to reach conclusions which were more to my liking. … Our first result was that while not changing the model at all, we could reach opposite conclusions from that of the Forrester group.
- We went on to build other models which more accurately recorded our own prejudices and points of view. But after a while, the point we had made began to sink in. If these models really represented little more than we could say in words, why not leave out the computer?
- The construction of this sort of computer model seemed to be a rather pointless endeavor. For this reason, and others, I moved away from urban studies.

Leo P. Kadanoff 1993



Computer Simulation & Excessive Realism

- Moral of this story. There are two ways to model nature
  - Excessive realism
    - Everything is computed as accurately as possible, with many levels of description and fitting parameters
  - Minimal models
    - Everything is computed as sloppily as possible, with as few levels of description as possible, and as few fitting parameters as possible

Computer Simulation & Excessive Realism

- Moral of this story. There are two ways to model nature
  - Excessive realism
    - Compute to get numerical information that verbal arguments cannot address
  - Minimal models
    - Compute to find emergent phenomena – an outcome of the dynamics that is not mandatory and usually collective

Computer Simulation & Excessive Realism

- Moral of this story. There are two ways to model m
  - Exces
    - Con that verl
  - Minir
    - Con – an outcome of the dynamics that is not mandatory and usually collective

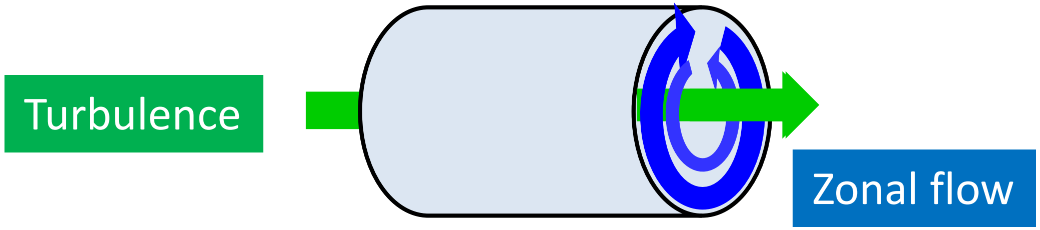
Example: the Hamiltonian of a fluid and a solid are identical, but only for low temperatures, can there be a non-zero shear modulus

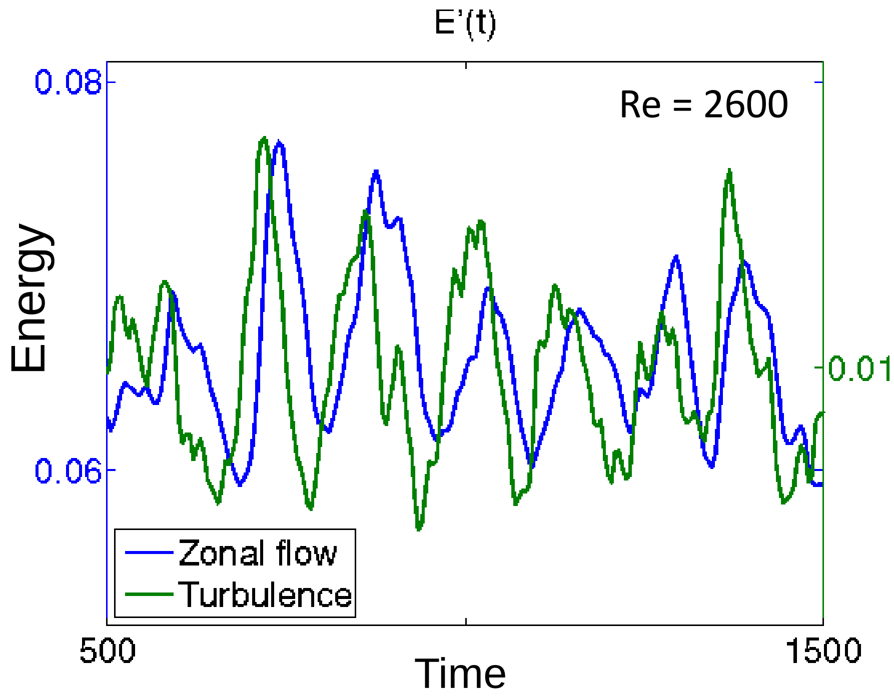
DNS of 3D Navier-Stokes equations

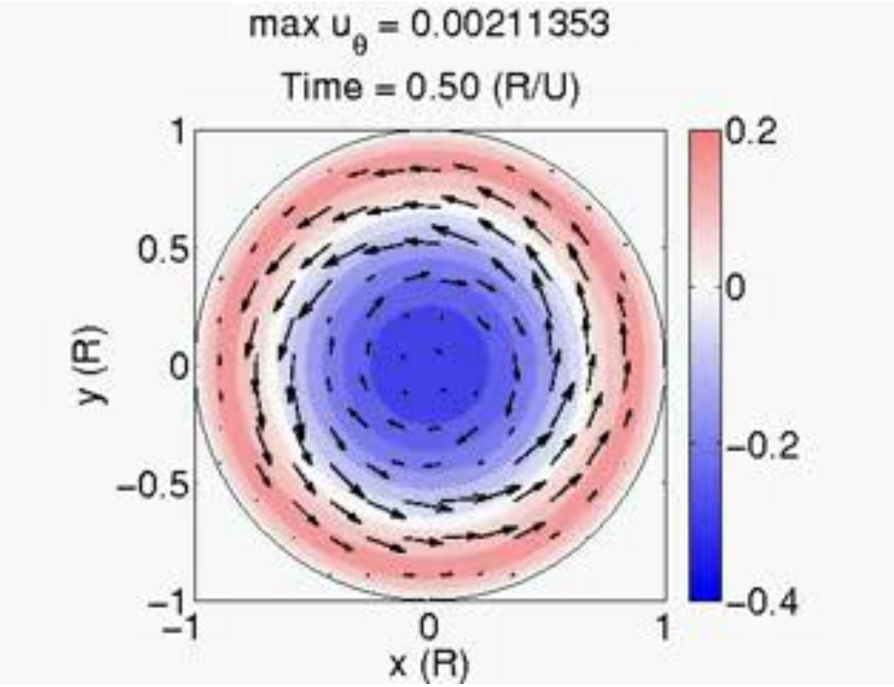
- Use pseudospectral method to deal with both derivatives and nonlinear terms in streamwise direction and azimuthal directions
- Use Chebyshev polynomials to resolve large gradients in the boundary layer, in the radial direction
- 60 grid points in the radial (r) direction,
- 32 Fourier modes in the azimuthal (θ) direction and 128 modes in the axial (z) direction
- The spatial resolutions were chosen such that the resolvable power spectra span over six orders of magnitude.
- The pipe length L is 10 times its diameter D, with periodic boundary conditions in the z direction

Simulation based on the open source code by Ashley Willis: openpipeflow.org

Predator-prey oscillations in pipe flow







Simulation based on the open source
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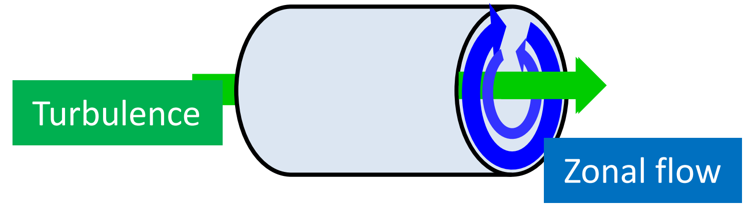
What drives the zonal flow?

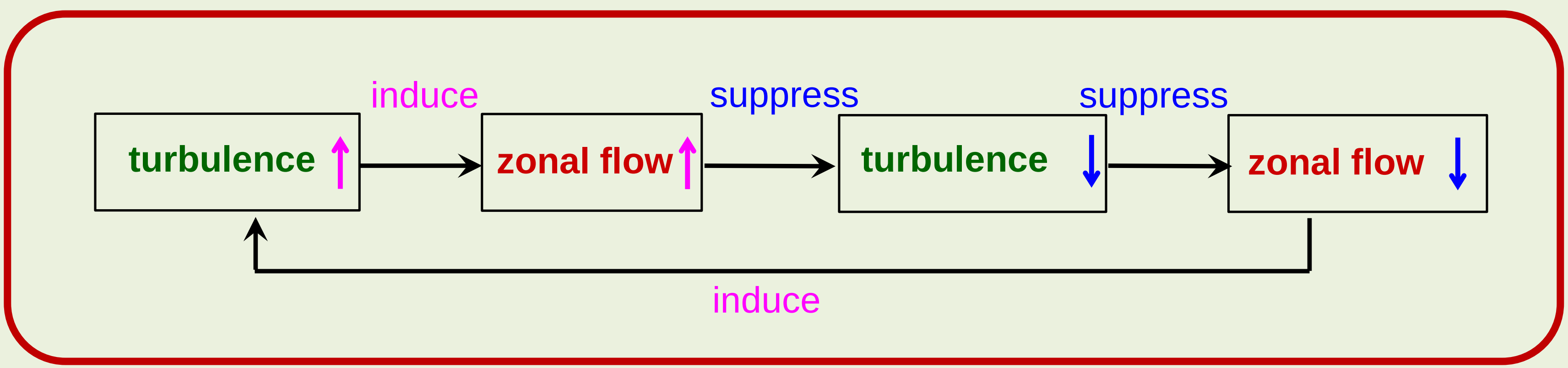
- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0): induced by turbulence and creates shear to suppress turbulence



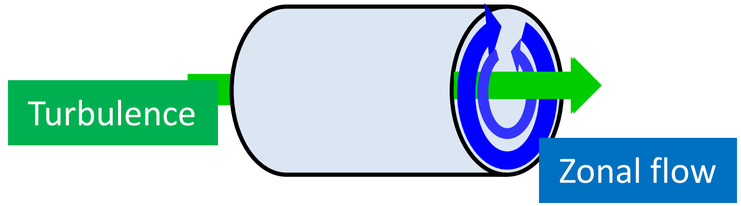
1) Anisotropy of turbulence creates Reynolds stress which generates the mean velocity in azimuthal direction

$$\partial_t \langle v_\theta \rangle = -\partial_r \langle (\widetilde{v}_\theta \cdot \widetilde{v}_r) \rangle - \mu \langle v_\theta \rangle$$

2) Mean azimuthal velocity decreases the anisotropy of turbulence and thus suppress turbulence



What drives the zonal flow?



- **Interaction in two fluid model**
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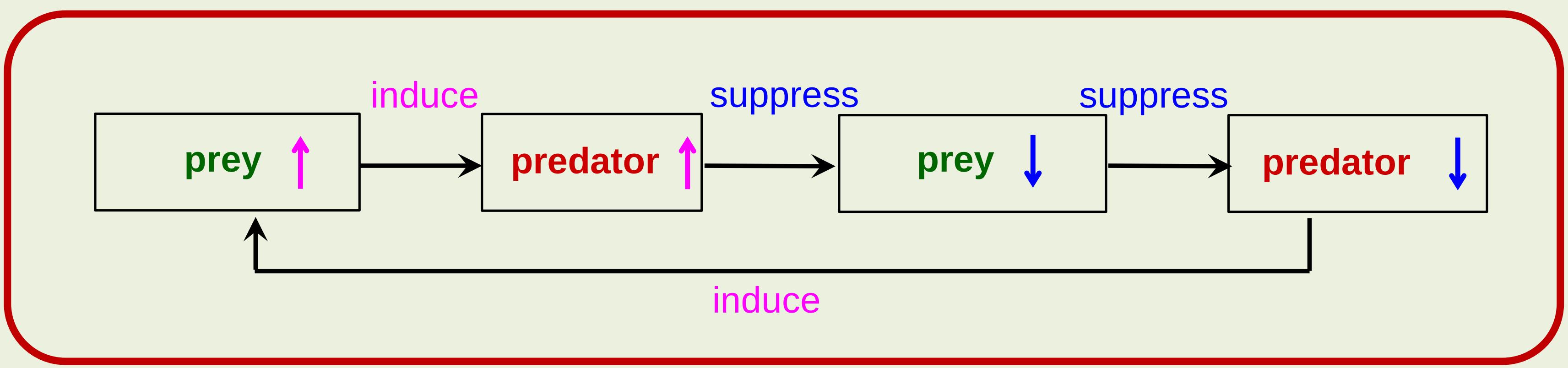
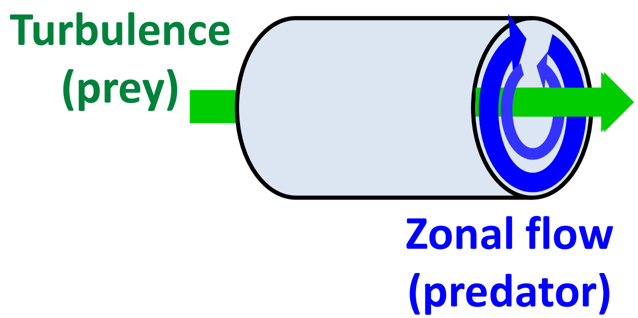
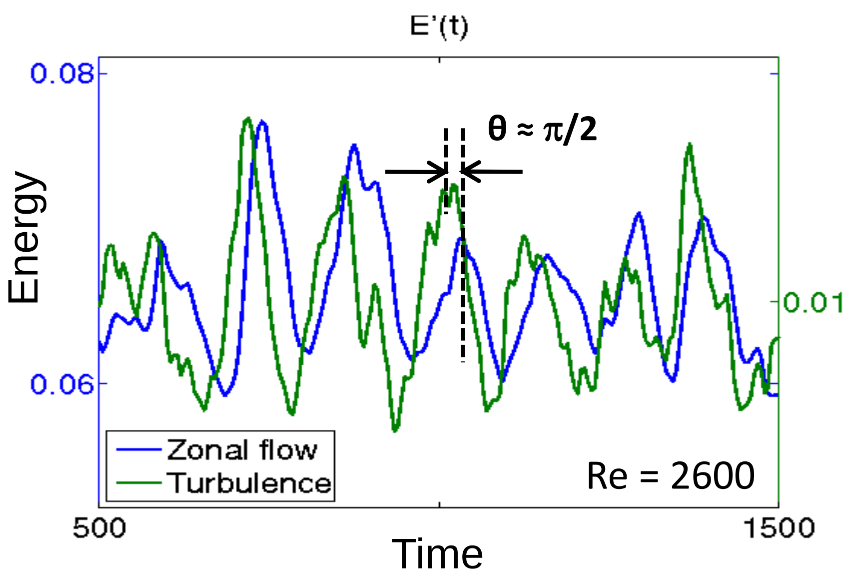
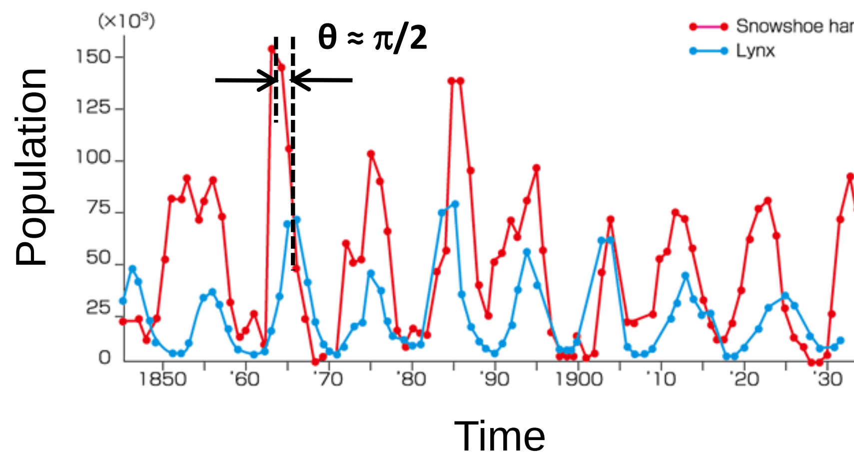


Pipe flow near transition to turbulence $\stackrel{?}{=}$ **Predator-prey ecosystem**



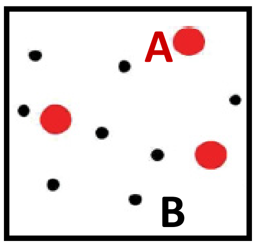




Stochastic model of predator-prey dynamics

- **Stochastic individual-level model**
  fluctuations in number → demographic stochasticity that induces **quasi-cycles**

A = predator
B = prey
E = food or available space

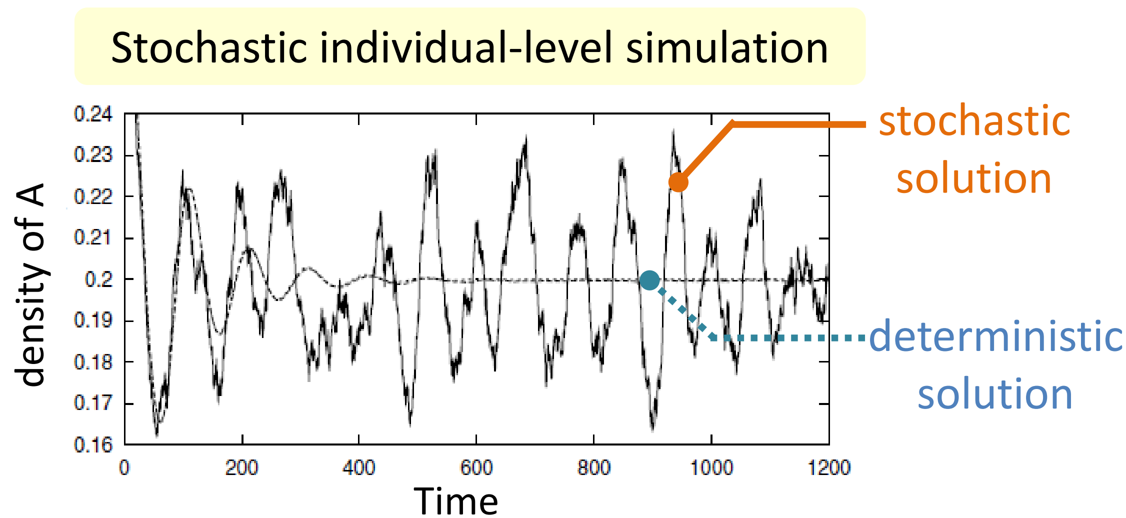


$$B + E \xrightarrow{b} B + B$$

$$B + B \xrightarrow{c} B + E$$

$$A + B \xrightarrow{p} A + A$$

$$A \xrightarrow{d_A} E \qquad B \xrightarrow{d_B} E$$



McKane & Newman (2005) Butler & Goldenfeld (2009)

Landau theory for transitional pipe turbulence is stochastic predator-prey

Derivation of predator-prey equations

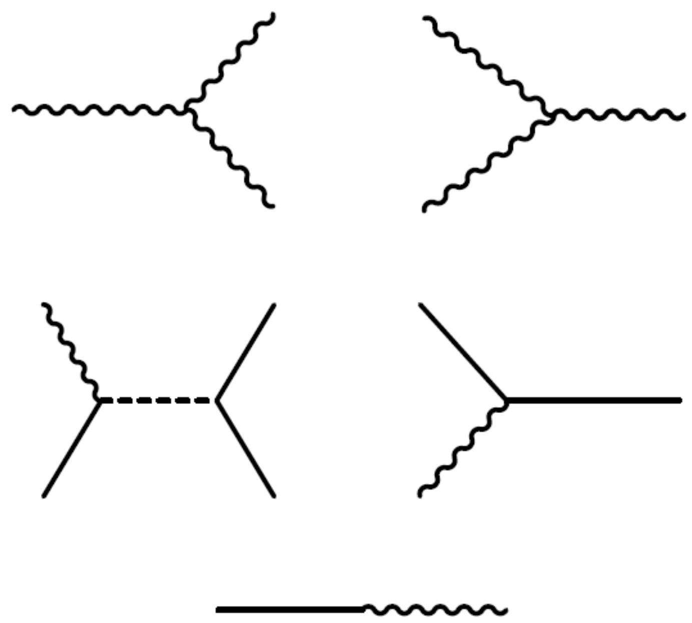
Zonal flow   Turbulence

Vacuum = Laminar flow

A = predator B = prey

E = food/empty state

Zonal flow-turbulence	Predator-prey



$$B + E \xrightarrow{b} B + B$$

$$B + B \xrightarrow{c} B + E$$

$$A + B \xrightarrow{p} A + A$$

$$A + B \xrightarrow{p'} A + E$$

$$B \xrightarrow{m} A$$

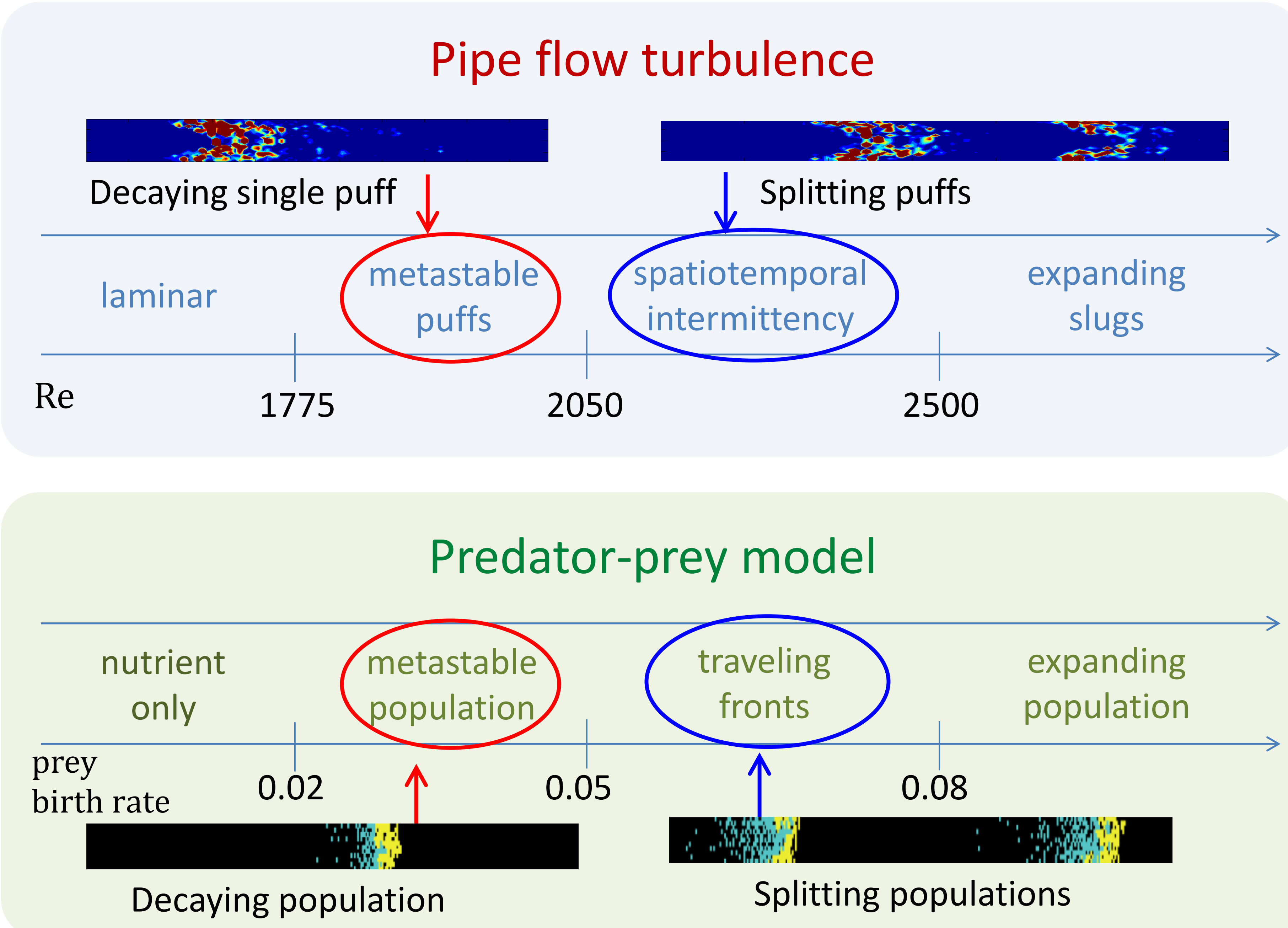
$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

Stochastic predator-prey recapitulates turbulence data

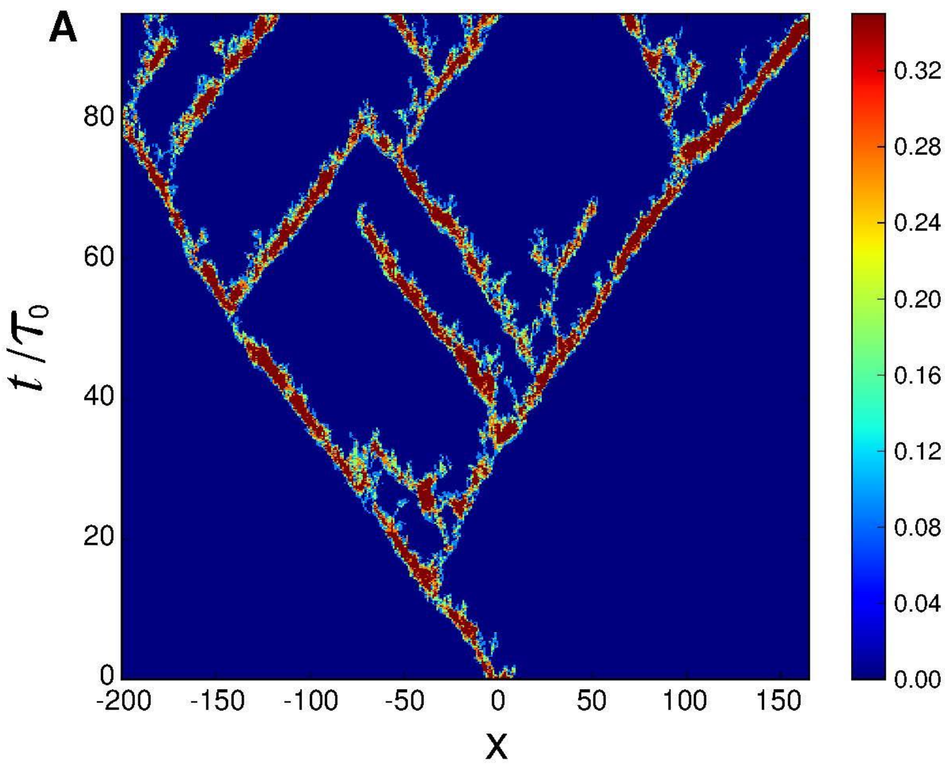
Phase diagram

Lifetime statistics

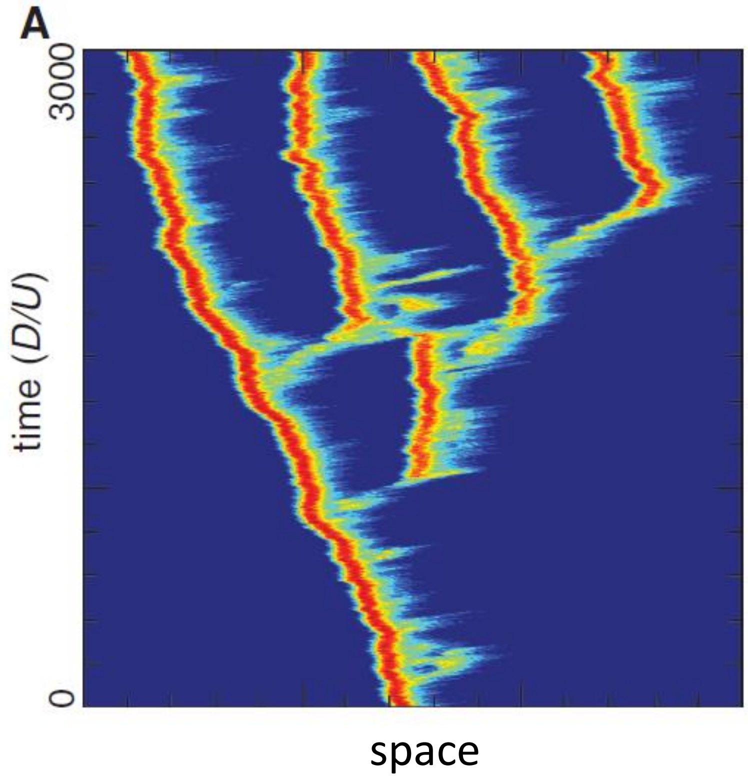
Universality class prediction


Pipe flow turbulence
Decaying single puff
Splitting puffs
laminar
metastable puffs
spatiotemporal intermittency
expanding slugs
Re
1775
2050
2500
Predator-prey model
nutrient only
metastable population
traveling fronts
expanding population
prey birth rate
0.02
0.05
0.08
Decaying population
Splitting populations

"Puff splitting" in predator-prey systems

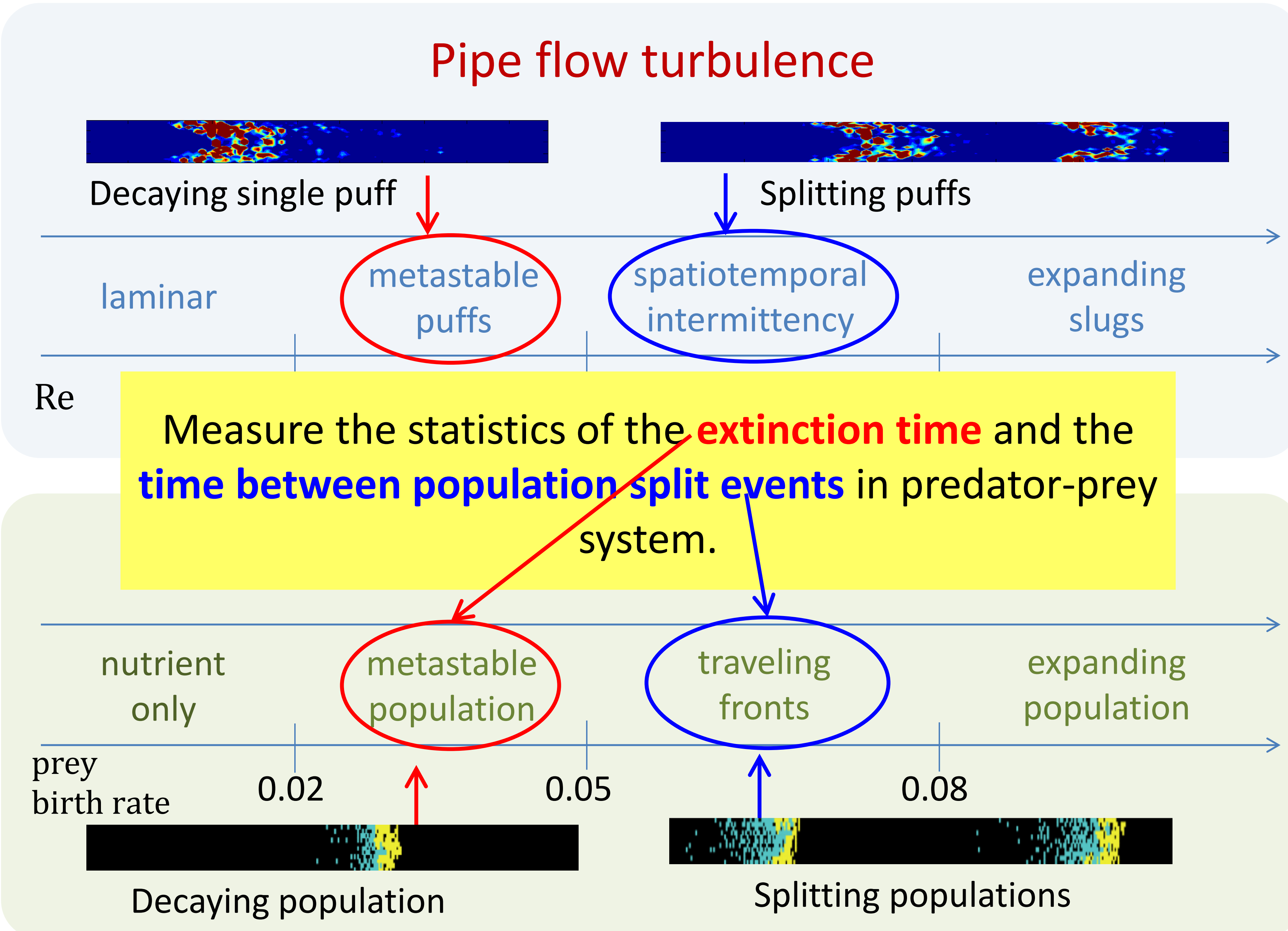


Population-splitting in predator-prey ecosystem



Puff-splitting in pipe turbulence

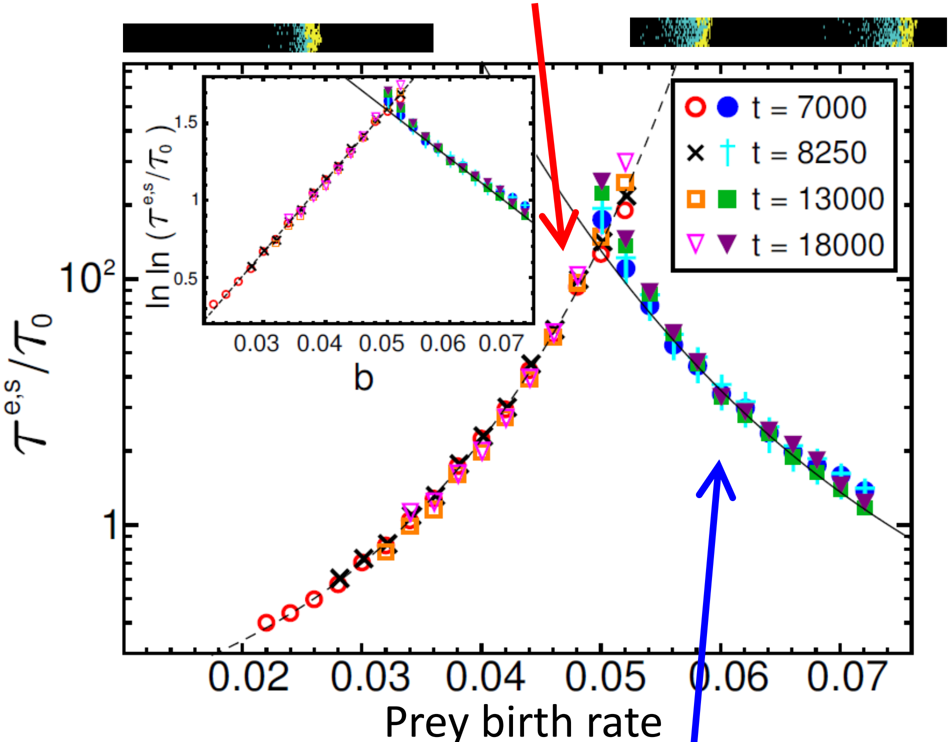
Avila et al., Science (2011)

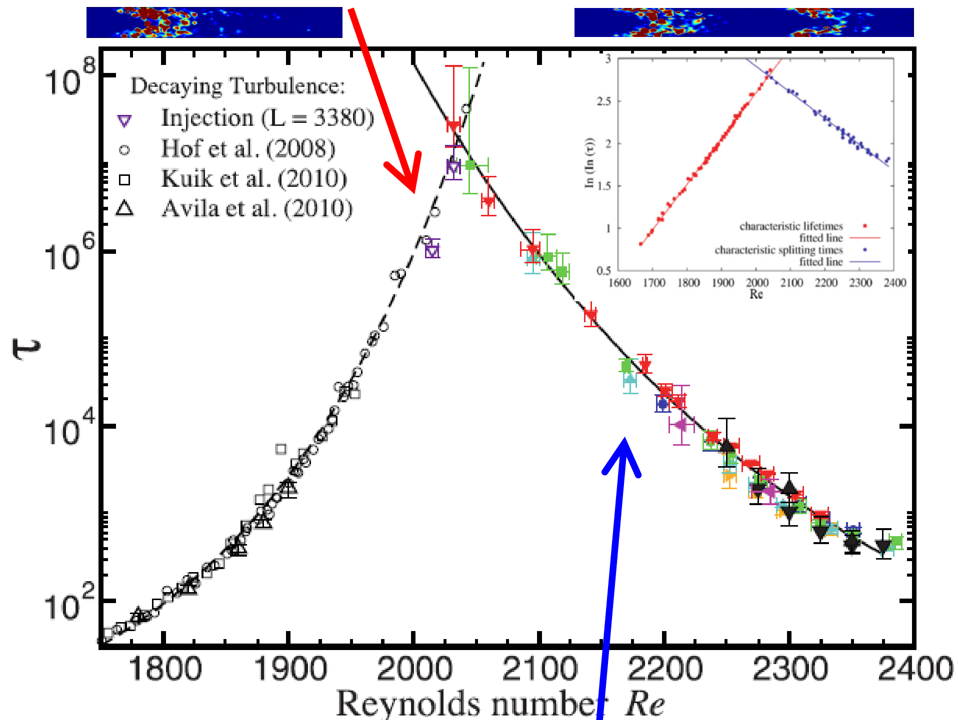

Pipe flow turbulence
Decaying single puff
Splitting puffs
laminar
metastable puffs
spatiotemporal intermittency
expanding slugs
Re
Measure the statistics of the **extinction time** and the **time between population split events** in predator-prey system.
nutrient only
metastable population
traveling fronts
expanding population
prey birth rate
0.02
0.05
0.08
Decaying population
Splitting populations

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

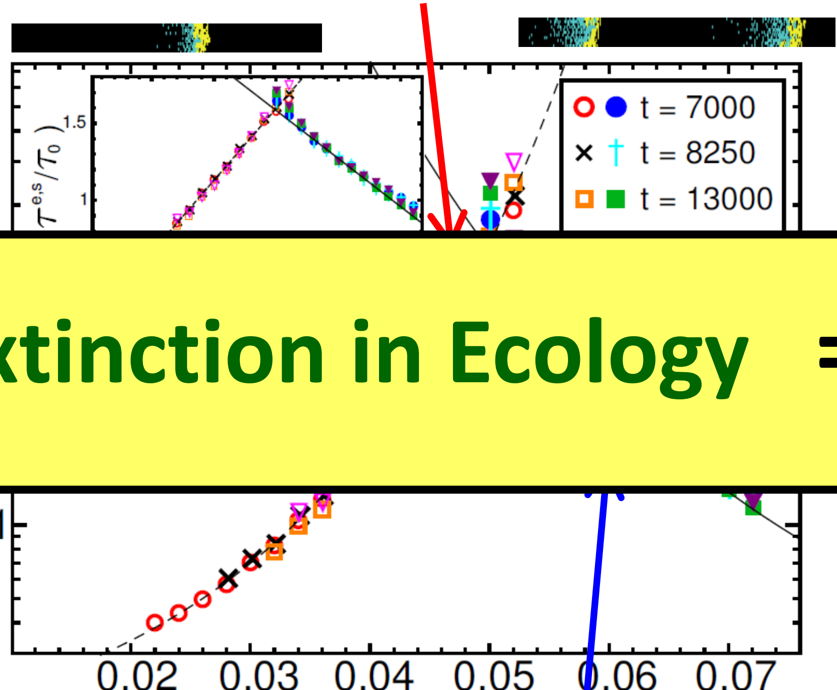
Mean time between puff split events

Shih, Hsieh and Goldenfeld
Nature Physics **12**, 245 (2016)

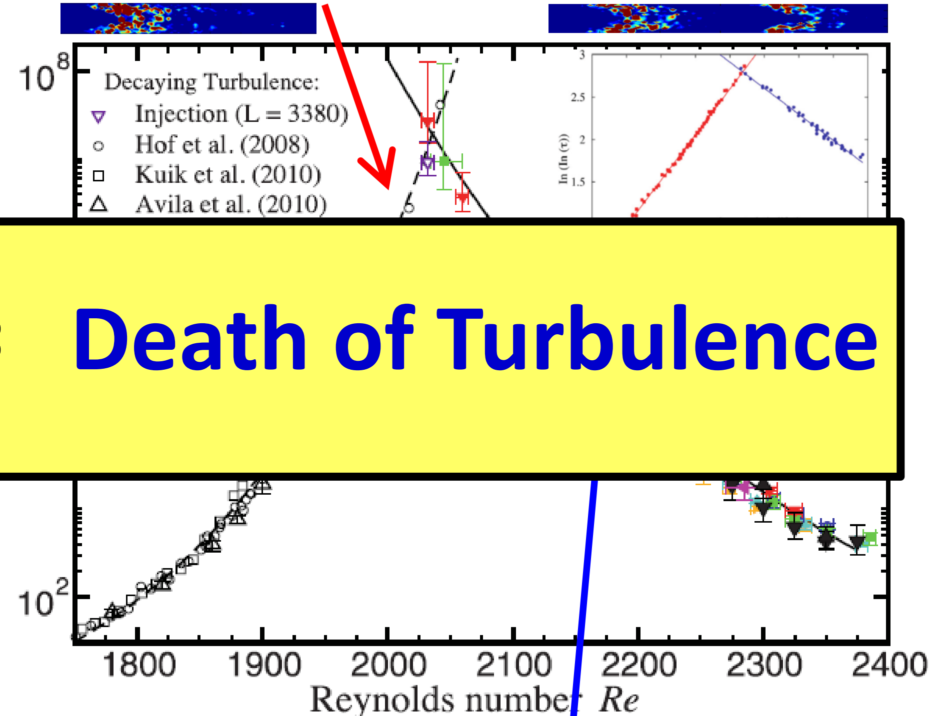
Avila *et al.*, *Science* **333**, 192 (2011)
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

Predator-prey vs. transitional turbulence

Prey lifetime



Turbulent puff lifetime



Extinction in Ecology = Death of Turbulence

Mean time between population split events

Mean time between puff split events

Shih, Hsieh and Goldenfeld *Nature Physics* **12**, 245 (2016)

Avila *et al.*, *Science* **333**, 192 (2011)

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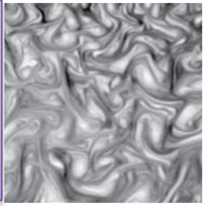
Roadmap: Universality class of laminar-turbulent transition

Universality class

?

(Boffetta and Ecke, 2012)

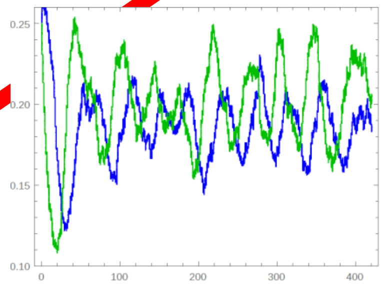
(Classical) Turbulence



Direct Numerical Simulations of Navier-Stokes

Two-fluid model

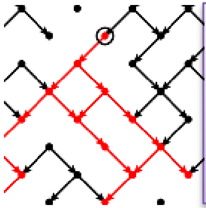
Turbulence + Zonal flow



Predator-Prey

(Pearson Education, Inc., 2009)

Roadmap: Universality class of laminar-turbulent transition



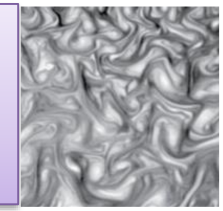
(Wikimedia Commons)

Directed Percolation

?

(Boffetta and Ecke, 2012)

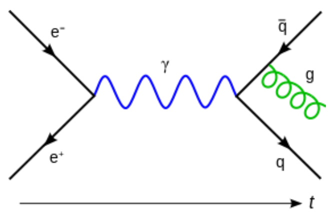
(Classical) Turbulence



Reggeon field theory (Janssen, 1981)

Direct Numerical Simulations of Navier-Stokes

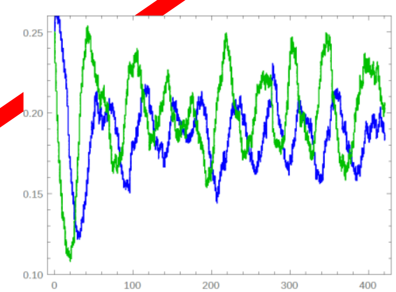
Field Theory



(Wikimedia Commons)

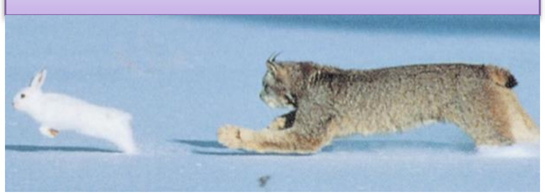
Two-fluid model

Turbulence + Zonal flow



Extinction transition (Mobilia et al., 2007)

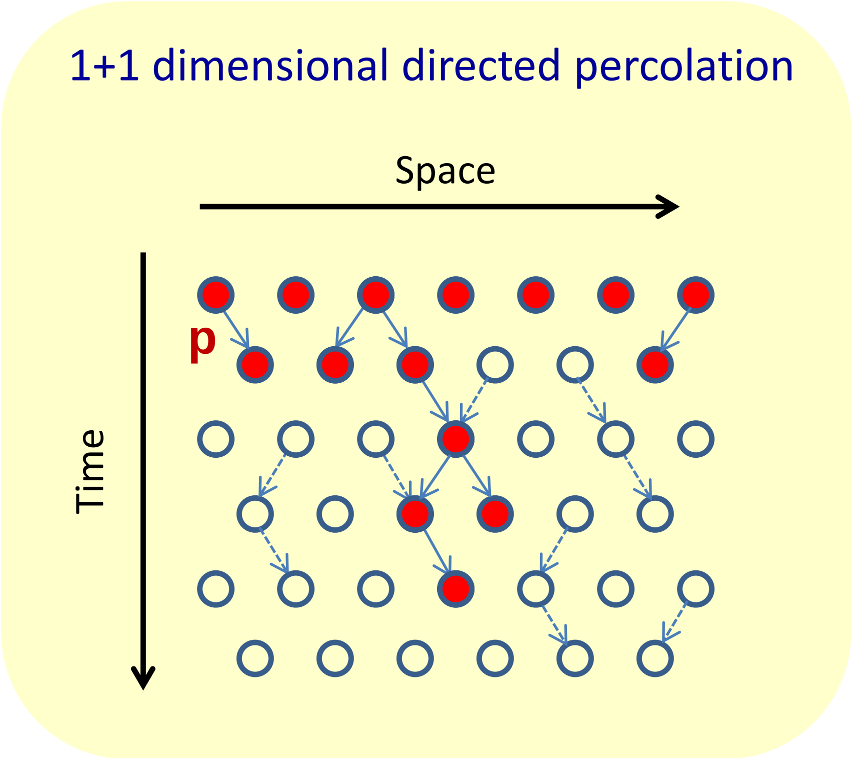
Predator-Prey

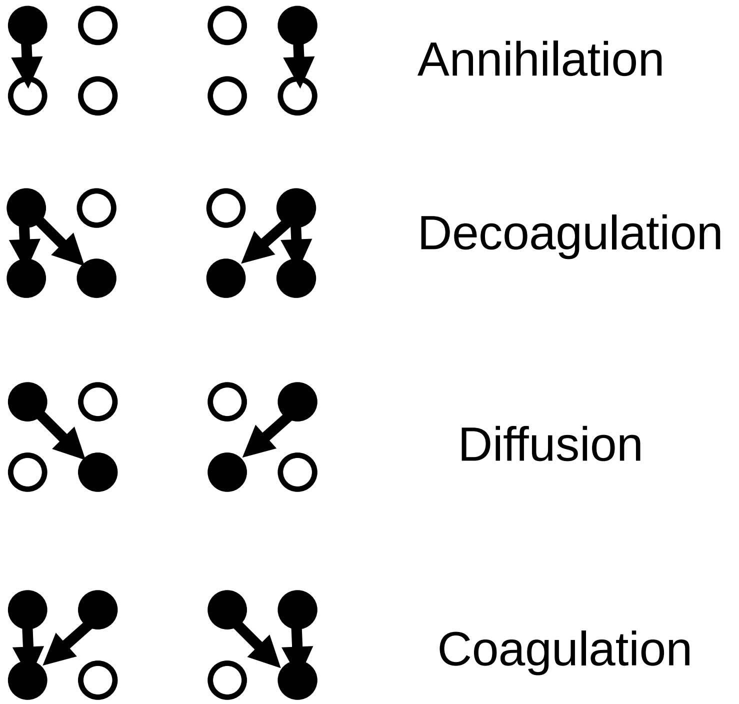


(Pearson Education, Inc., 2009)

Directed percolation & the laminar-turbulent transition

- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)

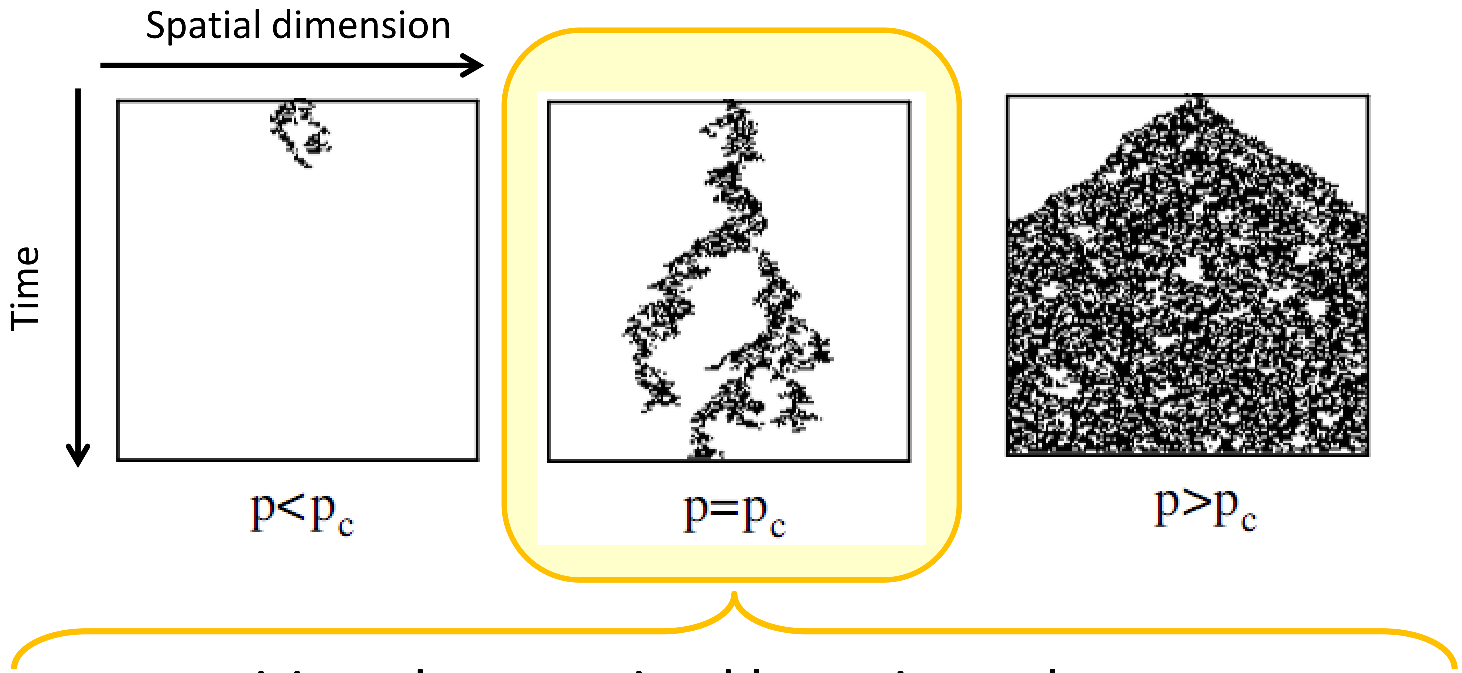




Hinrichsen, Adv. in Physics (2000)

Directed percolation transition

- A continuous phase transition occurs at p_c.

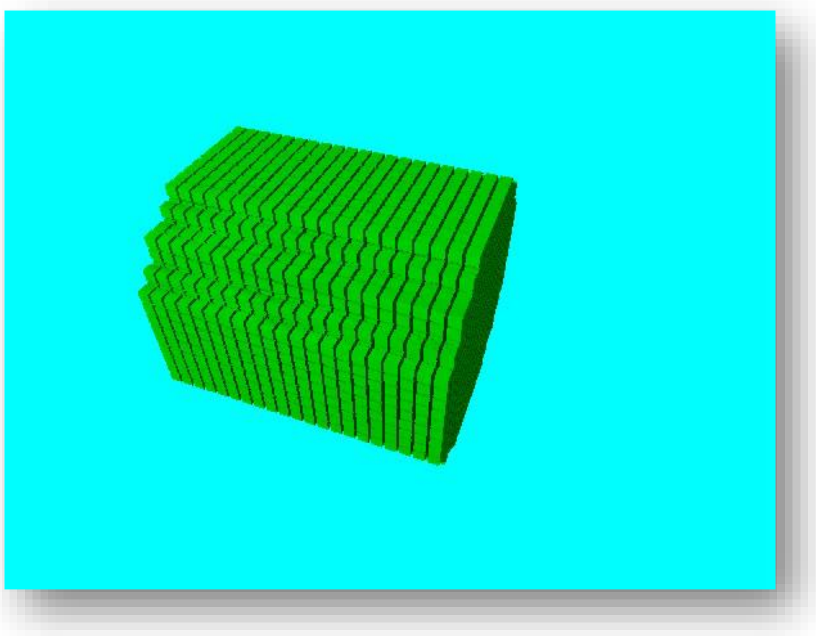


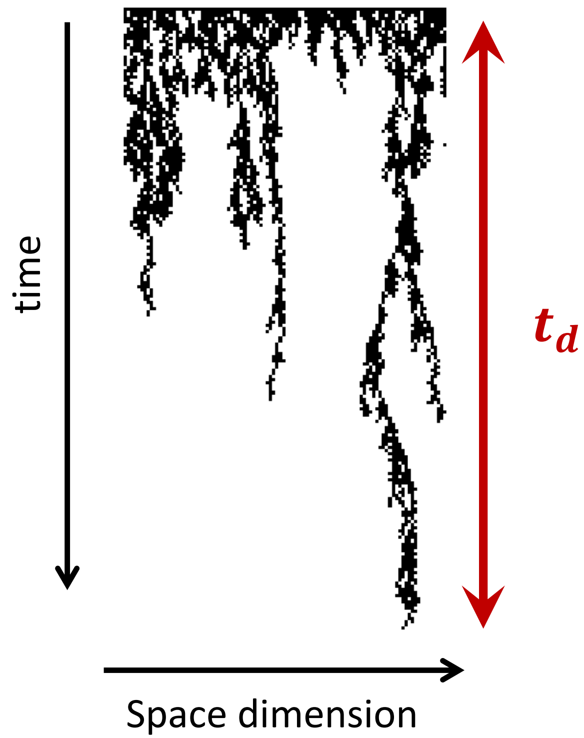
- Phase transition characterized by universal exponents:

$$\rho \sim (p - p_c)^{\beta} \qquad \xi_{\perp} \sim (p - p_c)^{-\nu_{\perp}} \qquad \xi_{\parallel} \sim (p - p_c)^{-\nu_{\parallel}}$$

Hinrichsen, Adv. in Physics (2000)

DP in 3 + 1 dimensions in pipe





Puff decay 3 + 1

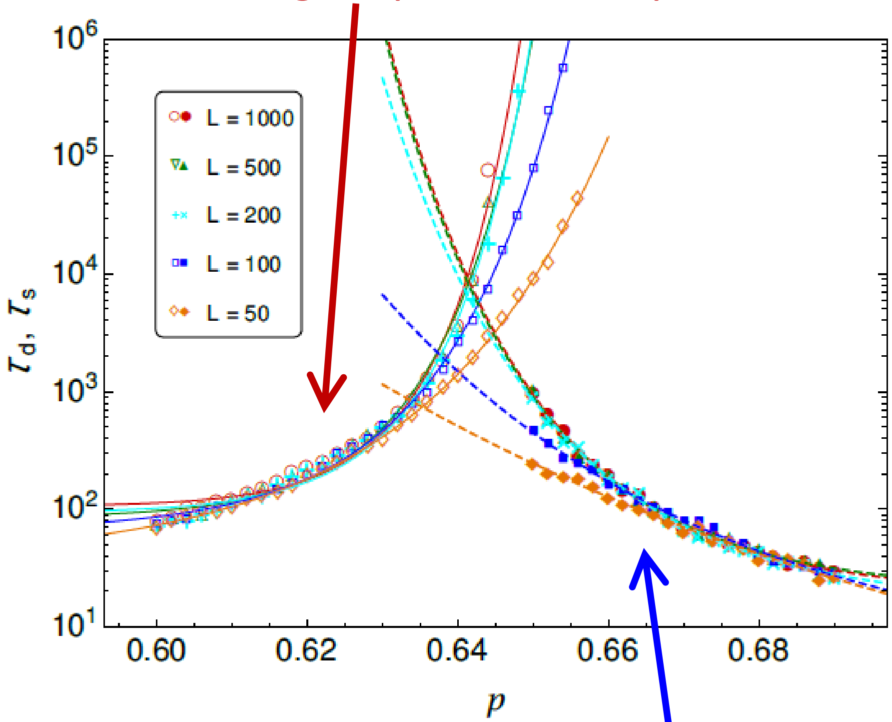
Puff decay 1 + 1

Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

Directed percolation vs. transitional turbulence

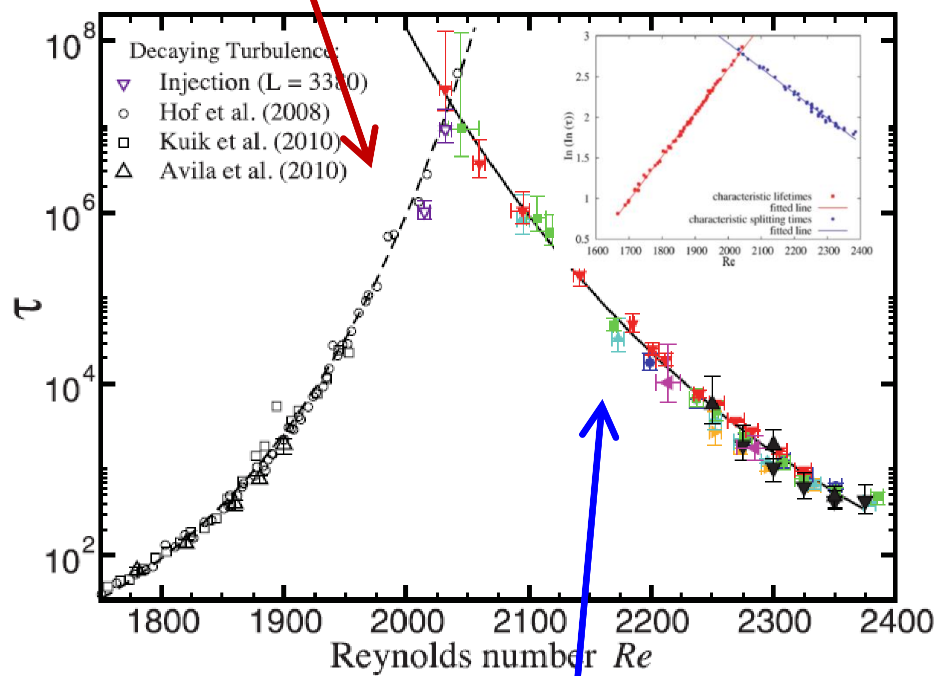
Survival probability $P(\mathrm{Re}, t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

Longest percolation path



Longest length of empty site

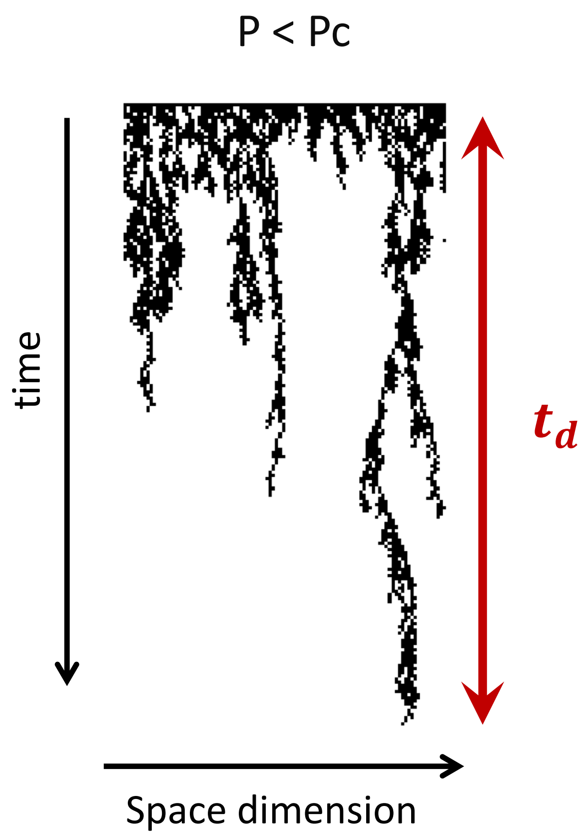
Turbulent puff lifetime



Mean time between puff split events

Avila *et al.*, *Science* **333**, 192 (2011)
Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Origin of superexponential scaling

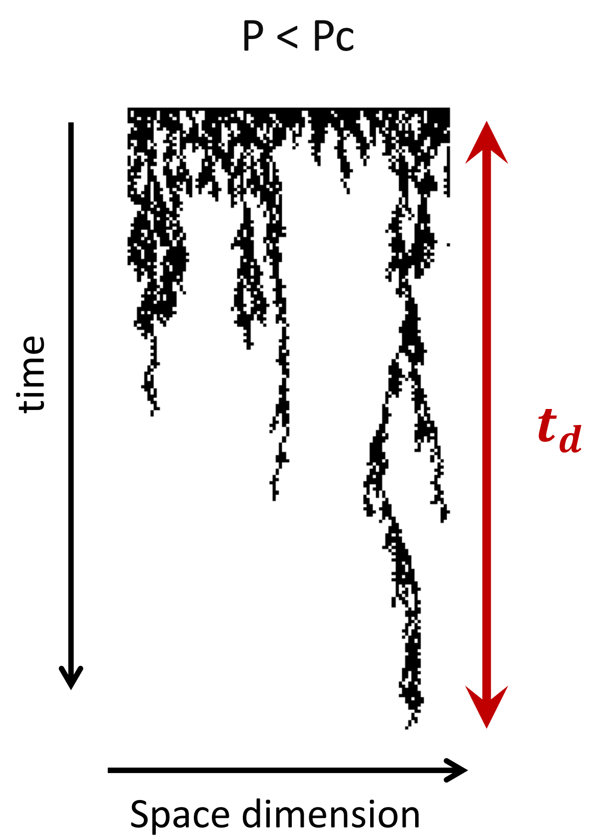


- Active state persists until **the most long-lived** percolating "strands" decay.
- In other words, **the longest path** = **the maximum** among **independent** random percolating paths

→ **Extreme value statistics**

Goldenfeld et al. (2010); Sipos & Goldenfeld (2011); Shih & Goldenfeld (in preparation)

Origin of superexponential scaling



- Active state persists until **the most long-lived** percolating "strands" decay.
- In other words, **the longest path** = **the maximum** among **independent** random percolating paths

→ **Extreme value statistics**

$$X_{\max} \propto \max(\{X_i\})$$

$$P(X_{\max} < y) = \exp(-\exp(-y))$$

- Near phase transition: correlated fluctuations are NOT i.i.d.

Goldenfeld et al. (2010); Sipos & Goldenfeld (2011)

Universality class of predator-prey system near extinction

Basic individual processes in predator (A) and prey (B) system:

Death $B_i \xrightarrow{d_B} E_i$ $\quad A_i \xrightarrow{d_A} E_i$ $\quad B_i \xrightarrow{m} A_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $\quad A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ $\quad A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j$

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $B_i \xrightarrow{d_B} E_i$ $\quad A_i \xrightarrow{d_A} E_i$ $\quad B_i \xrightarrow{m} A_i$

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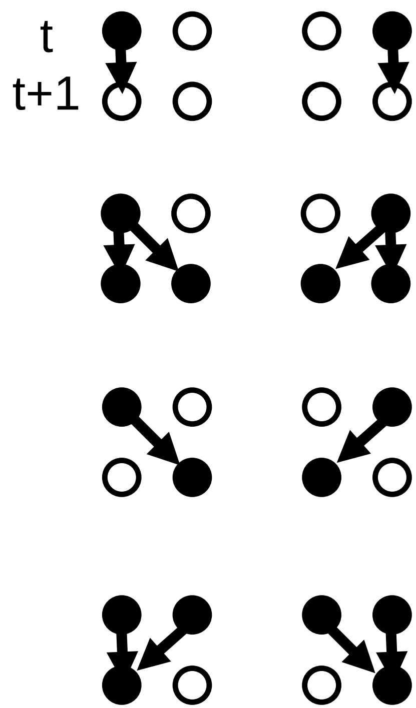


Annihilation

Decoagulation

Diffusion

Coagulation

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) populatio… n
survive; …

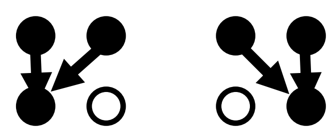
Near the extinction transition, stochastic predator-prey dynamics reduces to directed percolation

Death $B_i \xrightarrow{d_B}$ … ation

Birth $B_i + E$ … ulation

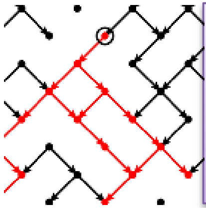
Diffusion $B_i + E_j$ … usion

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$



Coagulation

Summary: universality class of transitional turbulence

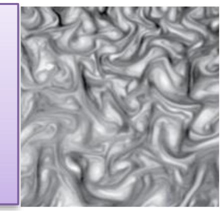


(Wikimedia Commons)

Directed Percolation

(Boffetta and Ecke, 2012)

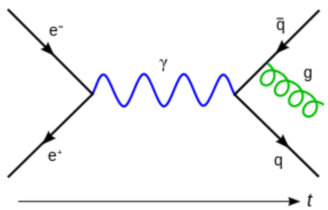
(Classical) Turbulence



Reggeon field theory
(Janssen, 1981)

Direct Numerical Simulations
of Navier-Stokes

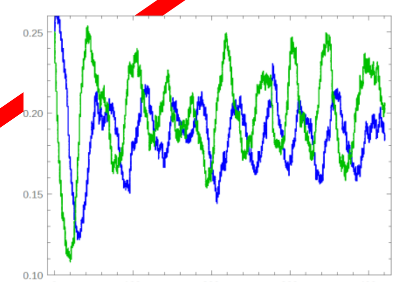
Field Theory



(Wikimedia Commons)

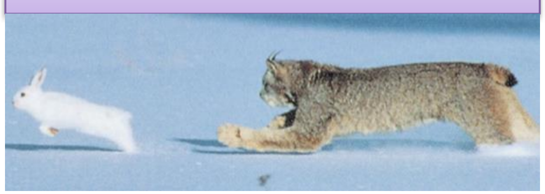
Two-fluid model

Turbulence + Zonal flow



Extinction transition
(Mobilia et al., 2007)

Predator-Prey

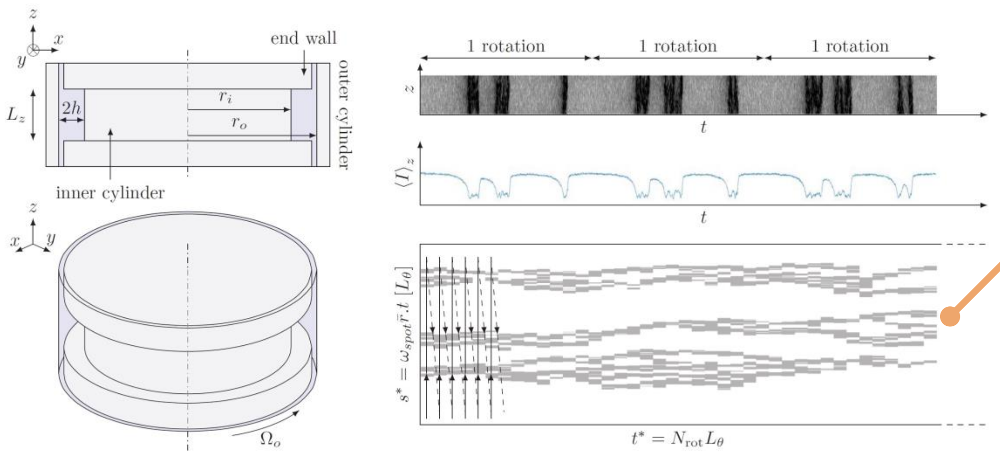


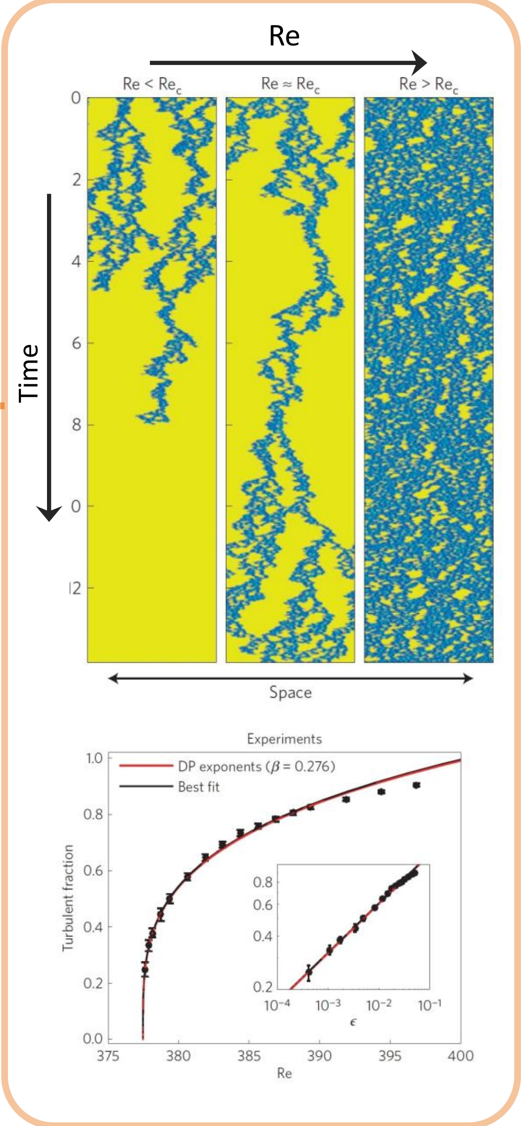
(Pearson Education, Inc., 2009)

Experimental test of DP in transitional turbulence

Directed percolation in turbulence experiments

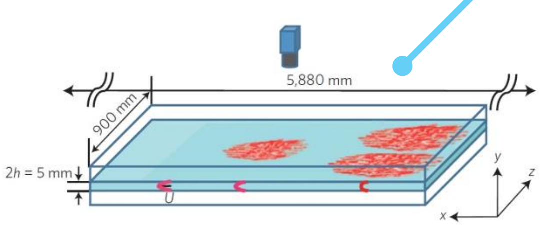
- Turbulent strands in **Couette flow** form long-time dynamics like (1+1) DP

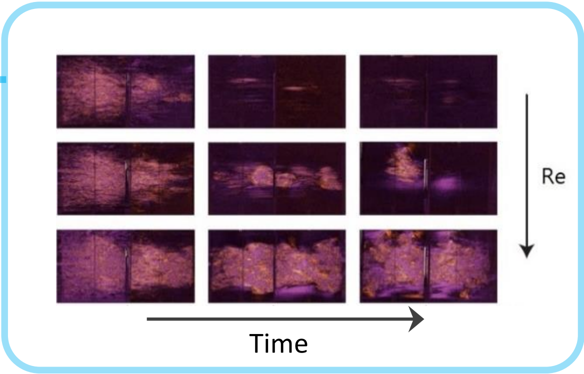




Lemoult et al., *Nature Physics* (2016)

- Turbulent patches in **channel flow** form clusters like (2+1) DP

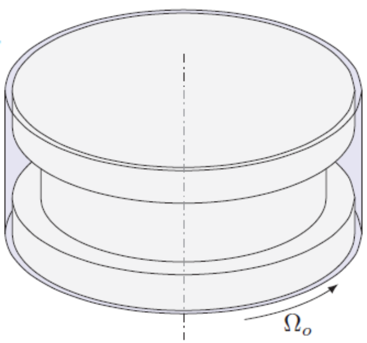




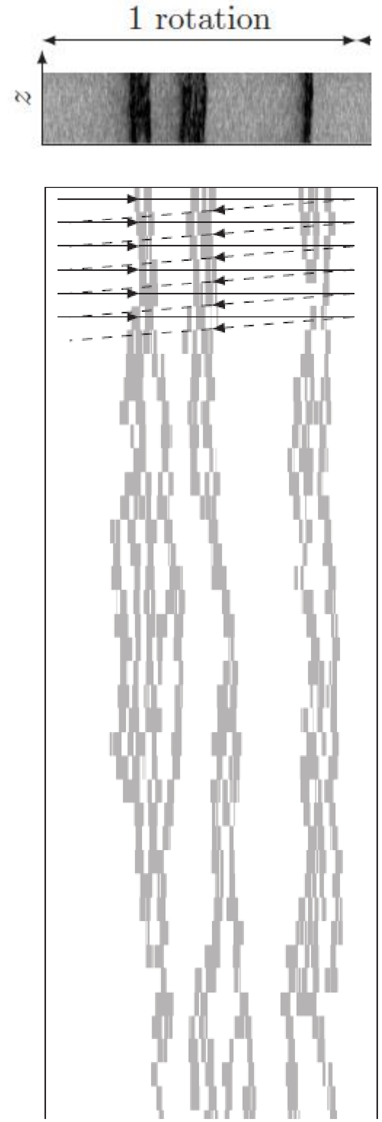
Sano & Tamai, *Nature Physics* (2016)

Turbulence and Ecology map to DP

Couette

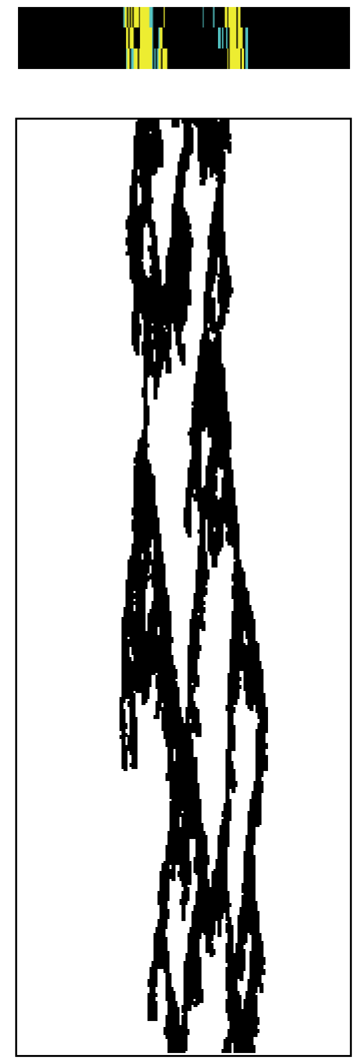


space



time

space

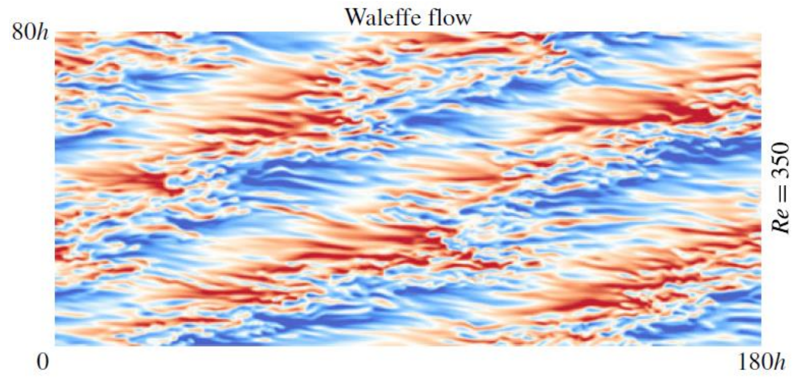


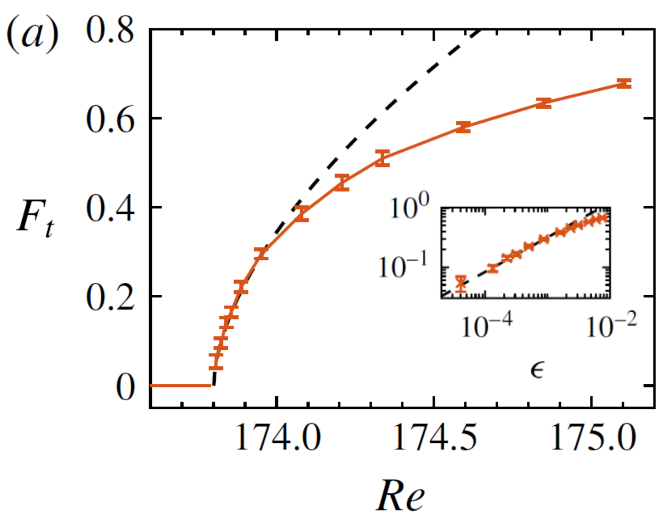
time

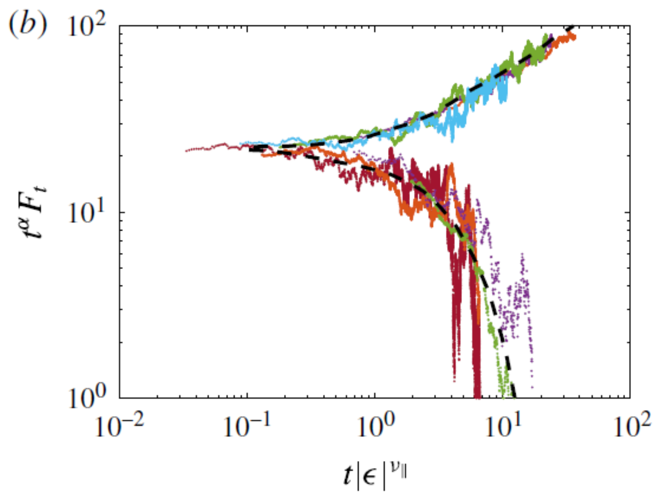
Ecology

2+1 dimensional DP in Navier-Stokes simulations

- Unprecedented large-scale direct numerical simulation of a **stress-free planar shear flow** (Waleffe flow) exhibits (2+1) DP scaling



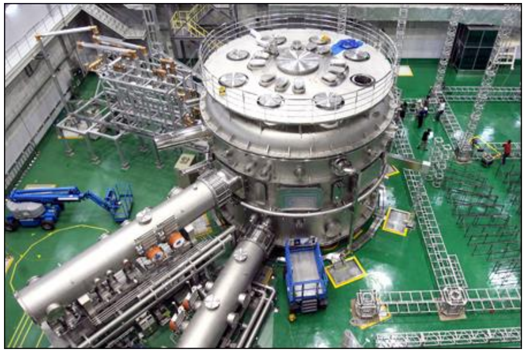


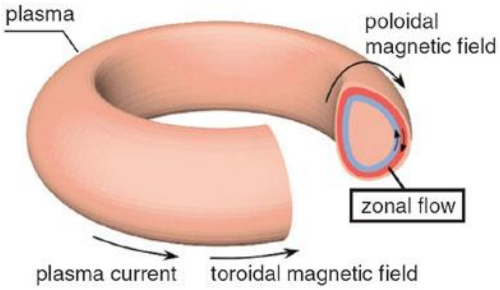


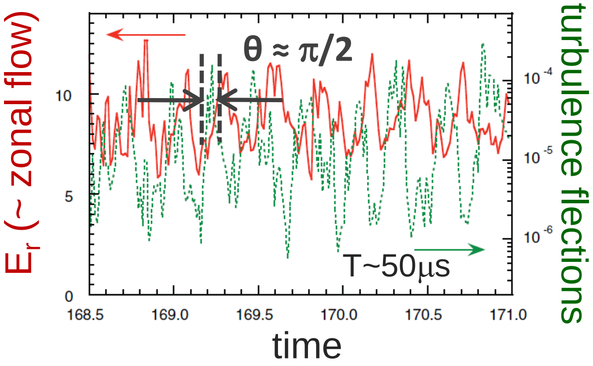
Chantry, Tuckerman & Barkley, *J. Fluid Mech.* (2016, 2017) Goldenfeld, *J. Fluid Mech.* (2017)

Observation of predator-prey dynamics in magneto-hydrodynamics

- Low-High confinement transition in fusion plasmas in a tokamak
- Interplay between drift-wave turbulence along the axis of the tokomak and an azimuthal zonal flow that emerges from the turbulence and simultaneously suppresses it
- Observations follow theoretical predictions by P. Diamond and collaborators







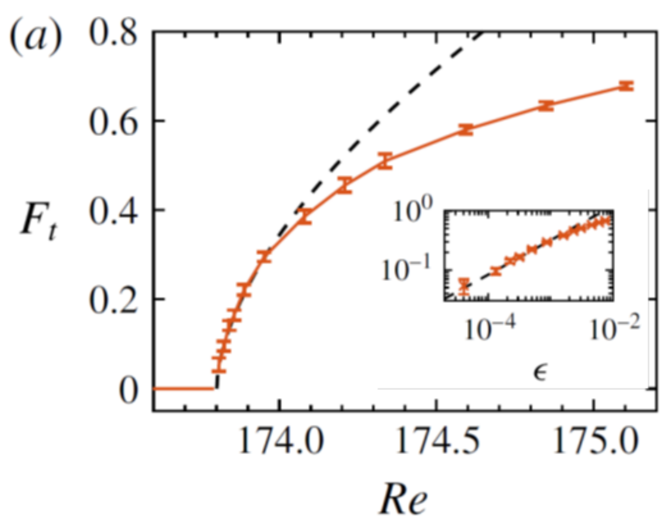
Estrada et al. *EPL* (2012)

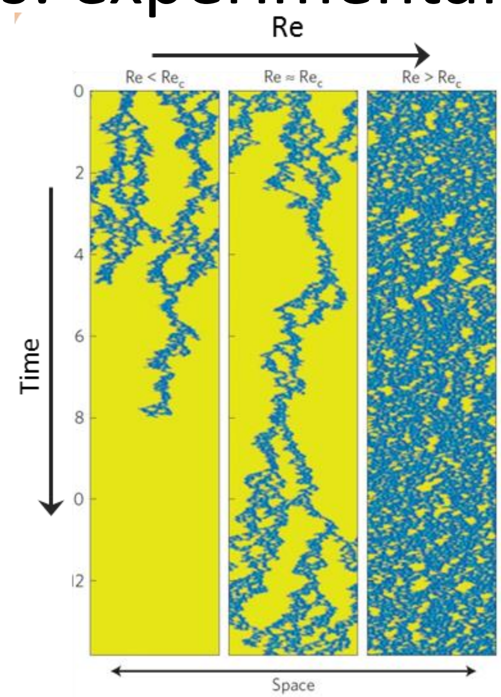
http://alltheworldstokamaks.wordpress.com/gallery-of-external-views/kstar-completed/

Summary of transitional turbulence

Preliminary theoretical results predict that the laminar-turbulence transition is a non-equilibrium critical point in the universality class of directed percolation.

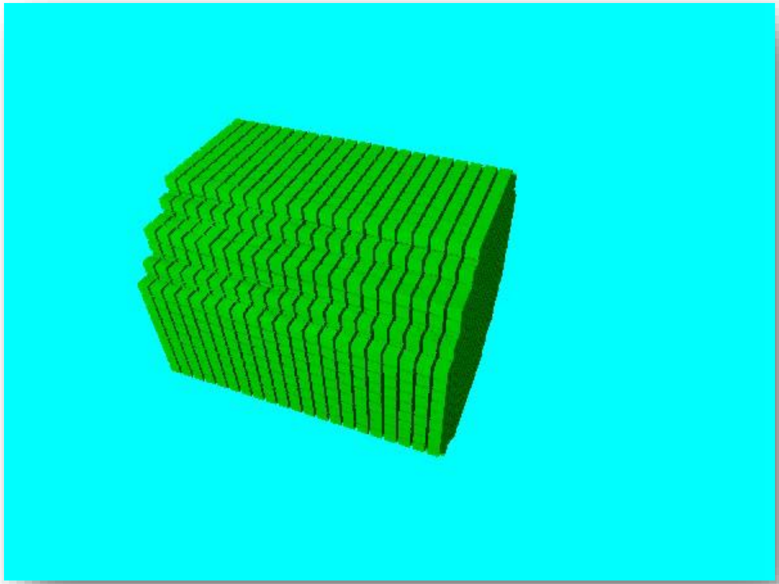
Confirmed in two specific flows: experimentally and computationally.

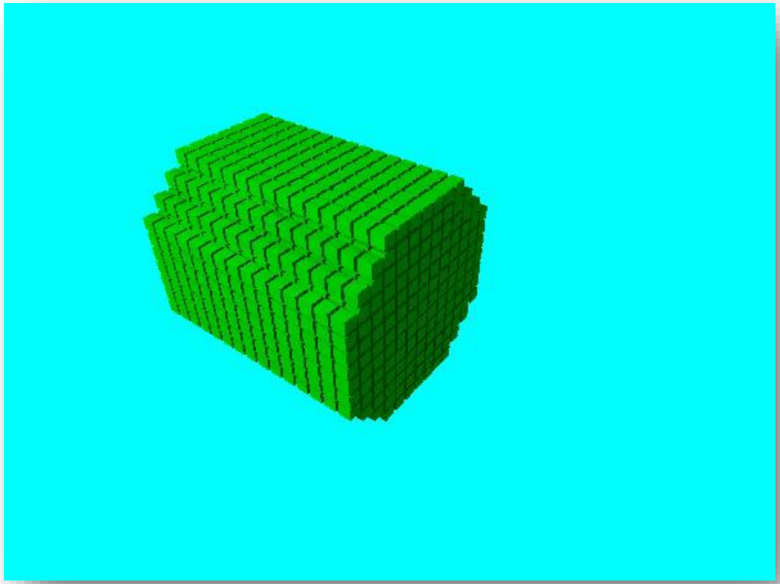




Ongoing work

DP in 3 + 1 dimensions in pipe



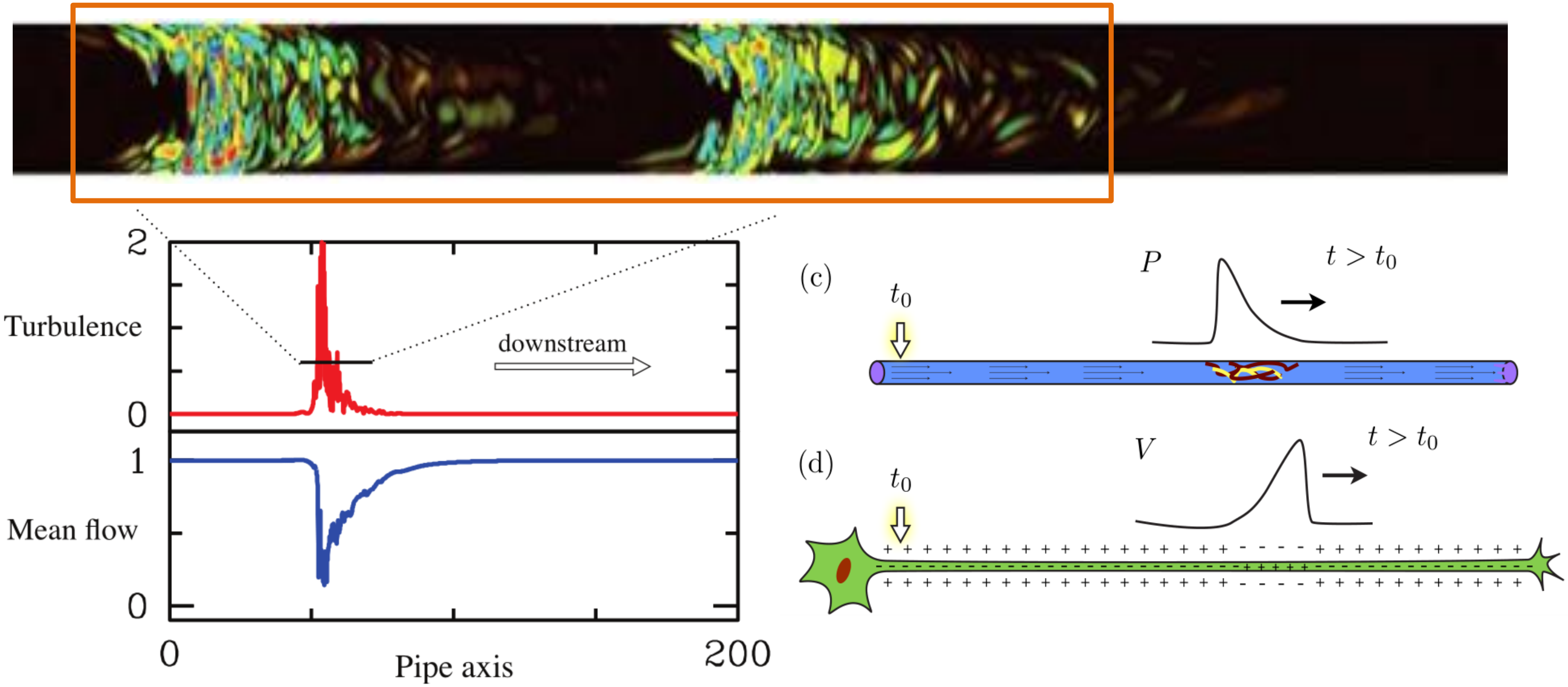


Puff decay
Re < 2040

Slug spreading
Re > 2250

Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

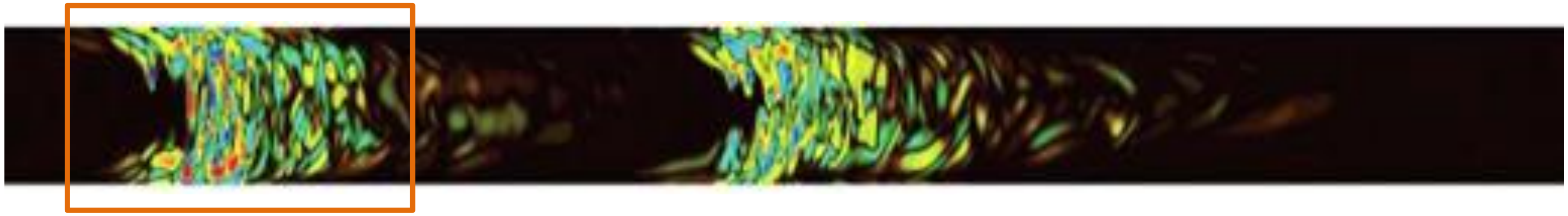
Transitional turbulence

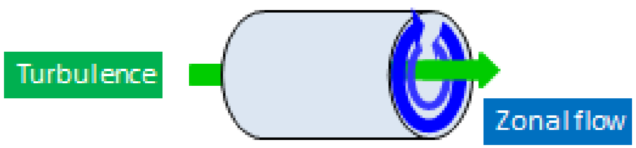


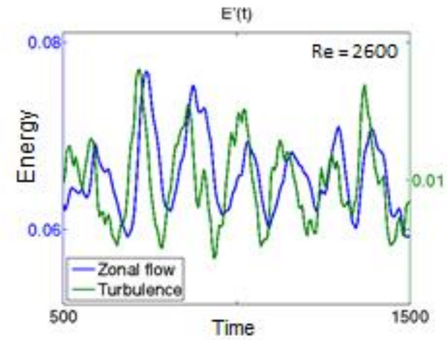
- Turbulent patches behave as an excitable medium (such as a nerve fibre), interacting with mean shear
  - Turbulence is like the nerve voltage
  - Mean flow is like the ion channel recovery

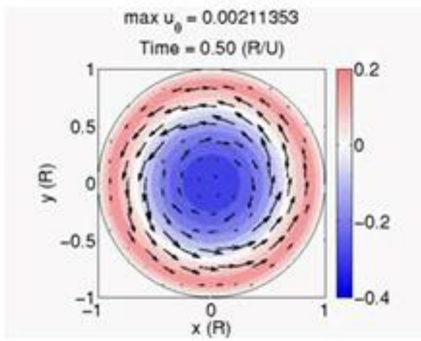
D. Barkley (2011)

Transitional turbulence





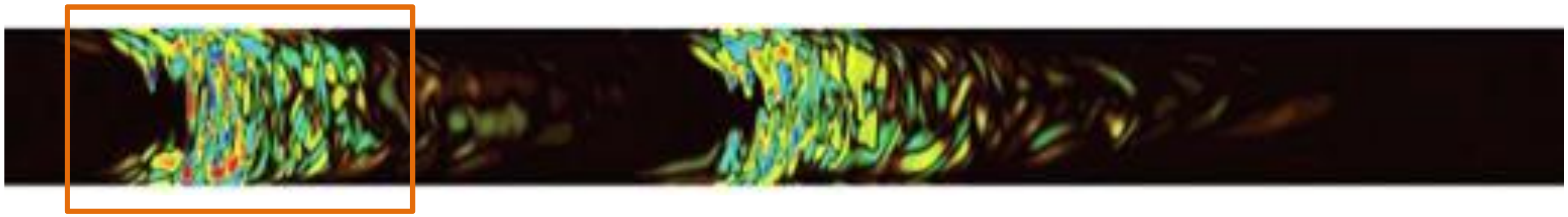


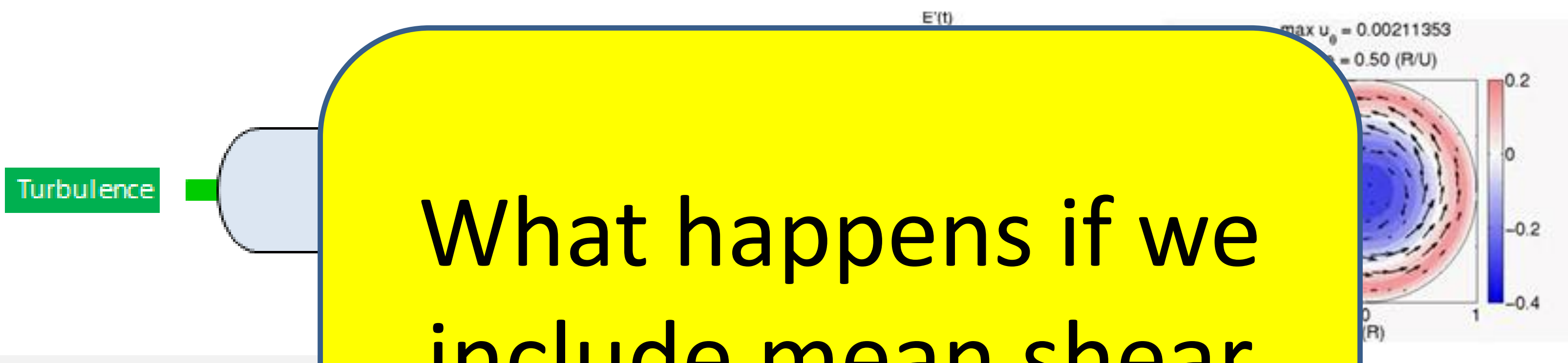


- Dynamics of single puff
  - Nascent turbulence generates an emergent mean flow (zonal flow)
  - Zonal flow inhibits the turbulence by shearing
  - “Predator-prey” oscillations
  - Long-wavelength fluctuations of soft mode -> second order phase transition
- Single puff spatial extent is actually determined by the transfer of laminar shear into turbulence

H.-Y. Shih et al. (2016)

Transitional turbulence



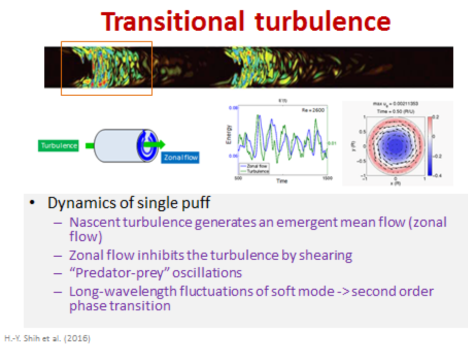


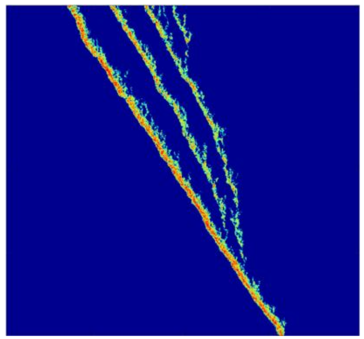
What happens if we include mean shear as well?

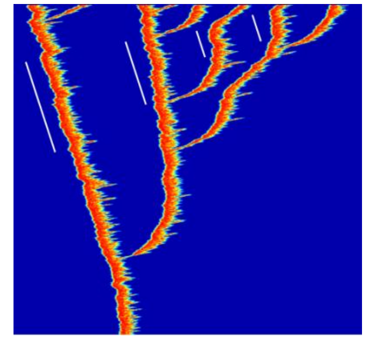
- Dynamics of
  - Nascent t ... flow)
  - Zonal flow
  - “Predator
  - Long-wavele ... phase transition
- Single puff spatial extent is actually determined by the transfer of laminar shear into turbulence

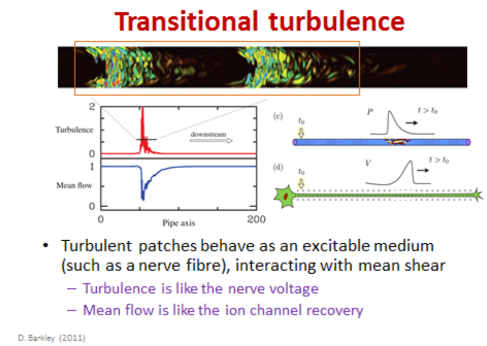
H.-Y. Shih et al. (2016)

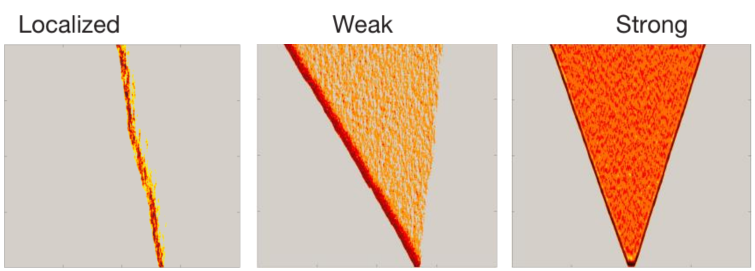
Transitional turbulence

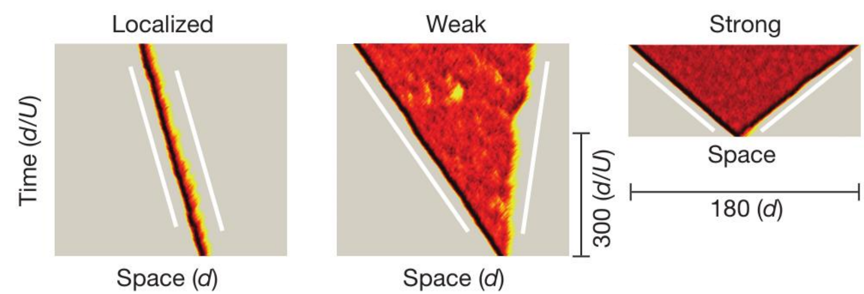












- Predator-prey and excitable media descriptions give equivalent predictions
  - Phase diagram
  - Kinematics of puffs and slugs

D. Barkley (2015) Wang et al. (unpubl.)

Fluctuations and dissipation

Nikuradse's pipe experiment (1933) to measure the friction factor *f*

$$f = \Delta P / l \rho U^2$$

Pipe diameter is 25-100 mm
Pipe length is ~ 70 diameters

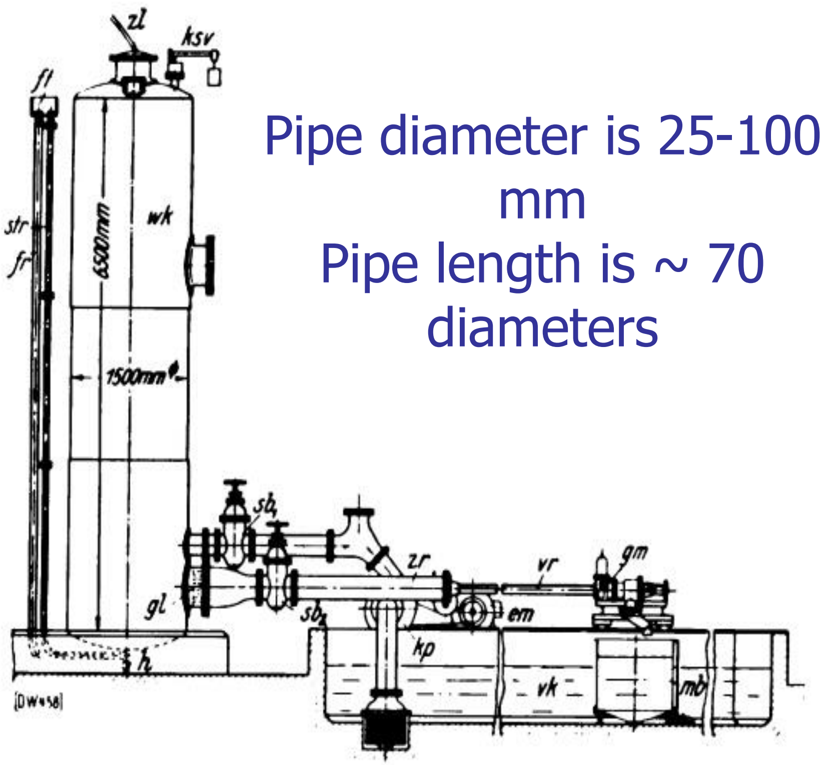


Figure 3.- Test apparatus.

em = electric motor		h = outlet valve	
kp = centrifugal pump		zr = feed line	
vk = supply canal		mb = measuring tank	
wk = water tank		gm = velocity measuring device	
vr = test pipe		ksv = safety valve on water tank	
zl = supply line		sb_1 = gate valve between wk and kp	
str = vertical pipe		sb_2 = gate valve between wk and zr	
fr = overflow pipe		gl = baffles for equalizing flow	
ft = trap			

Monodisperse sand grains 0.8mm glued to sides of pipe

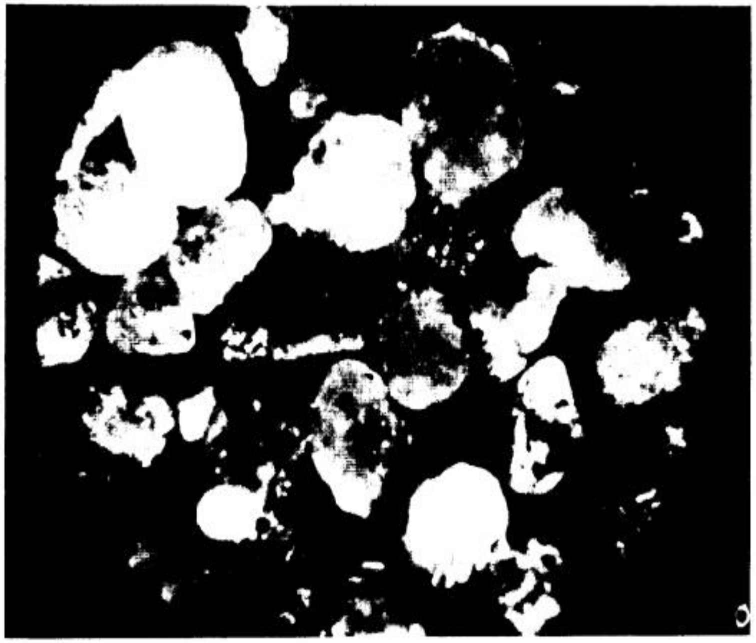


Figure 4.- Microphotograph of sand grains which produce uniform roughness. (Magnified about 20 times.)

Friction factor in turbulent rough pipes

Laminar

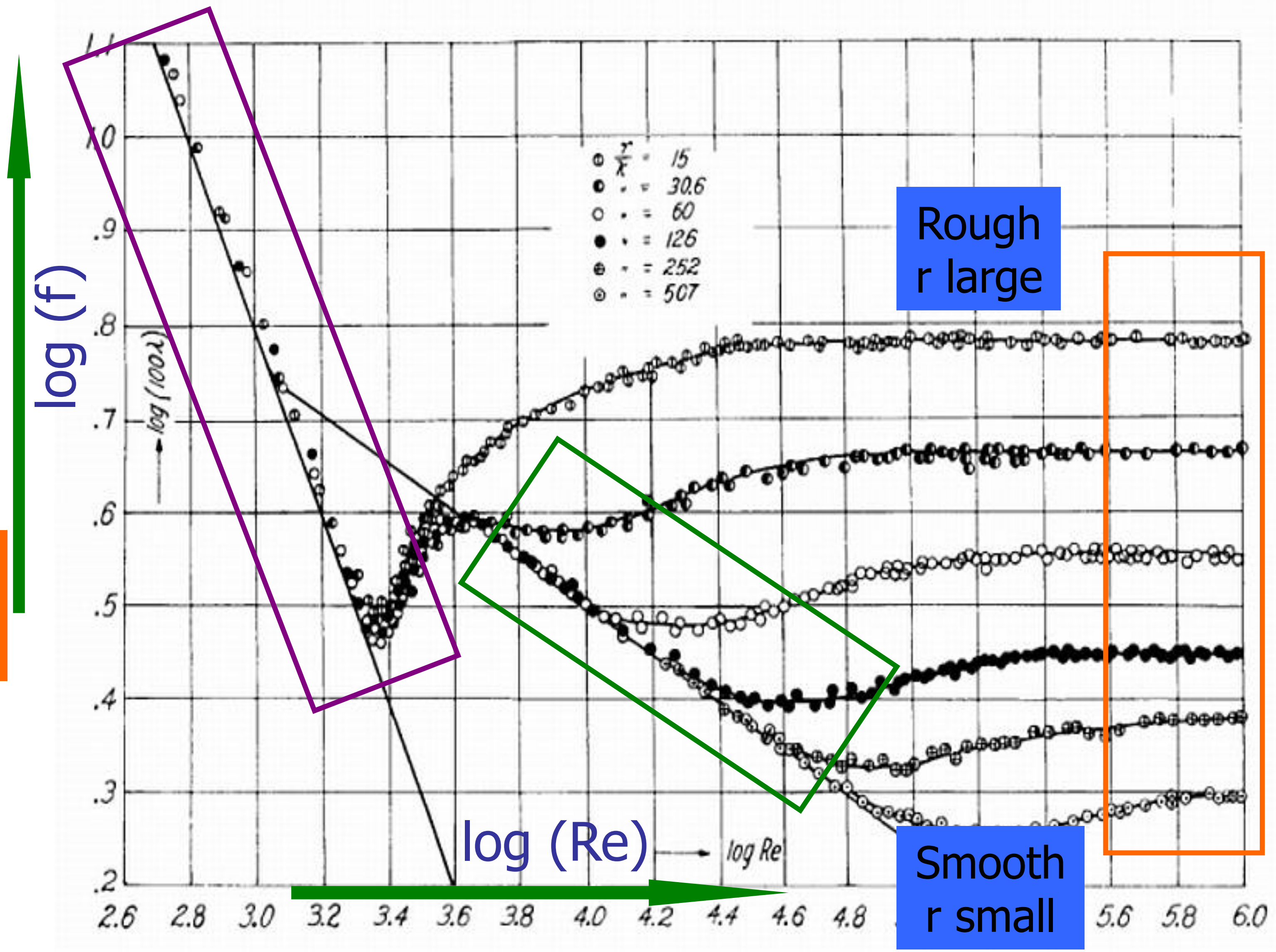
$f \sim 12/Re$

Blasius

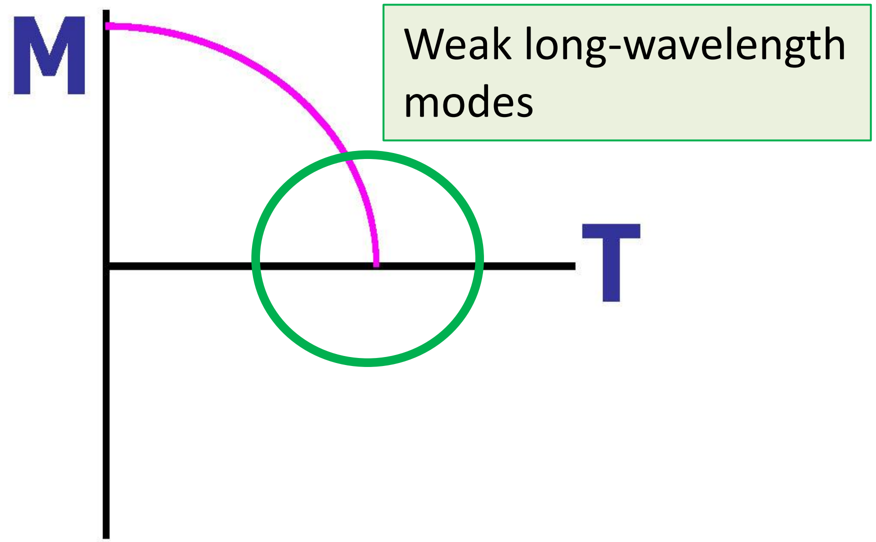
$f \sim Re^{-1/4}$

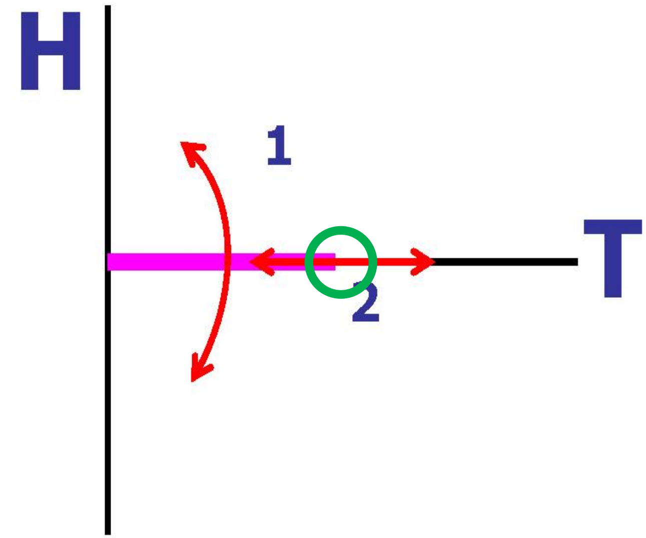
Strickler

$f \sim (r/D)^{1/3}$



Critical phenomena in magnets





$M \sim M_0[|T - T_c|/T_c]^\beta$ for $H = 0$ as $T \to T_c$

Critical isotherm: $M \sim H^{1/\delta}$ for $T = T_c$

- Widom (1963) pointed out that both these results followed from a *similarity formula*:

$$M(t,h) = |t|^\beta f_M(h/t^\Delta)$$

where $t \equiv (T - T_c)/T_c$ for some choice of exponent Δ and scaling function $f_M(x)$

Critical phenomena in magnets

$$M(t,h) = |t|^\beta f_M(h/t^\Delta)$$

where $t \equiv (T - T_c)/T_c$ for some choice of exponent Δ and scaling function $f_M(x)$

- **To determine the properties of the scaling function and unknown exponent, we require:**
  - $f_M(z)$ = const. for z = 0
  - This gives $M \sim M_0[|T - T_c|/T_c]^\beta$ for $T < T_c$
  - For large values of z, i.e. non-zero h, and t →0, t dependence must cancel out so that $M \sim H^{1/\delta}$

  Thus $f_M(z) \sim z^{1/\delta}, z \to \infty$.
  Calculate Δ: t dependence will only cancel out if $\beta - \Delta/\delta = 0$

$$M = |t|^\beta f_M(h/|t|^{\beta\delta})$$

- **This data collapse formula connects the scaling of correlations with the thermodynamics of the critical point**

Universality at a critical point

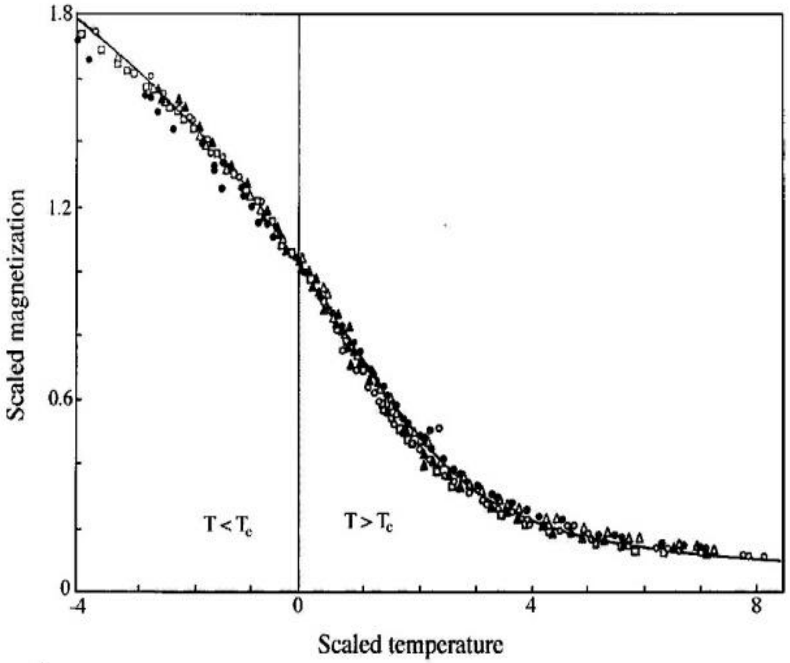


FIG. 1. Experimental *MHT* data on five different magnetic materials plotted in scaled form. The five materials are $CrBr_3$, EuO, Ni, YIG, and Pd_3Fe. None of these materials is an idealized ferromagnet: $CrBr_3$ has considerable lattice anisotropy, EuO has significant second-neighbor interactions. Ni is an itinerant-electron ferromagnet, YIG is a ferrimagnet, and Pd_3Fe is a ferromagnetic alloy. Nonetheless, the data for all materials collapse onto a single scaling function, which is that calculated for the $d=3$ Heisenberg model [after Milošević and Stanley (1976)].

Stanley (1999)

- Magnetization M of a material depends on temperature T and applied field H
  - M(H,T) ostensibly a function of two variables
- Plotted in appropriate scaling variables get ONE universal curve
- Scaling variables involve critical exponents

Critical phenomena and turbulence

	Critical phenomena	Turbulence
Correlations	$G(k) \sim k^{-2}$	$E(k) \sim k^{-5/3}$
Large scale thermodynamics	$M(h,t) = \|t\|^{\beta} f_M(ht^{-\beta\delta})$	?

Scaling of Nikuradse's data

	Critical phenomena	Turbulence
Temperature control	$t \to 0$	$1/Re \to 0$
Field control	$h \to 0$	$r/D \to 0$

Scaling of Nikuradse's Data

- In the turbulent regime, the extent of the Blasius regime is apparently roughness dependent.
  - $f \sim Re^{-1/4}$ as $r/D \rightarrow 0$
- At large Re, f is independent of Reynolds number.
  - $f \sim (r/D)^{1/3}$ for $Re \rightarrow \infty$
- Combine into unified scaling form
  - $f = Re^{-1/4}\, g([r/D]\, Re^{\alpha})$
    - Determine α by scaling argument: Re dependence must cancel out at large Re to give Strickler scaling
  - Exponent $\alpha = 3/4$ and the scaling function $g(z) \sim z^{1/3}$ for $z \rightarrow \infty$
- $f = Re^{-1/4}\, g([r/D]\, Re^{3/4})$

Data collapse in Nikuradse's data

PRL **96**, 044503 (2006) PHYSICAL REVIEW LETTERS week ending 3 FEBRUARY 2006

Roughness-Induced Critical Phenomena in a Turbulent Flow

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(Received 16 September 2005; published 30 January 2006)

PHYSICAL REVIEW E **77**, 055304(R) (2008)

Intermittency and rough-pipe turbulence

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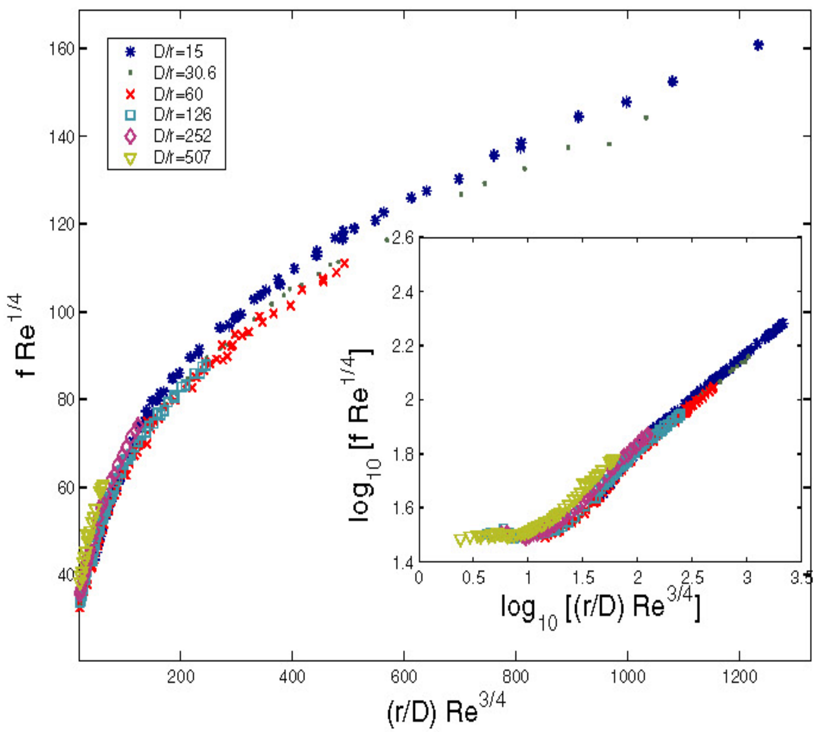
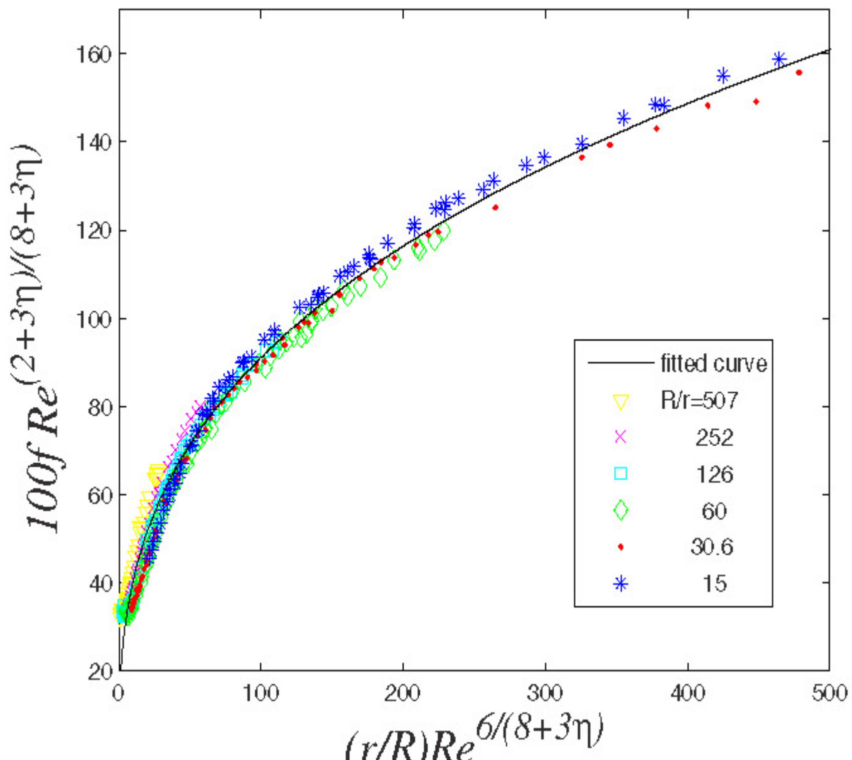
(Received 23 February 2008; published 15 May 2008)



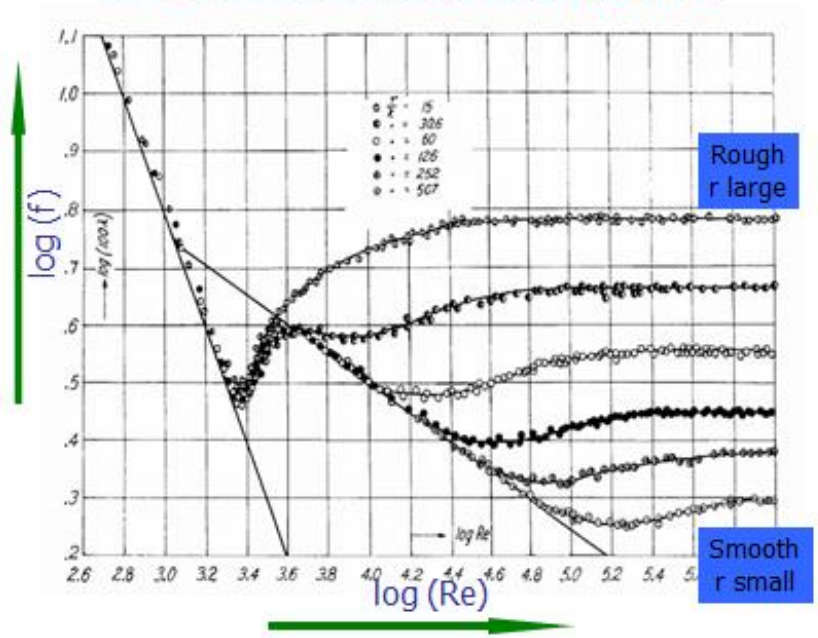
Mean field (Kolmogorov 1941) exponents

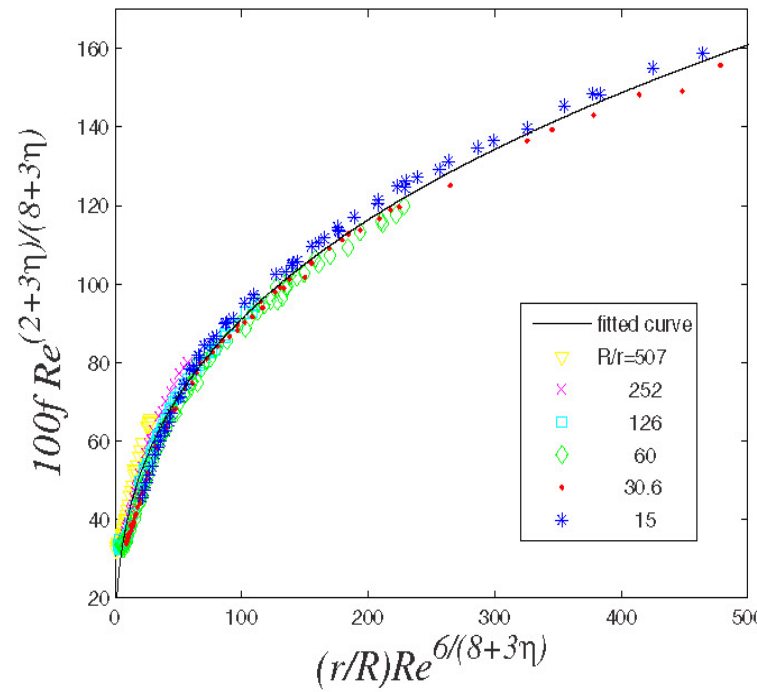


Intermittency corrections included
Value of η ~ 0.02 consistent with spectral estimates

Data collapse of friction factor

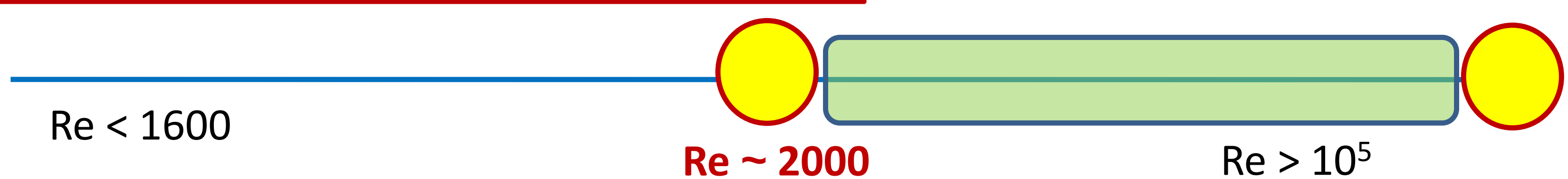
Friction factor in turbulent rough pipes





$$f = \mathrm{Re}^{-(2+3\eta)/(8+3\eta)}\, g\left(\frac{r}{R}\,\mathrm{Re}^{6/(8+3\eta)}\right)$$

Intermittency corrections included
Value of η ~ 0.02 consistent with spectral estimates



Goldenfeld, Phys. Rev. Lett. (2006); Mehrafarin & Pourtolami, Phys. Rev. E. (2008)

By simply measuring the pressure drop across a pipe, Nikuradse in 1933 measured the anomalous spectral exponents (intermittency corrections) 8 years before Kolmogorov's mean field theory!

Friction factor → intermittency corrections to velocity fluctuations

Connection between the velocity fluctuations at small scales and the friction factor or dissipation is called the "spectral link" and is an example of a fluctuation-dissipation relation

Calculating scaling exponents

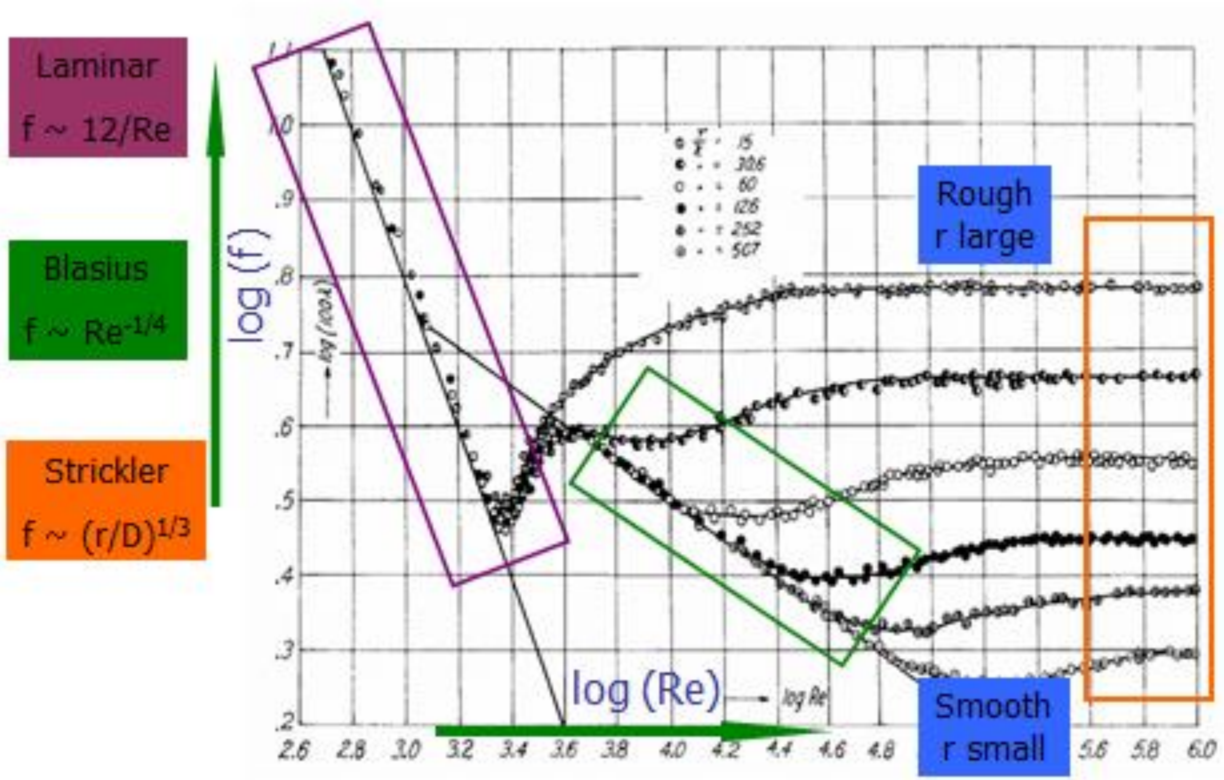
- Basic idea: $f \sim \rho V u_s / \rho V^2$
  - u_s is the RMS velocity at a characteristic scale s determined by main source of dissipation
  - For weak turbulence dissipation comes from the flow, i.e. the Kolmogorov scale $s \sim Re^{-3/4}$
  - For strong turbulence, dissipation because fluid strongly affected by wall roughness $s \sim r$
  - Model this by $s \sim r + Re^{-3/4}$

$$f \propto u_s \propto \left(\int_{1/s}^{\infty} E(k)dk\right)^{1/2}$$

Dissipation **Fluctuation**

Gioia & Chakraborty PRL (2006); Guttenberg & Goldenfeld (2009); Gioia, Guttenberg, Goldenfeld, Chakraborty PRL (2010)

Friction factor in turbulent rough pipes



- **K41: Use $E(k) \sim k^{-5/3}$**

$$f \sim \left(\frac{r}{R} + ab\,\mathrm{Re}^{-3/4}\right)^{1/3}$$

- **Large Re: $s \sim r$ and $f \sim (r/D)^{1/3}$ Strickler law predicted!**
- **Small Re: $s \sim \eta$ and $f \sim \mathrm{Re}^{-1/4}$ Blasius law predicted!**

Gioia & Chakraborty (2006)

Experimental test in 2D

- Two cascades in 2D turbulence, an inverse one of energy, a forward one of enstrophy

$$E(k) \propto \lambda^{2/3} k^{-3}$$

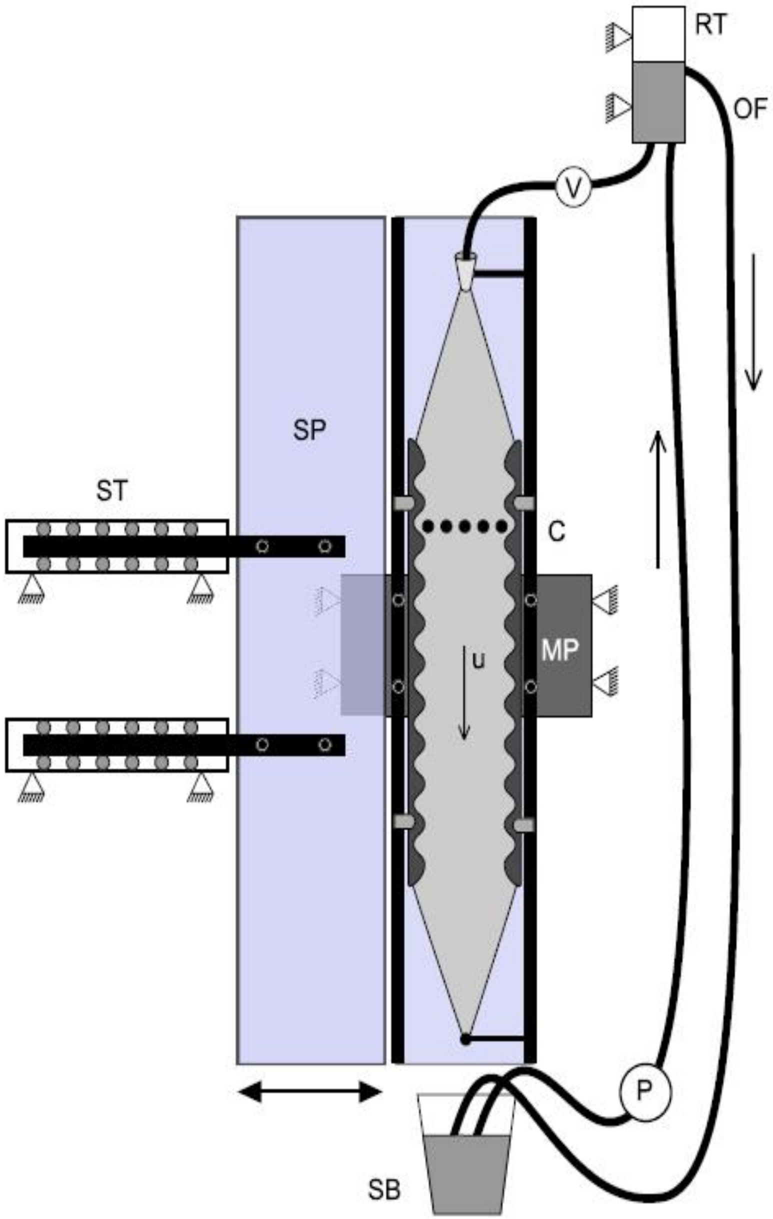
2D forward cascade

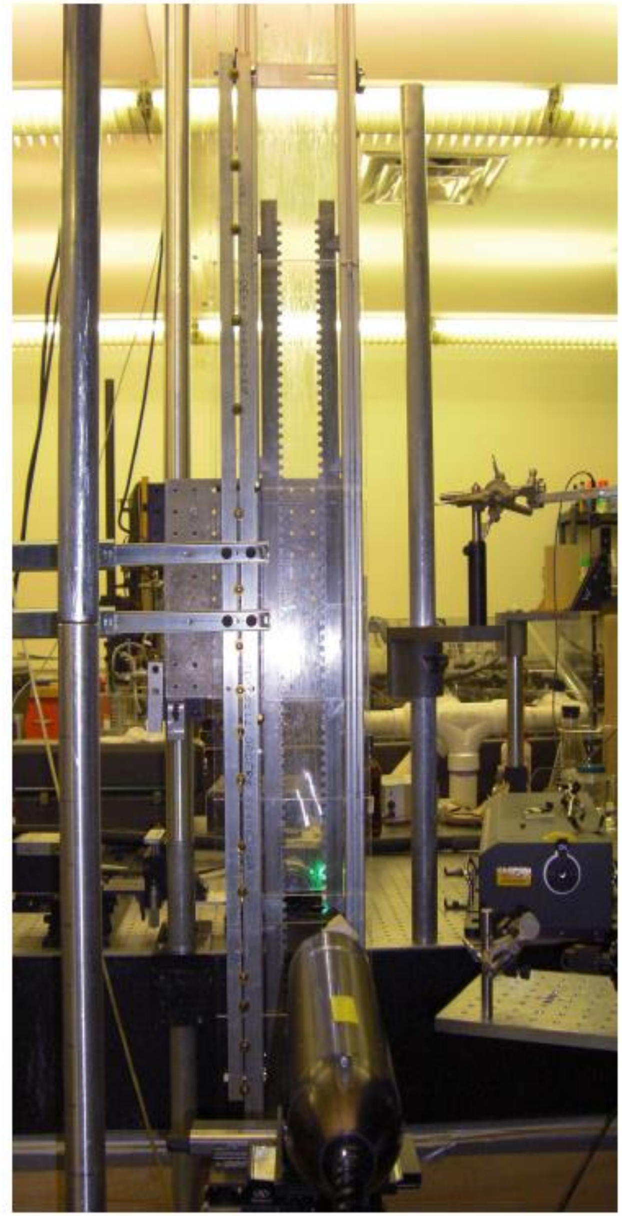
$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

2D inverse cascade

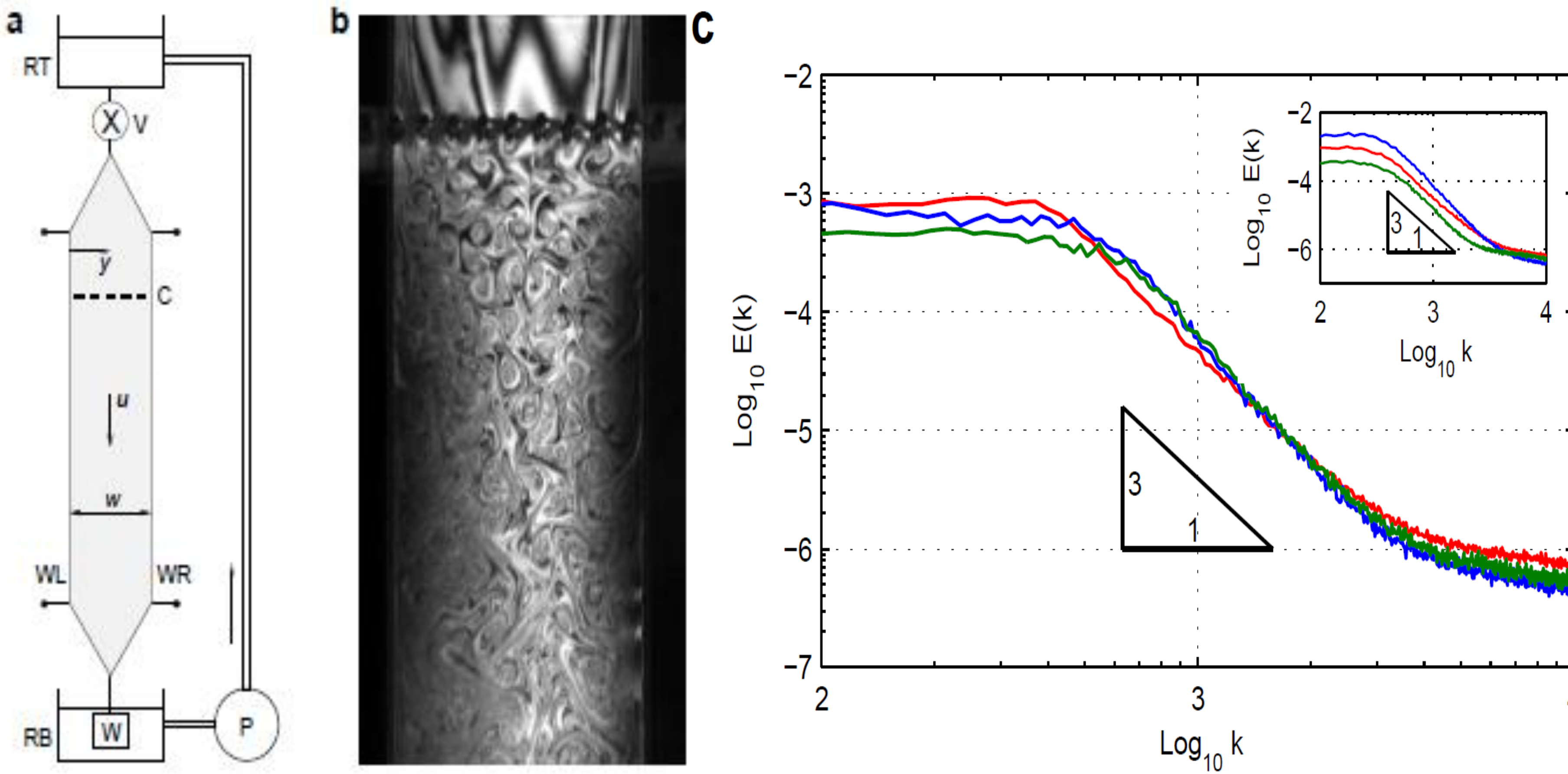
Enstrophy cascade
$f \sim Re^{-1/2}$ (Blasius)
$f \sim (r/D)$ (Strickler)

Inverse cascade
$f \sim Re^{-1/4}$ (Blasius)
$f \sim (r/D)^{1/3}$ (Strickler)



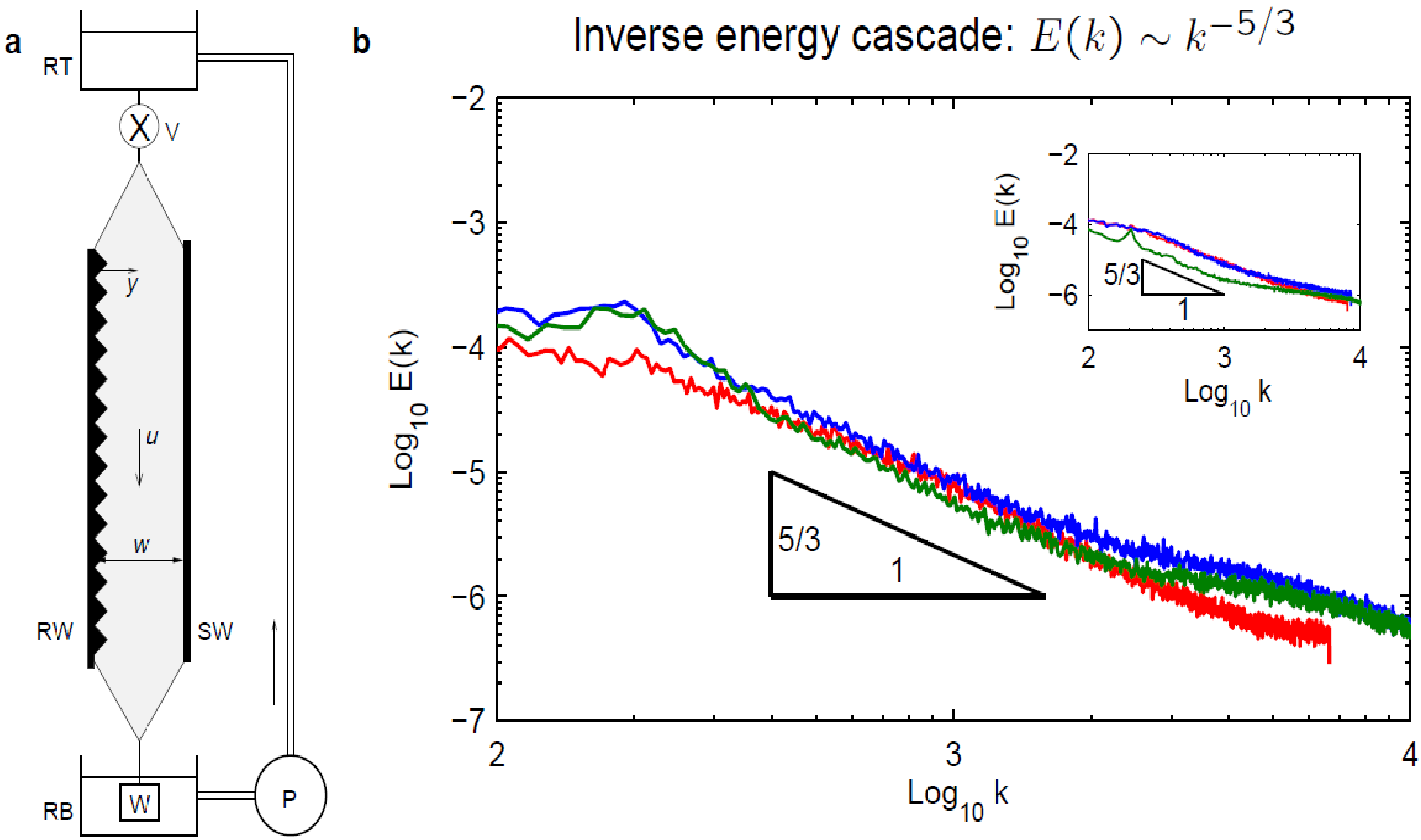


Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)



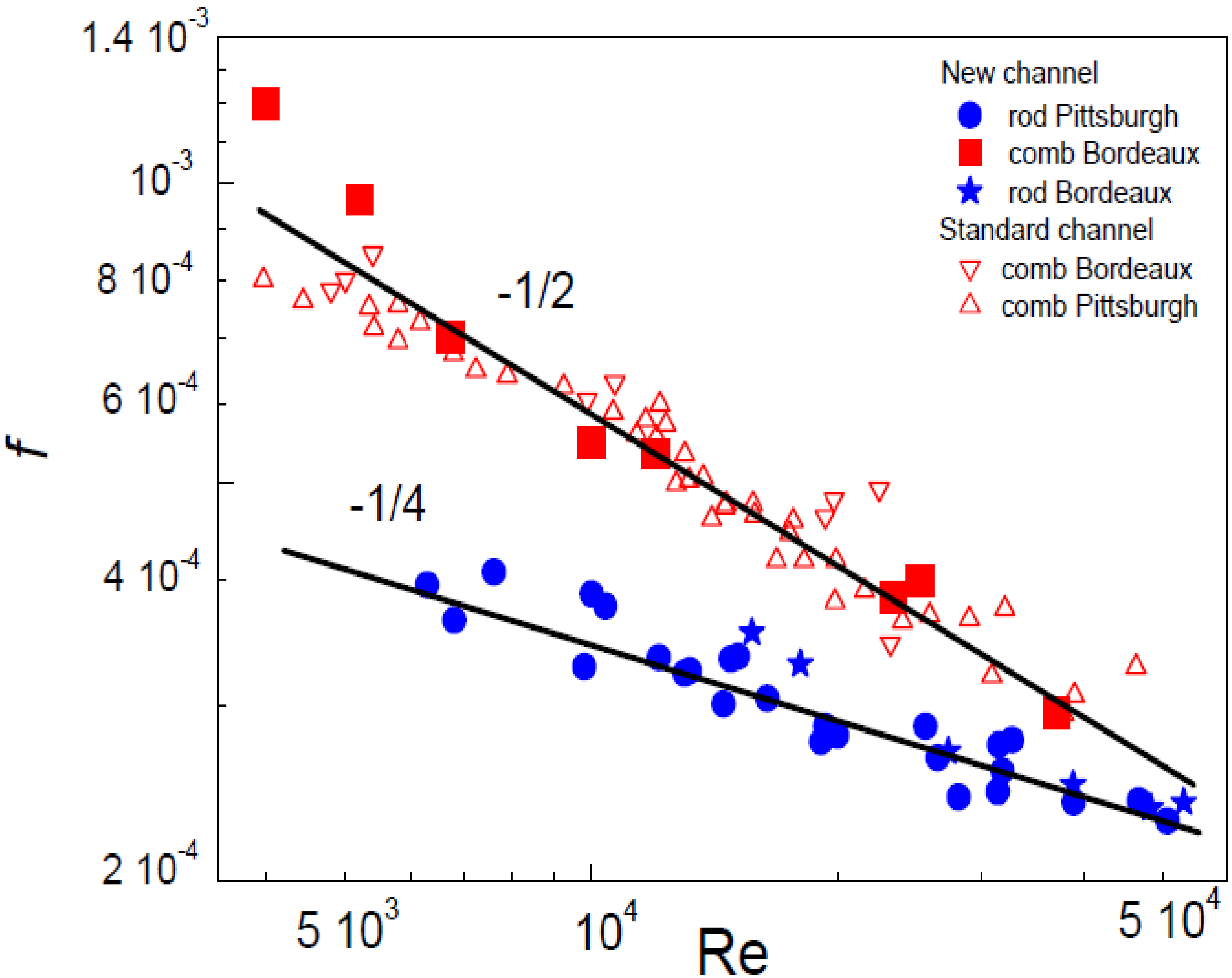
Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)

Inverse cascade in 2D soap films



Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)

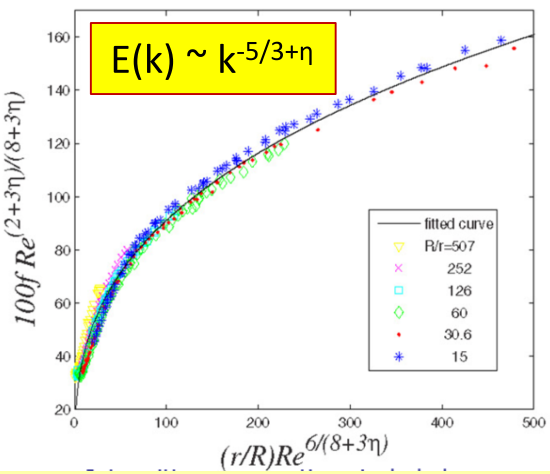
Friction factor depends on cascade



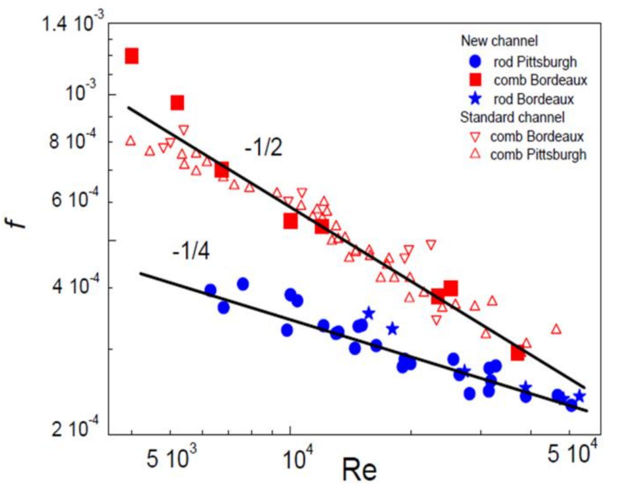
Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)

Fluctuations and Dissipation

The drag experienced at large scales reflects the very nature of the turbulent state at smaller scales. We make two predictions, both confirmed experimentally

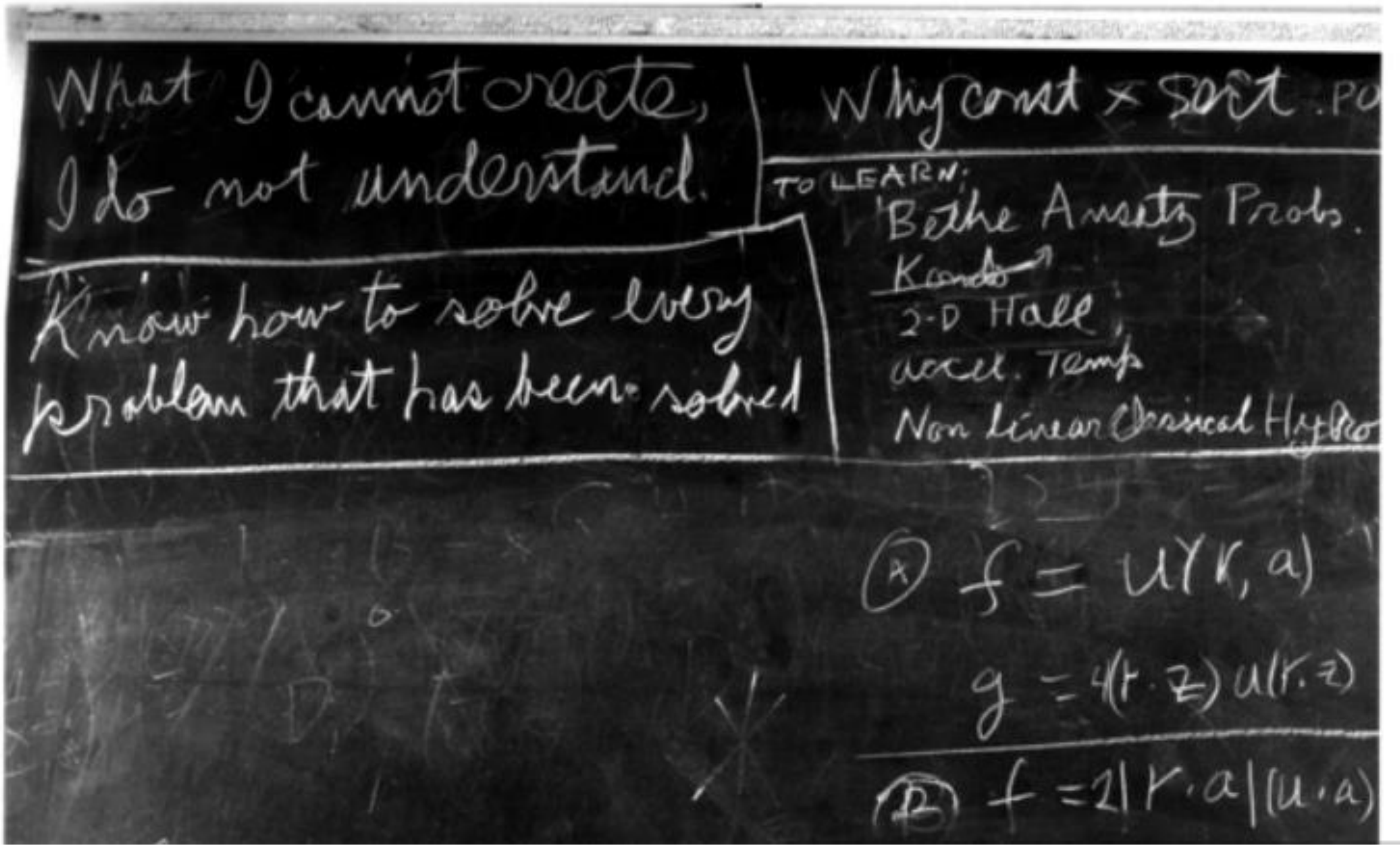


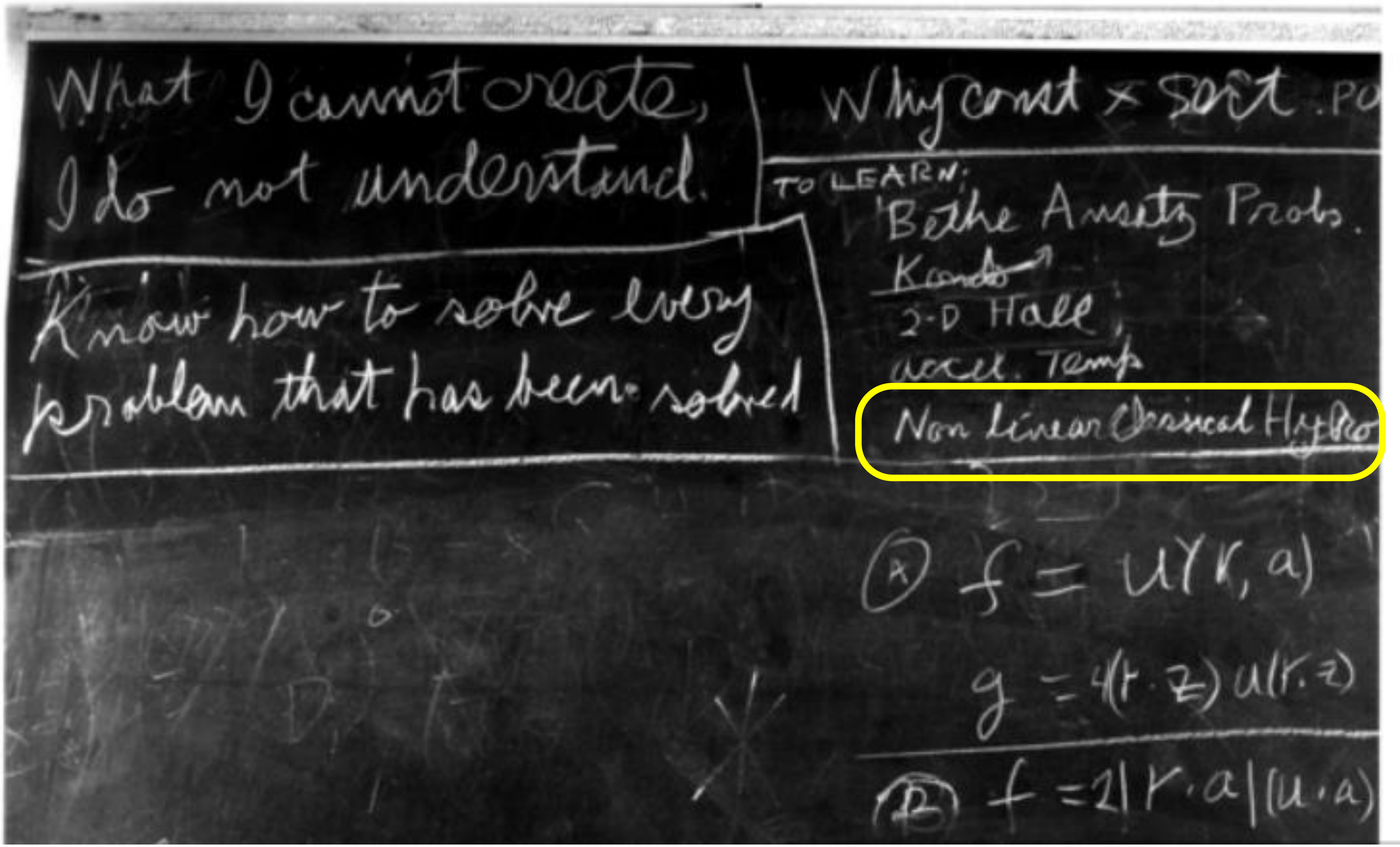
Anomalous scaling exponent from data collapse of friction factor



Friction factor depends on energy spectrum/cascade, measured in 2D

Goldenfeld, Phys. Rev. Lett. (2006); Mehrafarin & Pourtolami, Phys. Rev. E. (2008); Gioia & Chakraborty, Phys. Rev. Lett. (2006); Kellay et al. Phys. Rev. Lett. (2012)


What I cannot create,
I do not understand.
Know how to solve every
problem that has been solved
Why const × sort
TO LEARN:
Bethe Ansatz Probs.
Kondo
2-D Hall
accel. Temp
Non Linear Classical Hydro


What I cannot create,
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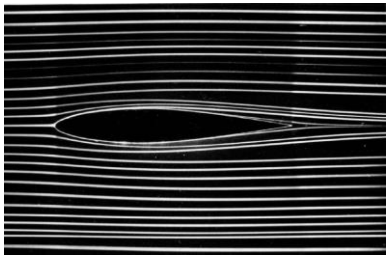
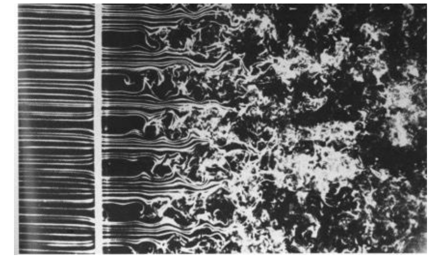
Why const × sort .PO
TO LEARN:
Bethe Ansatz Probs.
Kondo
2-D Hall
accel. Temp
Non Linear Classical Hydro
Know how to solve every
problem that has been solved
f = u(r, a)
g = 4(r · z) u(r · z)
f = 2|r · a|(u · a)

Take-home cartoon





laminar flow

steady

predictable

Re < 1600

laminar-turbulent transition

critical behavior

Re ~ 2000

fully-developed turbulence

fluctuating

unpredictable

Re > 10^5

Theory

Expt

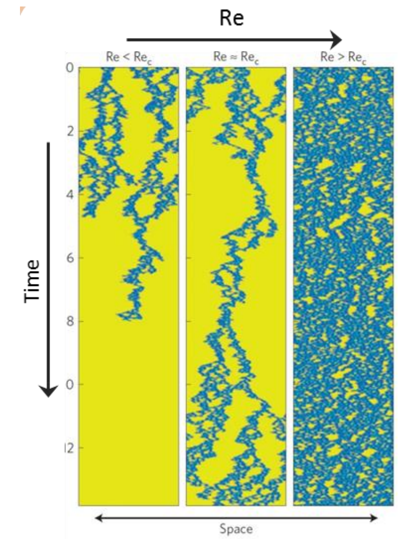
Conventional approach

Our approach

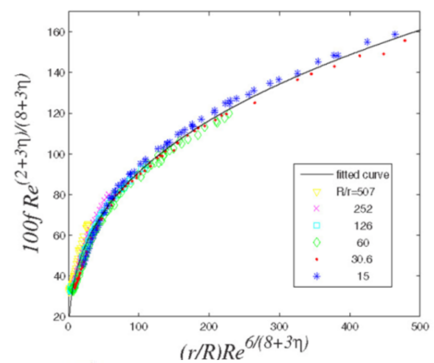
Critical points with their own scaling laws, crossovers and universality classes

Unifying concepts of fluctuation-dissipation, rare events, mean-flow interactions

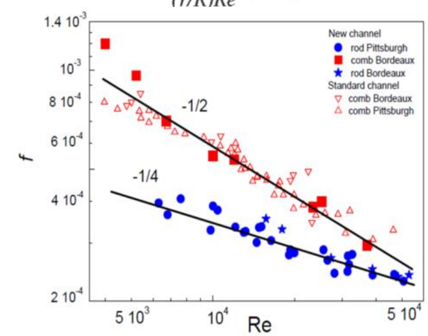
Summary



- The transition to turbulence seems to be a non-equilibrium phase transition in the universality class of directed percolation
  - Theory
  - Experiments in quasi-1D Taylor-Couette
  - DNS in 2D Waleffe flow

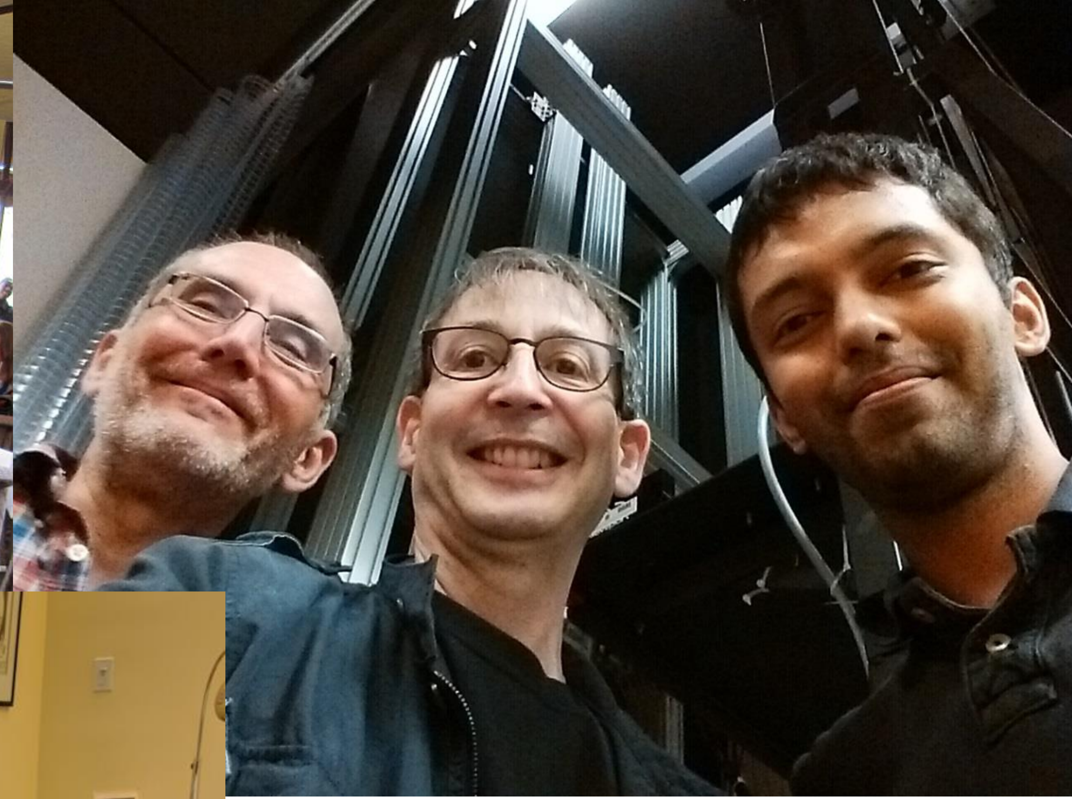


- Widom scaling of the friction factor suggests that fully-developed turbulence is controlled by a critical point and anomalous dimensions can be extracted from small Re data

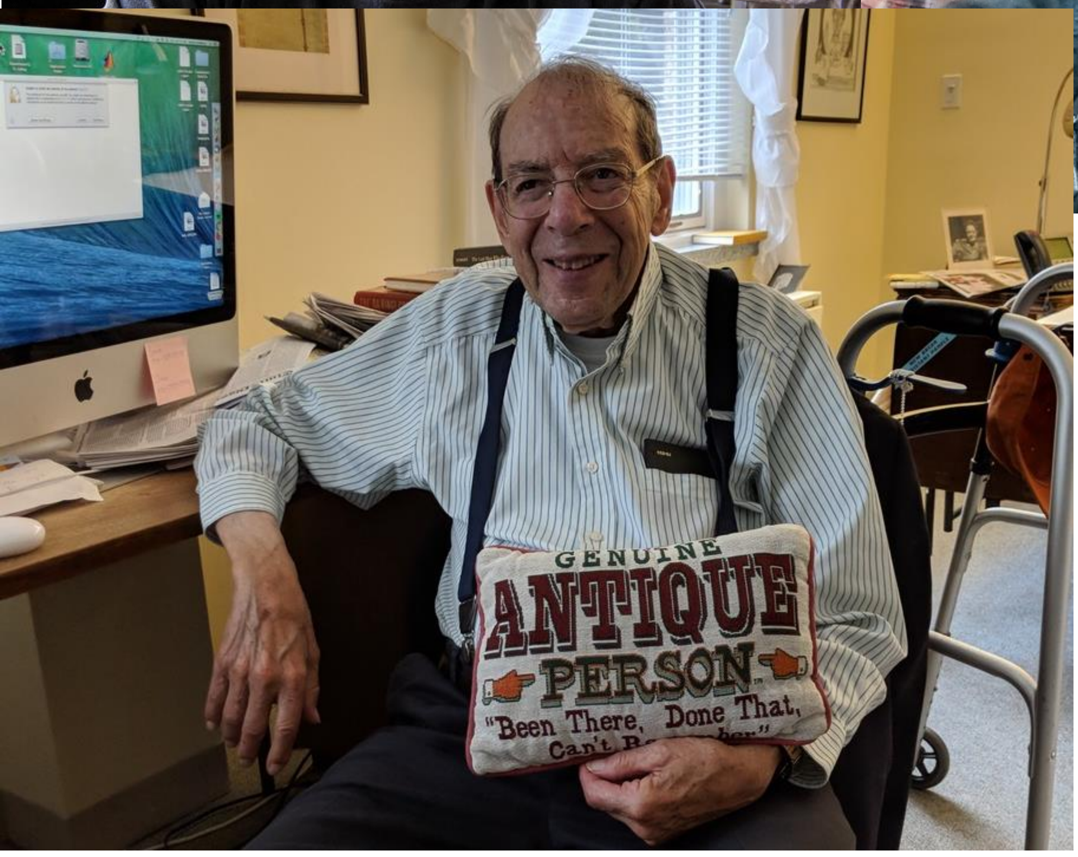


- Friction factor depends on the nature of the small scale velocity fluctuations
  - Predicted and observed dependence on spectrum in quasi-2D turbulence

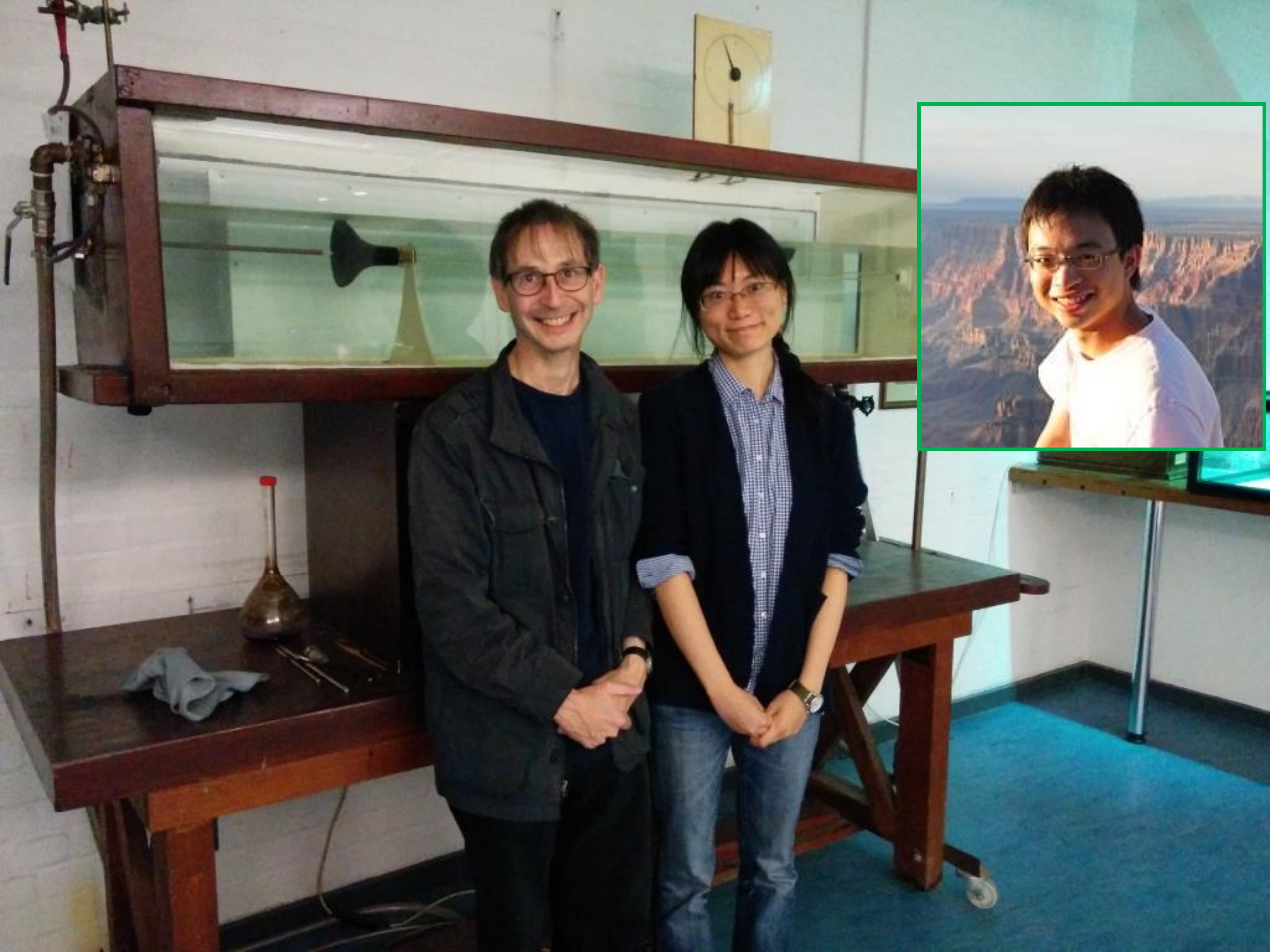
Hamid Kellay



Gustavo Gioia, NG, Pinaki Chakraborty



Walter Goldburg (1927-2018



Turbulence is a life force. It is opportunity. Let's love turbulence and use it for change.

Lucky Numbers 34, 15, 28, 4, 19, 20

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