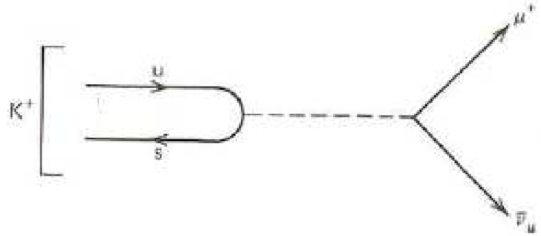
LECTURE 2

Motivation for Cabibbo Theory

1 Leptons and quarks participate in weak interactions through charged V-A currents Constructed from the pairs of left handed fermion states:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \text{ and } \begin{pmatrix} u \\ d \end{pmatrix}$$ couple with universal coupling constant, extend this to include heavier quarks $$\begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

BUT $K^+ \to \mu^+ \nu_\mu$ occurs and K^+ is made of u and $\bar{s}$ $\Rightarrow$ there has to be a weak current that couples a u and $\bar{s}$ quark



Contradicts this scheme, only u↔ d, c ↔s transitions

2

The strangeness changing hadronic weak currents appeared to be weaker than the strangeness conserving hadronic currents

Example: Decay rate for $K^+ \to \mu^+ \nu_\mu \; < \; \pi^+ \to \mu^+ \nu_\mu$

Cabibbo's proposal : Instead of introducing new couplings to accommodate the above, Keep universality, but modify the quark doublets

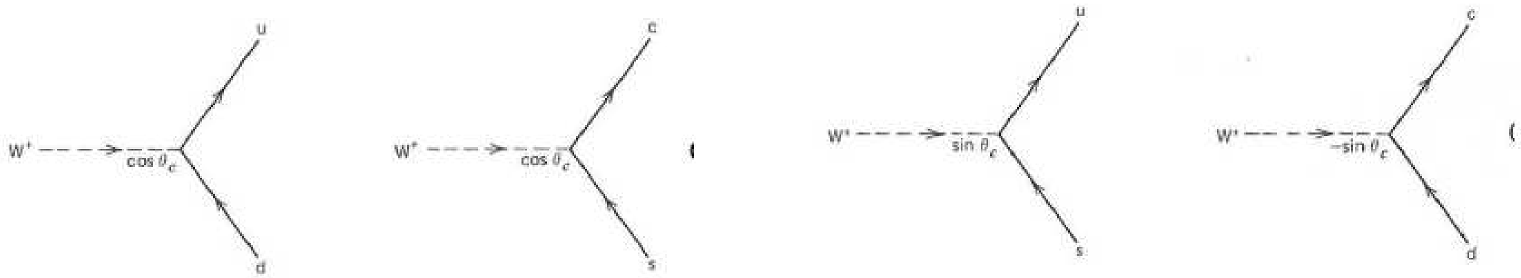
Charged current couples "rotated" quark states: $\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$ ......

where $d' = d\cos\theta_c + s\sin\theta_c$ $\quad$ θ_c is the quark mixing angle or the Cabibbo angle

$s' = -d\sin\theta_c + s\cos\theta_c$

The Cabibbo favored transitions are propotional to $\cos\theta_c$

The Cabibbo suppressed transitions are propotional to $\sin\theta_c$



Hence,

$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} \sim \sin^2\theta_c,$$

The hadronic current:

$$J^\mu = (\bar{u} \quad \bar{c}) \frac{\gamma_\mu (1-\gamma_5)}{2} U \begin{pmatrix} d \\ s \end{pmatrix}$$

$$U = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

Neutral K mesons and CP Violation

The Gell-Mann Nishijima formula, $Q = I_3 + (B+S)/2$ indicates - in addition to the charged kaons $K^{\pm}$ of $S=\pm1$, 2 neutral kaons to complete the I=1/2 doublets.

	I_3	
S	½	- ½
+1	K^+	K^0
-1	$\bar{K}^0$	K^-

K^0 and $\bar{K}^0$ are charge conjugates of one another and posess definite strangeness eigenvalues

Since strong and EM interactions conserve strangeness, these are the states to be considered in these interactions BUT since weak interactions do not conserve strangeness, these states do not posess definite lifetimes for weak decays.

If the weak interactions are "turned off", linear combinations of these states (K_L and K_S) are eigenstates of the Hamiltonian. When the weak interactions are "turned on", these combinations will be states with definite lifetimes, but will no longer have definite strangeness – leads to the phenomenon of strangeness oscillations.

The short lived K_S decays in only 2 significant modes : $\pi^+\pi^-$ and $\pi^0\pi^0$ and each of these final states has CP eigenvalue +1.

$C|\pi^+\pi^-\rangle = |\pi^-\pi^+\rangle$, BUT $P|\pi^+\pi^-\rangle = C|\pi^+\pi^-\rangle$

Since $P|\pi^+\pi^-\rangle = (-1)^l|\pi^+\pi^-\rangle \Rightarrow \eta_C(\pi^+\pi^-) = (-1)^l$

For, $|\pi^0\pi^0\rangle$, since the 2 particles are identical, l can only be even, $\eta_C(\pi^0\pi^0) = 1$

The long lived K_L decays to many known modes, including the fully allowed $\pi^+\pi^-\pi^0$
The three pions are in a relative S-state, due to the small Q value of the decay.

The $\pi^+\pi^-$ has CP +1, π^0 has C= +1, P = -1, and therefore CP = -1.
Hence the combined $\pi^+\pi^-\pi^0$ system has CP = -1
$\pi^0\pi^0\pi^0$ also has CP = -1: Any orbital angular momentum l, between any two pions has to be even by Bose symmetry; l value of the remaining pion about the dipion is also even, since initial J = 0. So net parity is the product of intrinsic pion parities, P = -1, also C = +1 $\Rightarrow$ CP = -1

2π state has CP = +1, while 3π state can have CP = -1 or +1, but CP = -1 is heavily favored kinematically.
Now, $CP|K^0\rangle = |\overline{K^0}\rangle$ and $CP|\overline{K^0}\rangle = |K^0\rangle$

Therefore,
$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad \text{CP = -1}$$
$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad \text{CP = +1}$$

Eigenstates of CP

If CP invariance is to hold, $|K_s\rangle = |K_2^0\rangle$ and $|K_L\rangle = |K_1^0\rangle$ and $K_L^0 \rightarrow \pi^+\pi^-$ and $K_L^0 \rightarrow \pi^0\pi^0$ are strictly forbidden. BUT in 1964, Christenson, Cronin, Fitch and Turlay discovered that the decay $K_L^0 \rightarrow \pi^+\pi^-$ actually occurs with small but finite probability and the transition $K_L^0 \rightarrow \pi^0\pi^0$ was also soon observed.

1980 Nobel Prize
Fitch and Cronin

In presence of CP violation, the states K_s and K_L (states of definite mass and lifetime) are:

$$|K_s^0\rangle = \frac{1}{\sqrt{2}\sqrt{(1+|\epsilon|^2)}}[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K^0}\rangle]$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2}\sqrt{(1+|\epsilon|^2)}}[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K^0}\rangle]$$

In the limit of CP conservation, $\varepsilon \to 0$ and K_s and K_l reduce to CP eigenstates

Meson Mixing

Formalism for mixing is based on a time-dependent perturbation theory analysis of a 2-state system $|P^0\rangle$ *and* $|\bar{P^0}\rangle$ together with set of states |f⟩ into which these particles can decay. The total hamiltonian $H = H_0 + H_w$

H_0 → Strong and EM

H_w → Weak interactions which induce $P^0 \leftrightarrow \overline{P^0}$, $P^0 \to f$, $\bar{P}^0 \to f$

Time evolution of any linear combination of the neutral meson flavor eigenstates $a|P^0\rangle + b|\bar{P^0}\rangle$ governed by the Schrodinger equation:

$$i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \mathbf{H} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

→ f's do not appear, H not hermitian, projection of the state space

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\boldsymbol{\Gamma}$$

$\mathbf{M}$ → Mass matrix

$\boldsymbol{\Gamma}$ → Decay Matrix

Virtual Intermediate States

Zero in SM, Δ S=2, Δ C=2 or Δ B=2 do not occur at first order (superweak - ruled out)

$$M_{ij} = m_0\delta_{ij} + \langle i|V|j\rangle + \sum_f \mathcal{P}\left(\frac{\langle i|V|f\rangle\langle f|V|j\rangle}{m_0 - E_f}\right)$$

$V = H_w$

$$\Gamma_{ij} = 2\pi\sum_f \langle i|V|f\rangle\langle f|V|j\rangle\delta\left(m_0 - E_f\right)$$

Real Intermediate States

where $M_{11}=M_{22}$, $\Gamma_{11}=\Gamma_{22}$ - CPT invariance.

M_{12}, $M_{21}=M_{12}^*$ -- due to 2nd order transitions via virtual states

Γ_{12}, $\Gamma_{21}=\Gamma_{12}^*$ -- on-shell intermediate states

The eigenstates of the Hamiltonian are given by:

$$|P_\pm\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}\,[\,p\,|P^0\rangle \pm q\,|\bar{P}^0\rangle\,]$$

where

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\frac{\Gamma_{12}^*}{2}}{M_{12} - i\frac{\Gamma_{12}}{2}}}$$

Comparison with previous expression
⇒ $q/p = (1-\varepsilon)/(1+\varepsilon)$

The eigenvalues are, $\mu_\pm = H_{11} \pm \sqrt{H_{12}H_{21}} = M_\pm - i\frac{\Gamma_\pm}{2}$ with

$$M_\pm = M_{11} \pm \mathrm{Re}(H_{12}H_{21})^{1/2}, \quad \Gamma_\pm = \Gamma_{11} \mp \mathrm{Im}(H_{12}H_{21})^{1/2} \ .$$

Eigenstates of the Hamiltonian evolve as $P_\pm(t) = P_\pm e^{-i\mu_\pm t}$

the time evolution of the $|P^0\rangle$ and $|\bar{P^0}\rangle$ states can therefore be determined:

$$|P^0(t)\rangle = g_+(t)|P^0\rangle + \frac{q}{p}\, g_-(t)|\bar{P^0}\rangle$$

$$|\bar{P^0}(t)\rangle = \frac{p}{q}\, g_-(t)|P^0\rangle + g_+(t)|\bar{P^0}\rangle \ ,$$

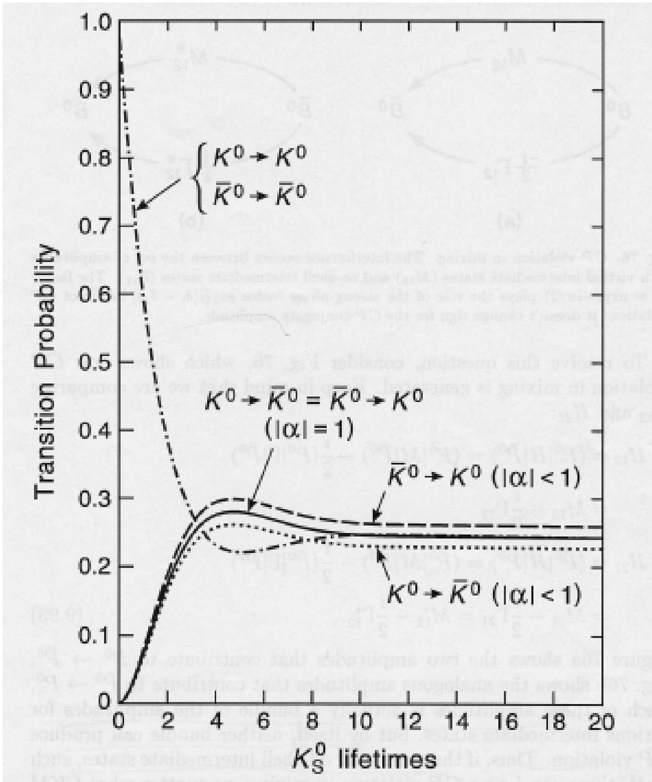
where,

$$g_+(t) = e^{-(\frac{\Gamma}{2}+iM)t}\cos\left[(\Delta M - i\frac{\Delta\Gamma}{2})\frac{t}{2}\right],$$

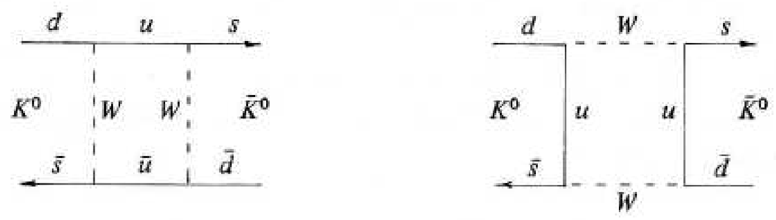
$$g_-(t) = e^{-(\frac{\Gamma}{2}+iM)t} i\sin\left[(\Delta M - i\frac{\Delta\Gamma}{2})\frac{t}{2}\right],$$

with, $\Delta\Gamma=\Gamma_- - \Gamma_+$, $\Delta M = M_- - M_+$, and Γ, M being the average width and masses. For $\Delta M>0$, we must associate, $M_- = M_H$ and $M_+ = M_L$.

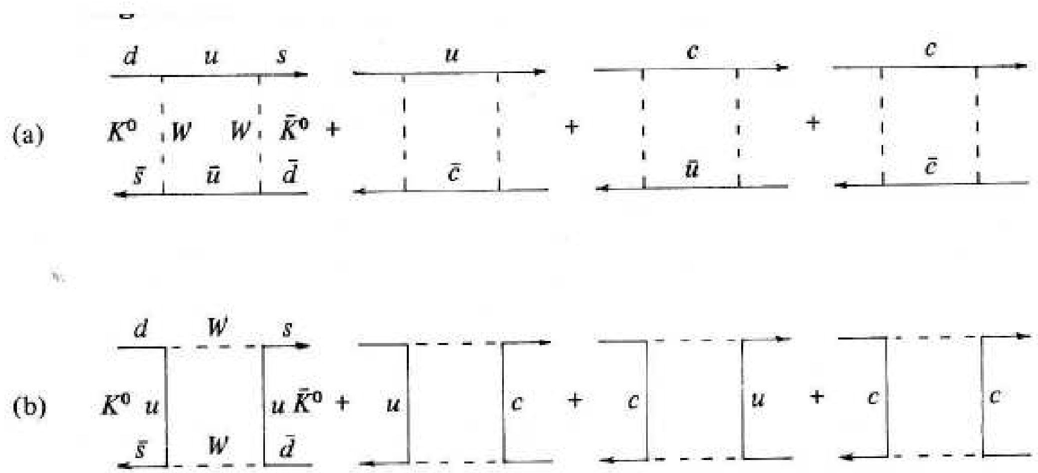
$$\mathrm{Prob}(P^0 \text{ at } t | P^0 \text{ at } t=0) = \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} + 2e^{-\Gamma t}\cos(\Delta M t)\right]$$
$$\mathrm{Prob}(\bar{P}^0 \text{ at } t | P^0 \text{ at } t=0) = \frac{1}{4}|\alpha|^2\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-\Gamma t}\cos(\Delta M t)\right]$$
$$\mathrm{Prob}(P^0 \text{ at } t | \bar{P}^0 \text{ at } t=0) = \frac{1}{4|\alpha|^2}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-\Gamma t}\cos(\Delta M t)\right]$$
$$\mathrm{Prob}(\bar{P}^0 \text{ at } t | \bar{P}^0 \text{ at } t=0) = \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} + 2e^{-\Gamma t}\cos(\Delta M t)\right],$$



In the SM Mixing comes from the Box diagrams:

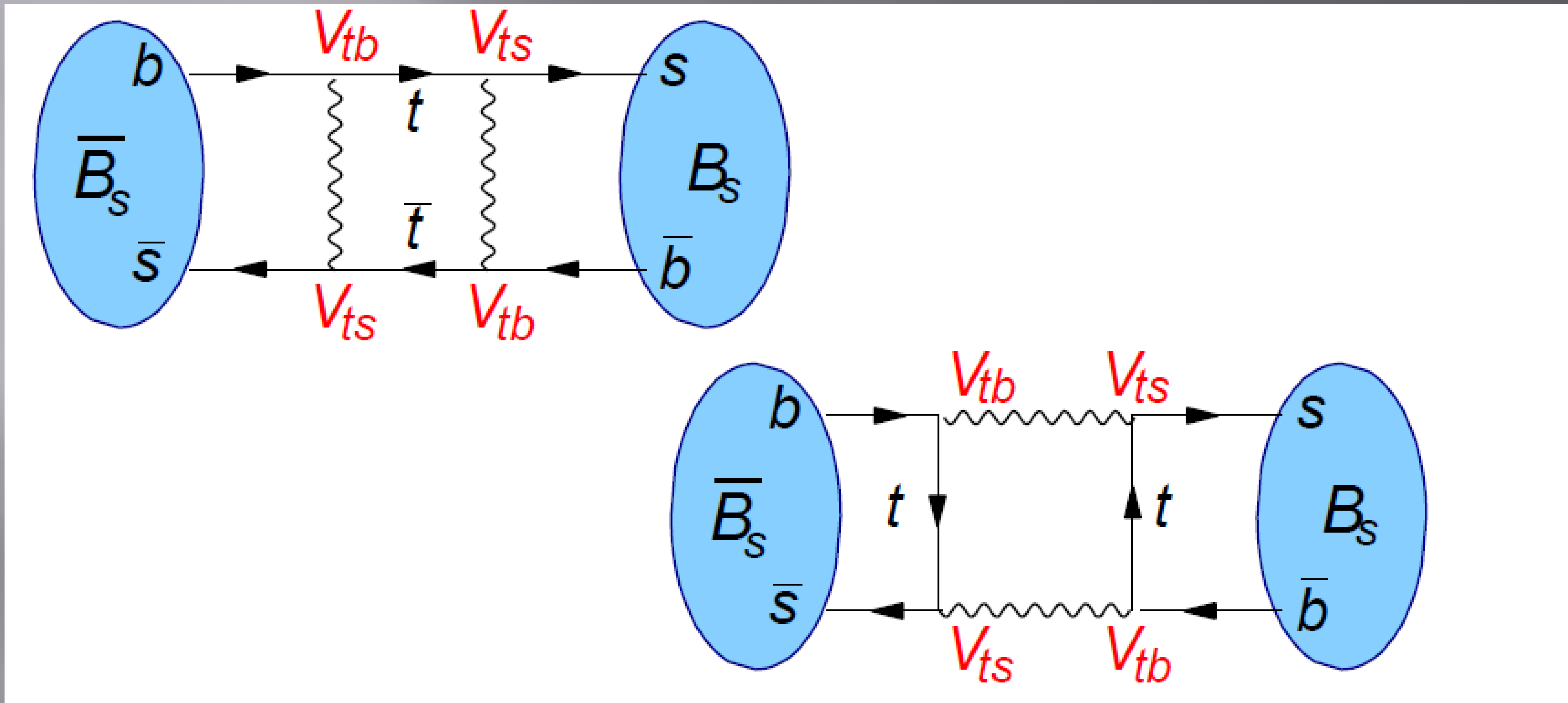


→ Internal u quark only



→ Contributions with charm

$s \rightarrow \mathrm{d} \quad \bar{s} \rightarrow \bar{d}$ for B_d



CP Violation in B Mixing

Model independent: CP violation in mixing $< \mathcal{O}(\frac{\Delta\Gamma}{\Delta m})$

	B_d	B_s
Δm = m_H - m_L	0.5 ps^{-1}	17.8 ps^{-1}
$\Delta\Gamma/\Gamma$ = $(\Gamma_L$ - $\Gamma_H)/\Gamma$	$\mathcal{O}$(0.01)	$\mathcal{O}$(0.1)
τ = $1/\Gamma$	1.5 ps	1.5 ps

$$\frac{\Delta\Gamma}{\Delta m} = \frac{\Delta\Gamma}{\Gamma}\frac{\Gamma}{\Delta m} = \frac{\Delta\Gamma}{\Gamma}\frac{1}{\tau * \Delta m}$$

$$B_d : \mathcal{O}(0.01)\frac{1}{1.5ps * 0.5ps^{-1}} \sim \mathcal{O}(0.01)$$

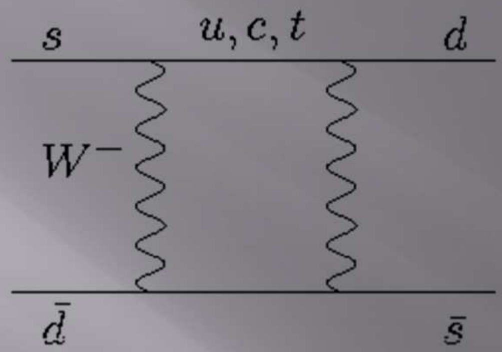
$$B_s : \mathcal{O}(0.1)\frac{1}{1.5ps * 18ps^{-1}} \sim \mathcal{O}(0.01)$$

CP violation in $B_{d/s}$ Mixing is negligible!

Table 1. Parameters of the four neutral oscillating meson pairs [9].

	$K^0/\overline{K}^0$	$D^0/\overline{D}^0$	$B^0/\overline{B}^0$	$B_s/\overline{B}_s$
τ [ps]	89.4 ± 0.1; 51700 ± 400	$0.413 \pm .003$	1.548 ± 0.021	1.49 ± 0.06
Γ [s^{-1}]	$5.61 \cdot 10^9$	$2.4 \cdot 10^{12}$	$(6.41 \pm 0.16) \cdot 10^{11}$	$(6.7 \pm 0.3) \cdot 10^{11}$
$y = \frac{\Delta\Gamma}{2\Gamma}$	-0.9966	$\|y\| < 0.06$	$\|y\| \lesssim 0.01^*$	$-(0.01 \ldots 0.10)^*$
Δm [s^{-1}]	$(5.300 \pm 0.012) \cdot 10^9$	$< 7 \cdot 10^{10}$	$(4.89 \pm 0.09) \cdot 10^{11}$	$> 15 \cdot 10^{12}$
Δm [eV]	$(3.49 \pm 0.01) \cdot 10^{-6}$	$< 5 \cdot 10^{-6}$	$(3.2 \pm 0.1) \cdot 10^{-4}$	$> 1.0 \cdot 10^{-2}$
$x = \frac{\Delta m}{\Gamma}$	0.945 ± 0.002	< 0.03	0.76 ± 0.02	$21 \ldots 40^*$

$K - \overline{K}$



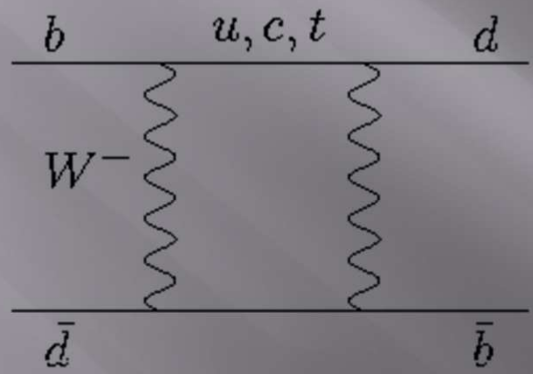
t: $(V_{ts}V^{*}_{td})^2 \sim \lambda^{10}$, m_t^2

c: $(V_{cs}V^{*}_{cd})^2 \sim \lambda^2$, m_c^2

u: $(V_{us}V^{*}_{ud})^2 \sim \lambda^2$, m_u^2

c dominates

$B - \overline{B}$



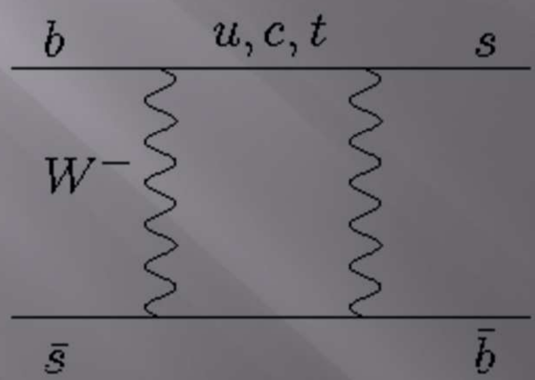
t: $(V_{tb}V^{*}_{td})^2 \sim \lambda^6$, m_t^2

c: $(V_{cb}V^{*}_{cd})^2 \sim \lambda^6$, m_c^2

u: $(V_{usb}V^{*}_{ud})^2 \sim \lambda^6$, m_u^2

top dominates

$B_s - \overline{B_s}$

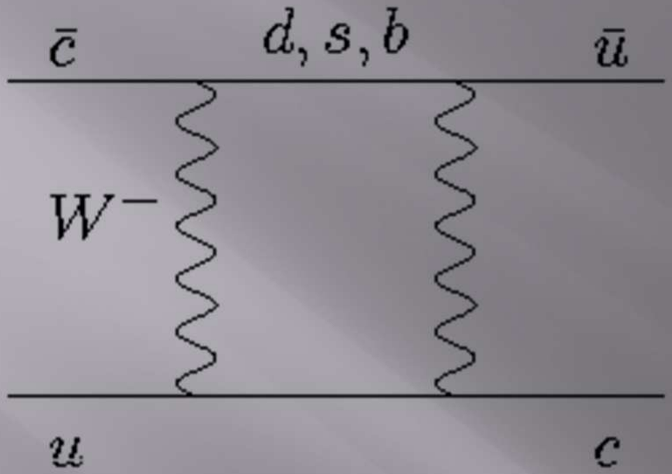


t: $(V_{tb}V^{*}_{ts})^2 \sim \lambda^4$, m_t^2

c: $(V_{cb}V^{*}_{cs})^2 \sim \lambda^4$, m_c^2

u: $(V_{ub}V^{*}_{us})^2 \sim \lambda^8$, m_u^2

$D-\bar{D}$



b quark not heavy enough to compensate for the large CKM suppression: $(V_{cb}V_{ub})^2 \sim \lambda^{10}$

Expect D mixing parameters to be small

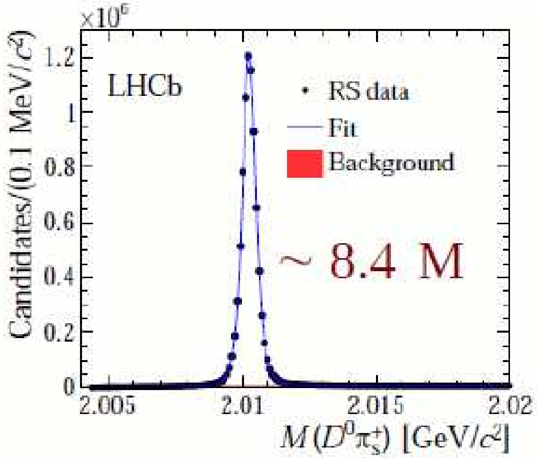
The contribution of b quark can be neglected $\Rightarrow$ D system essentially involves only 1st 2 generations $\Rightarrow$ no CPV

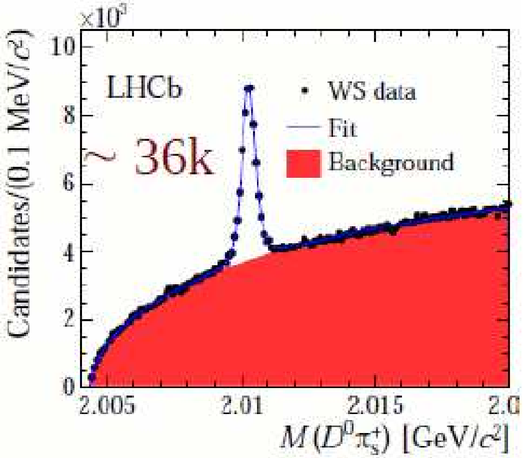
Hence mixing vanishes in the Flavor SU(3) limit

Has been shown to arise only at 2nd order in SU(3) breaking

other important result in charm: Observation of $D^0-\overline{D}^0$ mixing

measurement of time-dep ratio of $D^0 \to K^+\pi^-$ to $D^0 \to K^-\pi^+$ decay rates in D^{*+}-tagged events





[arXiv:1211.1230]
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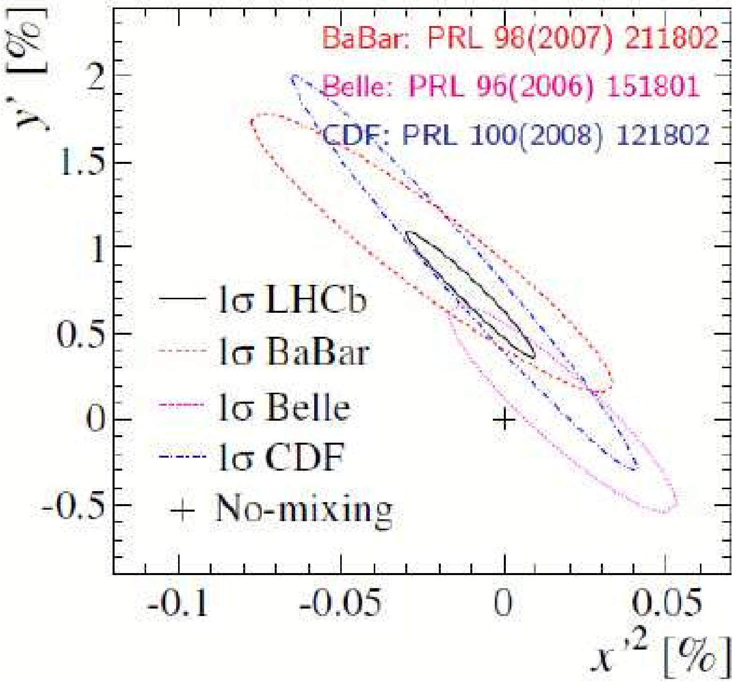


The ratio RS and WS events as a function of time gives access to the parameters x' and y'

$$R(t) \approx R_D + \sqrt{R_D}\, y' \frac{t}{\tau} + \frac{x'^2 + y'^2}{4} \left(\frac{t}{\tau}\right)^2$$

$$\mathbf{x'^2 = (-0.9 \pm 1.3) \times 10^{-4}}$$
$$\mathbf{y'^2 = (7.2 \pm 2.4) \times 10^{-3}}$$

No mixing hypothesis is excluded at 9 σ

1st observation of charm mixing from a single expt

Direct CP in charm decays (very hot topic)

pp collisions

$$A_{\rm raw}(f) = A_{CP}(f) + \cancel{A_{\rm D}(f)} + A_{\rm D}(\pi_{\rm s}) + A_{\rm P}(D^{*+})$$

Physics CP asymmetry

Detection asymmetry of D^0

Detection asymmetry of "slow" pions

Production asymmetry

To reduce systematics and (perhaps) enhance CP violation effect, LHCb measure

$$\Delta A_{CP} \equiv A_{CP}(K^- K^+) - A_{CP}(\pi^- \pi^+)$$

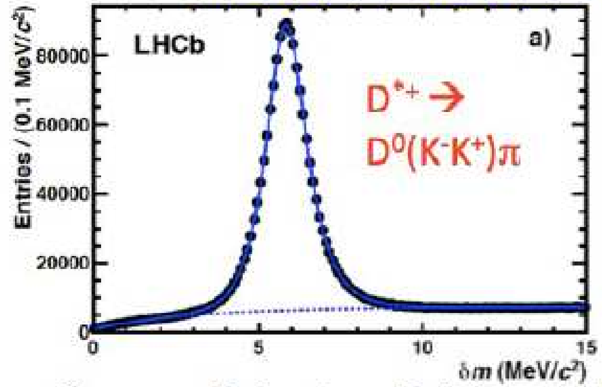
$$= [a_{CP}^{dir}(K^- K^+) - a_{CP}^{dir}(\pi^- \pi^+)] + \frac{\Delta \langle t \rangle}{\tau}\, a_{CP}^{ind}$$

$\Delta A_{CP} = (-0.82 \pm 0.21 \pm 0.11)\%$

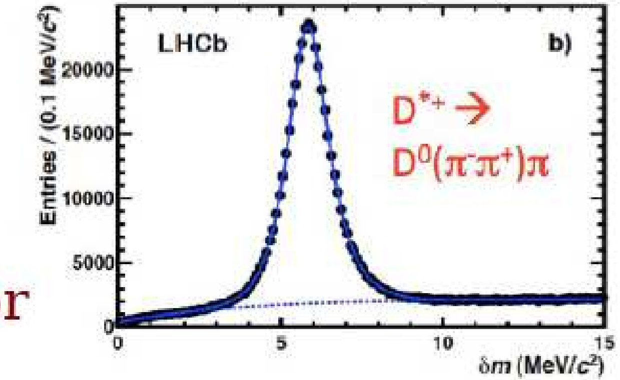
3.5σ away from zero

First evidence of CPV in the charm sector

PRL **108**, 111602 (2012)



$\delta m = m(hh\pi) - m(hh) - m(\pi)$



$$\Delta M = 2|M_{12}| \qquad \Delta\Gamma = \frac{2Re(M_{12}\Gamma_{12}^*)}{|M_{12}|}$$

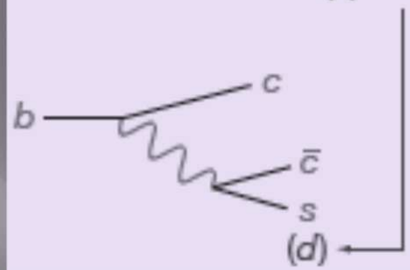
$$|\frac{\Gamma_{12}}{M_{12}}| \approx \frac{m_b^2}{m_t^2} \approx 10^{-3}$$

$|\Delta\Gamma| << \Delta M$

For B_s case :

Γ_{12} dominated by decay $b \to c\bar{c}s$
from decays into final states common
to both B_s^0 $(\bar{b}s)$ and $\bar{B}_s^0$ $(b\bar{s})$

For B_d^0 , analogous diagram
Cabibbo suppressed, Γ_{12} negligible



Measure of Phase in mixing can be made by rate asymmetry in semileptonic decays

$$\mathcal{A}_{\rm SL}^{(q)} \equiv \frac{\Gamma(B_q^0(t)\to l^-\overline{\nu}_l X) - \Gamma(\overline{B_q^0}(t)\to l^+\nu_l X)}{\Gamma(B_q^0(t)\to l^-\overline{\nu}_l X) + \Gamma(\overline{B_q^0}(t)\to l^+\nu_l X)} = \frac{|\alpha_q|^4 - 1}{|\alpha_q|^4 + 1} \approx \frac{|\Gamma_{12}^{(q)}|}{|M_{12}^{(q)}|}\sin\delta\Theta_{M/\Gamma}^{(q)}$$

$O(10^{-4})$

$O(m_b^2/m_t^2)$

$O(m_c^2/m_b^2)$

$$\Delta M_q = \frac{G_F^2}{6\pi^2}\eta_B m_{B_q}(B_{B_q}F_{B_q}^2)M_w^2 S_0(x_t)|V_{tq}|^2$$

Angle measurements cont….

Determination of α

Final state-$\pi^+\pi^-$. At the tree level, the amplitude involves a $b \longrightarrow u$ quark transition, $A \sim e^{-i\gamma}$, $\lambda = e^{2i\alpha}$. BUT there is Penguin Pollution. What one measures through a time dependent asymmetry here is not 2α

but some effective (POLLUTED) angle $2\alpha_{eff}$.
Problem resolved through an Isospin Analysis

GRONAU AND LONDON

isospin-Quantum number first ascribed to nucleons. In 1932, Heisenberg: neutrons, protons might be treated as different charge substates of the nucleon. The nucleon has $I = 1/2$, with I_3 values of $+1/2$ for the proton and $-1/2$ for the neutron.
Similar asignments for quarks u and $d \Rightarrow$ all mesons
pions: I=1, triplet of states π^+, π^- and π^0
Kaons, D-mesons and B-mesons $I = 1/2$, each have two doublets.

Isospin results in a simple relation between the Amplitudes for the $B^+ \to \pi^+\pi^0 \equiv A^{+-}$, $B_d^0 \to \pi^0\pi^0 \equiv A^{00}$ and $B_d^0 \to \pi^+\pi^- \equiv A^{+0}$

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0} .$$

With a similar relation for the conjugate amplitudes, $\bar{A^{+-}}$, $\bar{A^{00}}$ and $\bar{A^{+0}}$. In fact we could redefine the amplitudes such that the time dependent asymmetry in the $\pi^+\pi^-$ mode directly measures the angle between A^{+-} and $\bar{A}^{+-}$.

Just an isospin relation and geometry allows determination of α.

We decided to do even a little more geometry!

GRONAU, LONDON, N. SINHA AND R. SINHA Phy. Lett B 2000

Motivation: isospin analysis requires separate measurement of $BR(B_d^0 \to \pi^0\pi^0)$ and $BR(\overline{B_d^0} \to \pi^0\pi^0)$, and therefore suffers from potential practical complications:

- The branching ratio for $B_d^0 \to \pi^0\pi^0$ is expected to be smaller than $B_d^0 \to \pi^+\pi^-$.
- The presence of two π^0's in the final state means that the reconstruction efficiency is smaller.
- It will be necessary to tag the decaying B_d^0 or $\overline{B_d^0}$ meson, which further reduces the measurement efficiency.

Hence, we may only have, an actual measurement or an upper limit, on the sum of the branching ratios. In this case, a full isospin analysis cannot be carried out
QUESTION: assuming that we have, at best, only partial knowledge of the sum, $(BR(B_d^0 \to \pi^0\pi^0) + BR(\overline{B_d^0} \to \pi^0\pi^0))$, can we at least put bounds on the

size of penguin pollution? In the presence of penguin amplitudes, the CP asymmetry in $B_d^0(t) \to \pi^+\pi^-$ measures $\sin 2\alpha_{eff}$. Writing $2\alpha_{eff} = 2\alpha + 2\theta$, where 2θ parametrizes the effect of the penguin contributions, the more precise question: is it possible to constrain θ? Define new amplitudes $\tilde{A}^{ij} \equiv e^{-2i\alpha}\bar{A}^{ij}$.

Then, $\tilde{A}^{-0} = A^{+0}$, so that the A and $\tilde{A}$ triangles have a common base. First, we assign a coordinate system to the above figure, such that the origin is at the midpoint of the points X and Y. The points X, Y, W and Z correspond respectively to the coordinates $(+\ell, 0)$, $(-\ell, 0)$, (x_1, y_1) and (x_2, y_2). The goal of the exercise is to find the values of the coordinates (x_1, y_1) and (x_2, y_2). We then note that

$$B^{+-} = 2(x_1^2 + y_1^2) + 2\ell^2, \quad B^{+-}a_{dir}^{+-} = -4x_1\ell \,,$$
$$B^{00} = (x_2^2 + y_2^2) + \ell^2, \quad B^{+0} = (x_1^2 + y_1^2) + (x_2^2 + y_2^2) - 2x_1x_2 - 2y_1y_2 \,.$$

We therefore have four (nonlinear) equations in four unknowns, and we can solve for these coordinates as a function of ℓ. However, we must obtain *only real solutions* for x_2 and y_2, otherwise the triangles do not close. This puts a constraint on ℓ, which in turn, gives the following bound,

$$\cos 2\theta \geq \frac{\left(\frac{1}{2}B^{+-} + B^{+0} - B^{00}\right)^2 - B^{+-}B^{+0}}{B^{+-}B^{+0}y} \, .$$

This is the new lower bound on $\cos 2\theta$ (or upper bound on $|2\theta|$). The new bound contains the two previous bounds as limiting cases

ANOTHER INTERESTING CONSEQUENCE:

a lower limit on B^{00}/B^{+-}

$$\frac{1}{2} + \frac{B^{+0}}{B^{+-}} - \sqrt{\frac{B^{+0}}{B^{+-}}(1+y)} \leq \frac{B^{00}}{B^{+-}} \leq \frac{1}{2} + \frac{B^{+0}}{B^{+-}} + \sqrt{\frac{B^{+0}}{B^{+-}}(1+y)} \, .$$

Lower limit on B^{00}/B^{+-} useful, will give experimentalists some knowledge of the branching ratios for $B_d^0/\overline{B_d^0} \to \pi^0\pi^0$, help to anticipate the feasibility of the full isospin analysis.

γ

What About γ?

- The angle γ, phase of the V_{ub} element of the CKM matrix
  -is one of the most difficult to measure
- To get at γ alone, need to perform a direct asymmetry measurement.

- Need to look for decay modes with two weak amplitude contributions:

$$ae^{i\delta_a}e^{i\gamma} + be^{i\delta_b},$$

where the second term should not carry any weak phase.

Possible Modes

1. DK: the second contribution is from $b \to c$ tree
   $B^+ \to D^0(\bar{D}^0)K^+$ GRONAU, LONDON AND WYLER
   $B^+ \to D_{CP}K^+$ will involve interference of the above diagrams and hence determine γ.
   Method experimentally not feasible
   Measurement of $B^+ \to D^0K^+$ HARD.
   $B^+ \to D^0K^+ \to [K^-\pi^+]_{D^0}K^+ \sim B^+ \to \bar{D}^0K^+ \to [K^-\pi^+]_{\bar{D}^0}K^+$
   Improvement ATWOOD, DUNIETZ AND SONI
   Consider, $B^+ \to [f_i]_D K^+$, f_i are Doubly Cabibbo suppressed modes of $\bar{D}^0$.
   Can SOLVE for $B^+ \to D^0K^+$.
   Drawback-Need two Doubly Cabibbo suppressed BR's of D.
   RESOLVED N. SINHA AND R. SINHA, PRL 1998
   USE VECTOR-VECTOR FINAL STATES

DK methods

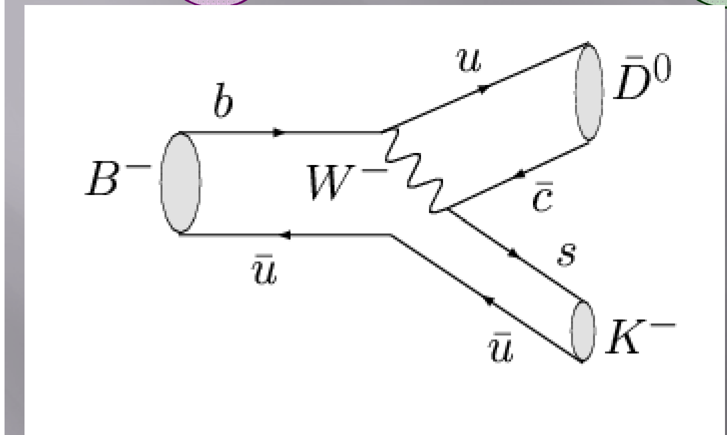
Need to look for decay modes with two weak amplitude contributions:

$$ae^{i\delta_a}e^{i\phi_3} + be^{i\delta_b},$$

b→ u

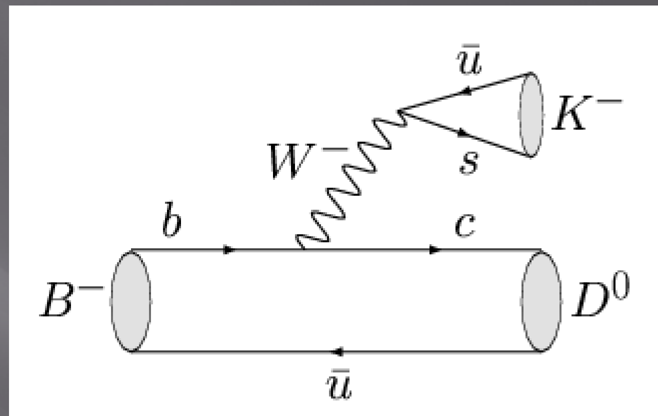
b→ c

No final state with 2 such tree contributions--BUT



color suppressed , B−→D0 K−~VubVcs

~**A** λ**3**(ρ+**i** η)



color allowed, B−→D0 K−~VcbVus

~**A** λ**3**

Among the DK methods, there is GLW, ADS, SS
Attention has been paid to the GGSZ, using Daltz plot analysis—but still large error.

γ cannot be measured using time dependent techniques

• Other methods developed:

----$K\pi$ Methods, interference of $b \to u$ tree and $b \to s$ penguin

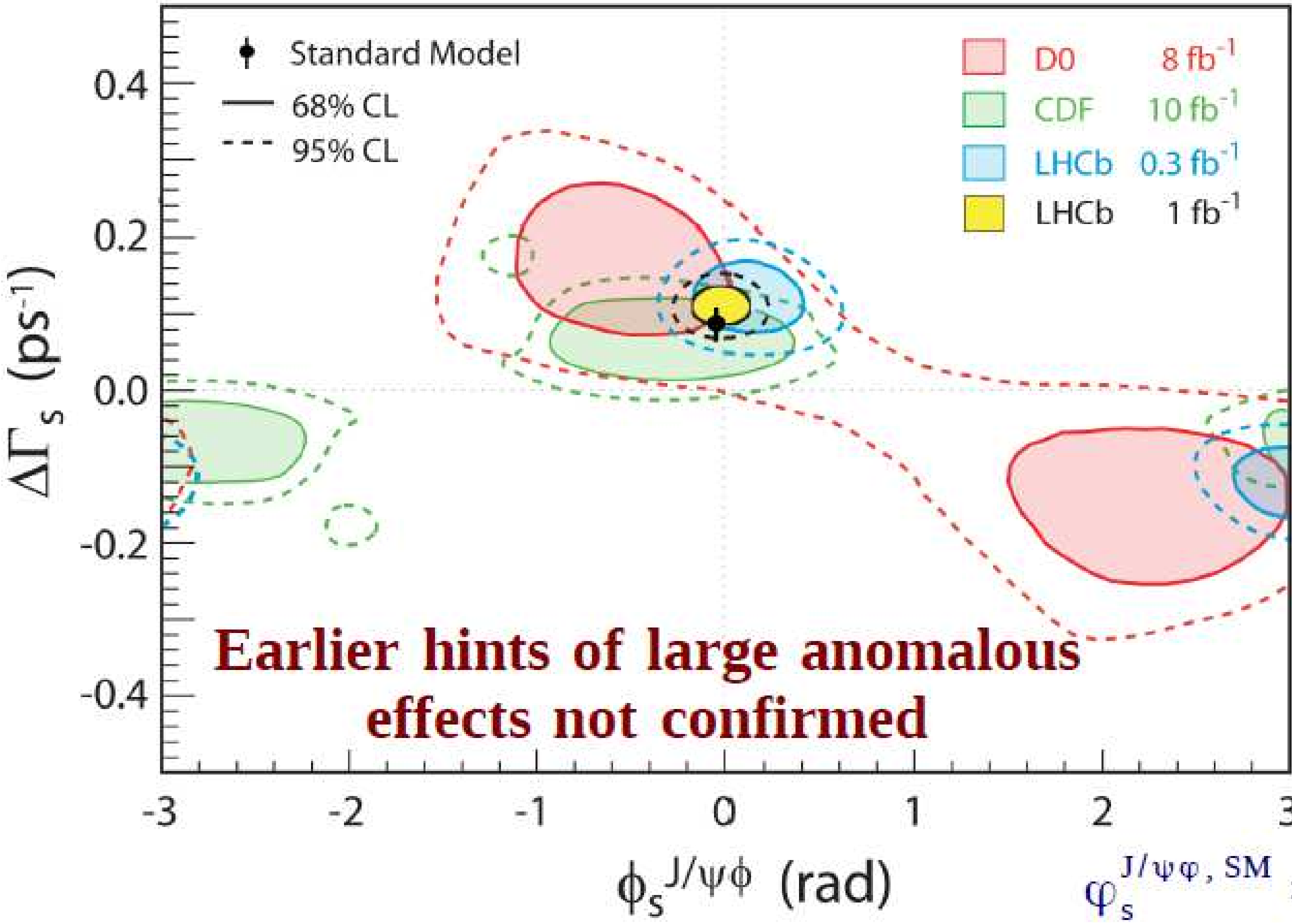
Tree is Cabibbo suppressed, Penguin contributions, including Electroweak Penguins cannot be neglected

Size of the tree and penguin unknown $\to$ hadronic uncertainties

β_s from $B_s \rightarrow J/\psi\varphi$, $J/\psi\pi\pi$ [LHCb-CONF-2012-002]

Significant improvements in precision ($\Delta\Gamma_s > 0$ now established)

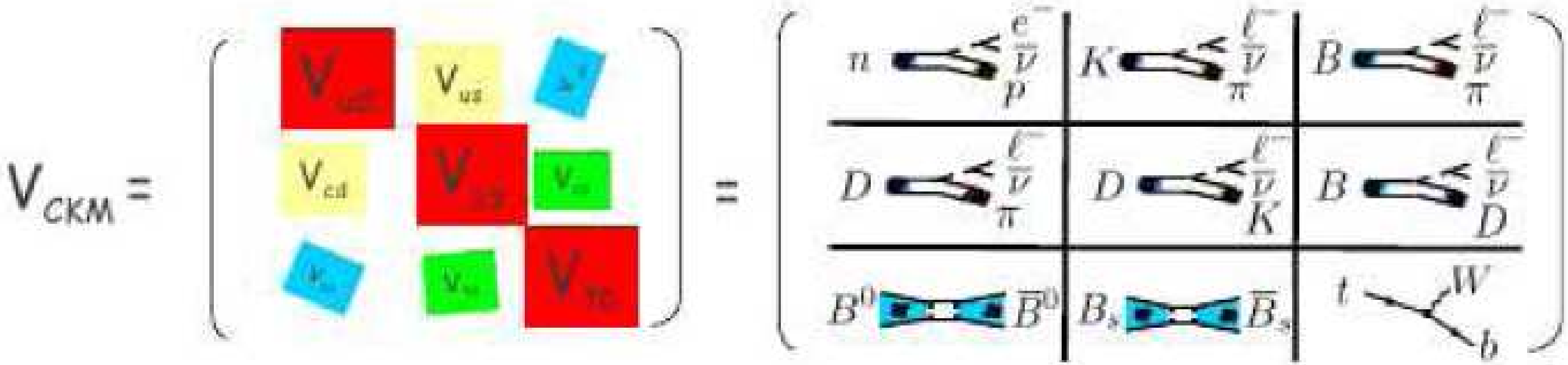
NB: $\sigma(\beta_s) \sim 4\sigma(\beta)$



$\varphi_s^{J/\psi\varphi,\,SM} = (-0.036 \pm 0.002)$ rad

$\sigma_{\phi_s}^{stat}$	LHCb ($1\ fb^{-1}$)	LHCb 2018	Upgrade ($50\ fb^{-1}$)	Theory uncertainty
$B_s^0 \rightarrow J/\psi\phi$	0.10	0.025	0.008	0.003
$B_s^0 \rightarrow J/\psi\pi\pi$	0.17	0.045	0.014	0.01

Side Measurements

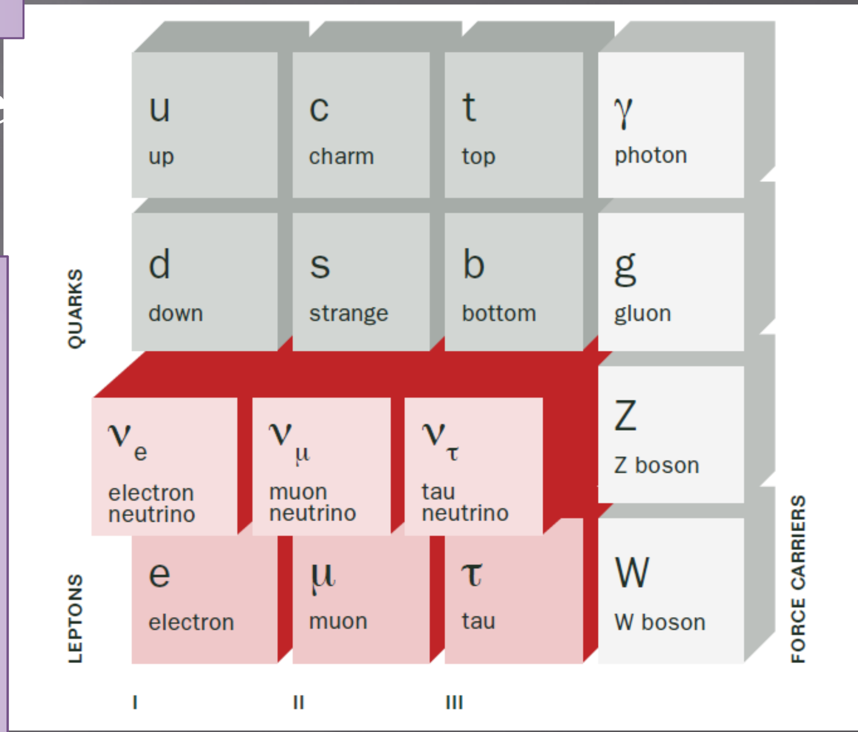


Neutrinos: What we know

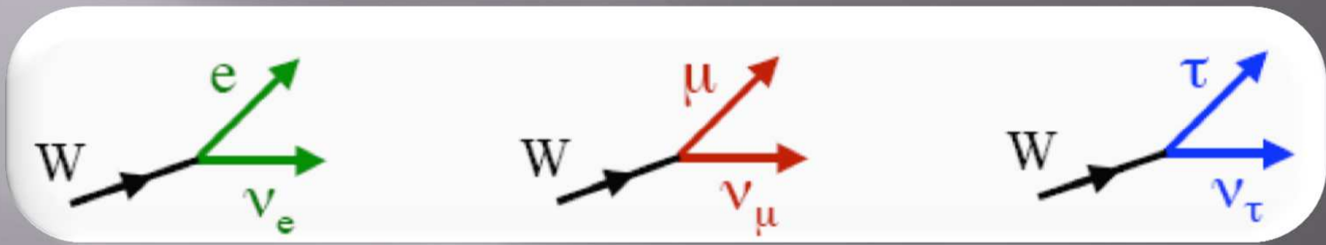
Neutral weakly interacting particles

In the last decade we have obtained compelling evidence that:

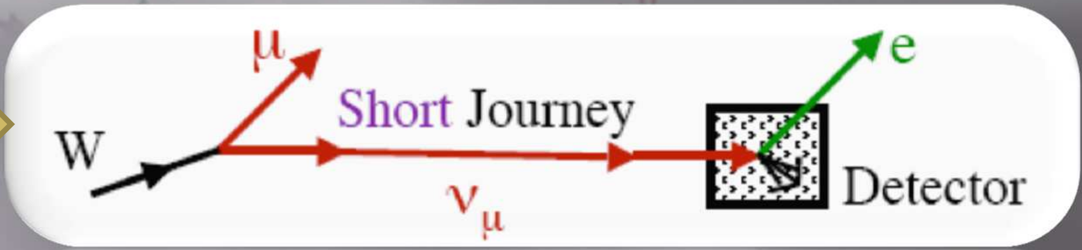
- Neutrinos have nonzero mass
- Mass is at least a million times lighter than that of an electron
- Neutrinos of different flavours mix
- Unlike quark mixing angles, two of the neutrino mixing angles are very large
- They could perhaps be their own antiparticles
- Their finite mass ⇒ *physics beyond the Standard Model*



The three flavours of neutrinos appear along with the corresponding lepton in the decay of the Weak Bosons



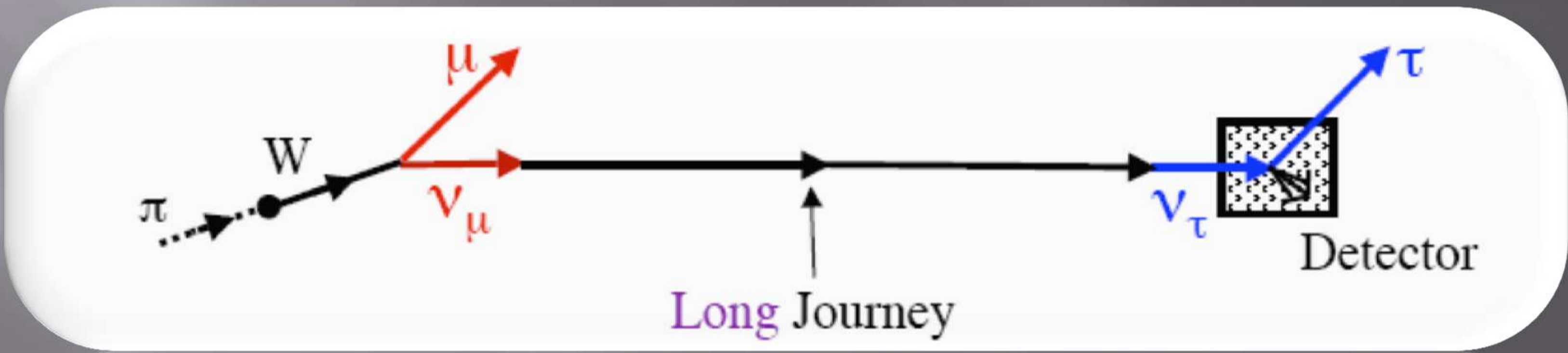
Does not happen



ν_α makes only ℓ_α

$\alpha = e, \mu, \tau$

If neutrinos have masses, leptons can mix and we can have,



$\nu_\mu \longrightarrow \nu_\tau$

Gives ν time to change character

Flavour change requires

- Neutrino Masses, Mass(ν_i)
- Lepton mixing

The neutrinos of definite flavour must be superpositions of the mass eigenstates

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \ .$$

Neutrino of flavor $\alpha = e, \mu, \text{or } \tau$

Neutrino of definite mass m_i

PMNS Leptonic Mixing Matrix

Mixing Matrix

Atmospheric, Cross-Mixing, Solar

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$

$s_{ij} \equiv \sin\theta_{ij}$

Probability for Neutrino Oscillation

In Vacuum

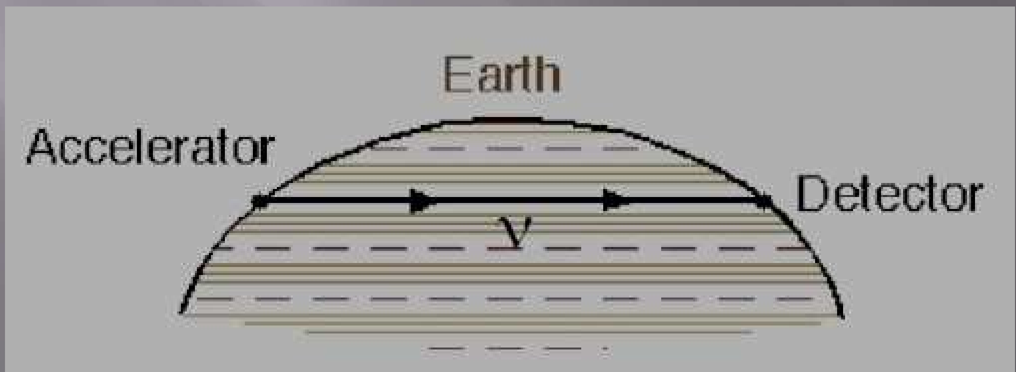
$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2(\Delta m^2_{ij}\frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin(\Delta m^2_{ij}\frac{L}{2E})$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$

In Matter



Long Baseline Experiments

Neutrinos......the broad picture of what we know...

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

↑ probed by LBL accelerator and atmospheric expts

↑ probed by accelerator and SBL reactor expts

↑ probed by solar & LBL reactor expts

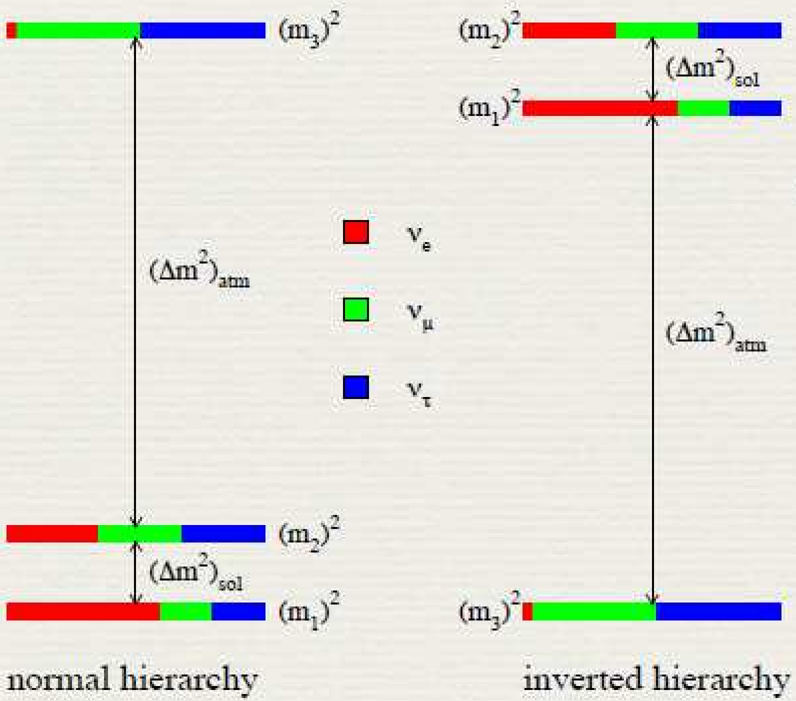
Neutrinos......the broad picture of what we know...

$\theta_{12} \approx 34^\circ$ ← from solar and Kamland (LBL reactor) →
$\theta_{23} \approx 45^\circ$ ← from atmos and accelerators →
$\theta_{13} \approx 9^\circ$ ← from reactors and accelerators

$$\Delta m^2_{sol} = 7.6 \times 10^{-5} \mathrm{eV}^2$$
$$|\Delta m^2_{atm}| = 2.4 \times 10^{-3} \mathrm{eV}^2$$

......and don't

What is the ordering of neutrino masses? (hierarchy)



Is there CP violation in the lepton sector?

i.e Is δ non-zero?

↑

← **Possible in next decade or so**

Are neutrinos their own anti-particles ? (Dirac or Majorana ?)

↑

More Difficult to answer soon

↓

What are their absolute masses ?

Large θ_{13} ....and consequences

Measurements of this parameter , hitherto known only upto an upper bound, have recently been made:

Recent result from Daya Bay : $\sin^2 2\theta_{13} = 0.089 \pm 0.010$ (stat) $\pm$ 0.005 (syst)

PRL., 108, 171803 (2012)

Recent result from RENO : $\sin^2 2\theta_{13} = 0.113 \pm 0.013$ (stat) 0.019(syst)

Recent result from DCHOOZ $\sin^2(2\theta_{13}) = 0.086 \pm 0.041$(stat) $\pm$ 0.030(syst)

PRL 108, 131801 (2012)

Result from T2K $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$

PRL 107, 041801 (2011)

Result from MINOS: $2 \sin^2\theta_{23} \sin^2 (2\theta_{13}) = 0.041$

PRL 107, 181802 (2011)

Measurements significantly impact the planning of future neutrino facilities.

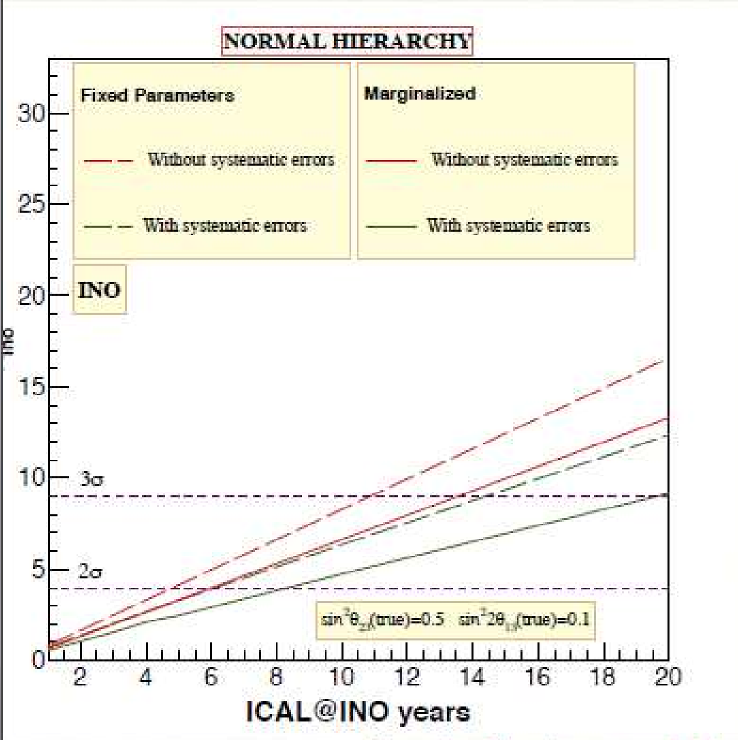
Hierarchy....prospects with upcoming expts...INO

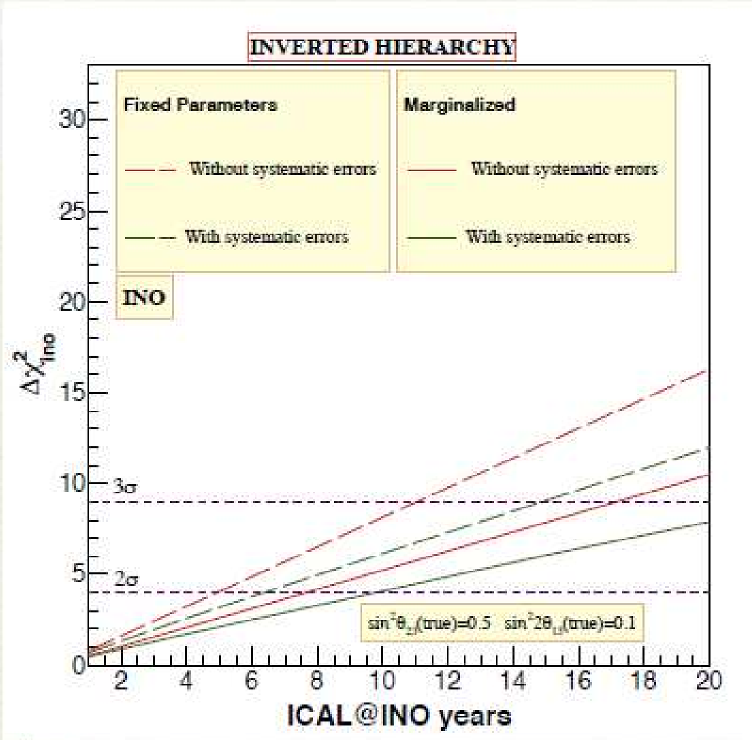
Salient Facts:

Large mass atmospheric neutrino detector in TN
50 kT iron calorimeter

Hierarchy determination via matter effects in muon survival

Magnetized, good charge and muon energy resolution



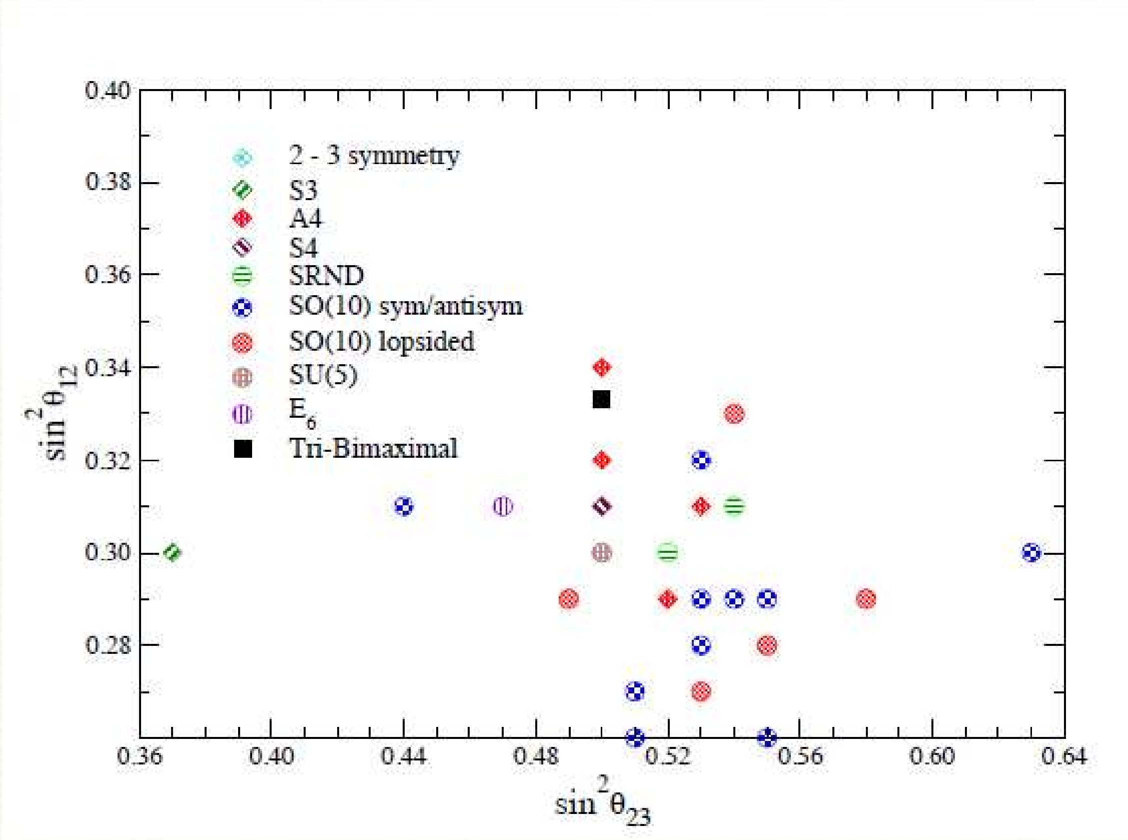


Possible construc-tion completi-on and start of data taking in 2017-18

Ghosh, Thakore and Choubey
arXiv 1212.1305

Bottom Line: 2 sigma determination in 10 yrs of running (~2028)

Octant of θ_{23} ....another BSM model discriminator



Albright, arXiv 0905.0146

CPV from Neutrino Mixing

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

The amplitude for $\nu_\mu \to \nu_e$ transitions can be written as

$$A_{\mu e} = U_{e2}^* U_{\mu 2} \left(e^{i\Delta_{12}} - 1\right) + U_{e3}^* U_{\mu 3} \left(e^{i\Delta_{13}} - 1\right)$$

where $\Delta_{1i} = \frac{\Delta m_{1i}^2 L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process can be written as

$$\bar{A}_{\mu e} = U_{e2} U_{\mu 2}^* \left(e^{i\Delta_{12}} - 1\right) + U_{e3} U_{\mu 3}^* \left(e^{i\Delta_{13}} - 1\right).$$

Large th_13......and consequences

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

At first glance, large value of s13 (as coefficient to CP phase) assists in attempts to measure presence of CP violation. E.g. it will enhance the number of electron appearance events events

Large θ_{13} and CP........

$$
\begin{aligned}
& P(\nu_\mu \to \nu_e) \\
= \quad & 4\frac{(\Delta m^2_{31})^2}{(\Delta m^2_{31}-a)^2} s^2_{23} s^2_{13} \sin^2\left(\frac{(\Delta m^2_{31}-a)L}{4E}\right) \\
+ \quad & 8 J_r \frac{\Delta m^2_{31}\Delta m^2_{21}}{a(\Delta m^2_{31}-a)} \sin\left(\frac{aL}{4E}\right) \\
\times \quad & \sin\left(\frac{(\Delta m^2_{31}-a)L}{4E}\right) \cos\left(\delta + \frac{\Delta m^2_{31} L}{4E}\right) \\
+ \quad & 4\left(\frac{\Delta m^2_{21}}{a}\right)^2 c^2_{12} s^2_{12} c^2_{23} \sin^2\left(\frac{aL}{4E}\right). \qquad (2)
\end{aligned}
$$

Atmospheric, large

CP violating

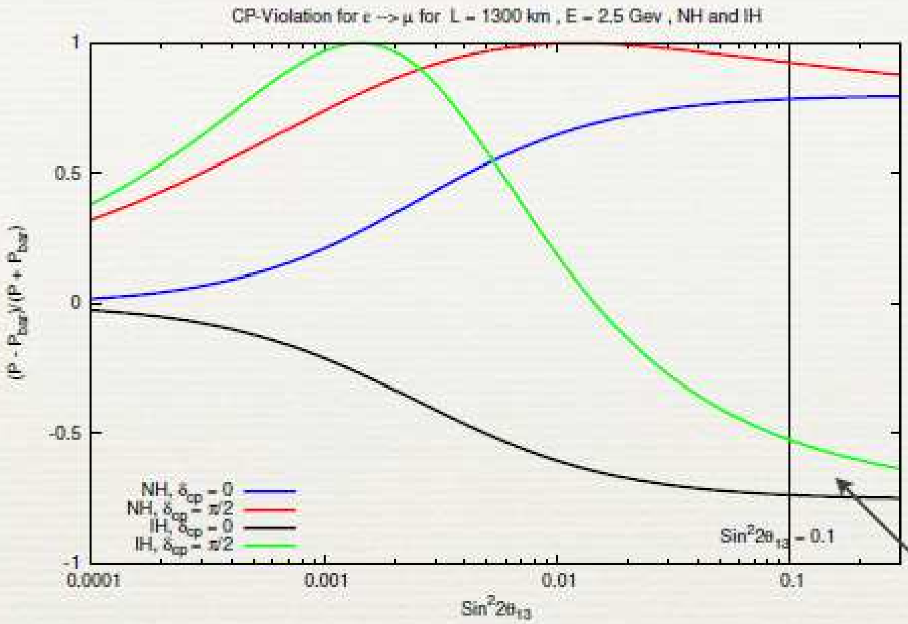
Solar, small

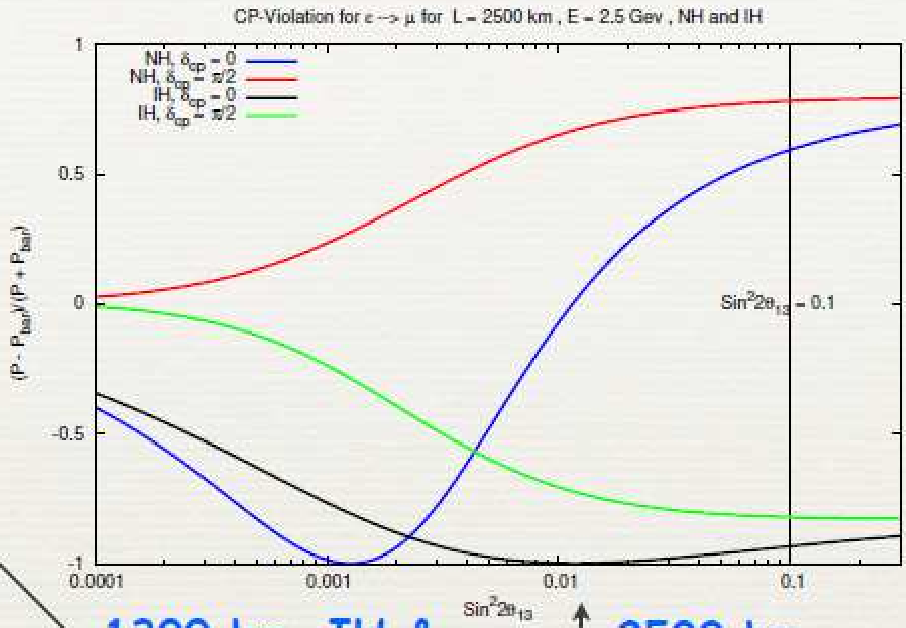
$$a \equiv 2\sqrt{2} G_F N_e(x) E$$

$$\Delta m^2_{ij} \equiv m^2_i - m^2_j, \; J_r \equiv c_{12} s_{12} c_{23} s_{23} s_{13}$$

$$A_{CP} = \frac{P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_e \to \bar{\nu}_\mu}}{P_{\nu_e \to \nu_\mu} + P_{\bar{\nu}_e \to \bar{\nu}_\mu}} \sim \frac{1}{sin\theta_{13}} \qquad (1)$$

Large θ_{13} and CP........



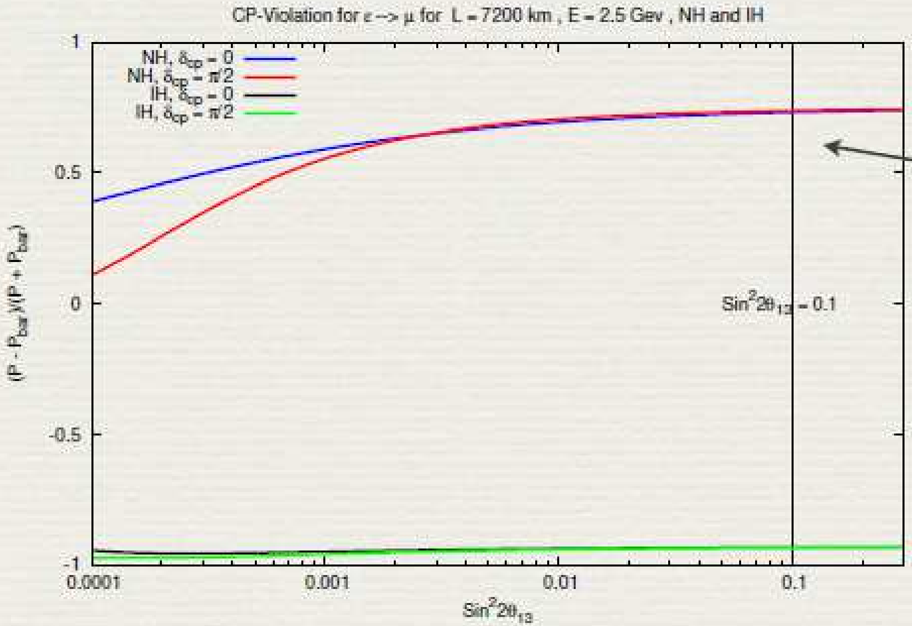


1300 km, IH & NH

δ =0, 90

2500 km, NH&IH

δ =0, 90



7200 km, IH & NH

δ =0, 90

Large θ_{13} does not mean enhanced CP measurement.

Overall "true CP" sensitivity depends on baseline and energy

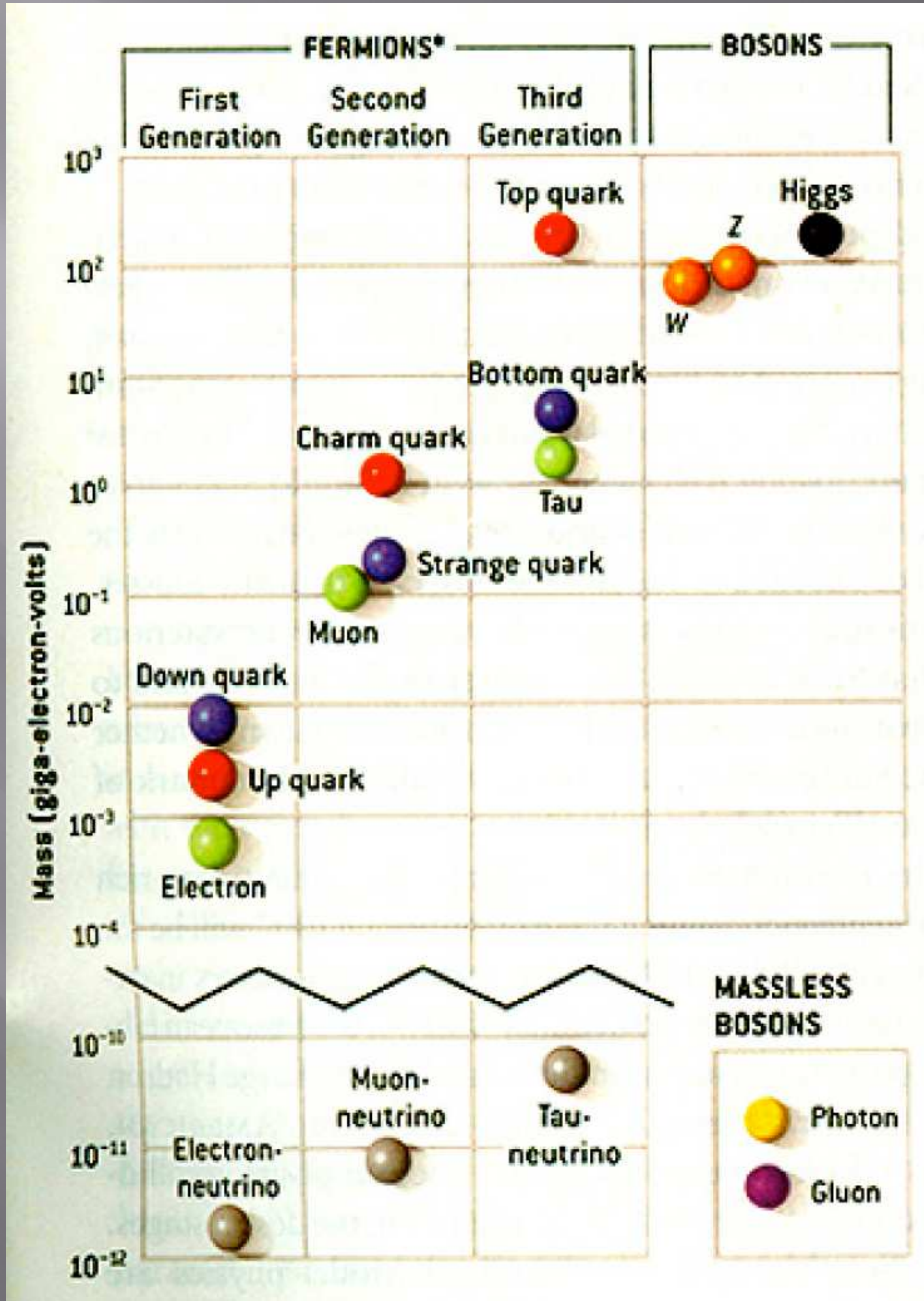
In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^{*}U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: $\Delta_{12}, \Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, we need $|U_{e3}| \neq 0$. ✓

Leptogenesis can explain the observed Baryon Number through CP-violating decays of heavy neutrinos in the **See-Saw** picture.

Leptogenesis is a very natural consequence of the **See-Saw** picture.


FERMIONS*
BOSONS
First Generation
Second Generation
Third Generation
Mass (giga-electron-volts)
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-10
10^-11
10^-12
Top quark
Higgs
Z
W
Bottom quark
Charm quark
Tau
Strange quark
Muon
Down quark
Up quark
Electron
MASSLESS BOSONS
Muon-neutrino
Tau-neutrino
Electron-neutrino
Photon
Gluon