LECTURE 1

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CP Violation

- CP is the discrete symmetry which takes a particle to an antiparticle with reversed “charges”.
- Universe was born with equal amounts of matter and antimatter.

  All evidence indicates that the universe is made up of matter alone.

  One necessary ingredient for this observed baryon asymmetry is CP Violation.
- Origin of CP violation not understood.
- CP violation is merely parameterized in the SM.
- CPV in SM not sufficient to explain the baryon asymmetry.

⇒New Physics

- CP violation has been observed in K meson and B meson decays and recently even in the D meson decays.

Baryon Asymmetry

1967

All evidence indicates that the universe is made up of matter
In the lab. possible to create antimatter-identical to matter, with opposite charges, matter+antimatter, annihilate.
Universe was born with equal amounts of both

$\Rightarrow$Matter and antimatter do not behave identically

1967 Sakharov gave the famous three conditions for generating a baryon number asymmetry in the universe:
(1) Baryon number violation, so that the antibaryon disappears.
(2) CP Violation, since only antibaryon has to disappear.
(3) Departure from thermal equilibrium, otherwise the created asymmetry will disappear.

Particle

$$CP\left|f(\vec{p},\lambda)\right\rangle = \left|\bar{f}(-\vec{p},-\lambda)\right\rangle$$

Momentum Helicity

If we sum over momenta and helicities, their reversal under CP won't matter. So, basically —

CP(Process) = (Same process with every particle replaced by its antiparticle)

For example —

$$B^+ \to \overline{D}^0\pi^+ \xrightarrow{CP} B^- \to D^0\pi^-$$

If these two processes have different rates, the difference is a violation of CP invariance.

Dynamic generation of Baryon asymmetry of the Universe

Starting with equal numbers of X (matter) and $\bar{X}$ (antimatter) particles, Baryon number violation in X decay would be exactly cancelled by an opposite sign baryon number violation in $\bar{X}$ decay if CP or C were conserved. Let X and $\bar{X}$ decay with branching fractions

$$f_i = B(X \rightarrow Y_i(\Delta B_i)) \qquad \bar{f}_i = B(\bar{X} \rightarrow \bar{Y}_i(-\Delta B_i))$$

The net baryon number resulting from these decays is

$$B_{net} \sim \sum_i B(X \rightarrow Y_i(\Delta Bi)) \,.\, \Delta B_i + \sum_i B(X \rightarrow \bar{Y}_i(-\Delta Bi))(-\Delta B_i)$$
$$\sim \sum_i (f_i - \bar{f}_i)\, \Delta B_i$$

Baryon number violation alone is not sufficient to generate the asymmetry.

A difference in the partial widths can be generated by C and CP violation.

The smaller symmetry violation, i.e., CP violation will determine the size of the baryon number asymmetry.

If the system is in thermal e quilibrium, all states with same energy will be equally populated, so particle and antiparticle populations will be the same.

Departure from thermal equilibrium is therefore another necessary condition for the baryon asymmetry.

CPV in Standard Model

$$\mathcal{L}_{SM} = \mathcal{L}_{kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

CP conserving

CP violating

- CPV due to complex Yukawa couplings
- Yukawa interactions→ mass terms

Product of Unitary matrix diagonalizing the mass matrices is the CKM

Within the SM the flavour changing weak interactions are described in terms of the weak current coupled to the W boson:

$$\mathcal{L}_{int} = -\frac{g}{\sqrt{2}}(\mathcal{J}^{\mu} W_{\mu}^{+} + \mathcal{J}^{\mu\dagger} W_{\mu}^{-})$$

$$\mathcal{J}^{\mu} = \sum_{i,j} V_{ij} J_{ij}^{\mu} = \sum_{i,j} \bar{u}_i \gamma^{\mu} \tfrac{1}{2}(1-\gamma_5) V_{ij} d_j \longrightarrow$$

For quark transitions

CPV arises in SM, due to the complex phase in the (CKM) -relates the weak eigenstates (d',s',b') to their mass eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$e^{-\imath\gamma}$

$e^{-\imath\beta}$

Elements describe charged-current couplings.

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \qquad V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \qquad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 \qquad V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \qquad V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$

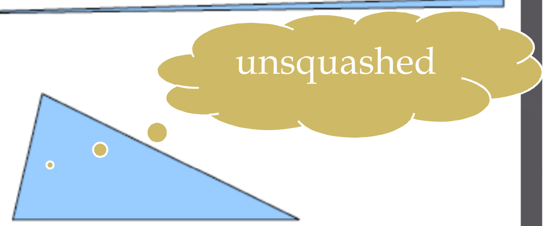
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \qquad V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \qquad V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$$

Unitarity triangles

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

$$\lambda \qquad \lambda \qquad \lambda^5$$

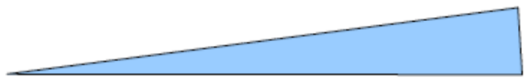


$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\lambda^3 \qquad \lambda^3 \qquad \lambda^3$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

$$\lambda^4 \qquad \lambda^2 \qquad \lambda^2$$



$N \geq 3$
Required for CPV

General N×N matrix	$2N^2$	parameters
Unitarity	N^2	conditions
Phases that may be absorbed by rephasing the quark fields	$2N-1$	
Free parameters	$(N-1)^2$	
N×N orthogonal matrix parametrized by	$N(N-1)/2$	rotation angles
Remaining parameters	$N(N-1)(N-2)/2$	physical phases

Lagrangian insensitive to phases of left-handed fields:

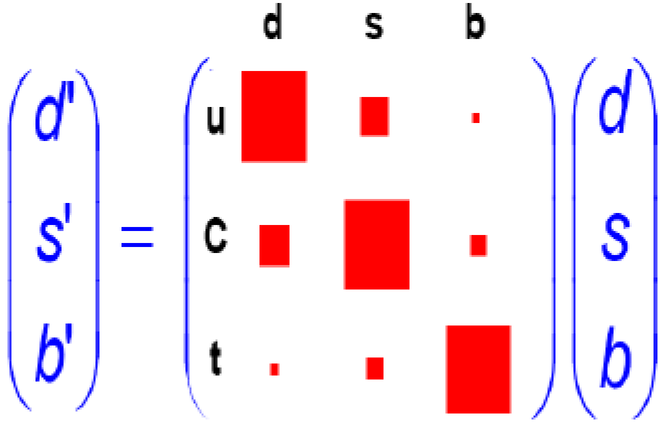
possible redefinition:

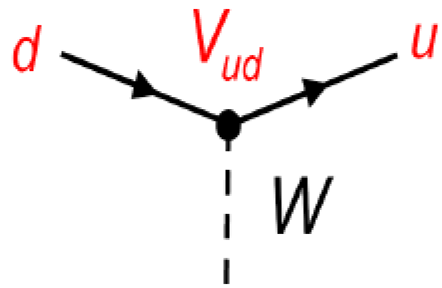
$$u_L \to e^{i\phi(u)} u_L \quad c_L \to e^{i\phi(c)} c_L \quad t_L \to e^{i\phi(t)} t_L$$
$$d_L \to e^{i\phi(d)} d_L \quad s_L \to e^{i\phi(s)} s_L \quad b_L \to e^{i\phi(b)} b_L$$

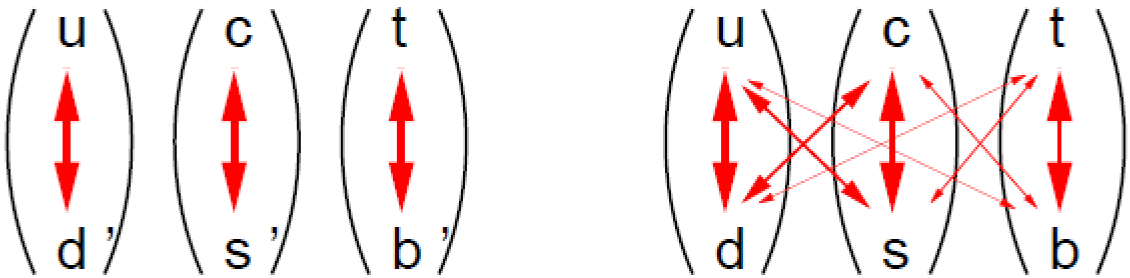
$\phi(q)$: real numbers

$$V = \begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

5 unobservable phase differences.







Diagonal elements of CKM matrix are close to one.

Only small of diagonal contributions.

Mixing between quark families is “CKM suppressed”.

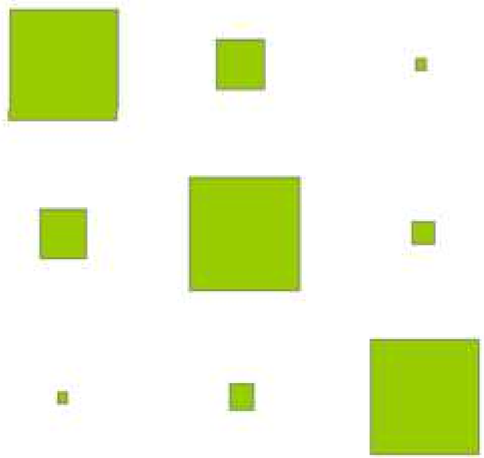
Heirarchy in Quark Mixing

Wolfenstein parametrization, $\lambda = \sin\theta_c$

$e^{-\iota\gamma}$

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$e^{-\iota\beta}$

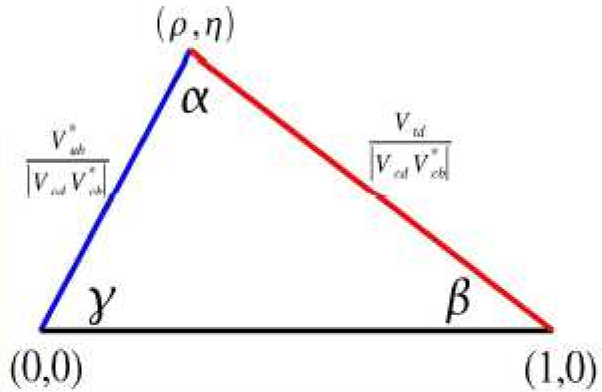


$\eta \neq 0 \Rightarrow$ CPV

CKM matrix has to be Unitary. "The unitarity triangle (UT)":

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\alpha = arg\left(\frac{-V_{tb}^*V_{td}}{V_{ub}^*V_{ud}}\right) \ \beta = arg\left(\frac{-V_{cb}^*V_{cd}}{V_{tb}^*V_{td}}\right) \ \gamma = arg\left(\frac{-V_{ub}^*V_{ud}}{V_{cb}^*V_{cd}}\right)$$



α, β, γ independent of the parametrization

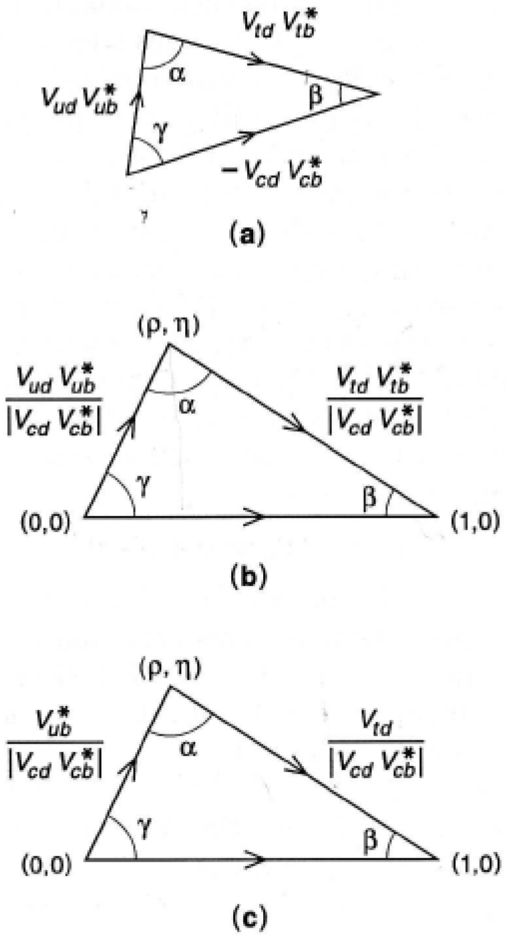
Are physical quantities that can be measured

α, β, γ nonzero $\Rightarrow$ CPV

Sides can indirectly measure the angles

BUT involves large hadronic uncertainities

CP violating asymmetries can directly measure these angles cleanly.


$V_{td}\,V_{tb}^*$
α
$V_{ud}\,V_{ub}^*$
β
γ
$-V_{cd}\,V_{cb}^*$
(a)
(ρ, η)
$\frac{V_{ud}\,V_{ub}^*}{|V_{cd}\,V_{cb}^*|}$
α
$\frac{V_{td}\,V_{tb}^*}{|V_{cd}\,V_{cb}^*|}$
γ
β
(0,0)
(1,0)
(b)
(ρ, η)
$\frac{V_{ub}^*}{|V_{cd}\,V_{cb}^*|}$
α
$\frac{V_{td}}{|V_{cd}\,V_{cb}^*|}$
γ
β
(0,0)
(1,0)
(c)

How do we test this parametarization?

CKM has a hierarchical structure, in the Wolfenstein parametrization:

$$\mathbf{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+A^2\lambda^5(\frac{1}{2}-\rho-i\eta) & 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8}(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2+A\lambda^4(\frac{1}{2}-\rho-i\eta) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

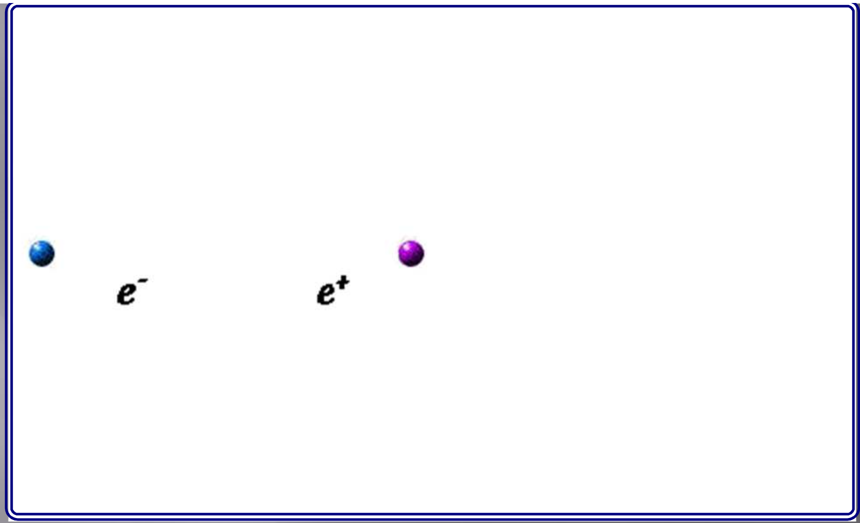
Non zero values of the phases; $\eta \neq 0$ CPV

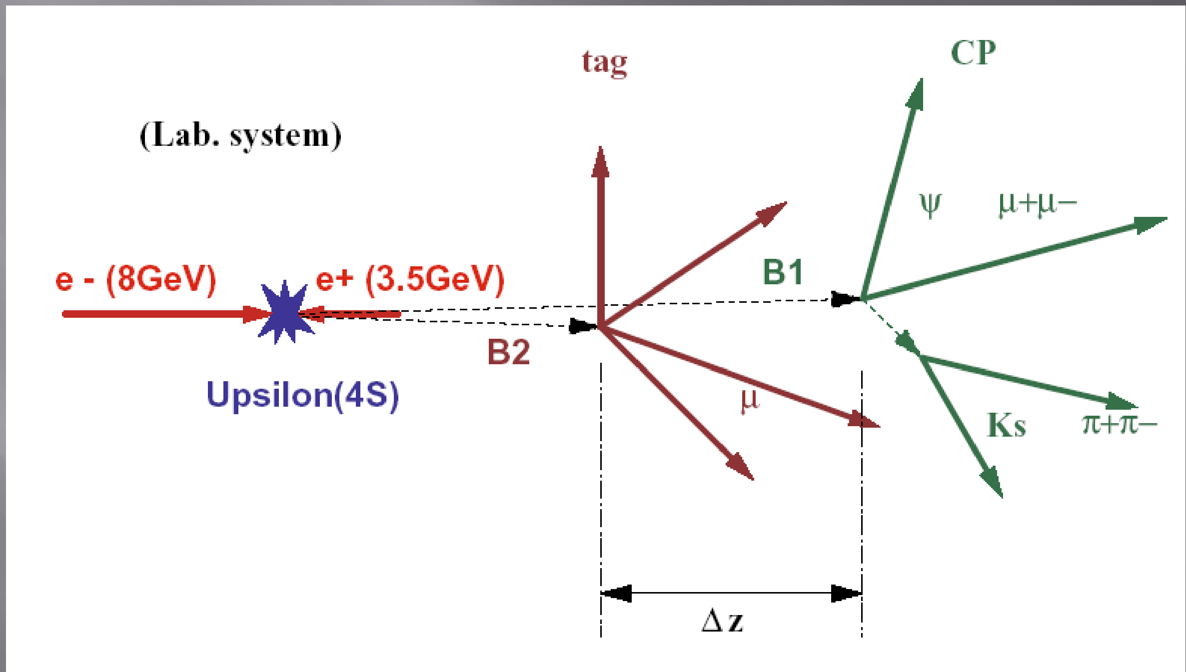
The B system is an excellent testing ground

Hence, there was a huge worldwide effort to measure CPV in B mesons.

Decays rare-require large number of B mesons – B-factories

B Factories

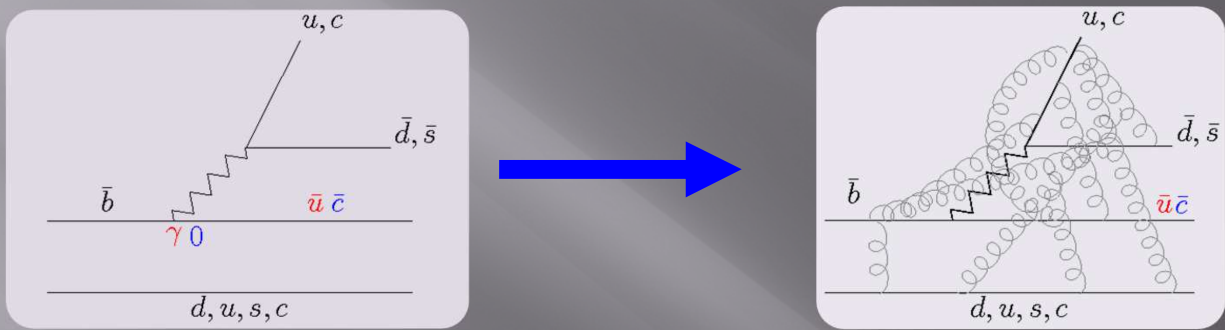




$$\Delta t \equiv t_{CP} - t_{tag} \sim \frac{\Delta z}{\beta\gamma\, c}$$

Extraction of Parameters

Weak decays of heavy quarks difficult to deal with because of hadronic or QCD effects which cannot be calculated reliably.



CP phases can be measured through certain asymmetries which free of hadronic uncertainties

- Phases are observable only through interference terms in modes to which 2 different amplitudes with distinct phases contribute
- The different ways in which the interference terms appear, result in the following categories of CPV

Question: how do we measure this phase?
Phases-elusive, only relative phases are observable through interference terms in modes to which two different amplitudes (with distinct phases) contribute.

⟶

The different ways in which the interference terms appear, result in the following categories of CPV

$$\left|\frac{q}{p}\right| \neq 1$$

CP violation in mixing

$$\left|\frac{\bar{A}}{A}\right| \neq 1$$

CP violation in decay (direct CPV)

$$\Im\left(\frac{q}{p}\frac{\bar{A}}{A}\right) \neq 0$$

CP violation in interference between mixing and decay

Direct CPV This will happen for decay amplitudes with two or more weak contributions,

$$A = A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2} \quad (1)$$

$$\bar{A} = A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2}. \quad (2)$$

$$A_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \quad (3)$$

$$= \frac{-2A_1 A_2 \sin(\Delta\phi) \sin(\Delta\delta)}{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\Delta\phi) \cos(\Delta\delta)} \quad (4)$$

Asymmetry is nonvanishing if $\Delta\phi = \phi_2 - \phi_1 \neq 0$ and $\Delta\delta = \delta_2 - \delta_1 \neq 0$

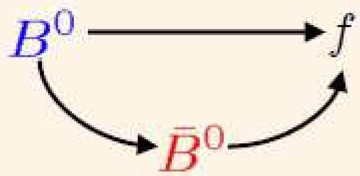
- CPV due to INTERFERENCE BETWEEN DECAYS WITH AND WITH-OUT MIXING

  Neutral B mesons can mix (induced by the box diagrams)

  Final states f into which both B^0 and $\bar{B}^0$ can decay, interfering amplitudes are provided by

  – $B^0 \to f$

  – $B^0 \to \bar{B}^0 \to f$



Similarly for other neutral mesons

The decay rate varies as a function of time

$$\Gamma(B^0(t) \to f) =$$
$$e^{-\Gamma t}\left[\frac{|A|^2+|\bar{A}|^2}{2} - \frac{|\bar{A}|^2-|A|^2}{2}\cos(\Delta M t) - \mathrm{Im}\left(\frac{q}{p}\frac{\bar{A}}{A}\right)\sin(\Delta M t)\right]$$

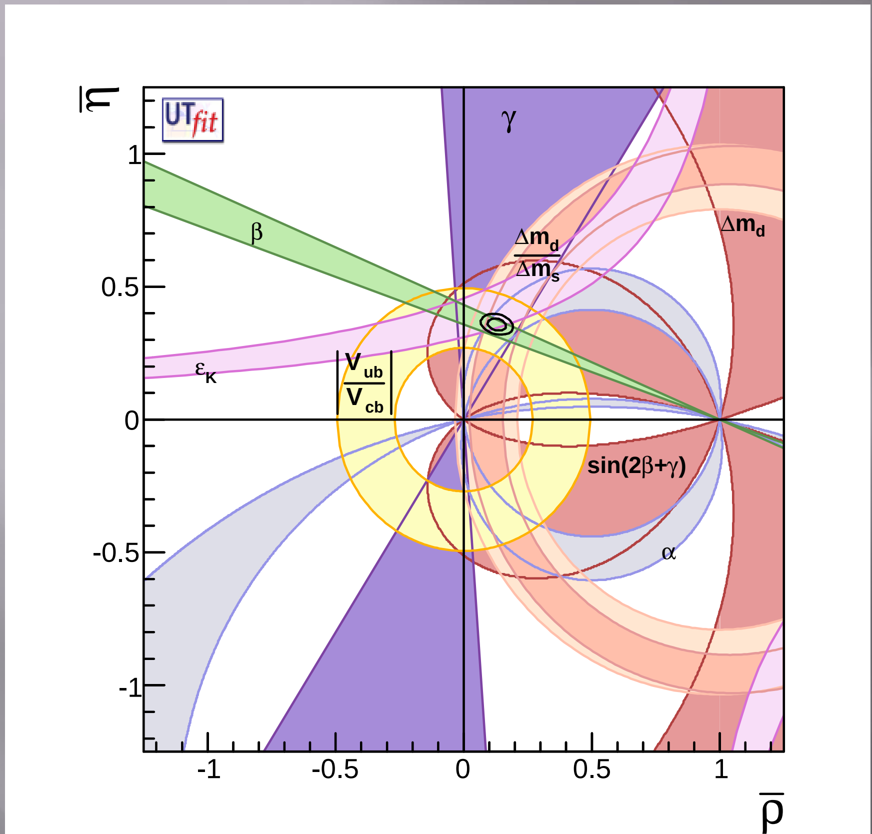
Golden Mode

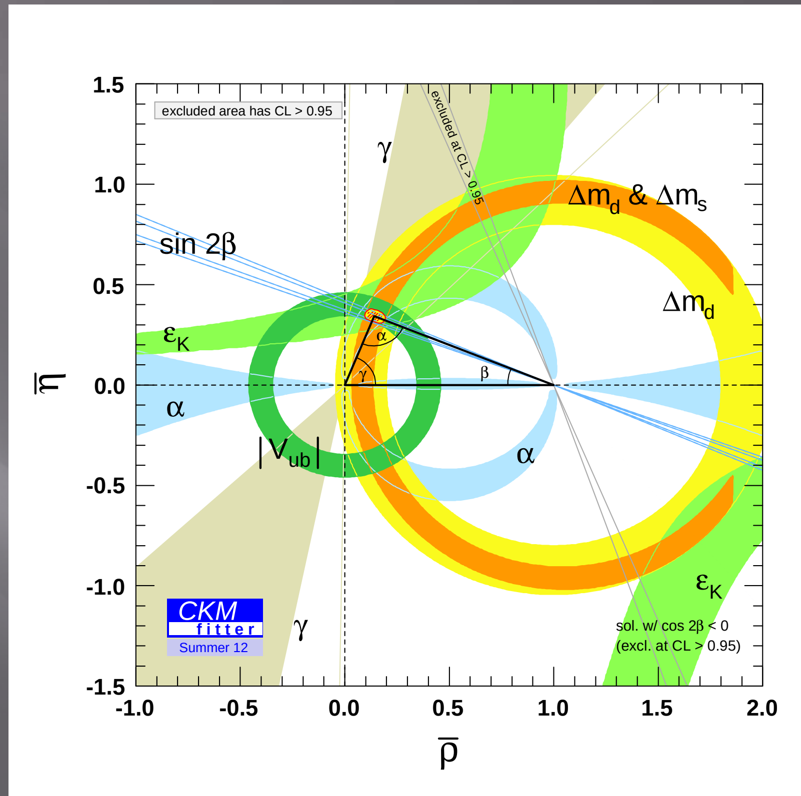
where, $\frac{q}{p} = e^{-2i\beta}$.

In B→ψK$_S$, the decay amplitude dominated by only one tree amplitude with a real CKM element, ⇒ $\bar{A} = A \;\to\; a_{CP} = \sin(2\beta)\sin(\Delta M t)$

Asymmetry completely free of hadronic uncertainities!!

Resulting Picture





Consistency, KM mechanism confirmed

Dominant

There is room for NP