Bootstrapping an $\mathcal{N} = 2$ fixed point

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Bootstrap techniques

There are several approaches to study crossing:

- **Virasoro symmetry.** In $2d$ the enhancement to Virasoro allows for an exact solution: the celebrated minimal models.
  [Belavin, Polyakov, Zamolodchikov]
- **The numerical bootstrap.** Powerful numerical techniques that constrain the low-lying spectrum.
  [Poland, Rattazi, Rychkov, Simmons-Duffin, Tonni, Vichi]
- **The analytical bootstrap.** Analytic constraints for operators with high spin.
  [Alday, Poland, Kaplan, Komargodski, Fitzpatrick, Simmons-Duffin, Zhiboedov]
- **Solvable truncation.** In supersymmetric theories there is a solvable truncation of the crossing equations.
  [C. Beem, M. Lemos, PL, W. Peelaers, L. Rastelli, B. van Rees]

Stress-tensor correlator

Correlators of $T(z)$ capture a truncation of $\langle J_R J_R J_R J_R \rangle$

$$\langle T(0) T(z) T(1) T(\infty) \rangle \sim 1 + \frac{z^4}{(1-z)^4} + \frac{8}{2c_{2d}} z^2 + \dots$$

We can expand it in $4d$ conformal blocks

$$\sum_{\mathcal{O}} \lambda_{\mathcal{O}}^2 G_{\mathcal{O}}(z, \bar{z})$$

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Assuming absence of higher-spin currents [Maldacena, Zhiboedov (2011)]

$$\lambda_{\mathcal{O}_0}^2 = \left(2 - \frac{11}{15 c_{4d}}\right)$$

A universal bound

Unitarity of the $4d$ theory implies

$$c_{4d} \geqslant \frac{11}{30}$$

The bound is quite universal, we had a minimal set of assumptions:

- $\mathcal{N} = 2$ superconformal symmetry.
- A stress-tensor or locality.
- That the theory is interacting.

The canonical rank one theories

There is a "canonical" set of theories, they have the simplest Coulomb branch geometries (can also be constructed in F-theory)

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c	$\frac{11}{30}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{7}{6}$	$\frac{13}{6}$	$\frac{19}{6}$	$\frac{31}{6}$
k	—	$\frac{8}{3}$	3	4	6	8	12

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Most of them are *strongly interacting* with no known Lagrangian description. We need the bootstrap!

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- The index exhibits "Cardy behavior" [Di Pietro, Komargodski (2014)]

$$\lim_{q\rightarrow 1} \mathcal{I}_{Schur}(q) \sim e^{\frac{8\pi^2}{\beta}(c_{4d}-a_{4d})} \quad \rightarrow \quad a_{4d} = \frac{43}{120}$$

Moduli space

Supersymetric theories have interesting moduli spaces of vacua: the "Higgs" branch and the "Coulomb" branch.

For the simplest Argyres-Douglas theory we have the following:

- It has no Higgs branch.
- Its Coulomb branch is parameterized by the vev of a single scalar ϕ.

$$\phi \text{ is chiral } (\mathcal{Q}_\alpha \phi = 0) \text{ with } \Delta_\phi = \frac{6}{5}$$

Bootstrapping the AD theory

Let us look at

$$\langle \phi(x_1)\bar{\phi}(x_2)\phi(x_3)\bar{\phi}(x_4)\rangle = \frac{1}{|x_{12}|^{\Delta_\phi}|x_{34}|^{\Delta_\phi}}\mathcal{G}(z,\bar{z})\,, \quad \Delta_\phi = \frac{6}{5}\,.$$

The superconformal block expansion reads

$$\mathcal{G}(z,\bar{z}) = \sum_{\Delta,\ell} \mathcal{G}_{\Delta,\ell}(z,\bar{z})\,,$$

where

$$\mathcal{G}_{\Delta,\ell}(z,\bar{z}) = g_{\Delta,\ell}(z,\bar{z}) + b_\pm\, g_{\Delta+1,\ell\pm1}(z,\bar{z}) + \dots$$

The blocks are known [Fitzpatrick et al. (2014)].

OPE selection rules

The antichiral OPE reads

$$\phi \times \bar{\phi} \sim 1 + T + \dots$$

Here T is the stress tensor.

The chiral OPE reads

$$\phi \times \phi \sim \phi^2 + \mathcal{C}_\ell + \dots$$

The operators $\mathcal{C}_\ell$ is a family of protected multiplets.

The OPE coefficients are not protected and there is always a gap.

Central charge bound

Let us be agnostic about the value of c

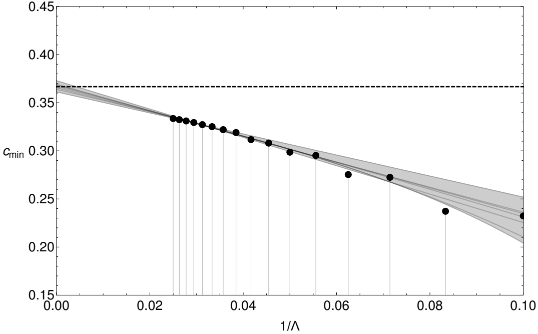

Figure: Minimum central charge for $\Delta_\phi = \frac{6}{5}$.

Bounds on ϕ^2

Let us put rigorous upper and lower bounds on the OPE coefficient of ϕ^2

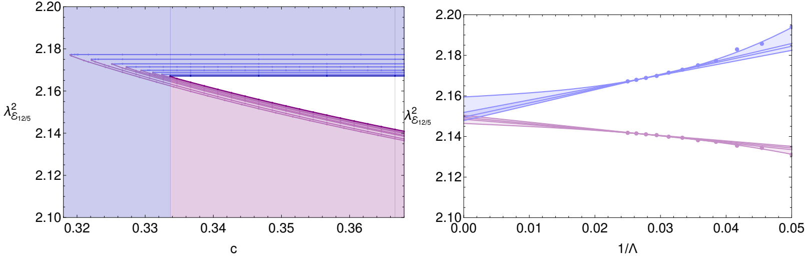

Figure: Numerical upper and lower bounds on the OPE coefficient squared of ϕ^2.

$$2.1418 \leqslant \lambda^2_{\phi^2} \leqslant 2.1672$$

The inversion formula

There is an inversion formula [Caron-Huot (2017)]:

$$\lambda(\Delta, \ell) \sim \int dDisc \left[\frac{(z\bar{z})^{\Delta_\phi}}{(1-z)^{\Delta_\phi}(1-\bar{z})^{\Delta_\phi}} \mathcal{G}(1-z, 1-\bar{z}) \right]$$

CFT data for s-channel provided we know the t-channel.

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$$\phi\times\bar{\phi} = 1 + T + \ldots$$

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The inversion formula gives us $\mathcal{C}_\ell$ at large spin ℓ (analytical bootstrap).

Numerics vs the inversion formula

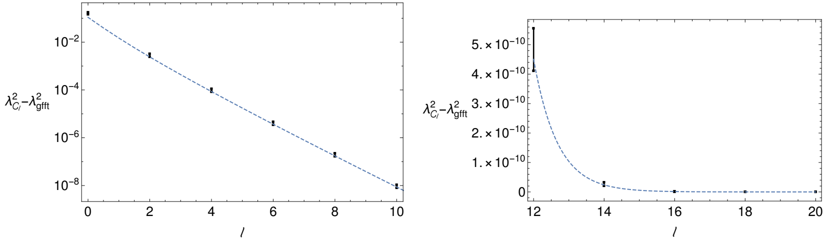

Figure: The dashed line shows the result of CH, where we considered only the contribution of the identity and stress-tensor operators in the non-chiral channel, and thus is an approximate result for sufficiently large spin.

Summary

- The $2d$ chiral algebra naturally selects the "simplest" $\mathcal{N} = 2$ model. Sits of the origin of the theory space, associated chiral algebra is the Yang-Lee model.
- We gave the first step towards bootstrapping the AD theory. Superconformal blocks are known, external dimension $\Delta = 1.2$ is a small number.
- Good agreement between numerics and analytics.
- To do: Improve numerics, improve analytics, develop block technology!

Thank you.