# Bootstrapping the space of $4d$ $\mathcal{N} = 2, 3$ SCFTs

Madalena Lemos

Nonperturbative and Numerical Approaches to Quantum Gravity, String Theory and Holography
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Based on:
1312.5344 w/ C. Beem, P. Liendo, W. Peelaers, L. Rastelli and B. van Rees
1511.07449 w/ P. Liendo
1702.05101 w/ M. Cornagliotto and V. Schomerus

# Outline

1. **The Superconformal Bootstrap Program**
2. **A solvable subsector**
3. $4d$ $\mathcal{N} = 3$ **SCFTs**
4. **Constraining the space of** $\mathcal{N} = 2$ **SCFTs**
5. **Summary and Outlook**

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→ See Pedro's talk!

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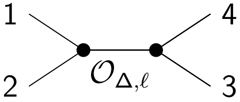

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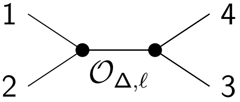
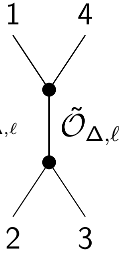



$$= \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda^2_{\tilde{\mathcal{O}}_{\Delta,\ell}}$$



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→ Meromorphic!

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$\hookrightarrow$ anti-holomorphic dependence drops out

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  - $\hookrightarrow$ $4d$ $\mathcal{N} = 3 \Rightarrow 2d$ $\mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]

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  - $\hookrightarrow$ $4d$ $\mathcal{N} = 3 \Rightarrow 2d$ $\mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]
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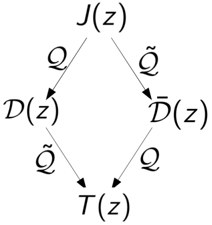

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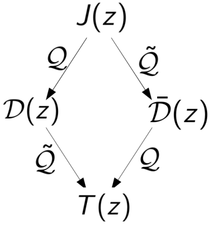
$2d$ $\mathcal{N} = 2$ **Stress tensor** $\mathcal{J}$



# $\mathcal{N} = 3$ Chiral algebra

- $4d$ $\mathcal{N} \geqslant 3$: some of the extra supercharges commute with $\mathbb{Q}$
  - $\hookrightarrow$ $4d$ $\mathcal{N} = 4 \Rightarrow 2d$ "small" $\mathcal{N} = 4$ chiral algebra
  - $\hookrightarrow$ $4d$ $\mathcal{N} = 3 \Rightarrow 2d$ $\mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]
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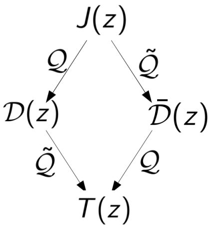
$\rightarrow$ Present in any local $\mathcal{N} = 3$ SCFT



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$2d$ $\mathcal{N}=2$ **Stress tensor** $\mathcal{J}$

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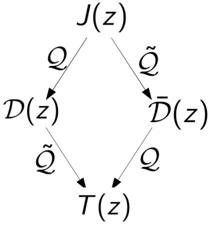
$\rightarrow$ A trivial statement in $2d$: $\langle\mathcal{J}\mathcal{J}\mathcal{J}\mathcal{J}\rangle$ is fixed in terms of $c_{2d}$



# Space of $\mathcal{N} = 3$ SCFTs

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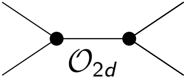
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

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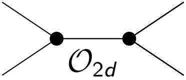
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 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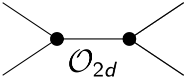
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 


$\rightarrow \lambda^2_{\mathcal{O}_{2d}} \rightsquigarrow \lambda^2_{\mathcal{O}_{4d}}$

assumptions: interacting theory, unique stress tensor

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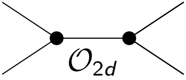
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 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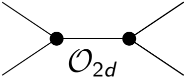
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 


$\rightarrow \lambda^2_{\mathcal{O}_{2d}} \rightsquigarrow \lambda^2_{\mathcal{O}_{4d}} \underbrace{\geqslant}_{4d \text{ unitarity}} 0 \Rightarrow$ New unitarity bound

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# Space of $\mathcal{N}=3$ SCFTs

**$2d$ $\mathcal{N}=2$ Stress tensor $\mathcal{J}$**

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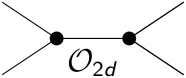
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 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$$\boxed{c_{4d} \geqslant \frac{13}{24}}$$ [Cornagliotto, ML, Schomerus]

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# Outline

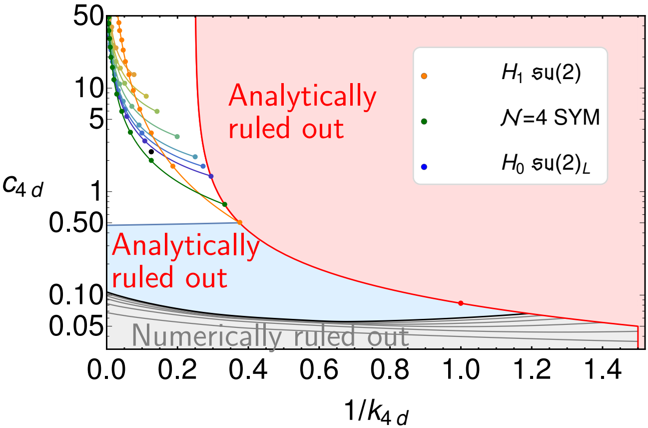
1. The Superconformal Bootstrap Program
2. A solvable subsector
3. $4d$ $\mathcal{N} = 3$ SCFTs
4. **Constraining the space of $\mathcal{N} = 2$ SCFTs**
5. Summary and Outlook

# $4d$ $\mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

$$\langle TTTT \rangle\,, \quad \langle J^a J^b J^c J^d \rangle\,, \quad \langle TTJ^a J^b \rangle$$

# $4d$ $\mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

$\langle TTTT \rangle$, $\langle J^a J^b J^c J^d \rangle$, $\langle TTJ^a J^b \rangle$




[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

# Outline

1. The Superconformal Bootstrap Program
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5. **Summary and Outlook**

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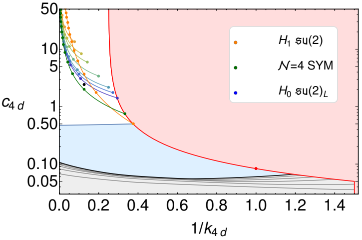

# Summary and Outlook

**New constraints on the space of allowed $\mathcal{N} > 1$ SCFTs**

$\rightarrow$ No "minimal" $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$

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**Numerically solving theories?**

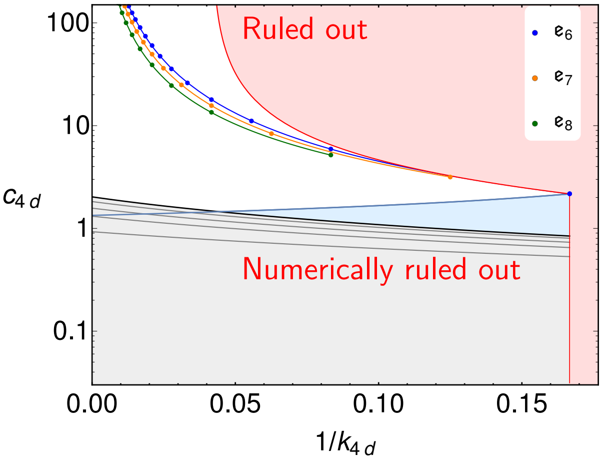
- Pedro's talk for the "simplest" $\mathcal{N} = 2$ SCFT

**Thank you!**

# Backup slides

# Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

## $E_6$ flavor symmetry




[Beem, ML, Liendo, Peelaers, Rastelli, van Rees] [ML, Liendo] [Beem, ML, Liendo, Rastelli, van Rees]