Lattice study of gauge/gravity duality in two-dimensional N=(8,8) SYM

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Giguere and D.K., JHEP 1505, 082 (2015).
Giguere and D.K., in preparation

Talk plan

1. Duality and Lattice studies

2. Sugino lattice action and SUSY WTI

3. Numerical test of Duality in 2D N=(8,8) SYM

4. Summary

1. Duality and Lattice studies

Gauge/gravity duality

- AdS/CFT [Maldacena, 1997] and more general duality conjecture states

  strongly coupled gauge theory

  = classical gravity on a curved space

- Application of duality

  (1) Non-perturbative formulation of string theory

  (2) BH information loss paradox

  (3) Holographic QCD, superconductor…

  ⇨ precision test of the gauge/gravity duality by lattice simulations is important.

Maximally supersymmetric Yang-Mills

- Action in p+1-dimensions

$$S = \frac{N}{\lambda}\int \mathrm{d}^{p+1}x\ \mathrm{tr}\Big\{\frac{1}{2}F_{\mu\nu}^2 + \frac{1}{2}(D_\mu X_i)^2 - \frac{1}{4}[X_i, X_j]^2 - \frac{i}{2}\psi_\alpha\gamma_\mu D_\mu\psi_\alpha + \frac{1}{2}\psi_\alpha\gamma_i[X_i,\psi_\alpha]\Big\}$$

A_μ : gauge field $\quad \mu = 0, \cdots, p$

X_i : scalar field $\quad i = p+1, \cdots, 9$

ψ_α : fermion $\quad \alpha = 1, \cdots, 16$

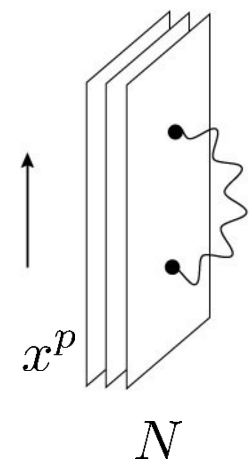
$\gamma_0 = i$

$\{\gamma_m, \gamma_n\} = \delta_{mn}$

$\lambda = g^2 N$ 't Hooft coupling constant

- dual to black p-brane solution in SUGRA

[Maldacena 1997, Itzhaki et al. 1998]



Numerical tests of duality in p+1 dimensions

- p=0 (BFSS model)

  momentum sharp cut-off:

  - Hanada et al (2007-2014)

  Hanada's lecture

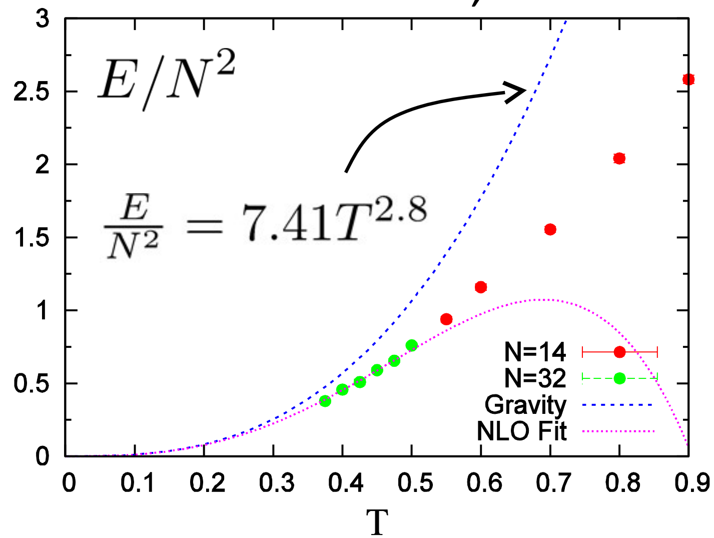
  lattice:

  - Catterall and Wiseman (2007-2010)
  - D.K. and Kamata (2012-2015)
  - Filev and O'Connor (2015)
  - Berkowitz, Rinaldi, Hanada, Ishiki, Shimasaki, Vranas (2016)

  …

Internal energy of Black hole (p=0)

D.K. and Kamata, 2015



$$\frac{E}{N^2} = 7.41T^{2.8} - CT^p + DT^{p+6/5}$$

	C	p	D
SUGRA	unknown	4.6	unknown
SYM			
Hanada et al. (2007)	5.55 ± 0.07	4.58 ± 0.03	—
D.K. – Kamata (2015)	9.0 ± 2.6	4.74 ± 0.35	—
Berkowitz et al. (2016)	10.2 ± 2.4	4.6 ± 0.3	6.2 ± 2.6

Lattice studies for $p \geq 1$ cases

- p=1

  Catterall, Joseph and Wiseman (2008-2010)

  Giguere and D.K. (2015-) ← This talk

  David Schaich's talk

- p=2

  Raghav G. Jha's talk

- p=3 (AdS/CFT)

  Catterall, Giedt, Schaich, Damgaard,
  Damgaard and Degrand (2014-)

This work

2D N=(8,8) SYM at finite T and large N	$\overset{?}{=}$	Thermodynamics of black string

- Sugino lattice action [Sugino, 2005]

  → exact two supersymmetries

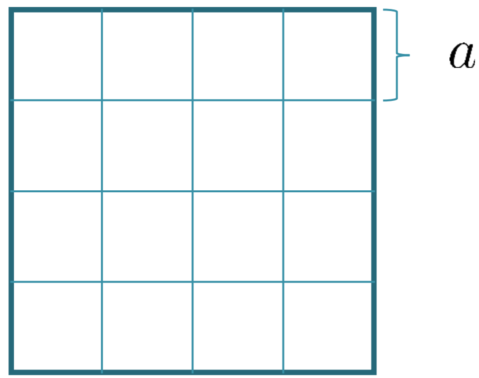
- Standard Monte-Carlo method (with quenching phase of fermion pfaffian)

2. Sugino lattice action and SUSY WTI

Lattice gauge theory

Wilson 1974

- Lattice



  - Fields live on sites or links
  - $\partial_\mu \longrightarrow$ difference operato∇_μ

- Lattice gauge field

continuum limit

$$P_{\mu\nu}(x) \sim 1$$

$$U_\mu(x, y) = e^{iaA_\mu(x,y)} \in G$$

$$S_G = \frac{N}{g^2} \sum_{x,\mu<\nu} \left\{ 1 - \frac{1}{N} \mathrm{Re}\,\mathrm{tr}\left(P_{\mu\nu}(x)\right) \right\} \longrightarrow \int d^4x \, \frac{1}{4} \mathrm{tr}\left(F_{\mu\nu}^2(x)\right)$$

$$P_{\mu\nu}(x) = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^{-1}(x + a\hat{\nu}) U_\nu^{-1}(x)$$

Lattice supersymmetry

- SUSY invariance in continuum theory

$$\partial(fg) = (\partial f)g + f(\partial g)$$

$$\Rightarrow \quad \delta S_{cont.} = \int d^4x\, \partial_\mu X_\mu(x) = 0$$

- SUSY is broken by the lattice cut-off

Dondi-Nicolai, Kato-Sakamoto-So, Bergner

$$\nabla(fg) = (\nabla f)g + f(\nabla g) + O(a)$$

$$\delta S_{lat.} = a^4 \sum_x \{\partial_\mu X_\mu(x) + O(a)\} = O(a)$$

+ UV divergences $O(1/a)$ $\Rightarrow$ SUSY is broken at quantum level

Lattice SYM with exact SUSY

- 2003-2004:

  Cohen-Kaplan-Katz-Unsal

  Sugino

  Catterall

  Catterall's lecture

- The other lattice formulations of SYM and applications:

  Giedt, Endres, Kawamoto, D'Adda, Kanamori, Kikukawa, Suzuki,

  Damgaard, Matsuura, Hanada, Joseph, O'Connor, Schaich, Jha,

  ⋯ . and D.K.

c.f. Lattice Wess-Zumino model and SUSY QM:

Dondi, Nicolai, Sakai, Sakamoto, Wipf, Wenger,
Bergner, Wozar, Baumgartner,…

Q-exact form of continuum N=(8,8) action

- Q-transformation

$$Q_\pm A_\mu = \psi_{\pm\mu}, \quad Q_\pm \psi_{\pm\mu} = iD_\mu \phi_\pm, \ldots$$

$$\Rightarrow \quad Q_\pm^2 = i\delta_{\phi_\pm}, \qquad \{Q_+, Q_-\} = -i\delta_C$$

δ_ω : gauge transformation

- Q-exact action

$$S = Q_+ Q_- \frac{N}{2\lambda} \int d^2x \ \mathrm{tr}\Big\{ -2iB_iG_i - \frac{2}{3}\epsilon_{ijk}B_iB_jB_k - \psi_{+\mu}\psi_{-\mu} - \chi_{+i}\chi_{-i} - \frac{1}{4}\eta_+\eta_- \Big\}$$

$$G_0 = D_0X_3 + D_1X_2$$
$$G_1 = D_1X_3 - D_0X_2$$
$$G_2 = i[X_2, X_3] + F_{01}$$

Sugino lattice action

[Sugino,2005]

- Q-exact lattice action

$$S = Q_+ Q_- \frac{N}{2\lambda} \sum_x \mathrm{tr}\Big\{ -2iB_i G_i - \frac{2}{3}\epsilon_{ijk} B_i B_j B_k$$
$$-\psi_{+\mu}\psi_{-\mu} - \chi_{+i}\chi_{-i} - \frac{1}{4}\eta_+\eta_- \Big\}$$

$$G_0 = \nabla_0^+ X_3 + \nabla_1^+ X_2$$
$$G_1 = \nabla_1^- X_3 - \nabla_0^- X_2$$
$$G_2 = i[X_2, X_3] + F_{01}^{lat}$$

lattice field strength

$\nabla_\mu^\pm$: forward/ backward difference operators

- Can define lattice Q-transformation satisfying

$$Q_\pm^2 = i\delta_{\phi_\pm}, \qquad \{Q_+, Q_-\} = -i\delta_C$$

⇨ Two exact SUSYs

Choice of gauge action

- A simple choice (double plaquette gauge action)

$$F_{01}^{lat}(x) = -\frac{i}{2}\left(P_{01}(x) - P_{01}^{\dagger}(x) - \frac{1}{N}\mathrm{tr}(P_{01}(x) - P_{01}^{\dagger}(x))\right)$$

- However, extra vacuum configurations exist:

$$S_B|_{X_i=0} = \frac{N}{2\lambda_0}\sum_{t,x}\mathrm{tr}\left\{-\frac{1}{4}\left(P_{01} - P_{01}^{\dagger} - \frac{1}{N}\mathrm{tr}(P_{01} - P_{01}^{\dagger})\right)^2\right\}$$

$$S_B = 0 \ \text{at} \ P_{01} = +1, \quad P_{01} = -1 \qquad SU(2)$$

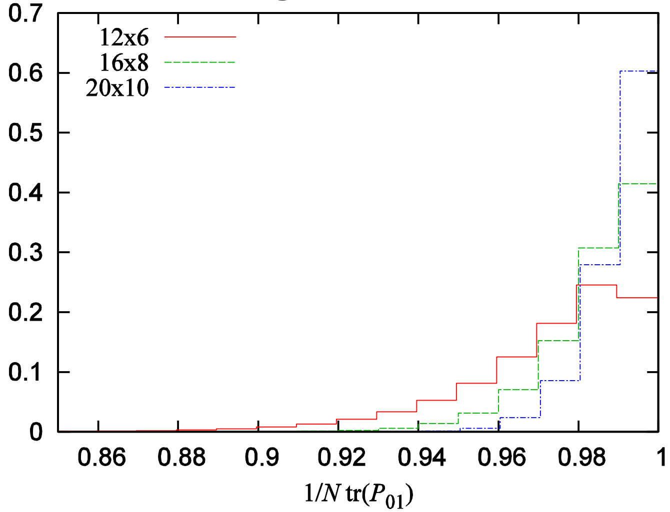
c.f. admissible gauge action [Sugino, 2004]

tangent-type action [Matsuura-Sugino,2014]

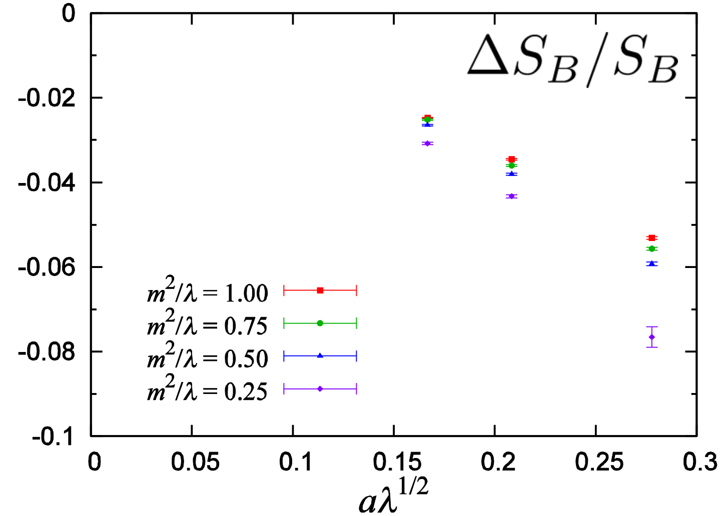
Test of boson part

Eric Giguere and D.K., 2015

Histogram of P_{01}



Higher order terms



$$\lambda_0 = \lambda a^2 \to 0$$

$$P_{01}(x) \to 1, \quad \Delta S_B \to 0.$$

Sugino lattice action of N=(8,8) SYM works for double plaquette action

SUSY Ward-Takahashi identity

- SUSY breaking source

  (1) finite T (anti-periodic boundary)

  (2) finite a ⟶ disappear in the continuum limit?

- SUSY WTI with supercurrent $J_\mu(x)$ holds in continuum SYM even at finite temperature

  (c.f. Kananori-Suzuki, 2007)

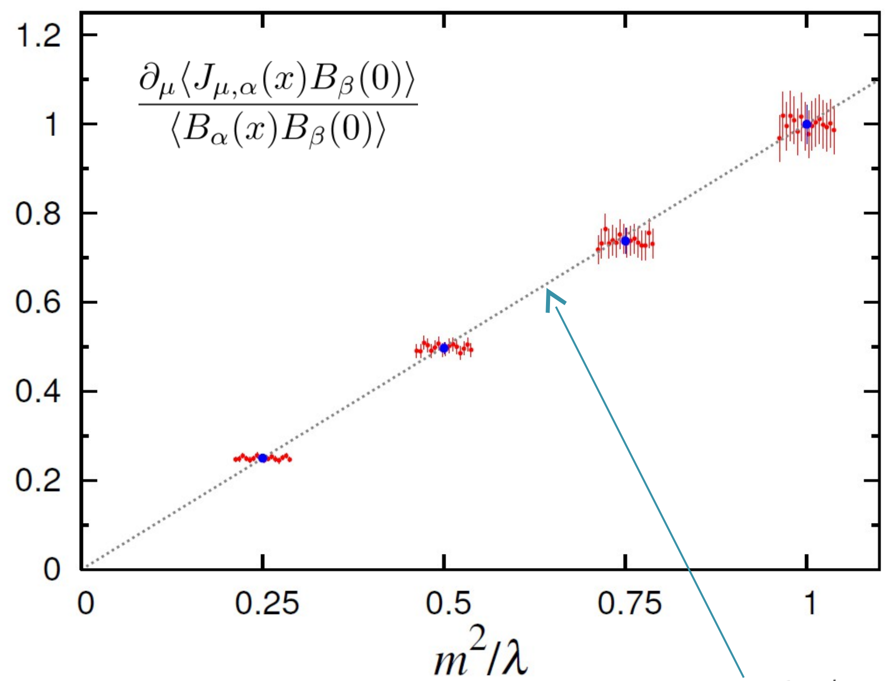
$$\langle \partial_\mu J_\mu(x) \mathcal{O}(y) \rangle = 0 \quad \text{for} \quad x \neq y$$

- Add mass term to the action for avoiding the problem of flat direction: $S \to S + \frac{N}{\lambda} \int d^2x \, \frac{m^2}{2} \mathrm{tr}(X_i^2)$

$$\langle \partial_\mu J_\mu(x) \mathcal{O}(y) \rangle = \frac{m^2}{\lambda} \langle B(x) \mathcal{O}(y) \rangle \quad \text{for} \quad x \neq y$$

Numerical test of Ward-Takahashi identity

Eric Giguere and D.K., 2015



SU(2)
T=1

$$\frac{\partial_\mu \langle J_{\mu,\alpha}(x) B_\beta(0)\rangle}{\langle B_\alpha(x) B_\beta(0)\rangle} = \frac{m^2}{\lambda}$$

SUSY WTI holds for all $(J_\mu(x))_\alpha, \alpha = 1, 2, \cdots, 16.$

All of the SUSYs are restored in the continuum limit.

3. Numerical test of Duality in 2D N=(8,8) SYM

SYM at finite temperature

- λ is only dimensionful parameter

$$S = \tfrac{N}{\lambda} Q_+ Q_- (\ldots) \qquad \dim[\lambda] = 2$$

- Scale-transformation $\qquad \dim\ [a] = -1$

$$\beta \equiv \frac{1}{T} = \beta_0 a, \qquad L = L_0 a, \qquad \lambda = \lambda_0 a^{-2}$$

$$a\frac{\partial}{\partial a}\log Z = a\frac{\partial\beta}{\partial a}\frac{\partial}{\partial\beta}\log Z + a\frac{\partial L}{\partial a}\frac{\partial}{\partial L}\log Z = -\beta L(\epsilon - p),$$

$$= 2\langle S\rangle$$

$$\Rightarrow \quad \epsilon - p = -\tfrac{2}{\beta L}\langle S\rangle \quad \xrightarrow{T \to 0} \quad 0$$

Thermodynamics of black string

- SUGRA

$$S = -\frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{g}\left[R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{12}e^{\phi}F^2_{\mu\nu\rho}\right],$$

- Black hole thermodynamics [Klebanov-Tseytlin, 1996]

$$E = \frac{2}{3}c_0 L N^2 T^3$$

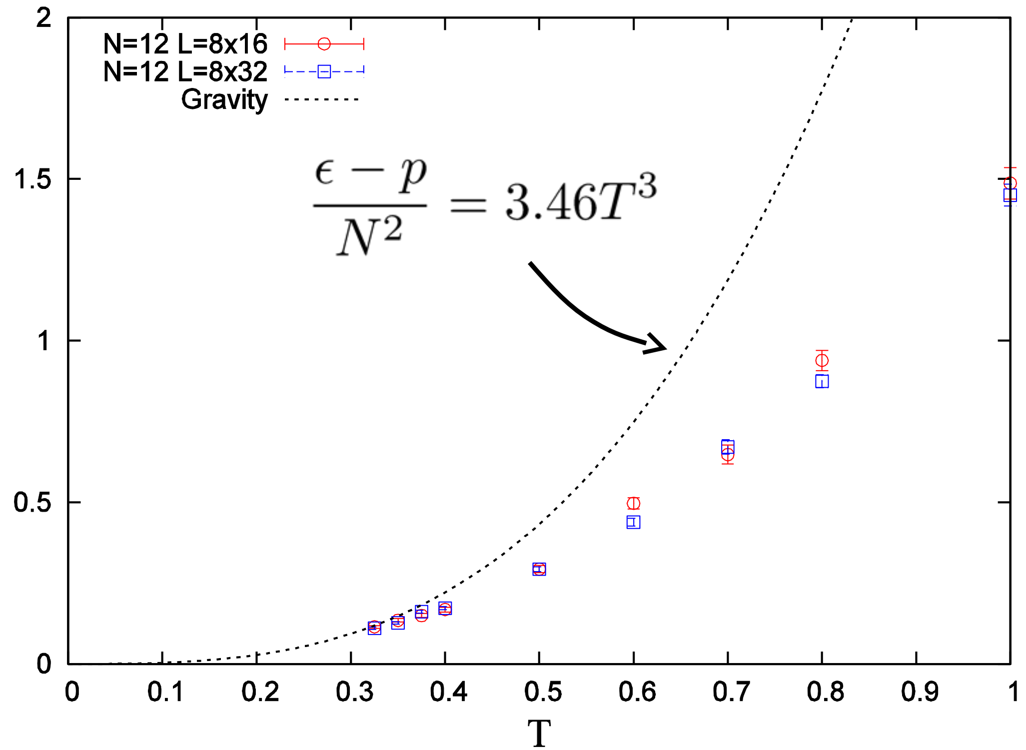
$$S = c_0 L N^2 T^2 \qquad c_0 = \frac{2^4\pi^{5/2}}{3^3} = 10.37....$$

$$E = TS - PV \quad \Rightarrow \quad \epsilon - p = \tfrac{1}{3}c_0 N^2 T^3$$

Lattice results of e-p

Giguere and D.K., in preparation

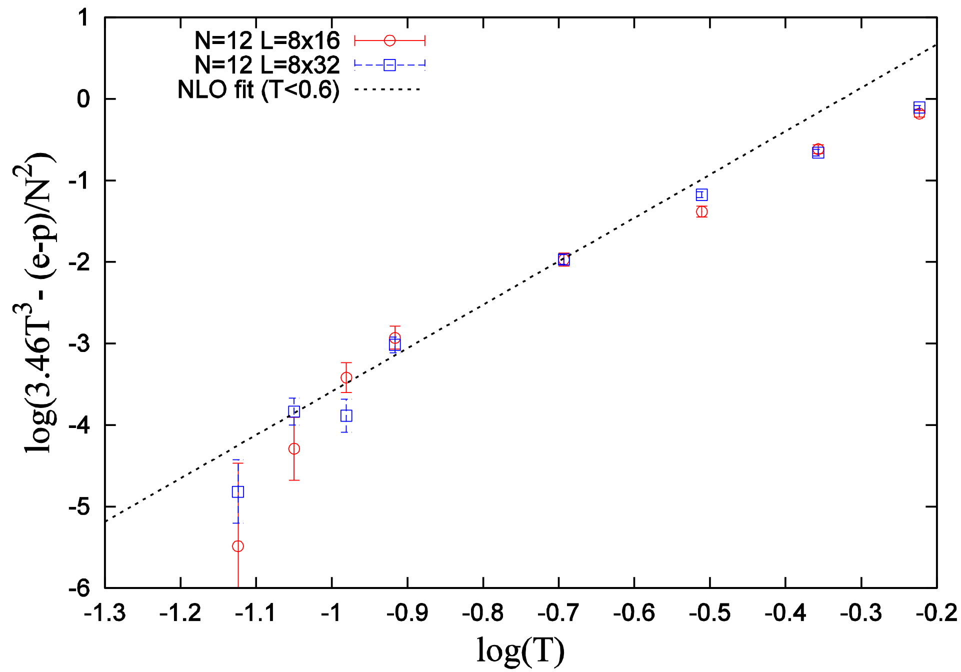
$$\frac{\epsilon - p}{N^2}$$



Gauge/gravity duality holds, at least, qualitatively.

α'-correction

Giguere and D.K., in preparation



$$\epsilon - p = 3.46T^3 - dT^\beta + \cdots$$

	d	β
SUGRA	unknown	5
SYM (Our result)	5.7 ± 1.3	5.32 ± 0.28

4. Summary

Summary

☑ SUSY Ward-Takahashi identity

Sugino lattice action works well for 2d SYM with 16 SUSY beyond PT.

☑ e-p of black string

Lattice results reproduce the prediction, and NLO result shows that the duality is true.

☐ Take continuum limit and large N limit of the results

☐ Need a method solving sign problem

Thank you.