Phase Transitions in the BMN Matrix Model

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2 Feb 2018 @ICTS

This talk is based on the collaboration with
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Outline

1. Introduction

2. Deconfinement phase transition

3. Lattice simulation

4. Summary and Discussion

1. Introduction

Motivation

Quantum theory of gravitation → String/M theory

But

String theory is defined based on perturbation theory.

We need non-perturbative formulation.

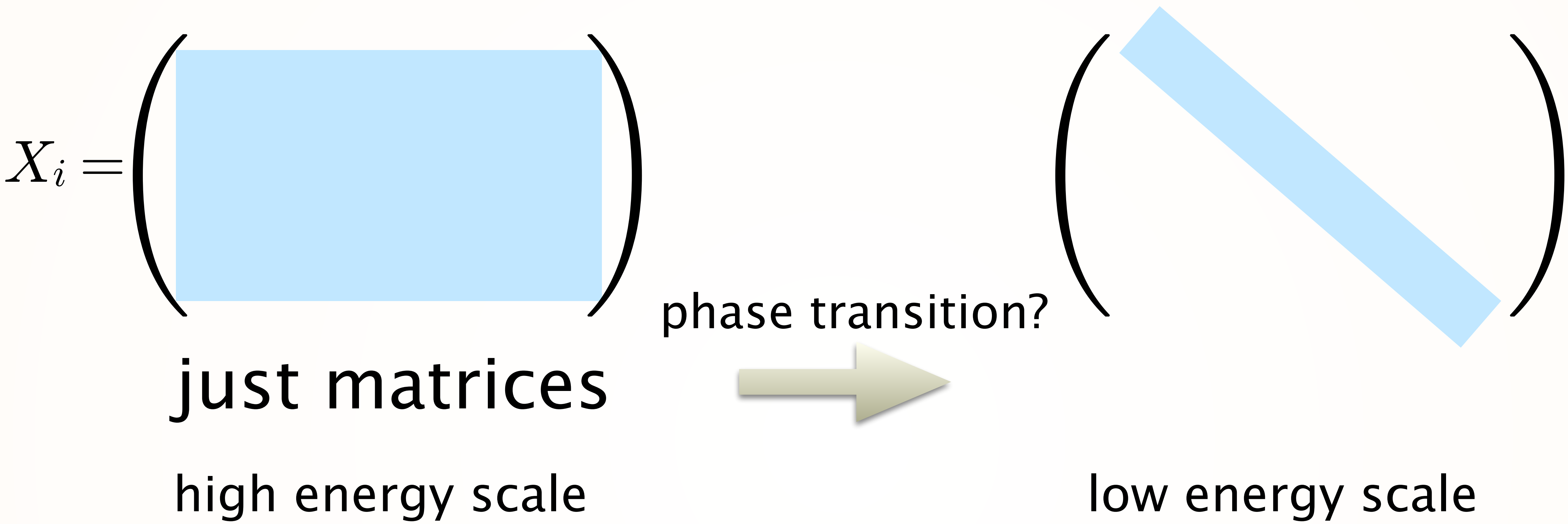
→ **Matrix Models** [Banks-Fischler-Shenker-Susskind '96, Ishibashi-Kawai-Kitazawa-Tsuchiya '96, …]

- The target space is regularised by matrices.
- Branes are naturally included.
- Some matrix models have gauge/gravity duality.

"Matrices = Strings"

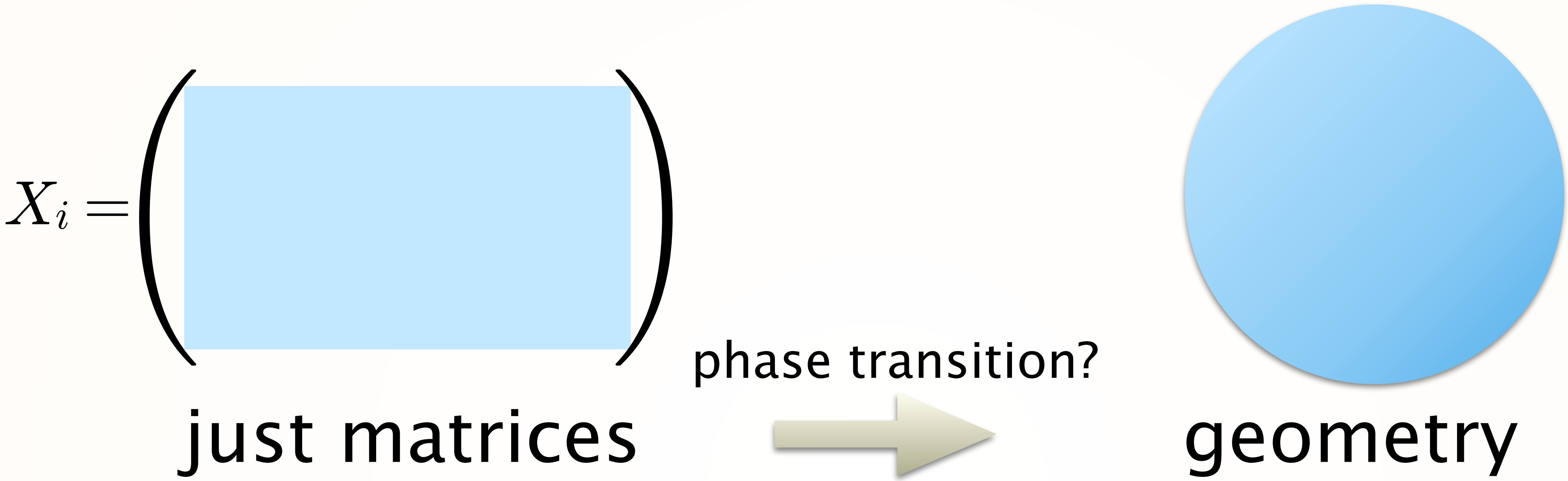
1. Introduction

Emergent geometry in matrix models



1. Introduction

Emergent geometry in matrix models



high energy scale

low energy scale

We choose the temperature as the energy scale.

In this talk, we focus on the thermal BMN matrix model

1. Introduction

The action of membrane theory:

[deWit-Hoppe-Nicolai '88]

$$S = -T_{\mathrm{M2}} \int d^3\sigma \sqrt{-\det[g_{MN}(X)\partial_\mu X^M \partial_\nu X^N]} + T_{\mathrm{M2}} \int C_3$$

membrane area

potential

plane-wave geometry:

$$g_{MN}dx^M dx^N = -2dx^+dx^- + dx^i dx^i - (\frac{\mu^2}{9}x^a x^a + \frac{\mu^2}{36}x^m x^m)dx^+ dx^+$$

$$C_3 = \frac{\mu}{6}\epsilon_{abc}x^a dx^b dx^c dx^+ \qquad i{=}1,\ldots,9 \qquad a{=}1,2,3 \qquad m{=}4,\ldots,9$$

1. Introduction

The action of membrane theory: [deWit-Hoppe-Nicolai '88]

$$S=\frac{p^+}{8\pi}\int d^3\sigma\left[(D_0X^i)^2-\frac{\mu^2}{9}(X^a)^2-\frac{\mu^2}{36}(X^m)^2-\frac{8\pi^2T_{\rm M2}^2}{(p^+)^2}\{X^i,X^j\}^2\right]$$

$$+T_{\rm M2}\int d^3\sigma\,\mu X^1\{X^2,X^3\}$$

$$D_0X^i=\partial_0X^i+\{A,X^i\}$$

Matrix regularisation

$$\frac{1}{4\pi}\int d^2\sigma\to\frac{1}{N}\mathrm{Tr}\qquad\{\,,\,\}\to\frac{-iN}{2}[\,,\,]$$

$$X^i(\sigma^\mu)\to\hat{X}^i(\sigma^0)\qquad A(\sigma^\mu)\to\frac{2}{N}\hat{A}(\sigma^0)$$

$$S=\frac{p^+}{2N}\int d\sigma^0\,\mathrm{Tr}\left[(D_0X^i)^2-\frac{\mu^2}{9}(X^a)^2-\frac{\mu^2}{36}(X^m)^2+\frac{c^2}{2}[X^i,X^j]^2-2ic\mu X^1[X^2,X^3]\right]$$

$$c=\frac{2\pi NT_{\rm M2}}{p^+}$$

$$D_0X^i=\partial_0X^i-i[A,X^i]$$

Bosonic BMN model

plane-wave geometry:

$$g_{MN}dx^Mdx^N=-2dx^+dx^-+dx^idx^i-\left(\frac{\mu^2}{9}x^ax^a+\frac{\mu^2}{36}x^mx^m\right)dx^+dx^+$$

$$C_3=\frac{\mu}{6}\epsilon_{abc}x^adx^bdx^cdx^+$$

$i=1,\dots,9$ $\quad a=1,2,3$ $\quad m=4,\dots,9$

1. Introduction

Rescale X^i and σ^0 to $\tilde{X}^i$ and t

$a,b=1,2,3, \quad m,n=4,\ldots,9$

Action of the BMN matrix model:

$$S = N\int dt\,\mathrm{Tr}\Big[\frac{1}{2}(D_t\tilde{X}^a)^2+\frac{1}{2}(D_t\tilde{X}^m)^2-\frac{1}{4}\left(\frac{\mu}{3}\epsilon_{abc}\tilde{X}^c - i[\tilde{X}^a,\tilde{X}^b]\right)^2+\frac{1}{2}[\tilde{X}^a,\tilde{X}^n]^2 +\frac{1}{4}[\tilde{X}^m,\tilde{X}^n]^2-\frac{\mu^2}{72}\tilde{X}^m\tilde{X}^m+\text{fermions}\Big]$$

- Symmetry: $S\widetilde{U}(2|4) \supset R\times SO(3)\times SO(6)$

[Berenstein-Maldacena-Nastase '02]

- Obtained by dimensional reduction of 4D $\mathcal{N}=4$ super Yang–Mills

$\longrightarrow$ 1D super quantum mechanics

Vacua: $SU(2)$ generators

$$\tilde{X}^a = -\frac{\mu}{3}(1_{N_2}\otimes L_a^{[N_5]})$$

$$\tilde{X}^m = 0$$

Number of M5–branes

Number of M2–branes

$L_a{}^{[N_5]}$: representation matrix of dim. N_5

N_2: multiplicity of this rep.

[Maldacena–SheikhJabbari–Raamsdonk '02]

1. Introduction

BMN matrix model

- Matrix regularisation of super-membrane theory on the plane-wave background

→ Nonperturbative formulation of M-theory (11D SUGRA)

[deWit-Hoppe-Nicolai '88, Banks-Fischler-Shenker-Susskind '96]

✦M2-brane realisation: fuzzy 2-sphere

✦M5-brane realisation: $SO(6)$ part (quantum effect)

A BPS sector realises their geometries at strong coupling.

[Y.A.-Ishiki-Shimasaki-Terashima '17]

$$\text{eigenvalues of } \phi \;\begin{cases} \xrightarrow{\text{large-}N_5} S^2 \\ \xrightarrow{\text{large-}N_2} S^5 \end{cases} \quad \text{uplift}$$

ϕ: a BPS operator considered to be the low energy moduli

[Goro's talk]

1. Introduction

BMN matrix model

[Lin-Lunin-Maldacena ’04, Lin-Maldacena ’05]

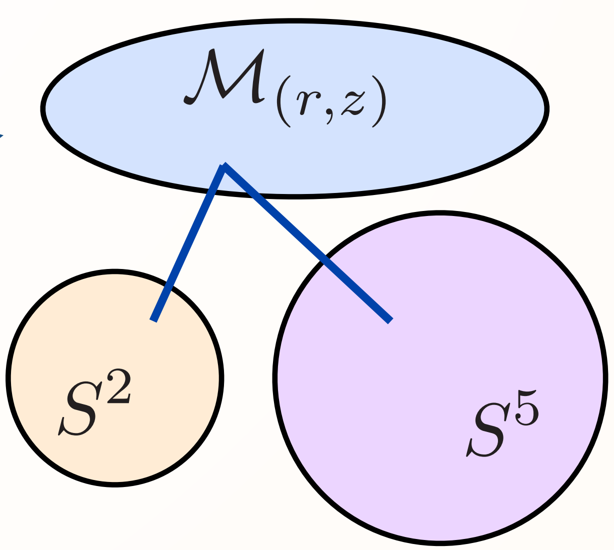
- Gauge/gravity dual to IIA SUGRA on bubbling geometries

Vacua BMN matrix model	⟷	Geometries IIA SUGRA
symmetry: $R{\times}SO(3){\times}SO(6)$	$\xleftrightarrow{SU(2\vert 4)}$	isometry: $R{\times}SO(3){\times}SO(6)$
vacua ($SU(2)$ rep.)		bubbling geometries
– dim. of irreducible rep.	⟷	– NS5 charge N_5
– multiplicity of irred. rep.	⟷	– D2 charge N_2

Part of Einstein equation was obtained by ϕ in the BMN model.

[Y.A.-Okada-Ishiki-Shimasaki ’14]

nontrivial part in terms of the isometry



1. Introduction

We have some understanding of the emergent geometries.

Can we see the emergence as we decrease the temperature?

Let’s look at phase transitions.

2. Deconfinement phase transition

There is a "deconfinement" phase transition at large-N.

[Furuuchi-Schreiber-Semenoff '03, Hadizadeh-Ramadanovic-Semenoff-Young '04]

$$\lim_{N\to\infty} \frac{F}{N^2} \begin{cases} = 0 & ;\ \text{confined} \\ \neq 0 & ;\ \text{deconfined} \end{cases}$$

F: free energy

At large μ, the theory becomes gauged harmonic oscillators.

One-loop integration

$$\beta F = \sum_{i,j} \left(3 \ln \left| 1 - e^{-\frac{\beta\mu}{3} + i\theta_{ij}} \right| + 6 \ln \left| 1 - e^{-\frac{\beta\mu}{6} + i\theta_{ij}} \right| - 8 \ln \left| 1 + e^{-\frac{\beta\mu}{4} + i\theta_{ij}} \right| \right) - \sum_{i,j\neq i} \ln \left| 1 - e^{i\theta_{ij}} \right|$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{n} \underline{\left\{ 1 - 3e^{-n\frac{\beta\mu}{3}} - 6e^{-n\frac{\beta\mu}{6}} + 8(-)^n e^{-n\frac{\beta\mu}{4}} \right\}} |u_n|^2 \right) - \sum_{n=1}^{\infty} \frac{N}{n}$$

$$\boxed{A = \mathrm{diag}\left(\frac{\theta_1}{\beta}, \cdots, \frac{\theta_N}{\beta}\right) \qquad \theta_{ij} := \theta_i - \theta_j \qquad u_n := \sum_{j=1}^{N} e^{in\theta_j}}$$

2. Deconfinement phase transition

$$\beta F = \sum_{n=1}^{\infty} \left(\frac{1}{n} \underline{\left\{ 1 - 3e^{-n\frac{\beta\mu}{3}} - 6e^{-n\frac{\beta\mu}{6}} + 8(-)^n e^{-n\frac{\beta\mu}{4}} \right\}} |u_n|^2 \right)$$

positive at low enough temperatures $\rightarrow$ $|u_n| = 0$, $F = 0$

Gross–Witten transition

$1 - 3e^{-\frac{\beta\mu}{3}} - 6e^{-\frac{\beta\mu}{6}} - 8e^{-\frac{\beta\mu}{4}} < 0$ $\rightarrow$ $|u_1| > 0$, $F \sim O(N^2)$

[Furuuchi–Schreiber–Semenoff '03]

Critical temperature of the deconfinement transition:

$$T_c = \beta_c^{-1} = \frac{\mu}{12 \ln 3} \left(1 + \underbrace{\frac{2^6 \cdot 5}{3\mu^3} + O(\mu^{-6})}_{\text{coming from higher loops}} \right)$$

[Spradlin–Raamsdonk–Volovich '04,
Hadizadeh–Ramadanovic–Semenoff–Young '04]

$P = u_1/N$ is the order parameter.
(Polyakov loop)

$$A = \mathrm{diag}\left(\frac{\theta_1}{\beta}, \cdots, \frac{\theta_N}{\beta}\right) \qquad \theta_{ij} := \theta_i - \theta_j \qquad u_n := \sum_{j=1}^{N} e^{in\theta_j}$$

2. Deconfinement phase transition

[Costa-Greenspan-Penedones-Santos '14]

At small μ and high temperatures,

the dual geometry is approximated by a non-extremal black-0 brane:

$$ds_{11}^2 = \frac{dr^2}{1-\frac{r_0^7}{r^7}} + r^2 d\Omega_8^2 + \frac{R^7}{r^7}dz^2 + \left(1-\frac{r_0^7}{r^7}\right)\left(2dz - \frac{r_0^7}{R^7}dt\right)dt$$

It must asymptote to the plane-wave geometry with $R\times SO(3)\times SO(6)$.

By regularity being imposed, the geometry dual to the thermal BMN

with $U(1)_M\times R\times SO(3)\times SO(6)$ isometry,

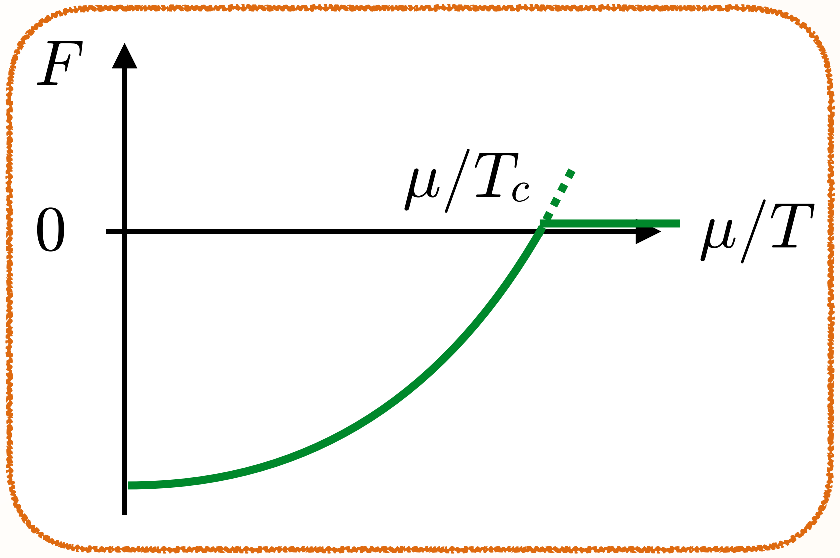
with perturbative μ-deformation, and

with the simplest horizon topology ($S^1\times S^8$)

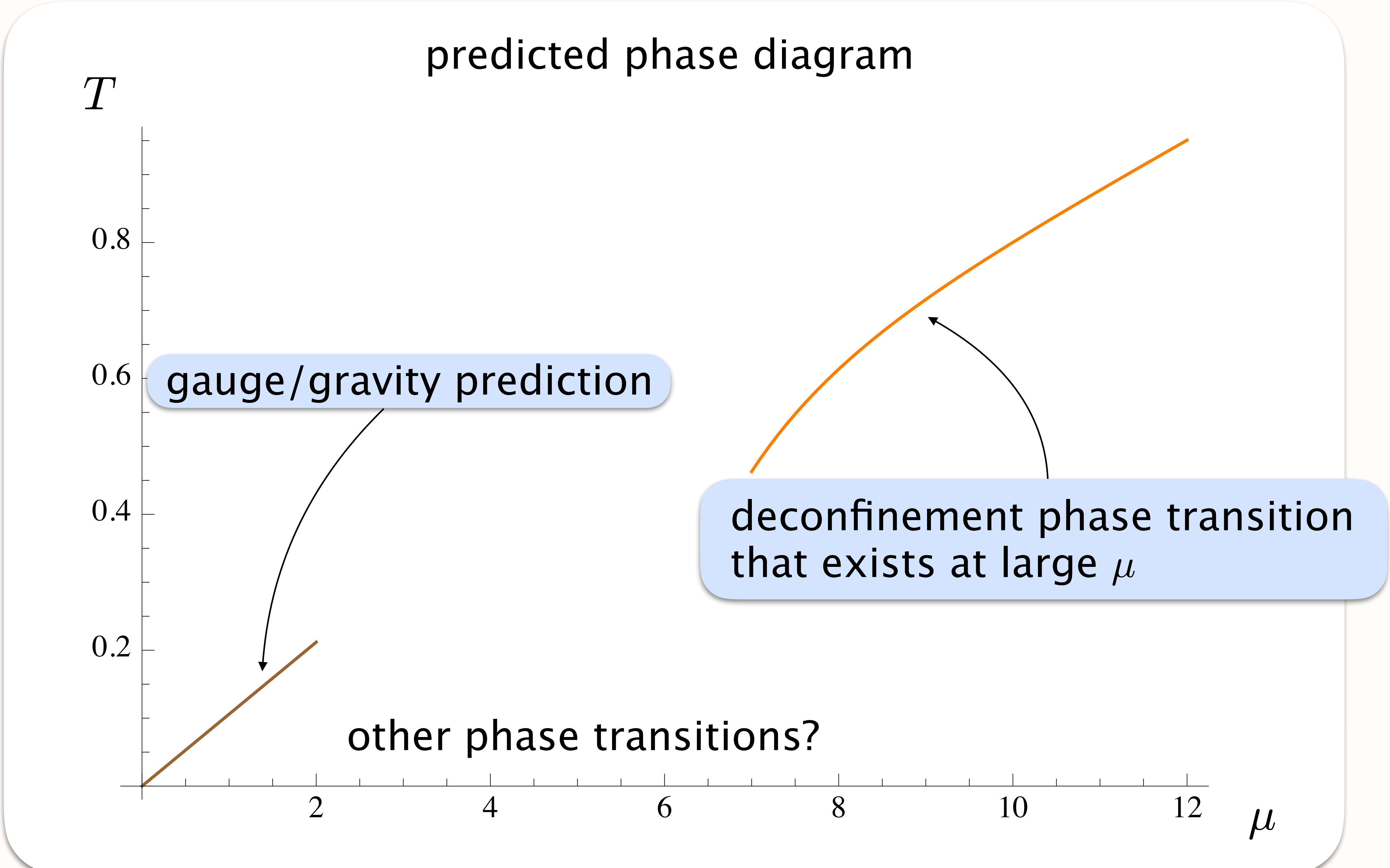
was computed. $\leftrightarrow$ trivial vac. $X_a=0$

Critical temperature from the gravity side:

$$\frac{T_c}{\mu} = 0.105905(57)$$



2. Deconfinement phase transition



It should have a rich structure at low temperatures, which should reflect geometrical information.

3. Lattice simulation

Computer simulations for Matrix theories

BFSS model ($\mu=0$)

- Consistency with $E \sim T^{14/5}$, no confinement phase transition
  [Anagnostopoulos–Hanada–Nishimura–Takeuchi '07, Catterall–Wiseman '07, '08]
- α' (low–T) correction (non–lattice) [Hanada–Hyakutake–Nishimura–Takeuchi '08]
- Quantum (1/N) correction (non–lattice) [Hanada–Hyakutake–Ishiki–Nishimura '13]
- Further consistency checks of gravity prediction (lattice)
  [Kadoh–Kamata '15, Filev–O'Connor '15]
- Reproduced the coeff. in the first term: $E = 7.41\ T^{14/5}$ (lattice)
  [Berkowiz–Rinaldi–Hanada–Ishiki–Shimasaki–Vranas '16]

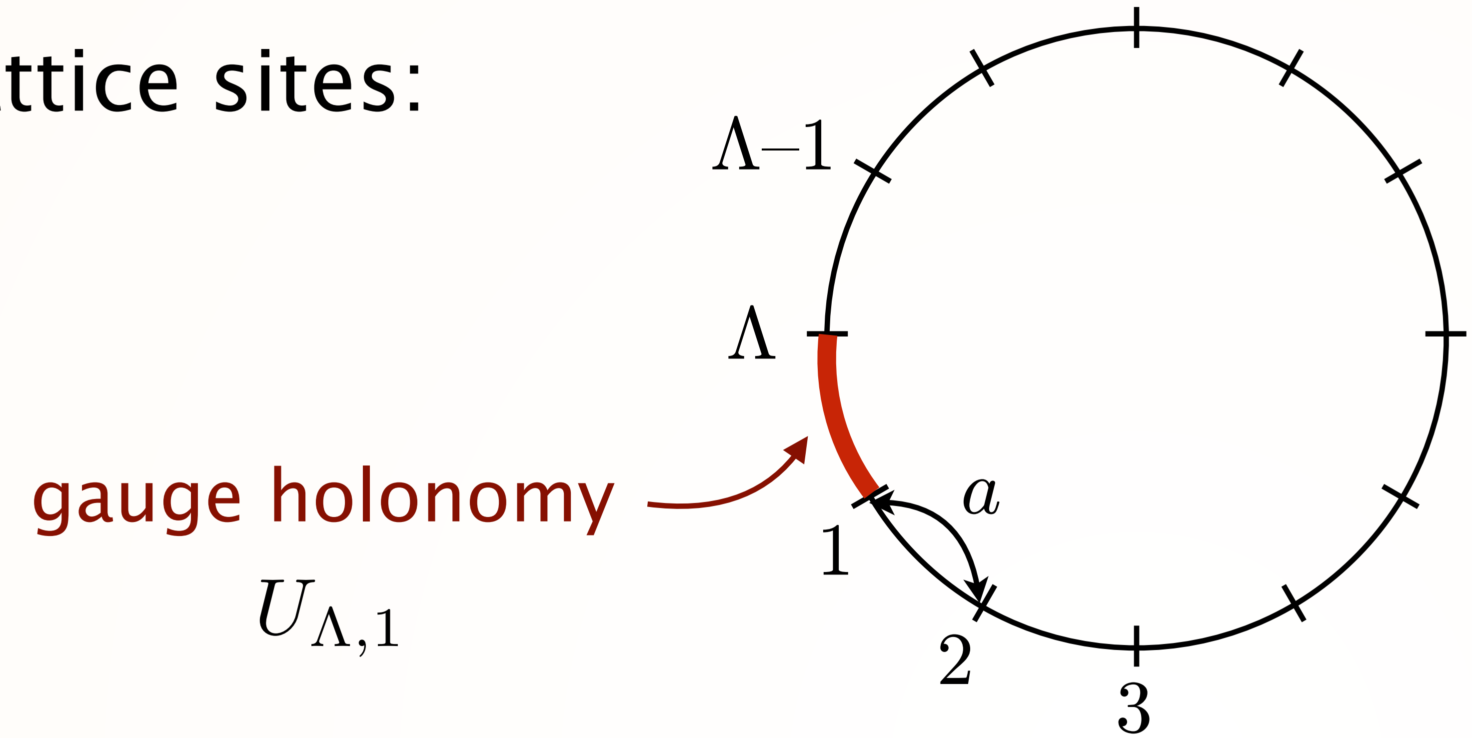
BMN model

- Observed the deconfinement phase transition [Catterall–Anders '10]

3. Lattice simulation

[Denjoe's talk]

Lattice sites:



The gauge field at the other links is 0.

$$U_{n,n+1} = \mathbf{1}$$

Discretisation of derivatives:

- Third-order accuracy.
- To avoid fermionic doublers, we use a Wilson mass term.

$$D_\tau + ra^{2n-1}\Sigma\triangle^n$$

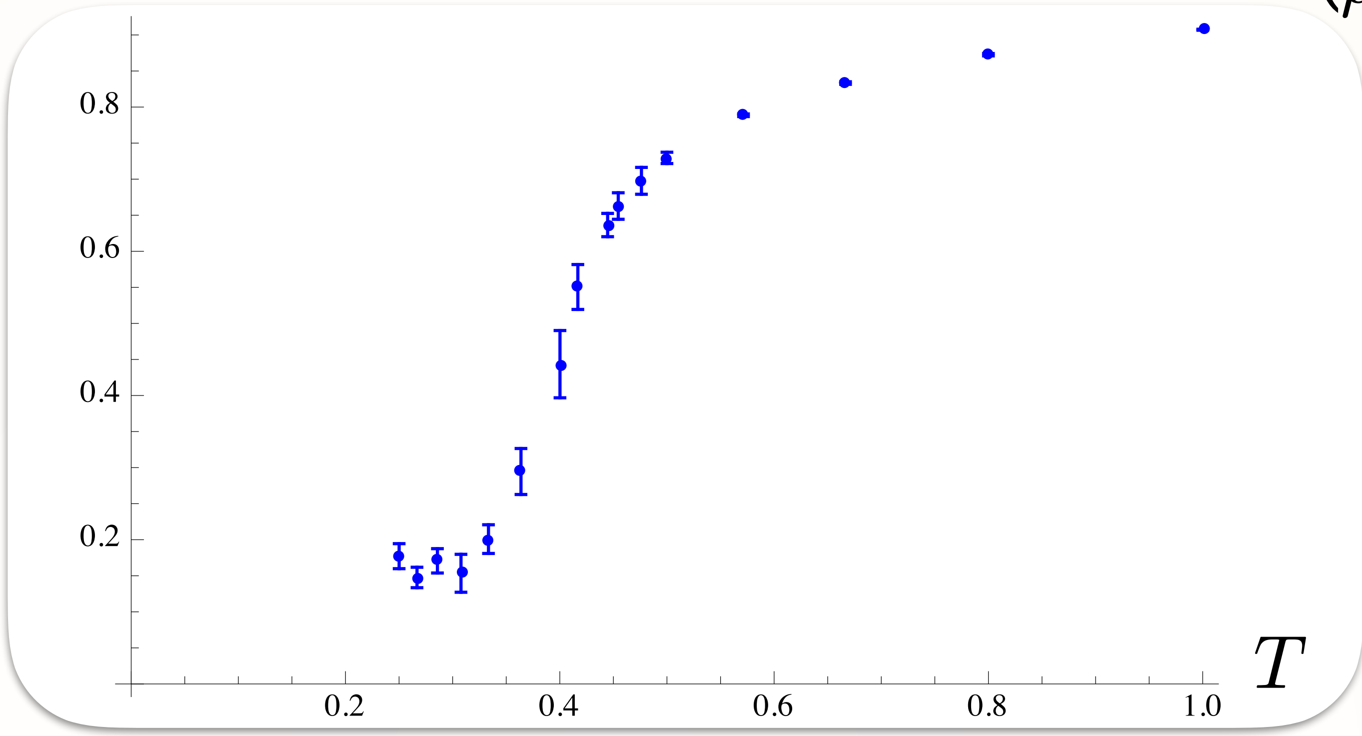
※ We choose $\Sigma = \mathrm{i}\gamma^{456}$ (not in the $SO(3)$-directions) so that the dispersion relation doesn't have a term linear in μ.

3. Lattice simulation

$(\mu=5,\ \Lambda=24,\ N=11)$

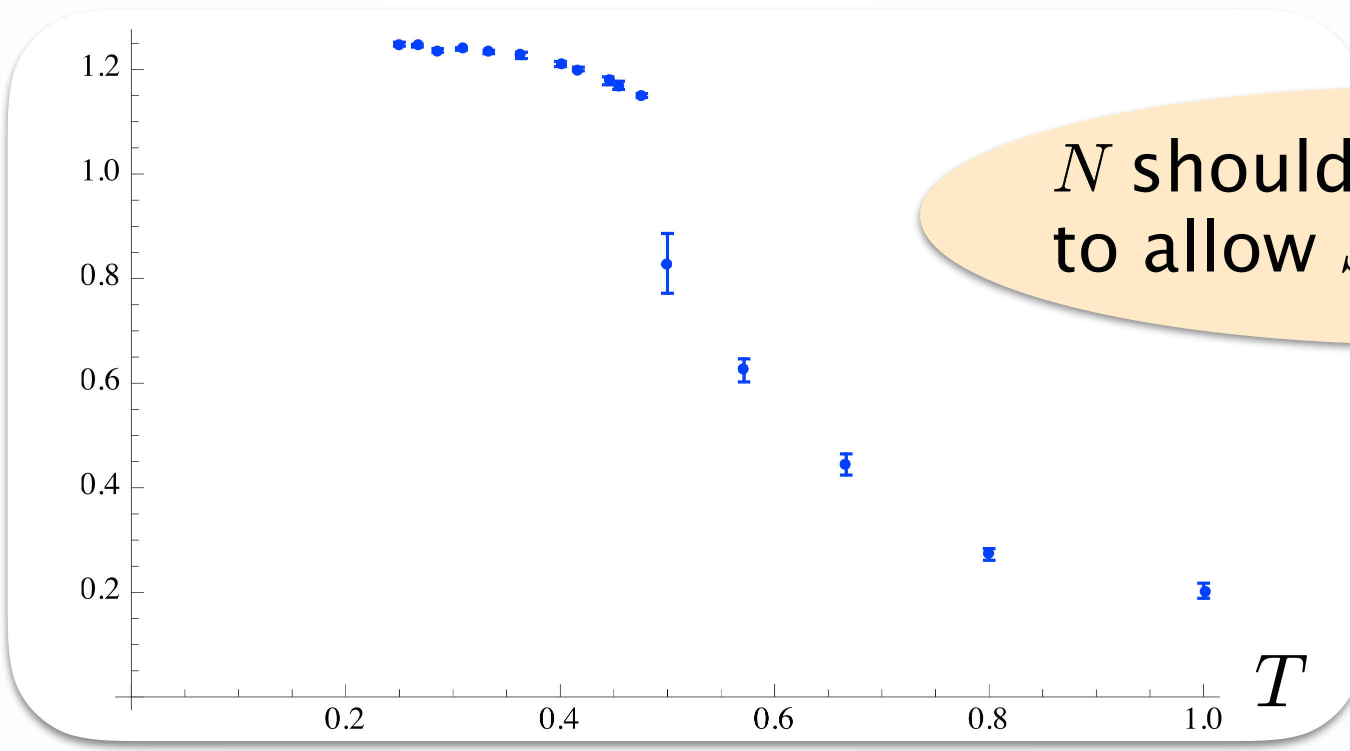
Polyakov loop:

$\langle |P| \rangle$



Myers term:

$\sim \langle \mathrm{Tr}\,(iX_1[X_2, X_3]) \rangle$

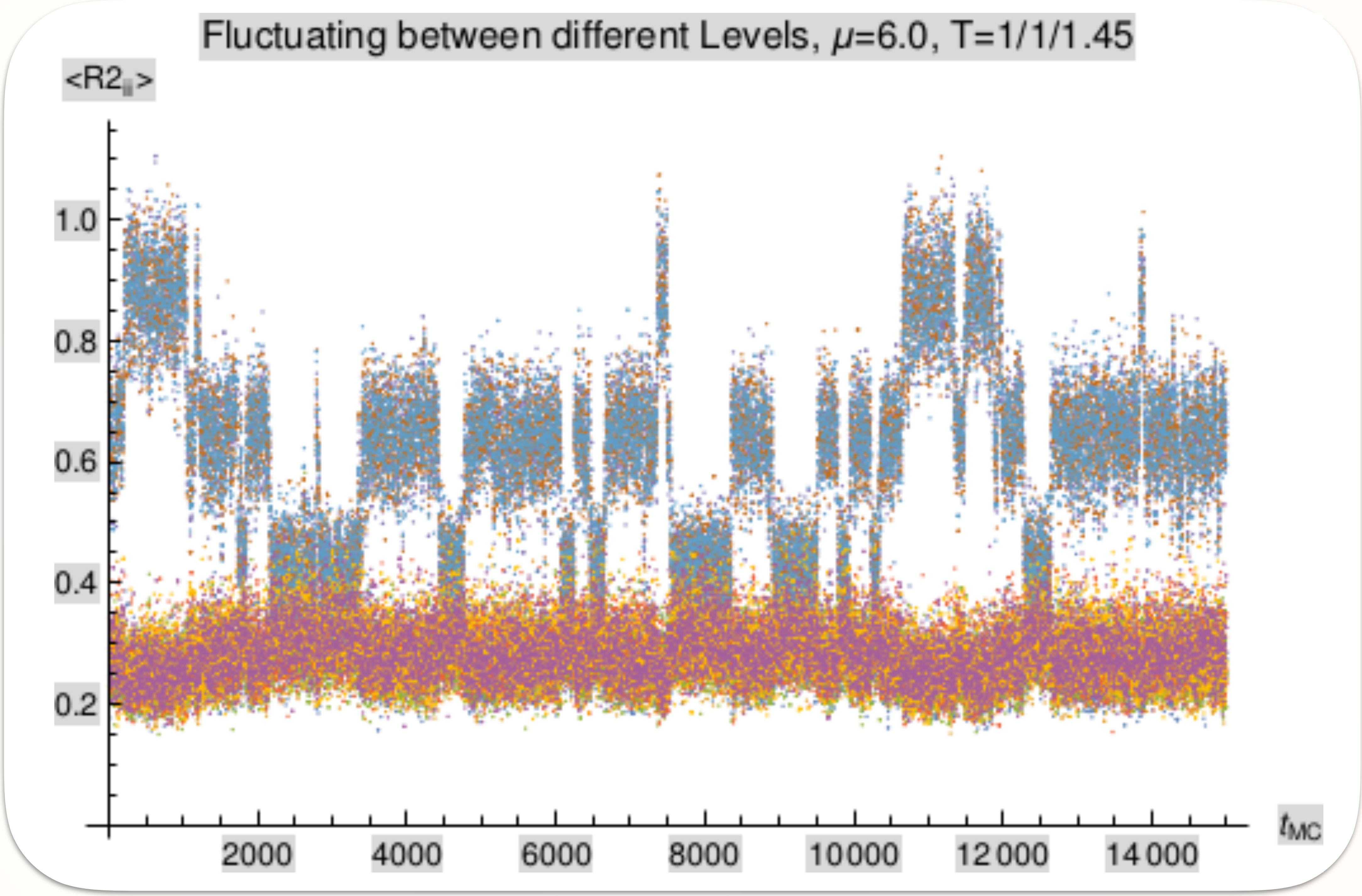


N shouldn't be so large to allow $SO(3)$ transition.

※ $SO(3)$ Casimir is also good to detect the transition.

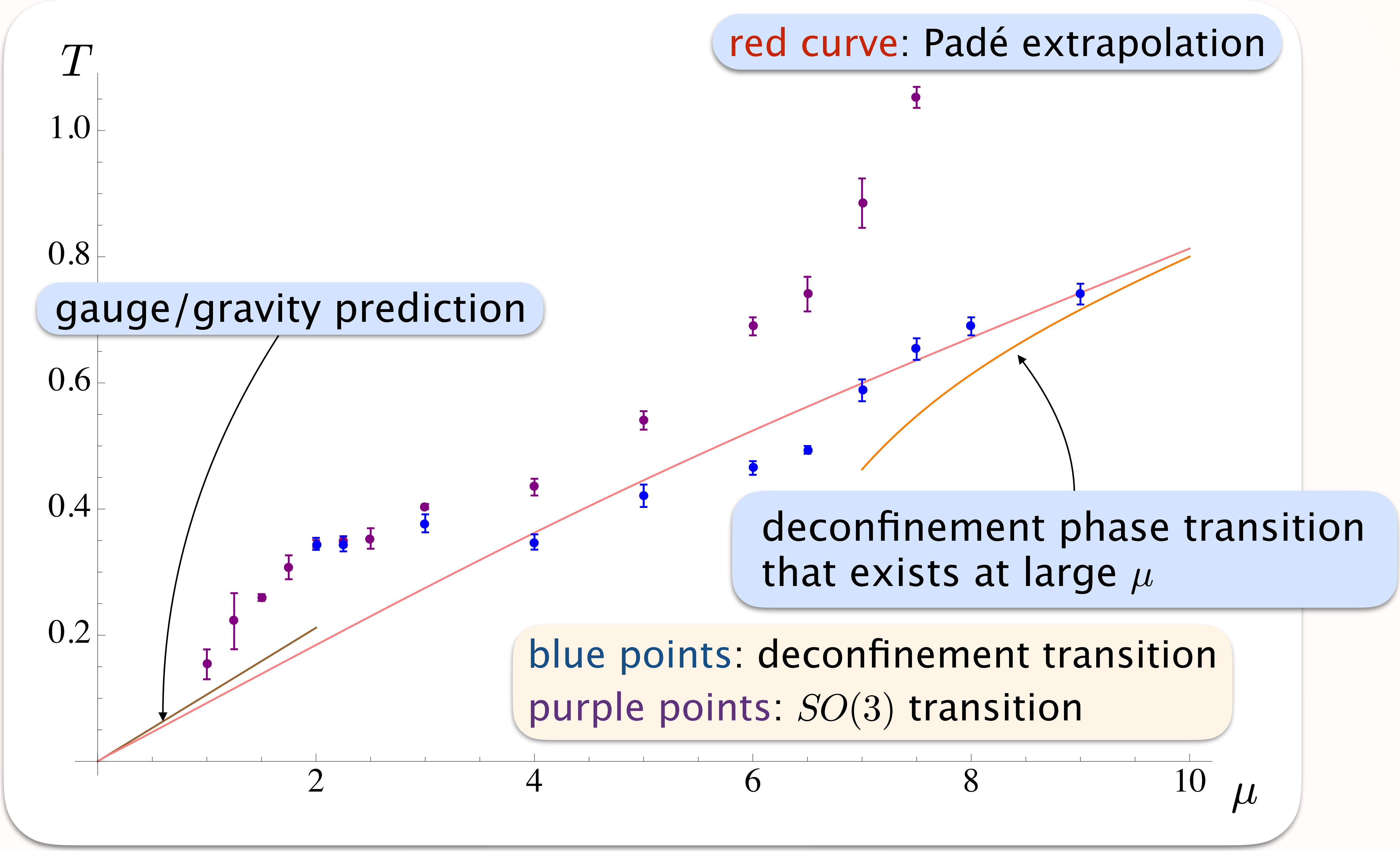
4. Lattice simulation

~ $\mathrm{Tr}[X_i X_i]/N$ $(\mu{=}6,\ \beta{=}1.45,\ \Lambda{=}24,\ N{=}8)$



3. Lattice simulation

($\Lambda=24,\ N=8$)



The simulation results agree with theoretical predictions.

4. Summary and Discussion

- We observed two phase transitions: the deconfinement transition and the $SO(3)$ transition.
  They don't merge at least finite Λ and N at $2 \lesssim \mu \leq 7.5$.

Geometrical interpretation:

- Is the $SO(3)$ transition "M5 → M2" or "no geometry → S^2"?

Gauge/gravity:

- The critical temperature of the deconfinement transition looks dependent on $SU(2)$ rep. By keeping the state at the trivial vacuum $X_a = 0$, we may get the deconfinement transition much closer to the gravity prediction.

- The gravity dual at zero temperature has many bubbling solutions, which correspond to vacua in the BMN model. We expect a richer structure at lower temperatures, which should reflect geometrical information.