New Universality in Chaos

Masanori Hanada

Kyoto/Boulder

Feb 02, 2018 @ ICTS

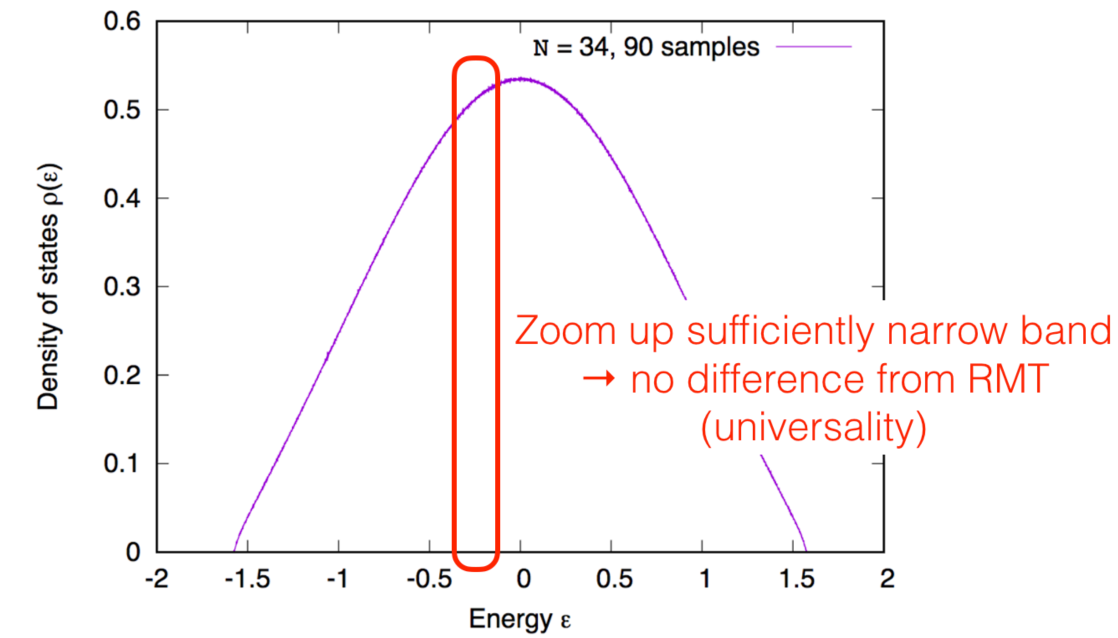
- Universality at late time
- Universality at early time **New**

Universality at late time

- Fine-grained energy spectrum should agree with Random Matrix Theory (RMT).

$$Z(\beta, t) \equiv \mathrm{Tr}\left(e^{-\beta M - iMt}\right)$$

SYK energy spectrum



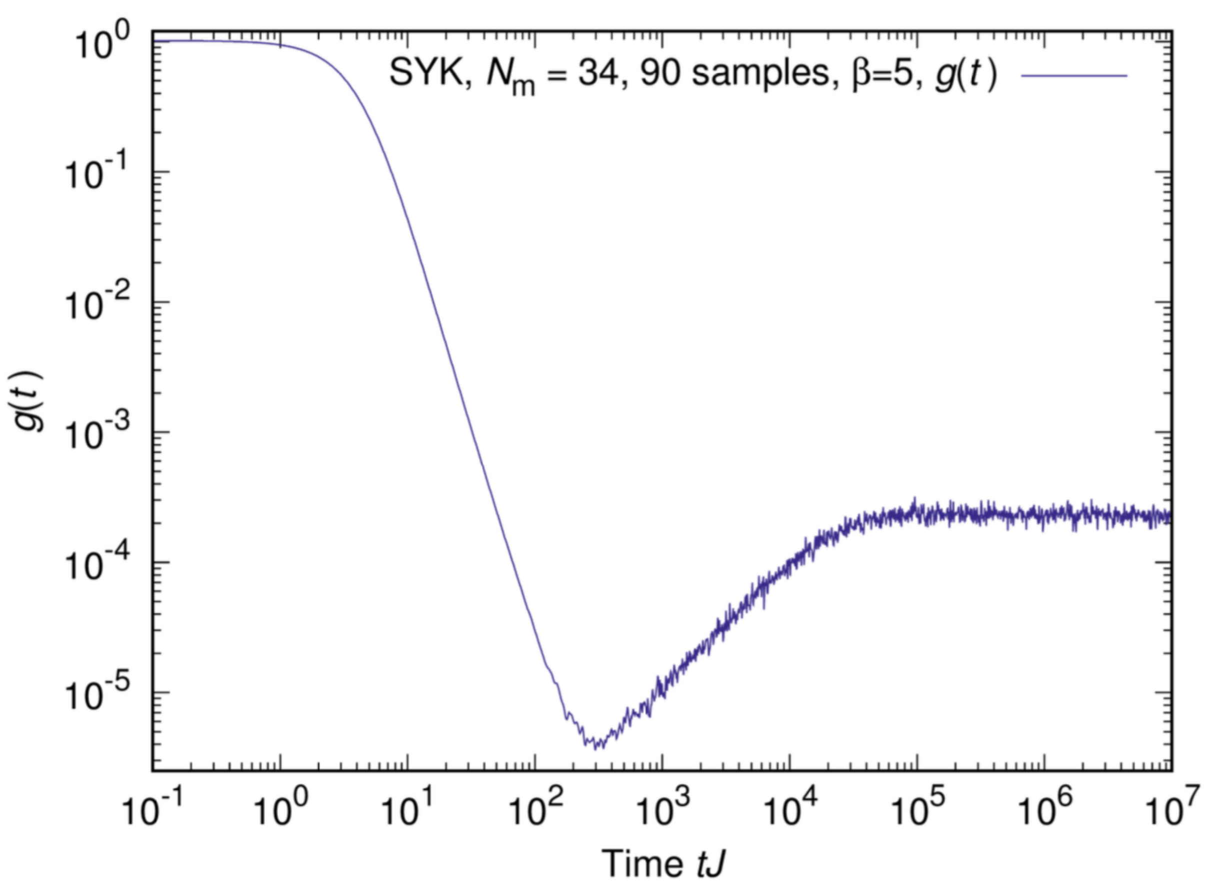
$$M \in \begin{cases} U(L) & (\mathrm{GUE}) \\ O(L) & (\mathrm{GOE}) \\ USp(L) & (\mathrm{GSE}) \end{cases}$$

$L=2^N$ for N-qubit system

- Universality of fine-grained energy spectrum
  = universality at late time

Spectral Form Factor

$$\sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t} = Z(\beta+it)Z(\beta-it) = Z(t)Z^*(t)$$



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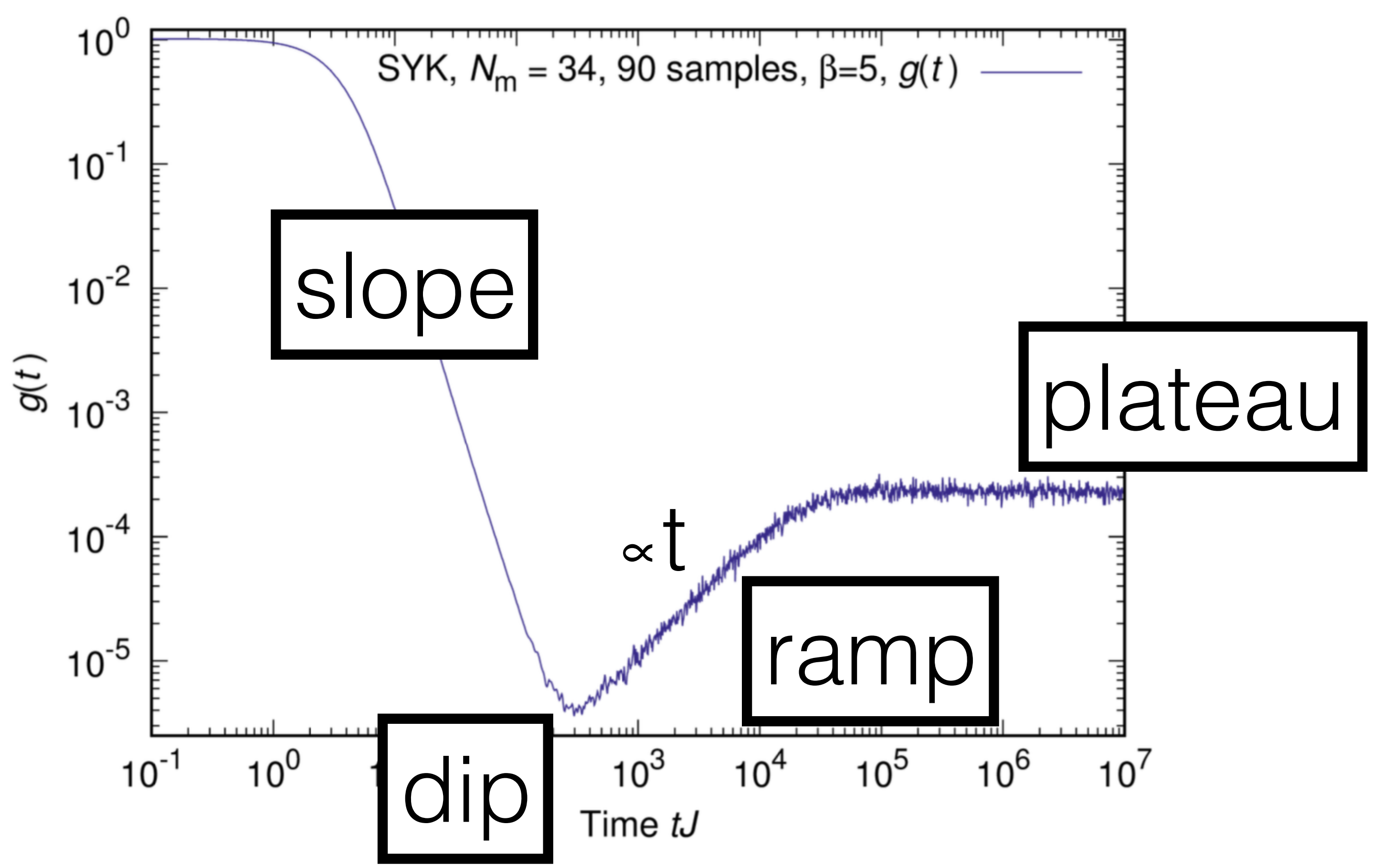
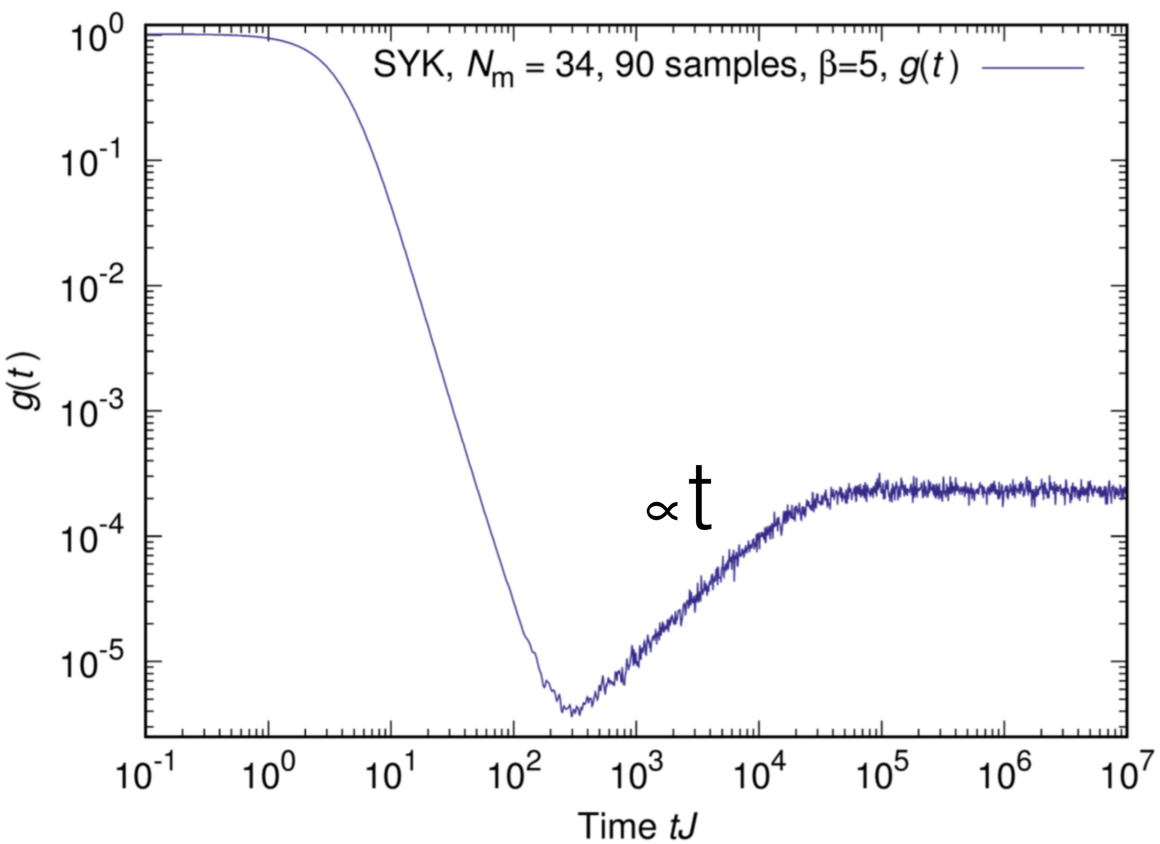
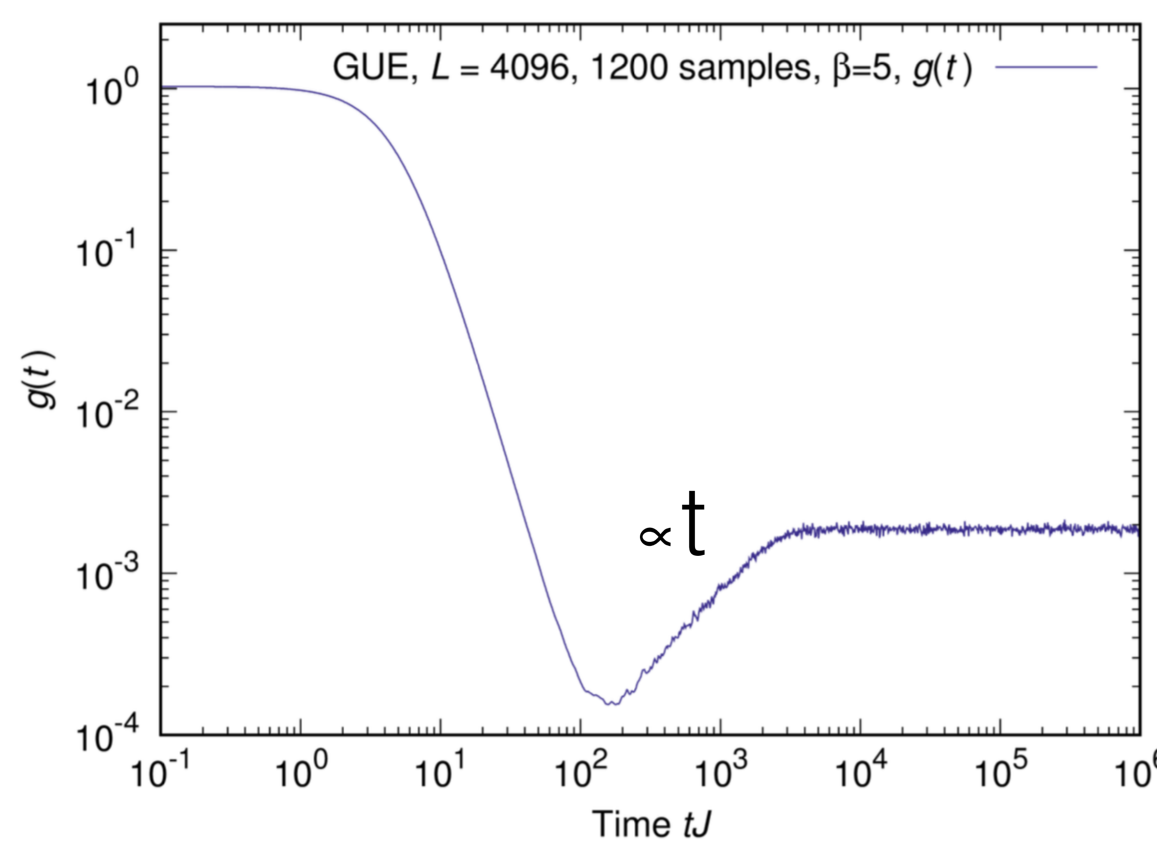


$$g(t;\beta) \equiv \frac{\langle Z(\beta,t)Z^*(\beta,t)\rangle_J}{\langle Z(\beta)\rangle_J^2}$$

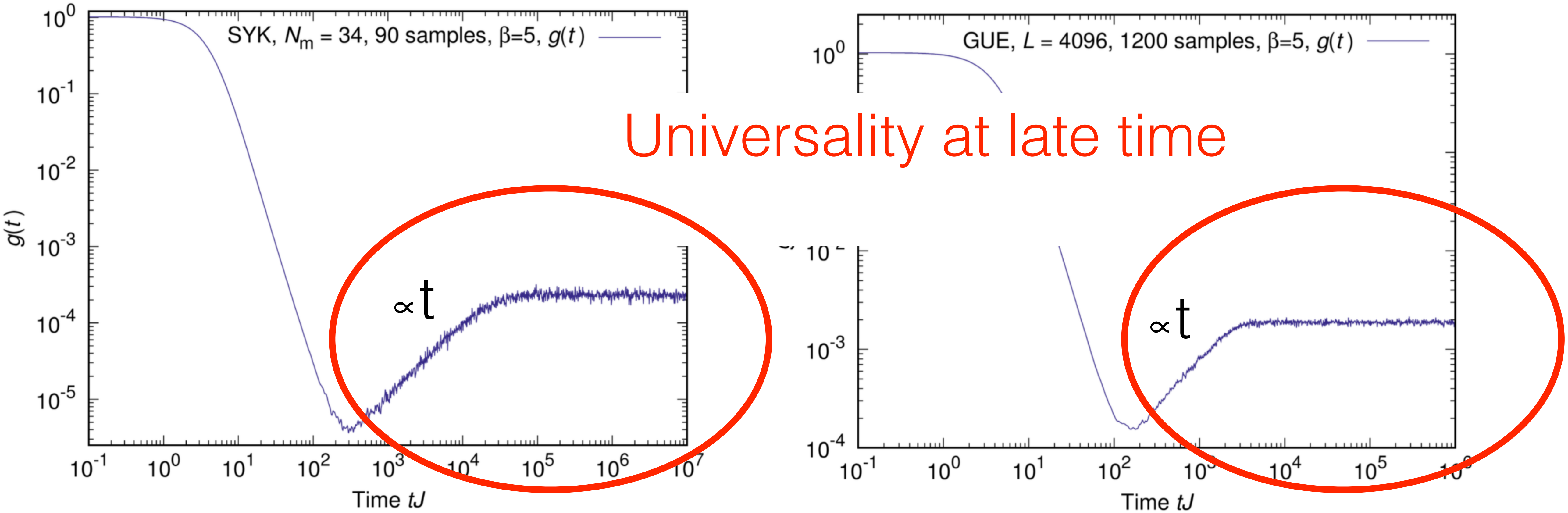




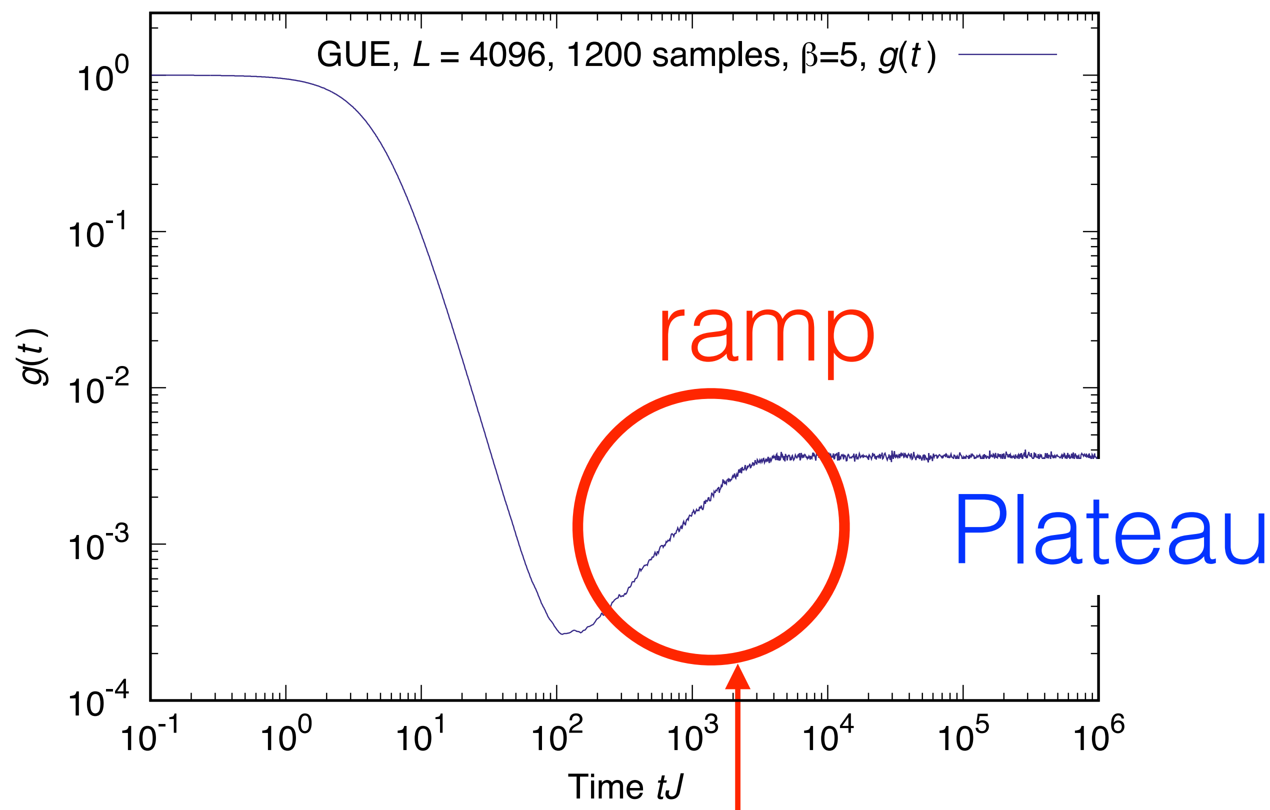
SYK

RMT

$$g(t;\beta) \equiv \frac{\langle Z(\beta,t)Z^*(\beta,t)\rangle_J}{\langle Z(\beta)\rangle_J^2}$$



SYK RMT

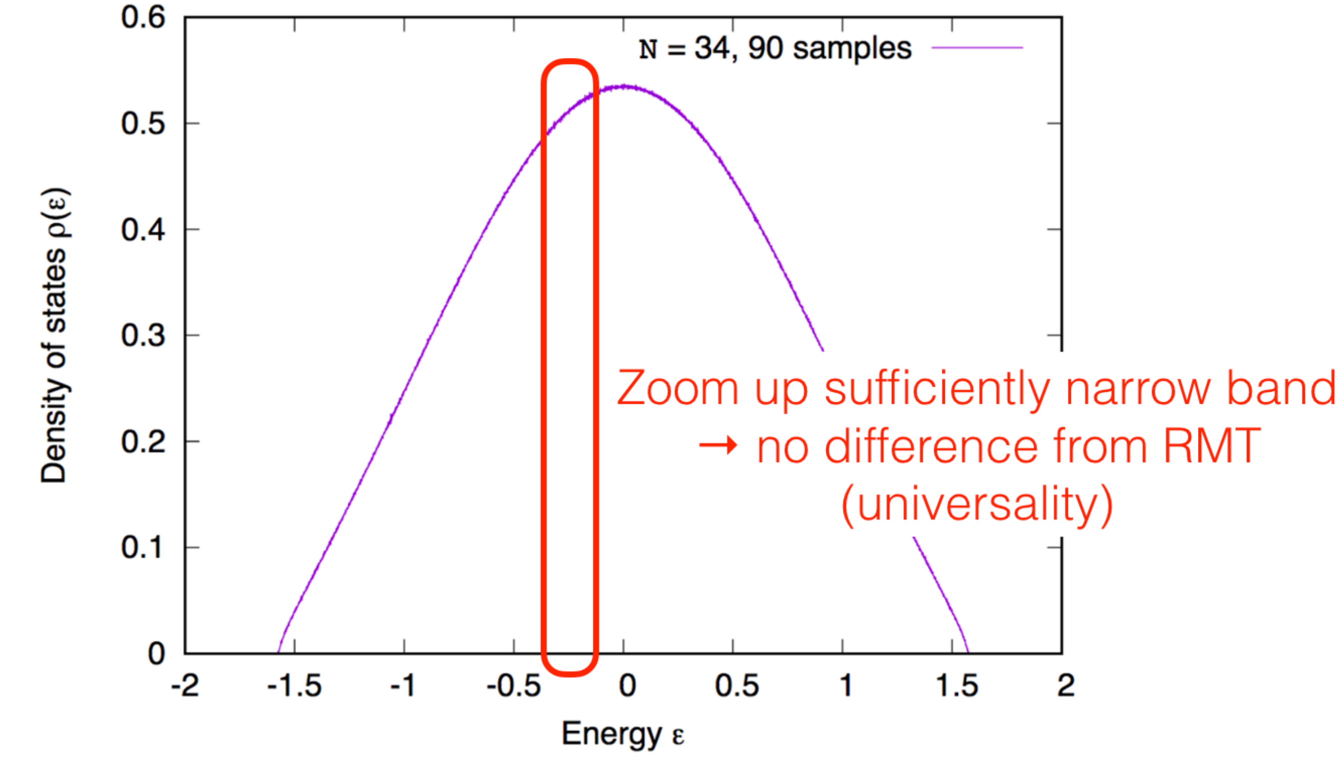


The correlation in the spectrum is important, not just the nearest-neighbor level spacing!

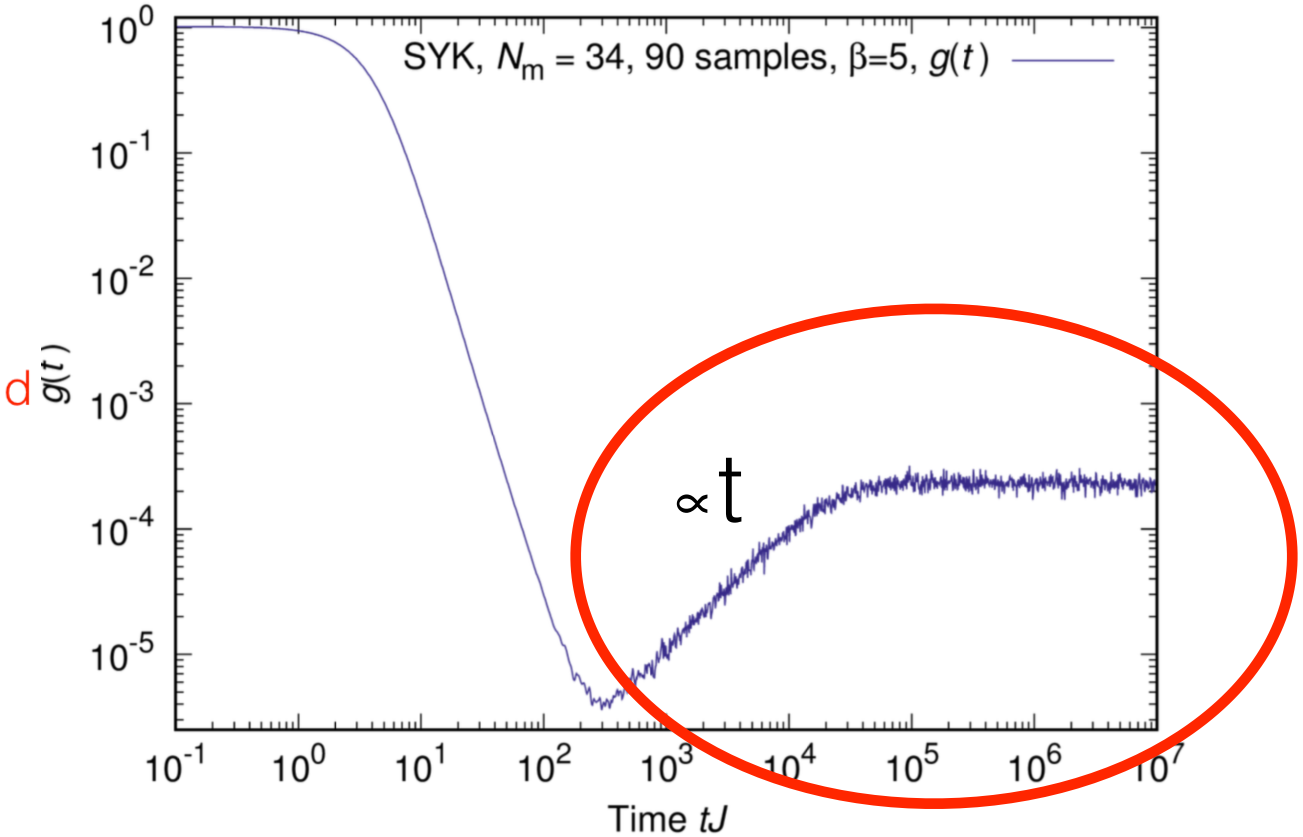
Plateau

$$\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4 < \lambda_5 < \lambda_6 < \lambda_7 < \lambda_8 < \lambda_9 < \ldots$$

SYK energy spectrum



SYK Spectral Form Factor



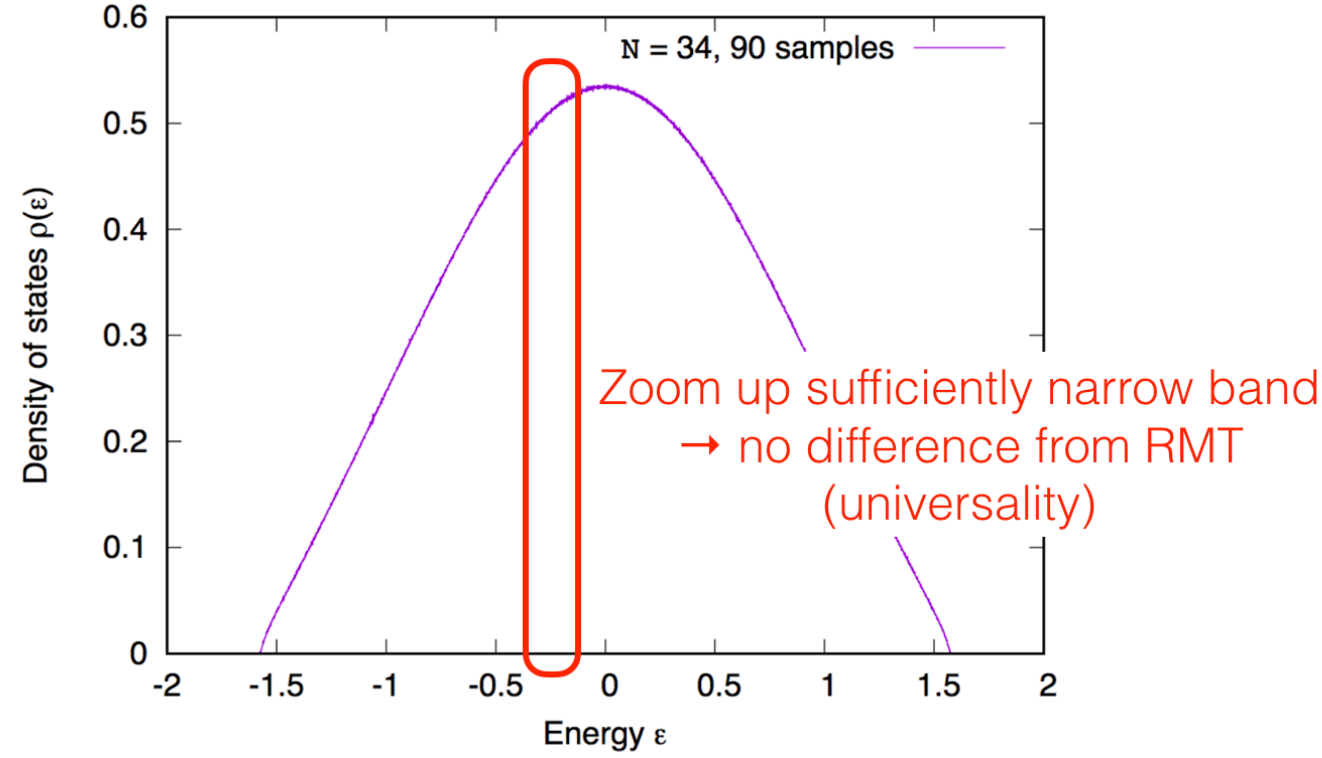
ΔE

$t > (\Delta E)^{-1}$

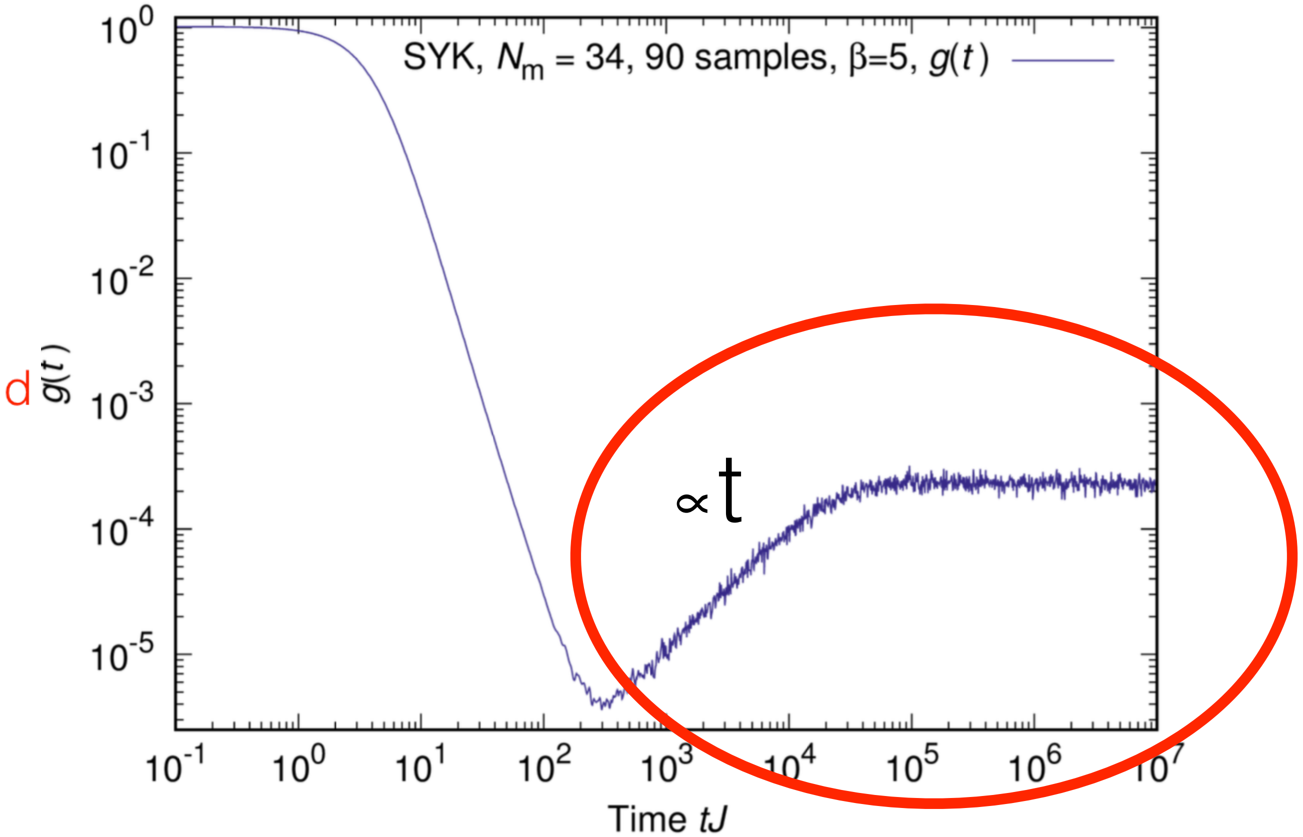
late-time universality
= loss of the individual features

scrambling?

SYK energy spectrum



SYK Spectral Form Factor



ΔE

$t > (\Delta E)^{-1}$

late-time universality
= loss of the individual features

Gharibyan, MH, Shenker, Tezuka, in preparation

~~scrambling?~~ diffusion

Late time is (a kind of) universally boring…

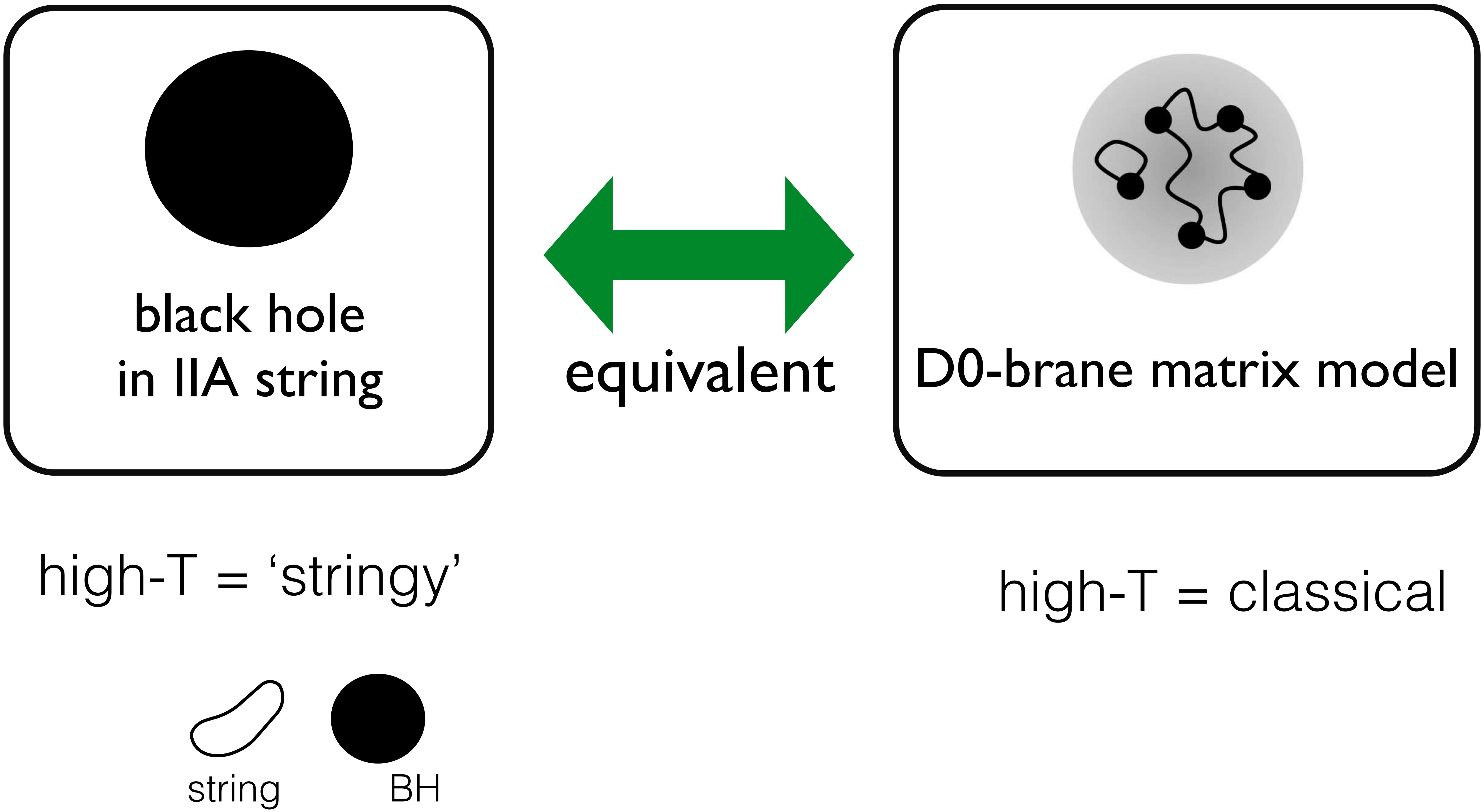
Early time behavior knows individual character of the system!

Early-time universality in classical chaos

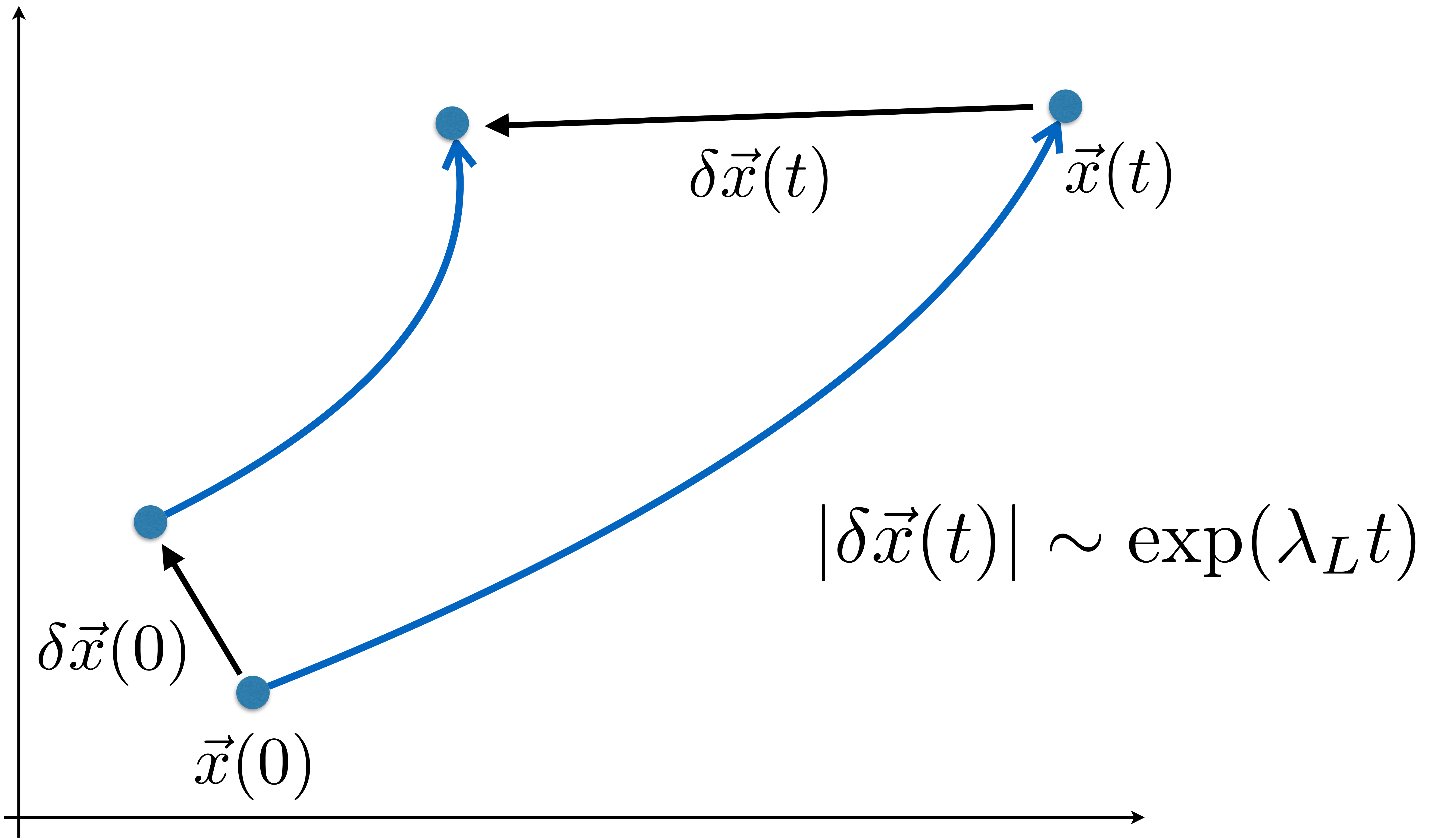
(quantum chaos will also be discussed later)

M.H.-Shimada-Tezuka, hep-th 2017

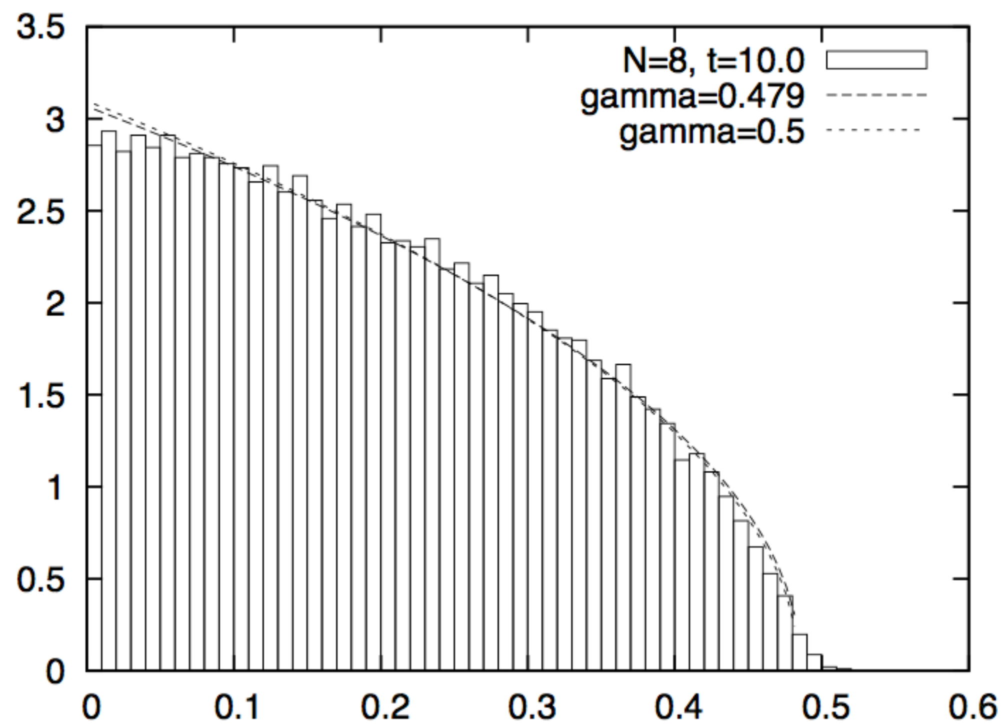
(see also Gur-Ari, M.H, Shenker, JHEP 2016)

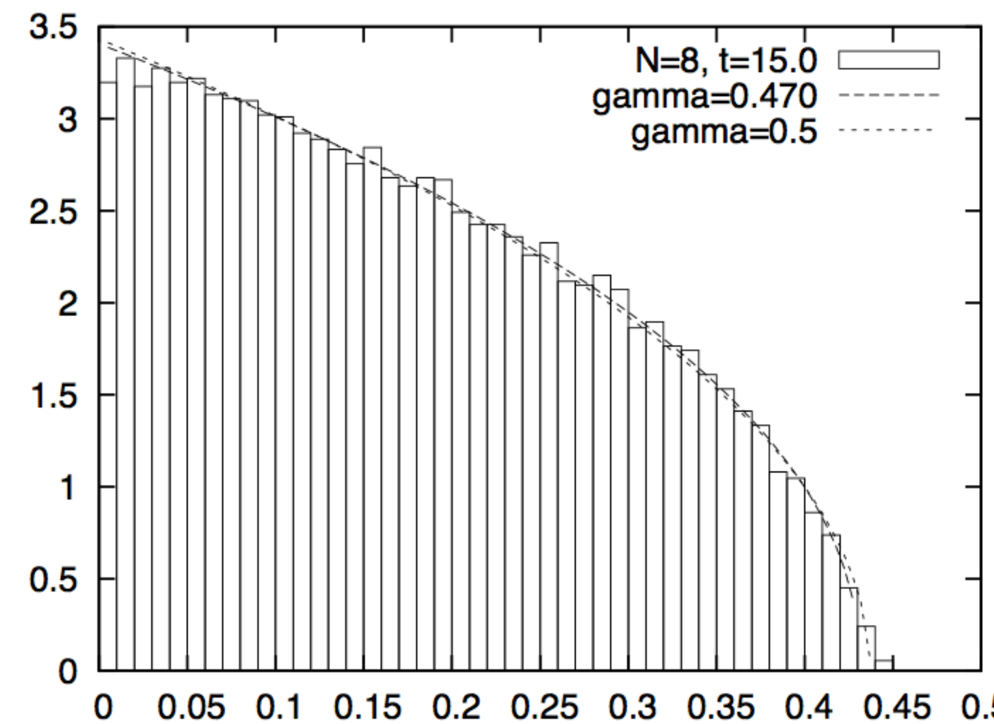

black hole
in IIA string
equivalent
D0-brane matrix model
high-T = 'stringy'
high-T = classical
string
BH

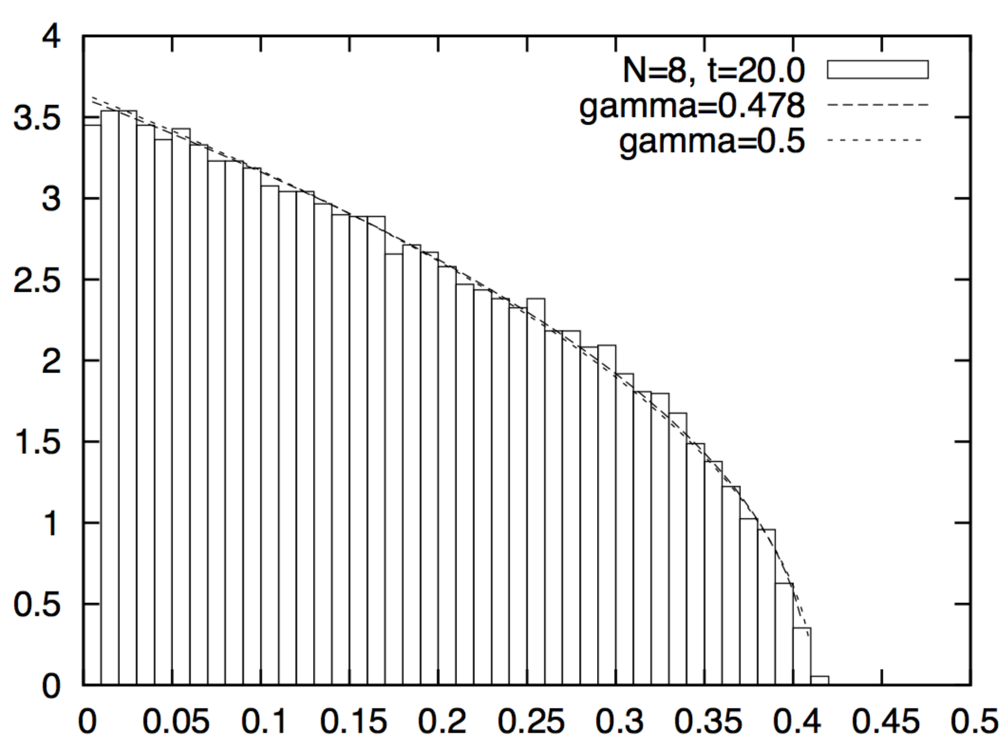
Lyapunov Exponent



Lyapunov Spectrum







$$\rho(\lambda) = \frac{(\gamma+1)(\lambda_{max}-\lambda)^{\gamma}}{\lambda_{max}^{\gamma+1}}$$

consistent with ‘semi-circle’

$\gamma = 0.5$

Gur-Ari, M.H, Shenker, JHEP 2016

RMT vs Classical Chaos

- Semicircle distribution → maybe Random Matrix Theory (RMT) is hidden behind it?
- If so, the correlation of the finite-time Lyapunov exponents should have a universal behavior.

$$\lambda_1 < \lambda_2 < \lambda_3 < \dots.$$

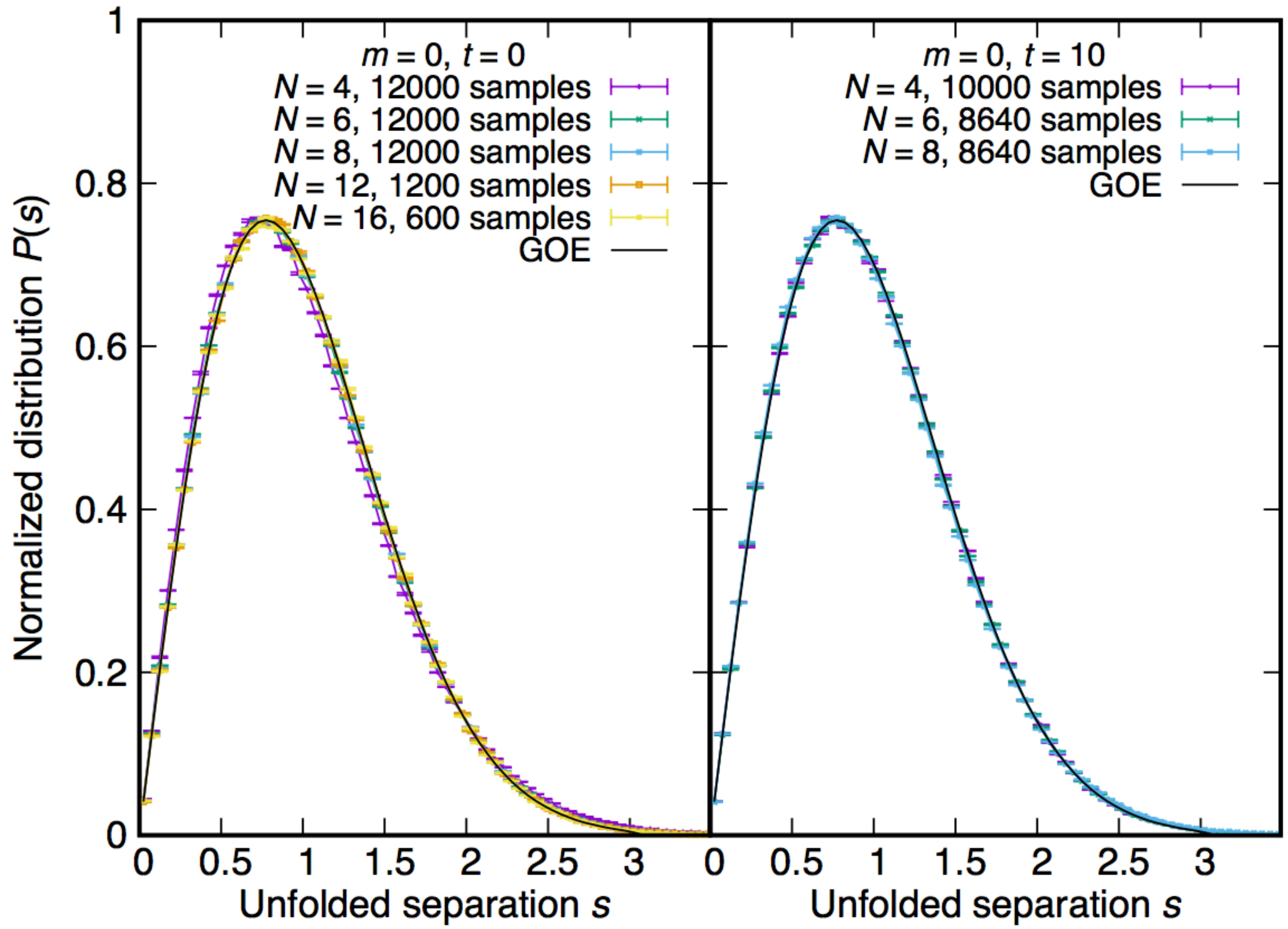
$$s_i = \lambda_{i+1} - \lambda_i$$

New

- $N \to \infty$ before $t \to \infty$

Somewhat surprisingly, this very natural limit has never been considered.

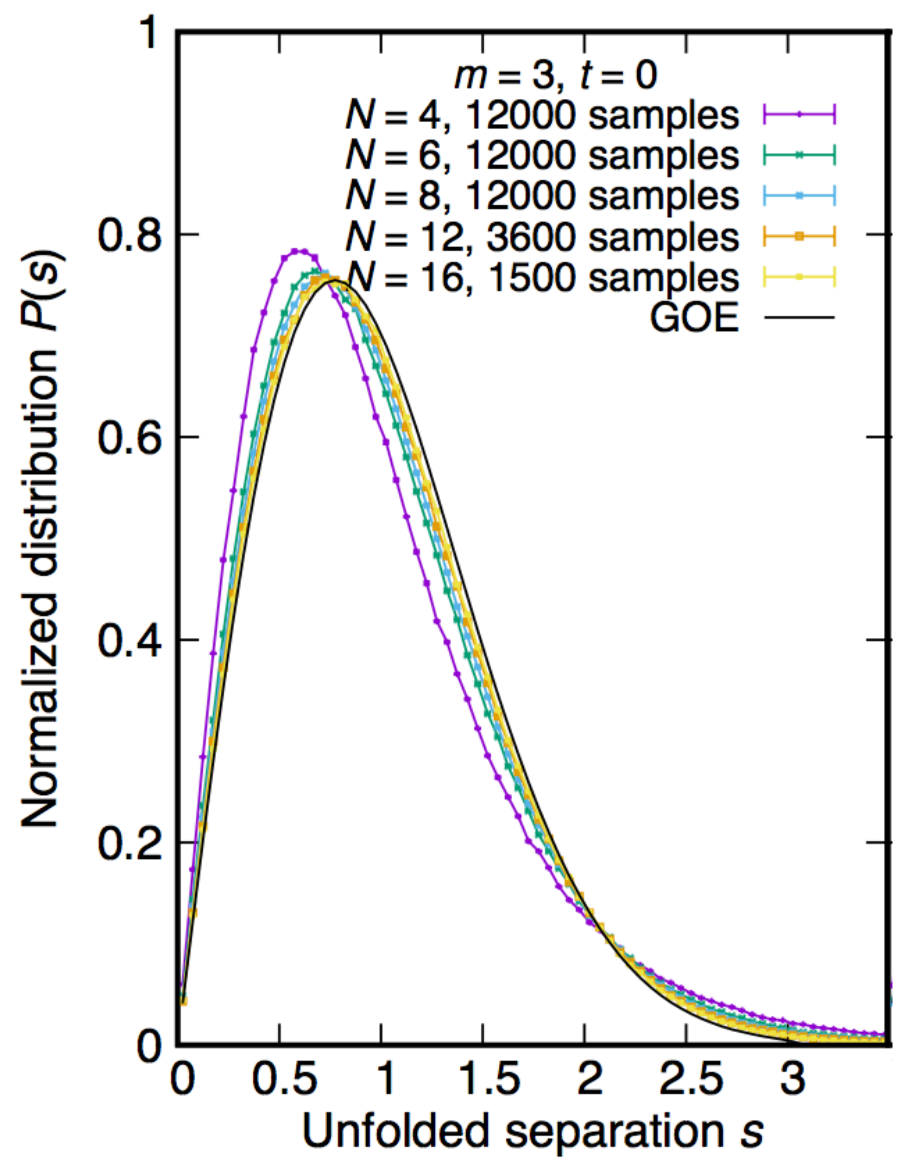
GOE-distribution at any time

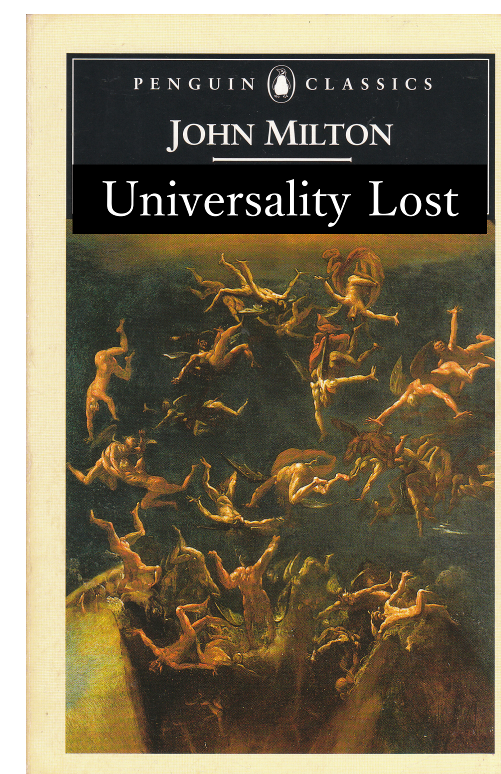


Lyapunov exponents are described by RMT

M.H.-Shimada-Tezuka, hep-th 2017

with a mass term (→no gravity interpretation), GOE is gone, at t=0.

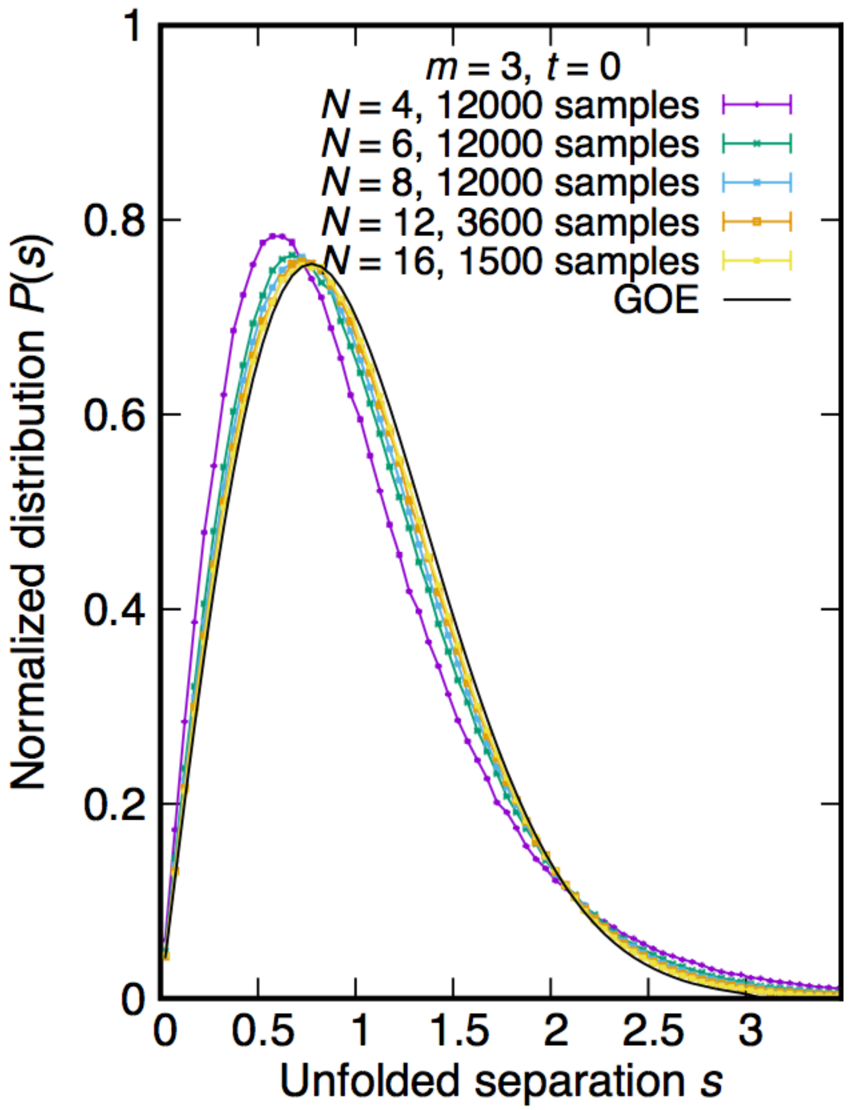


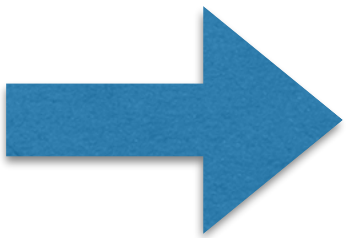


$$\Delta L = -(Nm^2/2)\Sigma X_M^2$$

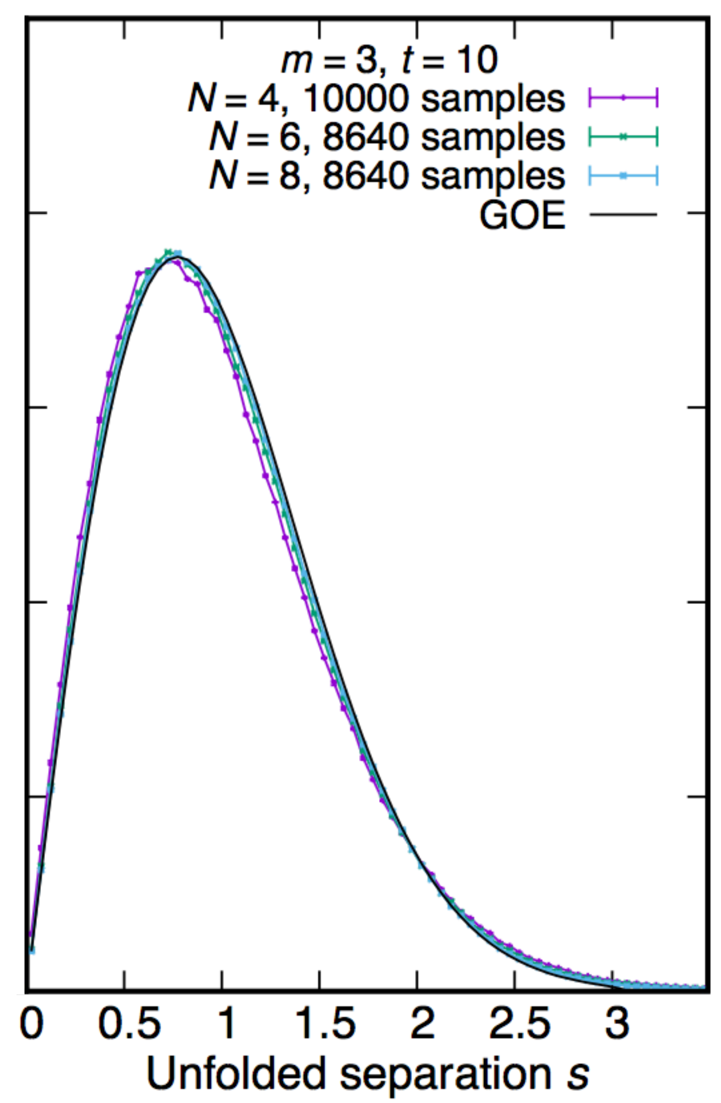
But GOE is back at late time

t=0





t=3



I shall return.

SFF from Lyapunov spectrum

$$Z(t) = \sum e^{iEt}$$

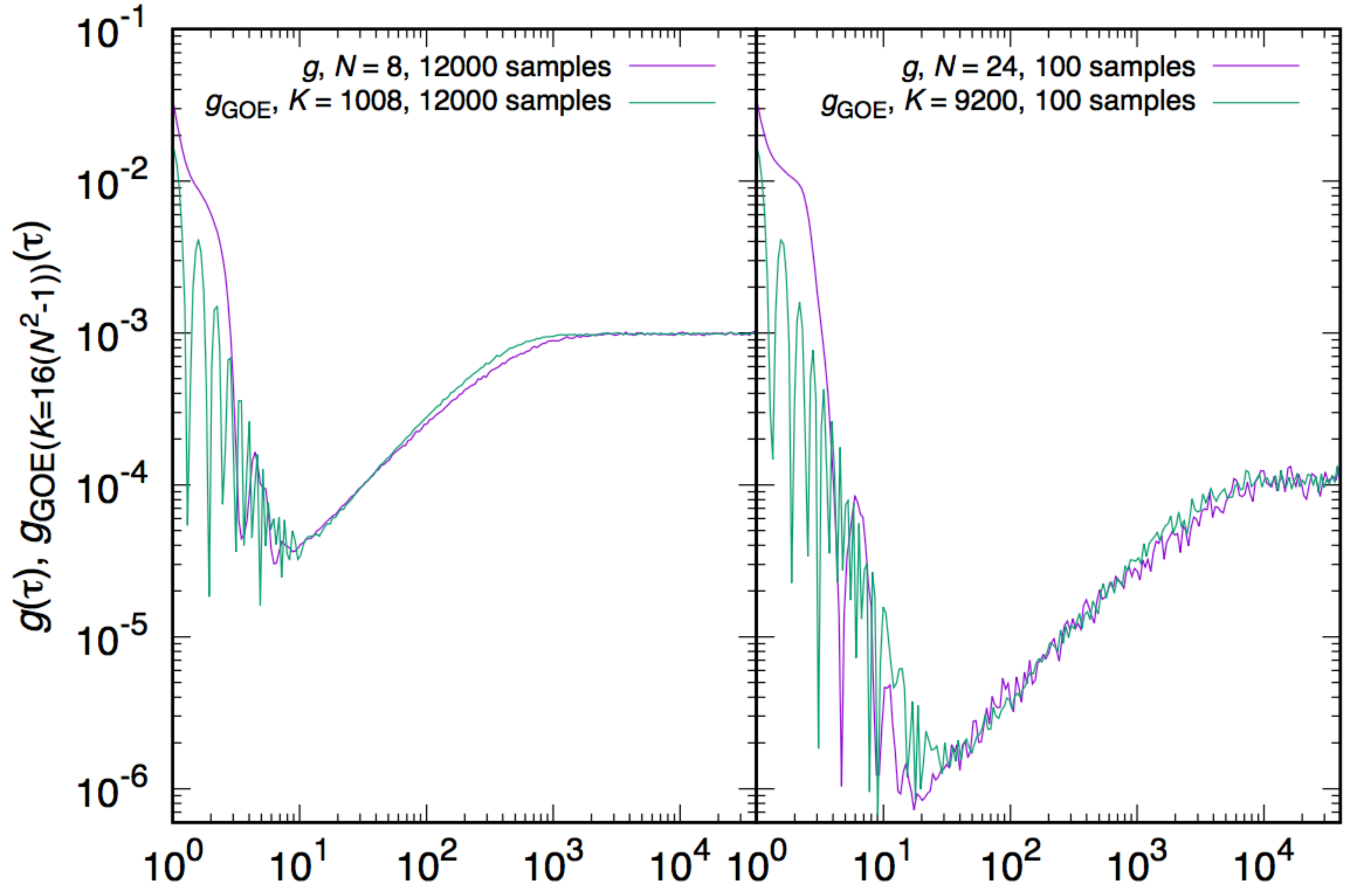
$$\mathrm{SFF} = |Z|^2$$

$$\rightsquigarrow \quad Z(\tau) = \sum e^{i\lambda\tau}$$

- $\lambda = \lambda(t)$: Lyapunov exponent at time t
- τ : ‘time' for SFF

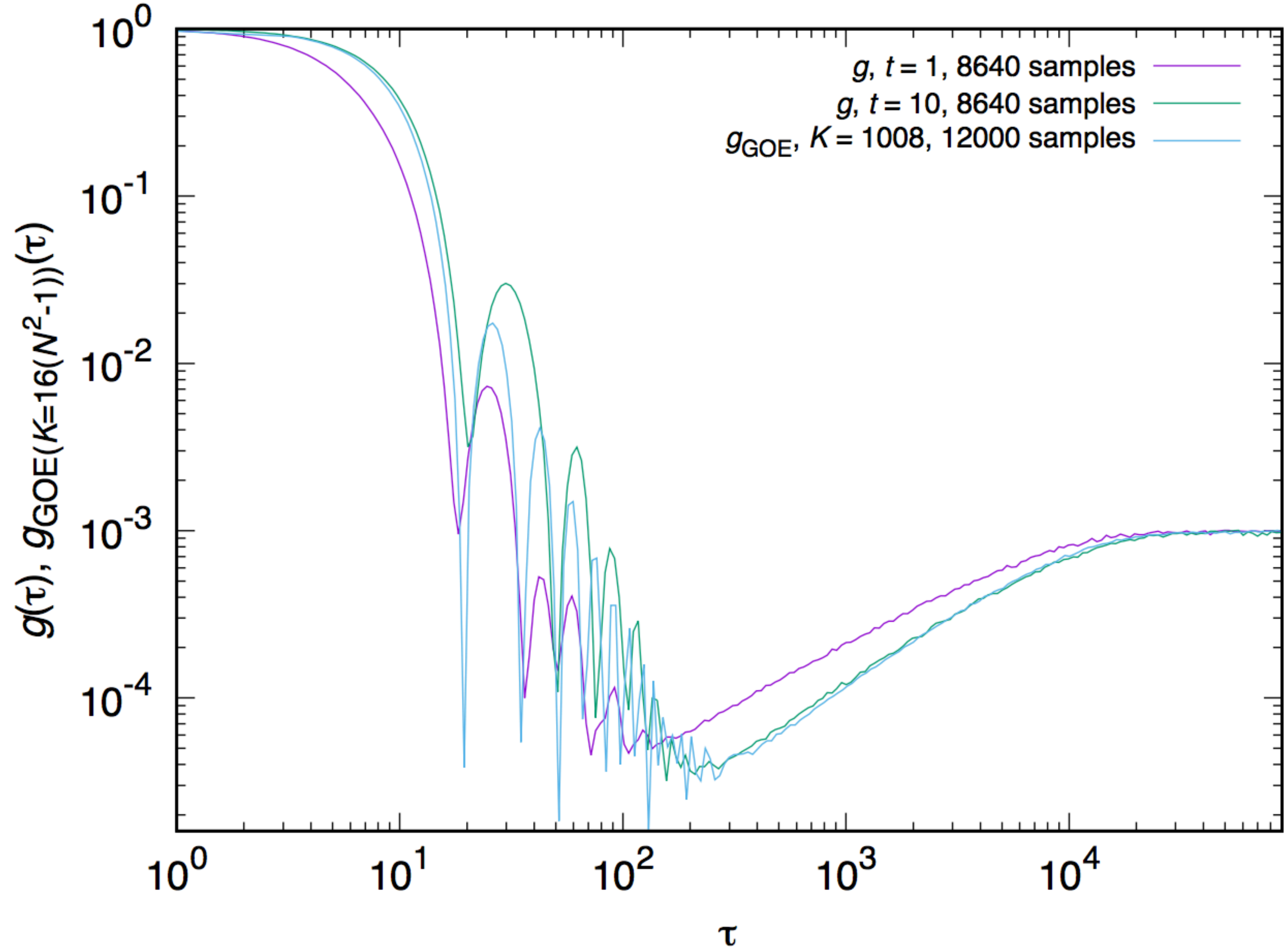
SFF captures the correlation beyond nearest neighbor

$|\Sigma e^{iEt}|$ from GOE vs. $|\Sigma e^{i\lambda\tau}|$ from matrix model (t=0)



universality beyond nearest neighbor

Mass deformed model, $\Delta L = -(Nm^2/2)\Sigma X_M^2$

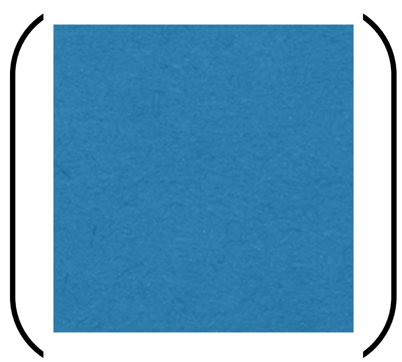


(m=3)

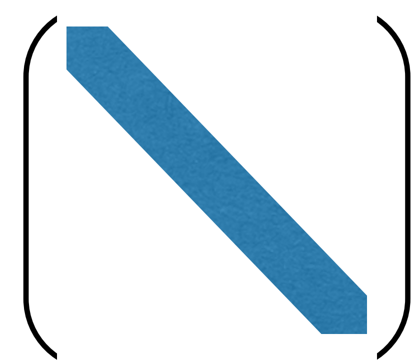
universality beyond nearest neighbor

Random Matrix Product

$$M(t) = M_t M_{t-1} \ldots M_1$$



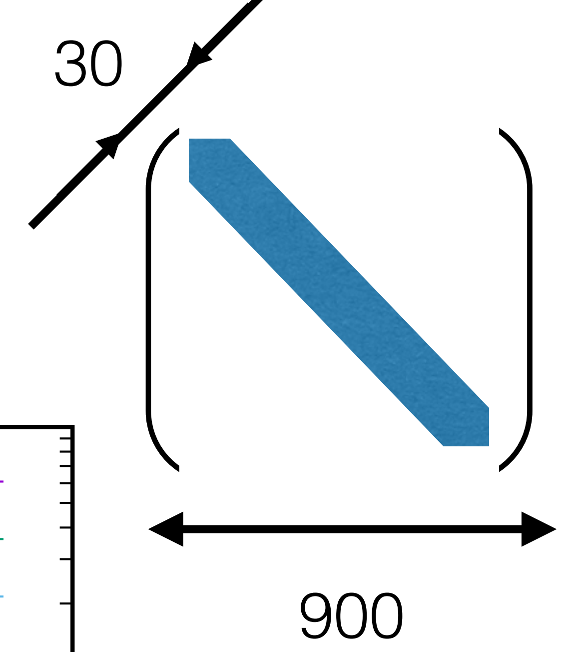
Gaussian Random Matrix

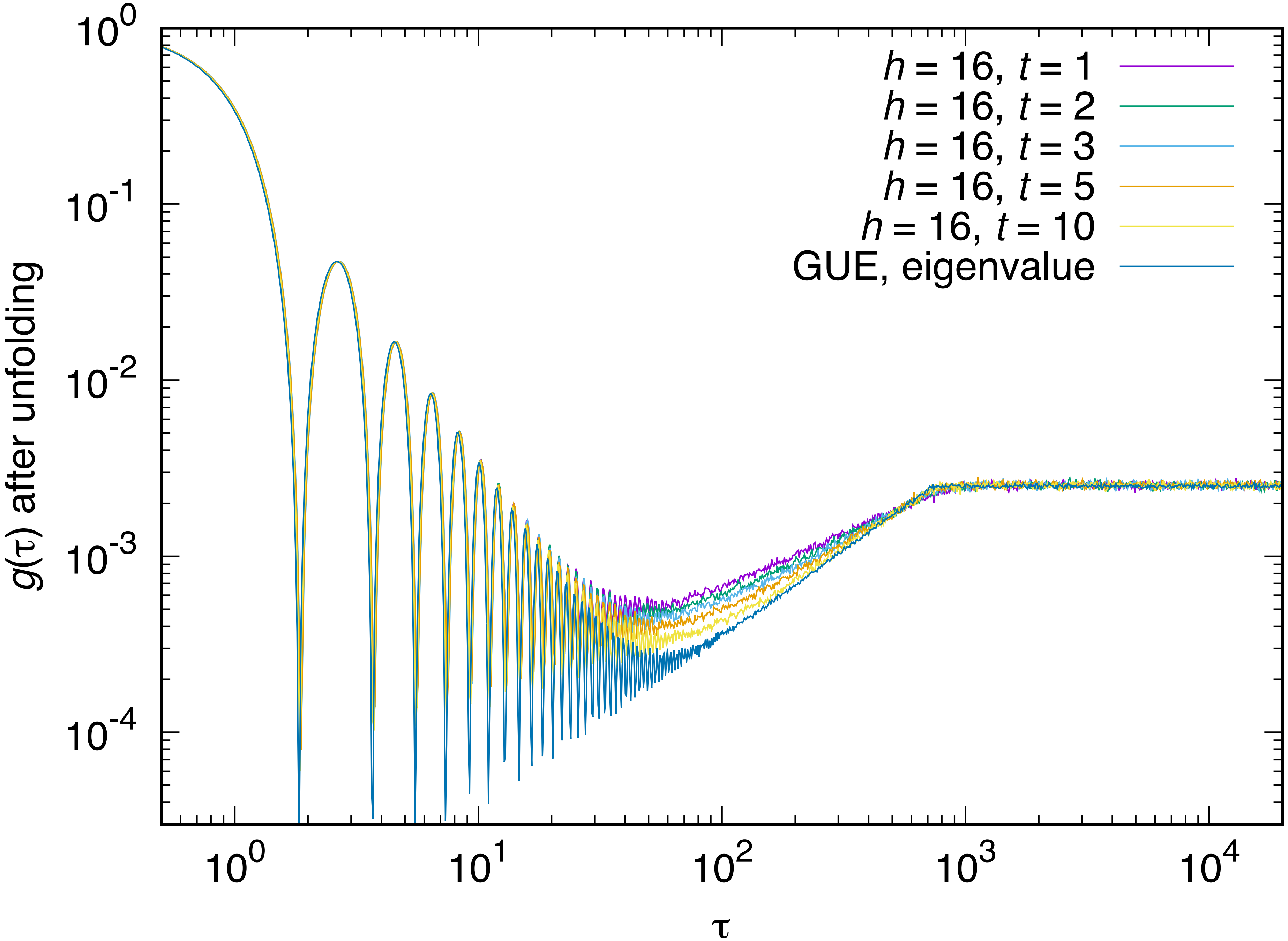


Gaussian Random Band Matrix

singular values = $\exp(\lambda_L t)$

Universality!

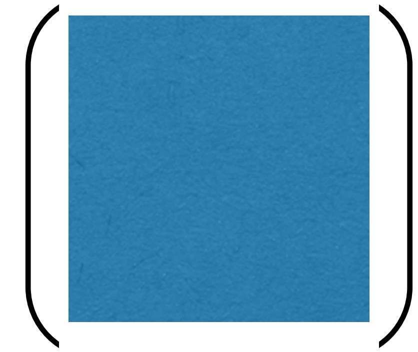




complex matrices → GUE

real matrices → GOE

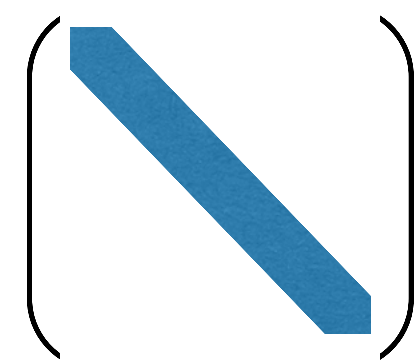
Numerical Observations



M.H.-Shimada-Tezuka, hep-th 2017

- **D0-brane matrix model & product of random Gaussian matrices — RMT already t=0**

Maybe a special property of quantum gravitational systems?



- **Other systems — not RMT at t=0, but converges to RMT at late time**

Likely to be a universal property in classical chaos.
Generalization to quantum theory?

Early-time universality in quantum chaos?

Gharibyan, MH, Swingle, Tezuka, in progress

- There is no consensus for the definition of 'quantum Lyapunov spectrum'

- Let's try the simplest choice:

z=(x,p)

$$M_{ij}(t) \equiv \frac{\delta z_i(t)}{\delta z_j(0)}$$

$$\hat{M}_{ij}(t) \equiv \sqrt{-1}[\hat{z}_i(t), \hat{\Pi}_j(0)]$$

$$\lambda_i(t) \equiv \frac{1}{t} \log s_i(t)$$

$$L_{ij}(t) = [M^\dagger(t)M(t)]_{ij} = M^*_{ki}(t)M_{kj}(t)$$

$$L^{(\phi)}_{ij}(t) \equiv \langle\phi|\hat{M}^*_{ki}(t)\hat{M}_{kj}(t)|\phi\rangle$$

$\hat{M}_{ij}(t)|\phi\rangle$ grows exponentially

$\langle\phi|\hat{M}_{ij}(t)|\phi\rangle$ cannot capture the growth

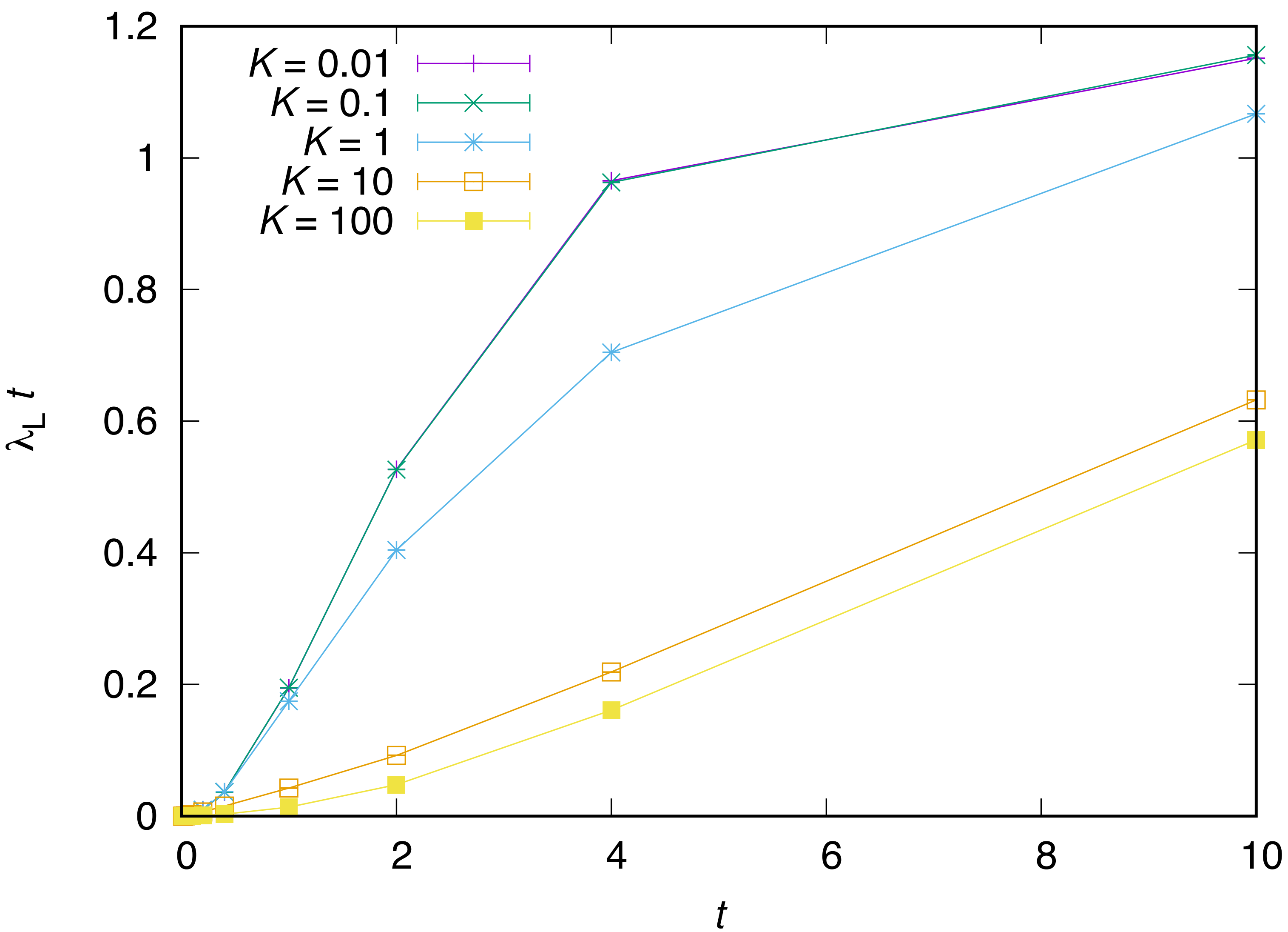
SYK model

$$\hat{H} = \sqrt{\frac{6}{N^3}} \sum_{i<j<k<l} J_{ijkl} \hat{\psi}_i \hat{\psi}_j \hat{\psi}_k \hat{\psi}_l + \frac{\sqrt{-1}}{\sqrt{N}} \sum_{i<j} K_{ij} \hat{\psi}_i \hat{\psi}_j$$

- N Majorana fermion with Gaussian Random Coupling
- Dual to AdS_2 gravity?
- Saturates the chaos bound (Maldacena-Shenker-Stanford bound)

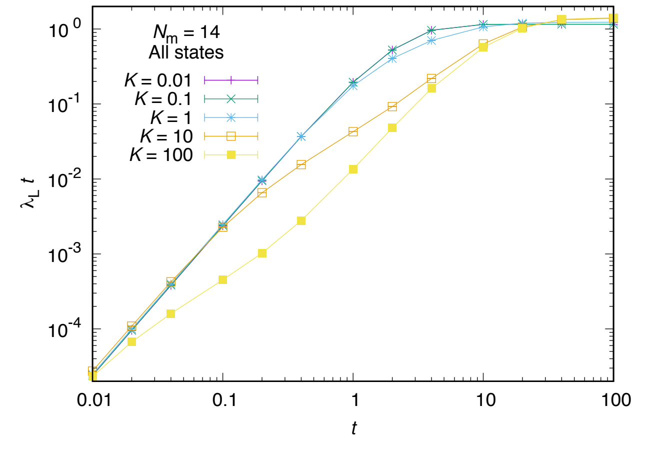
$$\hat{M}_{ij}(t) = \{\hat{\psi}_i(t), \hat{\psi}_j(0)\}$$

Lyapunov growth

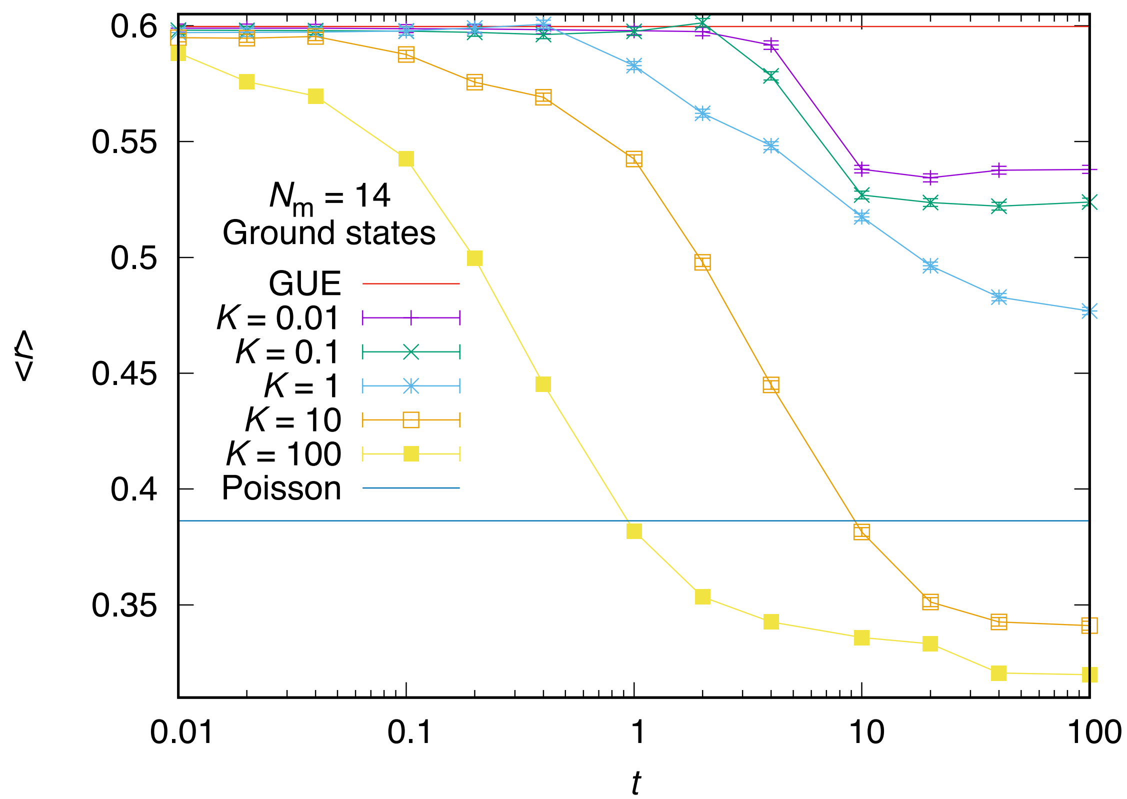


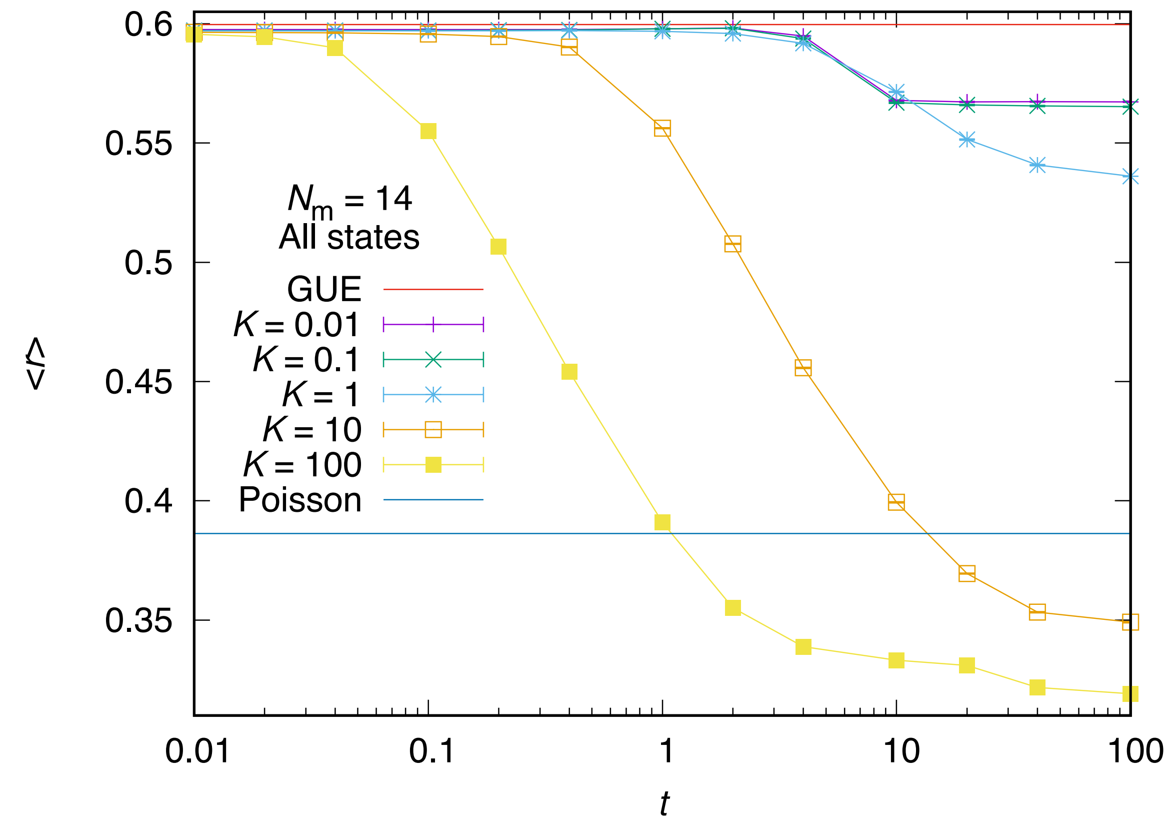
N=14

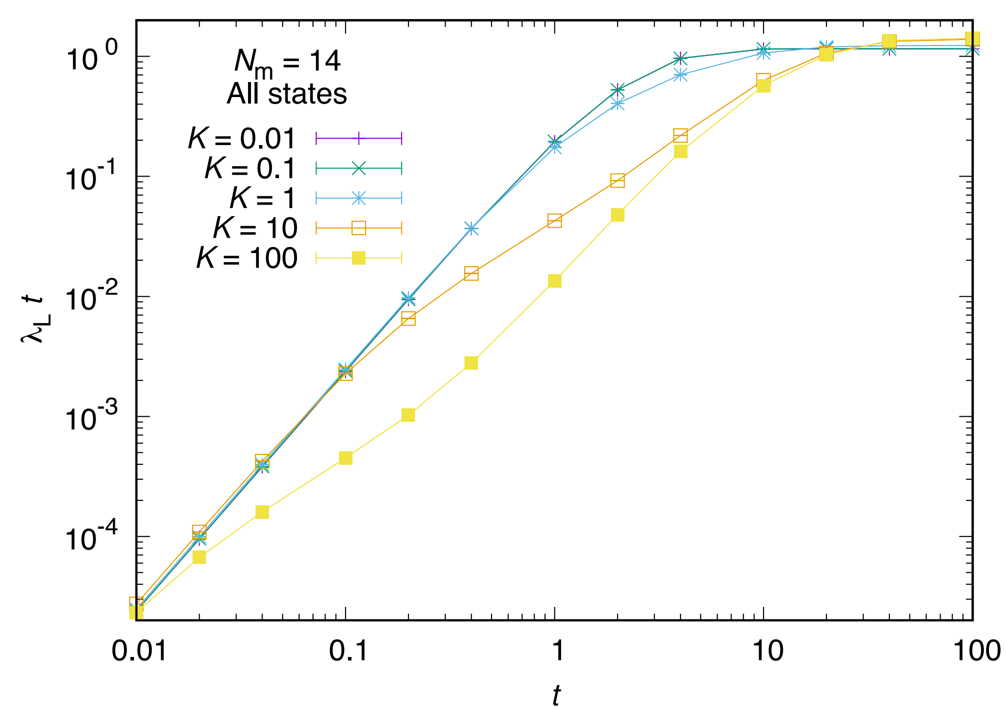
(T=∞)



RMT behavior

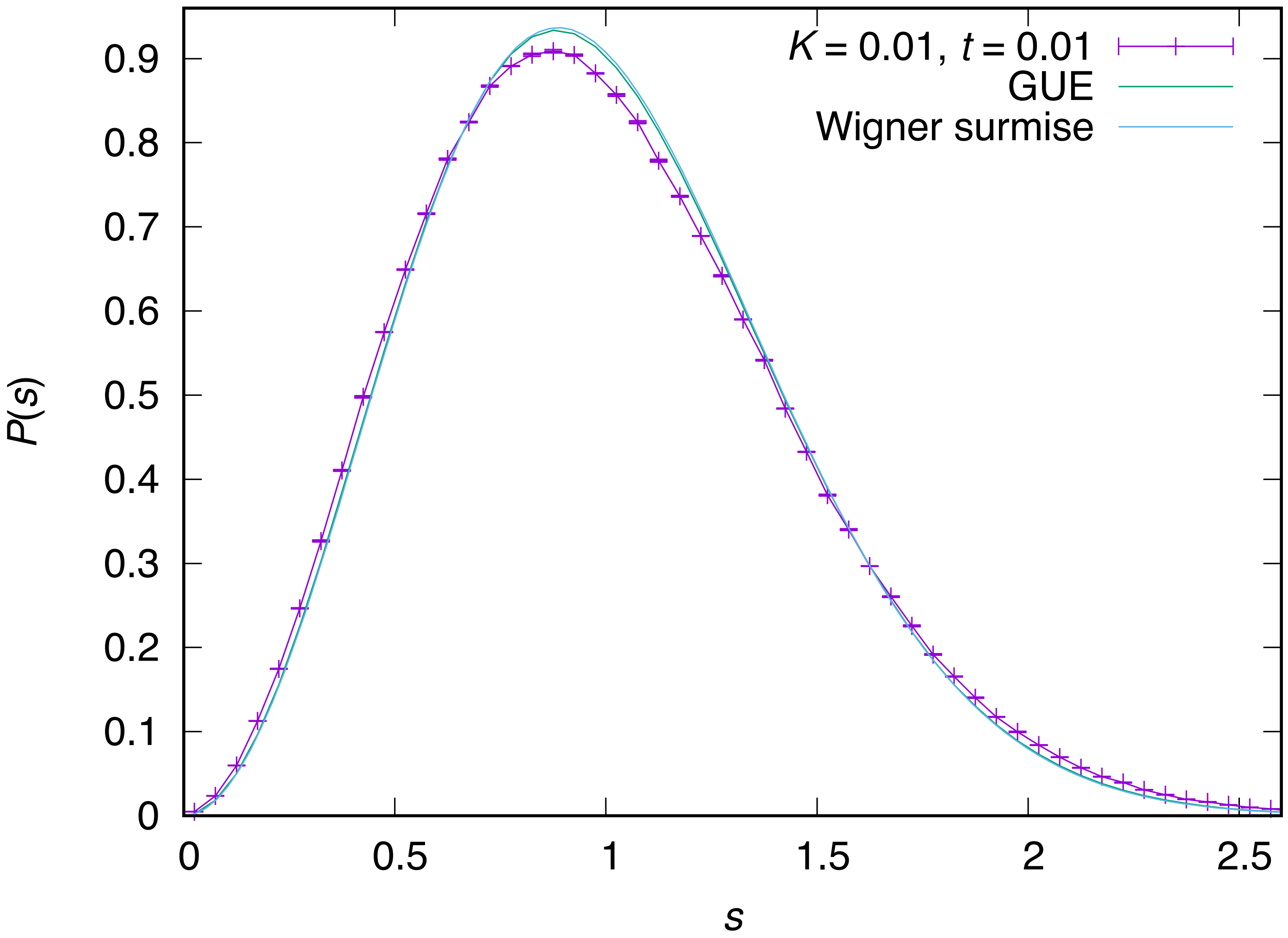






$$r = \frac{\min(\lambda_{i-1} - \lambda_i, \lambda_i - \lambda_{i+1})}{\max(\lambda_{i-1} - \lambda_i, \lambda_i - \lambda_{i+1})}$$

RMT behavior



nearest neighbor level separation

Conclusion

- Smoking gun of new universality.
- Both classical and quantum.
- Early time.
- Likely to characterize the strength of chaos.
- More numerical experiments & theoretical considerations will be needed.