Spherical transverse M5-branes from the plane wave matrix model

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- PRD96 (2017) no.12, 126003
- arXiv:1711.07681[hep-th] (to appear in JHEP)

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BFSS matrix model

[cf. IKKT, DVV]

$$S = \frac{1}{g^2}\int dt \mathrm{Tr}\left(\frac{1}{2}DX_\mu^2 + \frac{1}{4}[X^\mu, X^\nu]^2 + \text{fermions}\right)$$

$X^\mu : N \times N$ Hermitian matrices $(\mu = 1, 2, \cdots, 9)$

This model was originally introduced as
a regularized theory of a super membrane

[de Wit-Hoppe-Nicolai]

[BFSS, Susskind]

Conjecture

BFSS matrix model is not just
a regularized theory of a single M2-brane
but the second quantization of M-theory

Matrix Model = Second Quantization?

Does BFSS MM contain all states in M-theory?

$$|\text{a single M2}\rangle$$

$$|\text{multiple M2}\rangle$$

Relatively well understood in MM

$$|\text{a single M5}\rangle$$

$$|\text{multiple M5}\rangle$$

$$|\text{multiple M2/M5}\rangle$$

Longstanding problem
$\Rightarrow$ This talk

Why M5 difficult in MM?

- The world volume theory of M5-branes is not known.
- M5-brane charge is missing in SUSY algebra of MM.

  [Banks-Seiberg-Shenker]

No guiding principle to find M5-branes in MM

My talk

- Mass deformation of BFSS matrix model is very useful to understand this problem

Outline

BFSS MM ⇒ M-theory on $R^{1,10}$

↓ Mass deformation

Plane wave (BMN) MM ⇒ M-theory on pp-wave

[Berenstein-Maldacena-Nastase]

- M-theory on 11D pp-wave background admits spherical M2/M5 branes with vanishing light cone energy.
- This M5-brane should be described in PWMM as a vacuum state.

In finding this M5-brane in PWMM,
the target is restricted to the vacuum sector

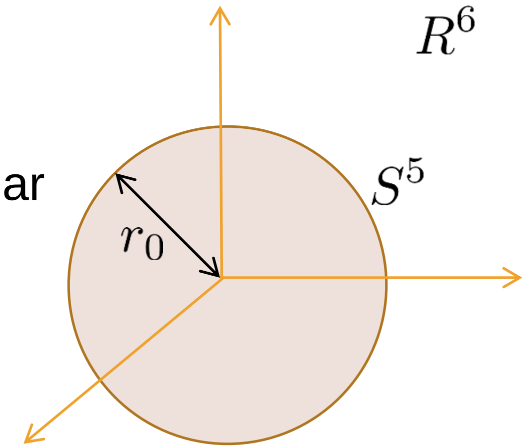
Conjecture [BMN, Maldacena-Sheiki-Jabbari-Raamsdonk] :

" The trivial vacuum (all fields=0) in PWMM corresponds to the spherical M5."

Our result

- To check this conjecture, we applied localization to PWMM.
- We found that in the appropriate large-N and strong coupling limit, the moduli distribution of SO(6) scalars in PWMM forms spherical shells.

Moduli distribution of SO(6) scalar



- For a single M5, the radius of the distribution agrees with the M5's radius.

$$r_0 = \left(\frac{\mu p^+}{6\pi^3}\right)^{1/4} = r_{M5}$$

- We also generalized this result to multiple M5-branes.

Plan of talk

1. Intoroduction
2. M-theory on pp-wave background
3. The conjecture for M5-branes in PWMM
4. M5-branes from PWMM
5. Summary and outlook

2. M-theory on pp-wave background

PP-wave background

- Maximally supersymmetric pp-wave solution in 11D SUGRA

$$ds^2 = -2dx^+dx^- + dx^M dx^M - \left(\frac{\mu^2}{9}x^i x^i + \frac{\mu^2}{36}x^a x^a\right) dx^+ dx^+$$

$$F_{123+} = \mu$$

$$\begin{cases} M = 1, \cdots, 9 \\ i = 1, 2, 3 \\ a = 4, \cdots, 9 \end{cases}$$

- Let us consider classical M2 and M5-branes on this geometry

M2-brane

- Dirac-Nambu-Goto action

$$S = -\int d^3\sigma \sqrt{-h} + \int C_3$$

$$\begin{cases} h = \det h_{\alpha\beta} \\ h_{\alpha\beta} = g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu \quad \text{(induced metric)} \\ X : \text{world volume} \to R^{1,10} \quad \text{(Embedding function)} \\ dC_3 = F_4 = \mu\, dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^+ \end{cases}$$

◆ After taking the light cone frame, fixing some gauge symmetries, the light cone Hamiltonan is reduced to

$$H = \frac{2\pi}{p^+}\left(P_i^2 + P_a^2 + \frac{1}{2}\{X_a, X_b\}^2 + \{X_a, X_i\}^2\right)$$

$$+\frac{p^+\mu^2}{288\pi}X_a^2 + \frac{p^+}{72\pi}\left(\mu X_i + \frac{6\pi}{p^+}\{X_j, X_k\}\right)^2$$

p^+: Light-cone momentum

$\{\ ,\ \}$: Poisson bracket on spatial world volume

$$\begin{cases} i = 1, 2, 3 \\ a = 4, \cdots, 9 \end{cases}$$

Spherical M2-brane

- Vaccum configuration

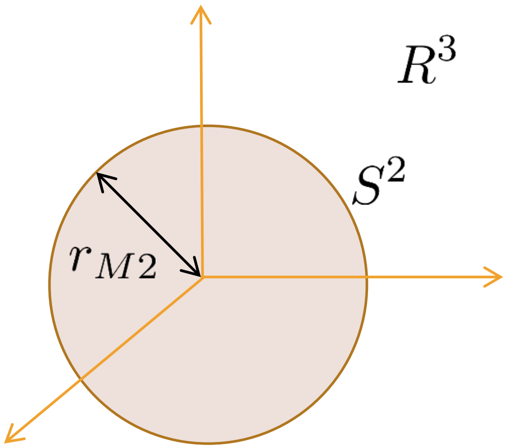
$$\mu X_i + \frac{6\pi}{p^+}\epsilon_{ijk}\{X^j, X^k\} = 0 \qquad X_a = 0 \ (a = 4, \cdots, 9)$$

This is solved by a spherical M2-brane

$$X_1^2 + X_2^2 + X_3^2 = r_{M2}^2$$

$$r_{M2} = \frac{\mu p^+}{12\pi}$$

(in the unit of $T_{M2} = 1$)



M5-brane

- We can repeat the same computation for a single M5-brane

$$S = -\int d^6\sigma\sqrt{-h} + \int C_6$$

$$\Downarrow$$

$$H = \frac{\pi^3}{p^+}\left(P_i^2 + P_a^2 + \frac{1}{4!}\{X_i, X_{a_1}, X_{a_2}, X_{a_3}, X_{a_4}\}^2 + \cdots\right) + \frac{p^+\mu^2}{18\pi^3}X_i^2 + \frac{p^+\mu^2}{72\pi^3}\left(X_{a_1}^2 + \frac{6\pi^3}{5!\mu p^+}\epsilon_{a_1 a_2 \cdots, a_6}\{X_{a_2}, \cdots, X_{a_6}\}\right)^2$$

$$\{f_1, \cdots, f_5\} = \epsilon^{a_1\cdots a_5}\partial_{a_1} f_1 \cdots \partial_{a_5} f_5$$

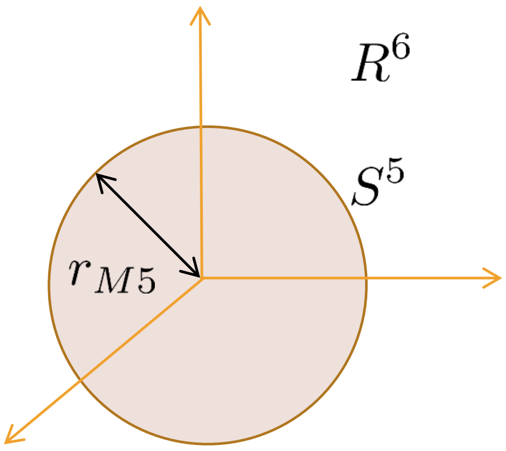
Spherical M5-brane

- The vacuum configuration = spherical M5

$$\begin{cases} X_i = 0 \ (i = 1, 2, 3) \\ \displaystyle\sum_{a=4}^{9} X_a^2 = r_{M5}^2 \end{cases}$$

$$r_{M5} = \left(\frac{\mu p^+}{6\pi^3} \right)^{1/4}$$

(in the unit of $T_{M5} = 1$)



Spherical M5-brane

Summary so far

In M-theory on pp-wave,
there exist spherical M2/M5-branes
with vanishing light cone energy

3. The conjecture for M5-branes in PWMM

Matrix model for M-theory on pp-wave

- PWMM (matrix regularization of M2-brane on pp-wave)

$$S_{PWMM} = \frac{1}{g^2}\int dt \mathrm{Tr}\left[\frac{1}{2}DX_M^2 - \frac{1}{4}[X_a, X_b]^2 - \frac{1}{2}[X_a, X_i]^2 - \frac{\mu^2}{8}X_a^2 - \frac{1}{2}\left(\mu X_i + \frac{i}{2}\epsilon_{ijk}[X_j, X_k]\right)^2 + \text{fermions}\right]$$

$$\begin{cases} X_i : \text{SO(3) scalar} \\ X_a : \text{SO(6) scalar} \end{cases}$$

- Conjectured to be second quantization of M-theory on pp-wave [BFSS, BMN].

- It must contain all states in M-theory. In particular, spherical M2 and M5 with zero light cone energy must be realized as vacuum states of PWMM.

Vacua of PWMM

- The potential terms vanish for

$$\mu X_i + \frac{i}{2}\epsilon_{ijk}[X_j, X_k] = 0, \quad X_a = 0$$

This is solved by any N-dimensional representation of SU(2) generator

$$X_i = \mu L_i \qquad [L_i, L_j] = i\epsilon_{ijk}L_k$$

Fuzzy sphere

- The fuzzy sphere is the regularization of S^2, not of S^5.
- We cannot see any spherical M5-brane like object…

Conjecture

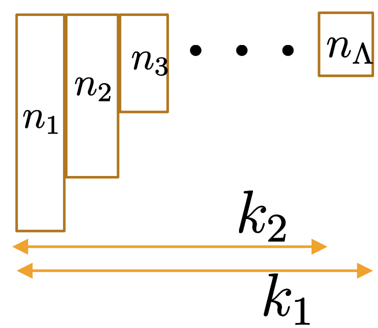
- "Trivial vacuum $(X_i = 0)$ corresponds to a single spherical M5". [BMN]
- More generally: Make an irreducible decomposition, [Maldacena-Sheiki-Jabbari-Raamsdonk]

$$X_i = \mu L_i = \mu \bigoplus_{s=1}^{\Lambda} L_i^{[n_s]}$$

n_s dim irrep

▪ Vacua are labelled by partitions of N
⇔ Young Tableau

▪ Let k_a be the length of a-th row



$n_s \to \infty$ is commutative limit of fuzzy sphere (M2-brane limit)
Dual limit $k_a \to \infty$ is conjectured to be the M5-brane limit

For the trivial vacuum ($n_s = 1$), they checked the conjecture by showing that the mass spectra of PWMM and a single M5-brane agree with each other.

Trivial Vacuum = Single M5 ?

- Trivial vacuum does not look like a spherical 1+5 dimensional object.

- However, recall that M-theory is supposed to be realized in the limit,

$$N \to \infty \quad \text{with} \quad p^+ = N/R \quad \text{fixed.}$$

This corresponds to a very strong coupling limit in PWMM

- SO(6) scalars may form the spherical M5-brane at strong coupling.

$$X^a = \underbrace{0 + \text{Large quantum fluctuation}}_{S^5?}$$

We checked this by using the localization!

4. M5-branes from PWMM

Decoupling limit

- The system we consider : M5-brane + bulk gravity
- We take the decoupling limit of M5-brane to focus only on the M5-brane

In terms of the parameters of PWMM, this limit is given by the strong coupling limit in the 't Hooft limit. [Maldacena-Sheiki-Jabbari-Raamsdonk]

Assumption

- We assume that in the strong coupling limit, the low energy moduli of the matrices are given by commuting matrices,

$$[X^A, X^B] \to 0$$

This condition will be needed to describe classical geometric objects of SUGRA.

Localization in PWMM

$$S_{PWMM} = \frac{1}{g^2}\int dt \mathrm{Tr}\left[\frac{1}{2}DX_M^2 - \frac{1}{4}[X_a, X_b]^2 - \frac{1}{2}[X_a, X_i]^2 \right.$$
$$\left. -\frac{\mu^2}{8}X_a^2 - \frac{1}{2}\left(\mu X_i + \frac{i}{2}\epsilon_{ijk}[X_j, X_k]\right)^2 + \text{fermions}\right]$$

- For simplicity, let us consider the simplest partition,

$$X_i = \mu L_i = \mu L_i^{[N_5]} \otimes \mathbf{1}_{N_2}$$ (N_5=1 ⇔ the trivial vacuum)

- After Wick rotation, we set the boundary condition as

All fields → The vacuum configuration $(\tau \to \pm\infty)$

◆ We consider a complex scalar field,

$$\phi(t) = X_3(t) + i(X_8(t)\sin t + X_9(t)\cos t)$$

There exist supersymmetries such that $Q\phi = 0$ (4 SUSYs, ¼ BPS)

At $t = 0$, $\phi = X_3 + iX_9$

Eigenvalue distribution of ϕ gives
the moduli distribution on the (X_3, X_9)-plane

The eigenvalue distribution of ϕ is computable by the localization

- Any correlation function made of ϕ can be computed by localization

$$\langle \mathrm{Tr}\phi^{n_1}(t_1)\mathrm{Tr}\phi^{n_2}(t_2)\cdots\rangle$$

- Localization

We add a nice Q-exact term tQV to the action
⇒ The partition function is shown to be independent of t
⇒ Taking the large-t limit, the saddle points of the Q-exact term dominate
⇒ Saddle point $\phi = X_3 + iX_9 = \mu L_3 + iM$

M : constant matrix commuting with L_i

- Our result of the localization

$$\langle \mathrm{Tr}\phi^{n_1}(t_1)\mathrm{Tr}\phi^{n_2}(t_2)\cdots\rangle = \langle \mathrm{Tr}(\mu L_3 + iM)^{n_1}\mathrm{Tr}(\mu L_3 + M)^{n_2}\cdots\rangle_{eff}$$

Effective action

$$S = \frac{1}{g^2}\sum_{i=1}^{N_2} m_i^2 + \sum_{J=0}^{N_5-1}\sum_{i\neq j}\log\left(\frac{((2J)^2+(m_i-m_j)^2)((2J+2)^2+(m_i-m_j)^2)}{((2J+1)^2+(m_i-m_j)^2)^2}\right)$$

Classical part

1-loop determinant

This also means that the spectrum of ϕ is determined by the effective action;

$$\langle \mathrm{Tr}(z-\phi)^{-1} \rangle = \langle \mathrm{Tr}(z-\mu L_3 + iM)^{-1} \rangle_{eff}$$

Under the assumption of the commutativity,
eigenvalue distribution of M = distribution of the low energy moduli of X^9

◆ In the large-N limit, the saddle points of S_{eff} dominate.

- Weak coupling limit → the saddle is trivial $(X_9 = M = 0)$
- Strong coupling limit → There is a non-trivial equilibrium

→ Moduli distribution of X_9 in the decoupling limit of M5-brane

$$\rho(x) = c(a-x^2)^{3/2} \qquad a, c \text{ : constants}$$

($M \gg L_3$ in this limit ⇒ only SO(6) scalars expand)

Uplift to SO(6) symmetric distribution

- Assuming SO(6) symmetry and the commutativity of moduli, we introduce the six dimensional distribution $\tilde{\rho}$ by [Filev-O'Connor]

$$\int dx_9 x_9^n \rho(x_9) =: \int dx_4 dx_5 \cdots dx_9 x_9^n \tilde{\rho}(r) \quad \text{for any } n$$

$$r = \sqrt{x_4^2 + \cdots + x_9^2}$$

- For $\rho(x) = c(a - x^2)^{3/2}$, we find $\tilde{\rho}(r) = \tilde{c}\,\delta(r - r_0)$

SO(6) scalars form a spherical shell in R^6

- Furthermore, the radius of the distribution agrees with the radius of M5,

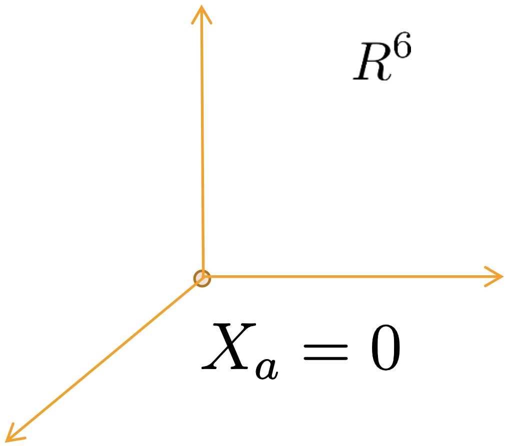
$$r_0 = \left(\frac{\mu p^+}{6\pi^3}\right)^{1/4} = r_{M5}$$

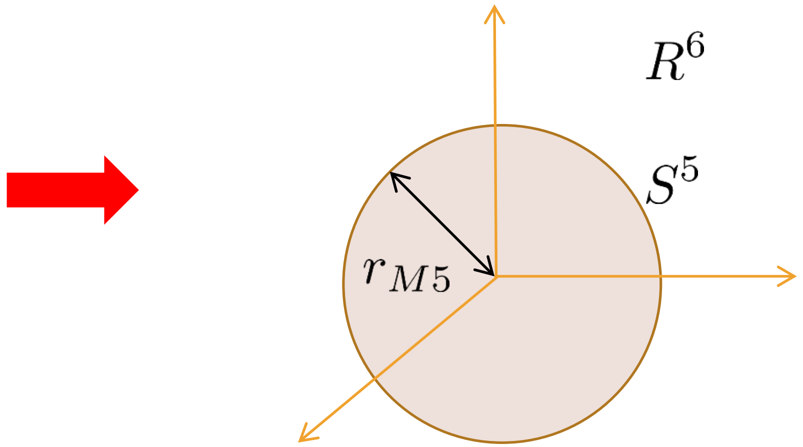
In the M5-brane limit, SO(6) scalars form a spherical M5-brane!

Emergent M5-branes

At weak coupling, vacuum configuration was trivial

At strong coupling, however, typical moduli configuration is a spherical shell





Trivial

Spherical M5-brane

Summary

- By applying localization, we derived a spherical distribution for low energy moduli of SO(6) scalars in the M5-brane limit of PWMM.
- The radius agrees with that of the spherical M5-brane in M-theory
- This result can be generalized to multiple concentric M5-branes
- (multiple) M5-brane states are indeed contained in PWMM

Nice evidence for PWMM to be the second quantization of M-theory!

Outlook

- Excited states? (Rotating M5 etc)
- Theory for multiple M5-branes?
- Numerical work?

The most general partition

For the most general partition. the eigenvalue integral is given by a multi-matrix model.

$$Z = \int \prod_{s=1}^{\Lambda} \prod_{i=1}^{N_2^{(s)}} dm_{si} Z_{1-loop} e^{-\frac{2n_s}{g^2} m_{si}^2}$$

$$Z_{1-loop} = \prod_{s,t=1}^{\Lambda} \prod_{J=|n_s-n_t|/2}^{(n_s+n_t)/2-1} \prod_{i=1}^{N_2^{(s)}} \prod_{j=1}^{N_2^{(t)}} \left[\frac{\{(2J+2)^2 + (m_{si}-m_{tj})^2\}\{(2J)^2 + (m_{si}-m_{tj})^2\}}{\{(2J+2)^2 + (m_{si}-m_{tj})^2\}^2} \right]^{1/2}$$

Recently, we found an exact solution to this model

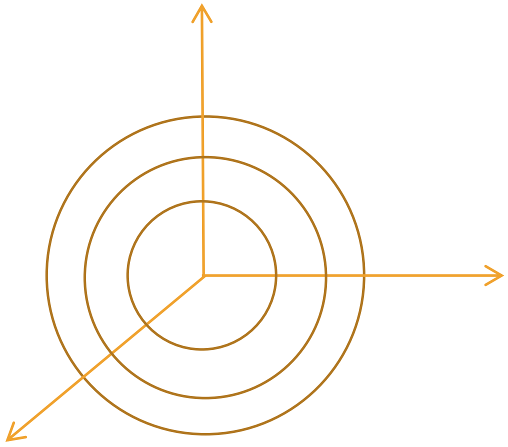
[Asano-Ishiki-Shimasaki-Terashima, to appear in JHEP]

Multiple M5-branes

Solution in the decoupling limit

$$\hat{\rho}_s(x) = \frac{8^{3/4} \sum_{r=1}^{s} N_2^{(r)}}{3\pi \lambda_s^{1/4}} \left[1 - \left(\frac{x}{x_s} \right)^2 \right]^{3/2} , \quad x_s = (8\lambda_s)^{1/4}, \quad \lambda_s = g^2 \sum_{r=1}^{s} N_2^{(r)}$$

The SO(6) symmetric uplift is Λ-stacks of spherical shells



Agrees with the claim of the conjecture!