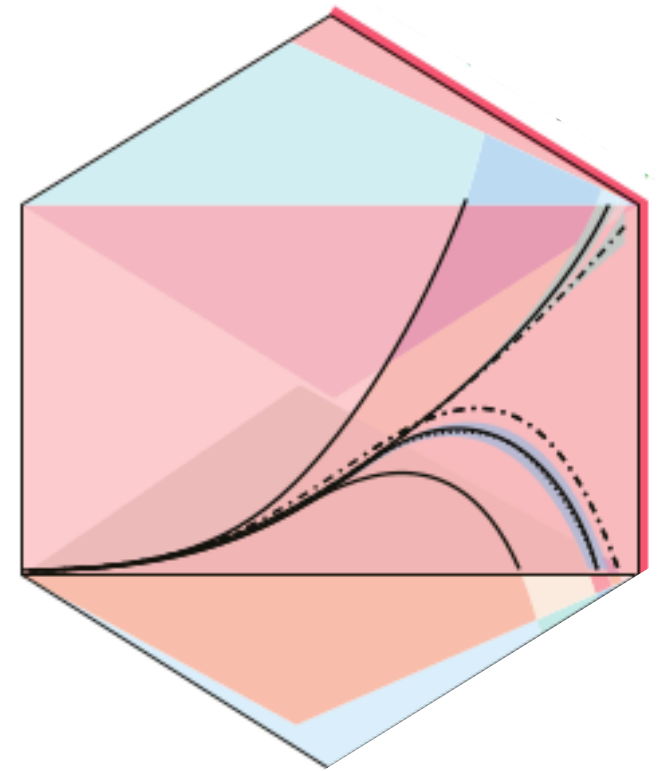




Black Holes in D0-brane Quantum Mechanics

Evan Berkowitz

MCSMC: Monte Carlo String + M-Theory Collaboration

Forschungszentrum Jülich

NUMSTRINGS 2018

2018-02-02

ICTS

1606.04948 1606.04951 EB, Enrico Rinaldi, Masanori Hanada, Pavlos Vranas,
Goro Ishiki, Shinji Shimasaki

1709.01932 Rinaldi, EB, Hanada, Maltz, and Vranas

+ forthcoming work

Parts of this work were performed under the auspices of the U.S. Department of Energy
by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344

Outline

- Gauge/Gravity Duality & Motivation
- The BFSS / D0 Matrix Model
- Monte Carlo, Fitting
- Tests of Holography
- [[Probe of Geometry]]

STRING THEORY SUMMARIZED:

I JUST HAD AN AWESOME IDEA.
SUPPOSE ALL MATTER AND ENERGY
IS MADE OF TINY, VIBRATING "STRINGS."

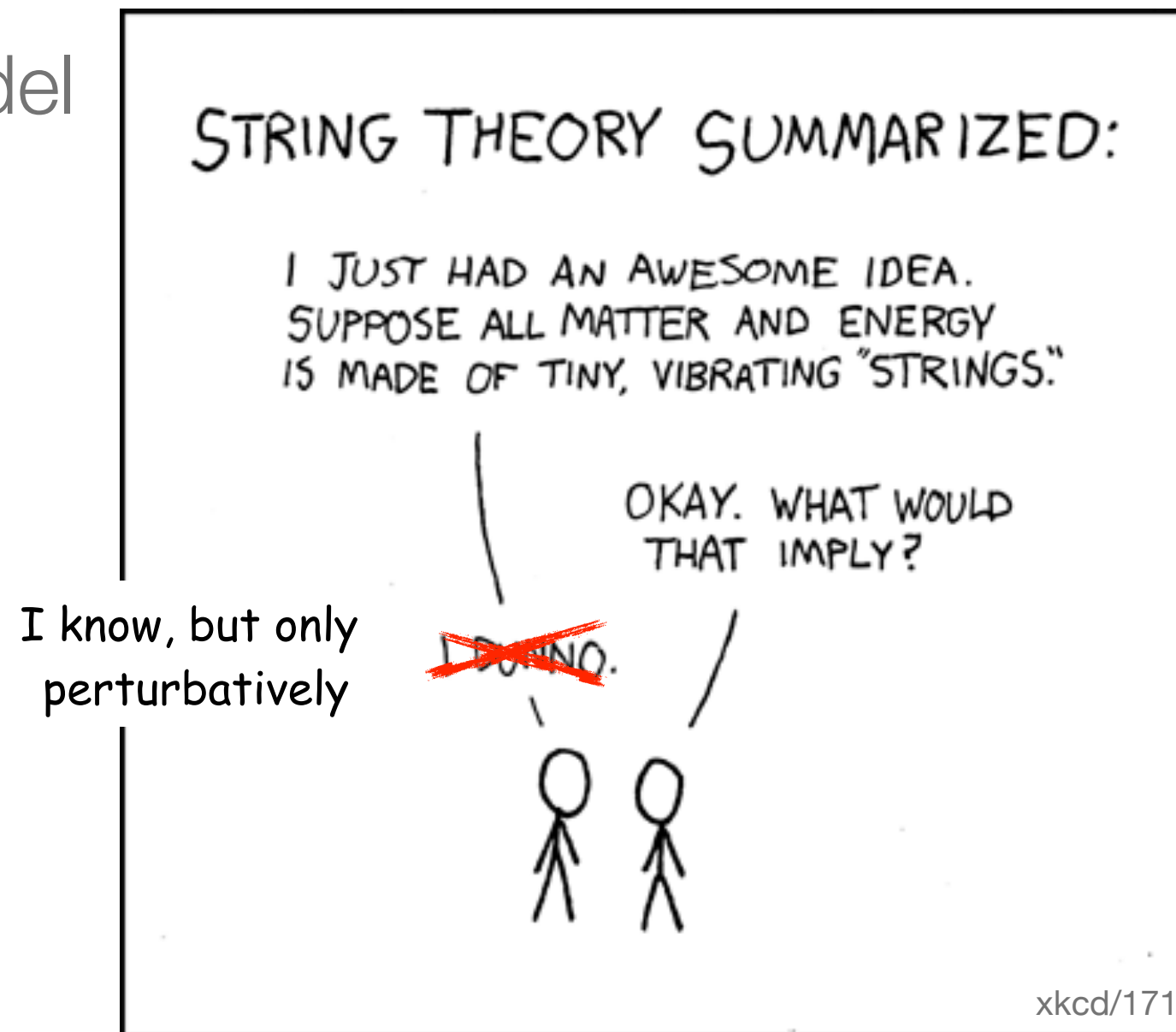
OKAY. WHAT WOULD
THAT IMPLY?

I DUNNO.



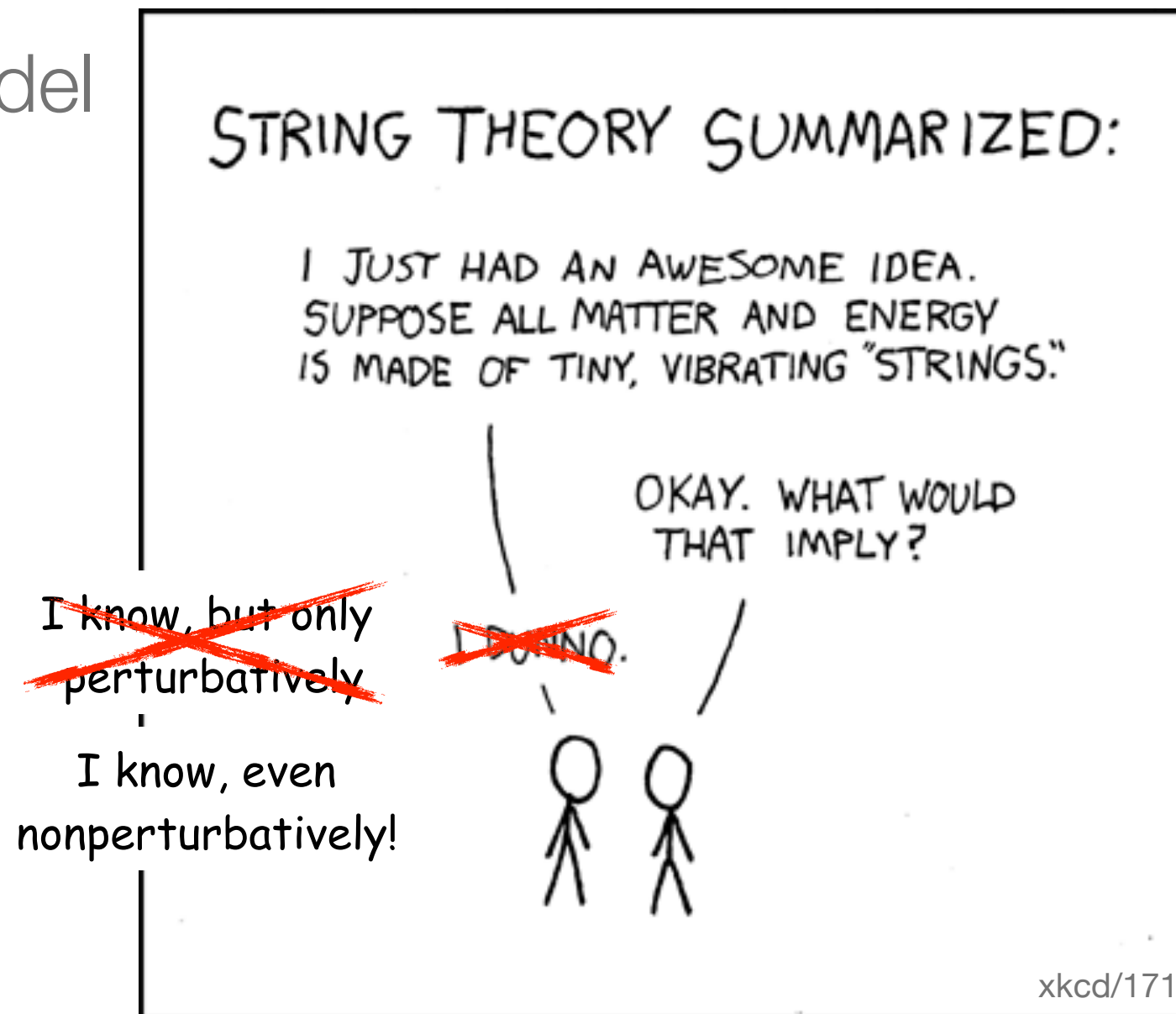
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0+1 D0 Brane QM / BFSS Matrix Model

Banks Fischler Shenker Susskind hep-th/9610043

$$L = \frac{1}{2g_{YM}^2} \text{Tr} \left\{ (D_t X_M)^2 + i \bar{\psi}^\alpha D_t \gamma_{\alpha\beta}^{10} \psi^\beta + \underbrace{\bar{\psi}^\alpha \gamma_{\alpha\beta}^M [X_M, \psi^\beta]}_{\text{Yukawa}} + \underbrace{[X_M, X_{M'}]^2}_{\text{Self-interaction}} \right\}$$

$$D_t \cdot = \partial_t \cdot - i[A_t, \cdot]$$

γ^M left-handed part of 9+1D γ s

X_M 9 bosonic

ψ^α 16 fermionic

$N \times N$ matrices

- Obvious nonperturbative definition (discretized quantum mechanics)
- Defined for all N and g_{YM}
- Manifestly unitary

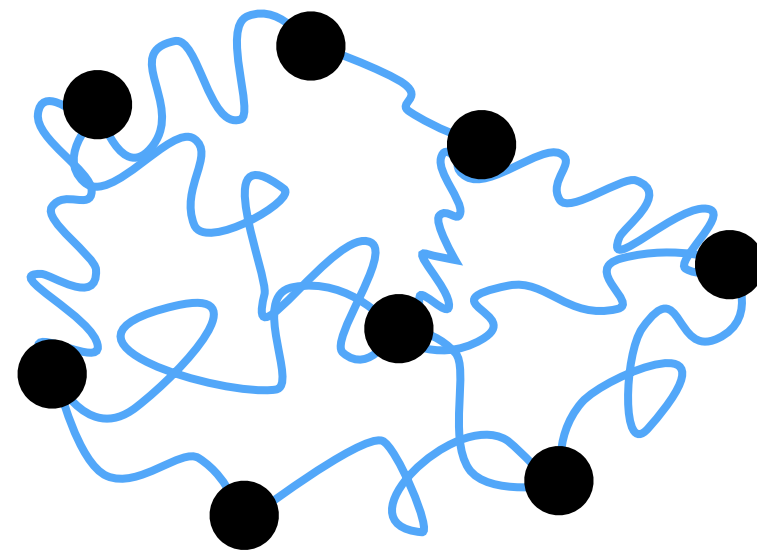
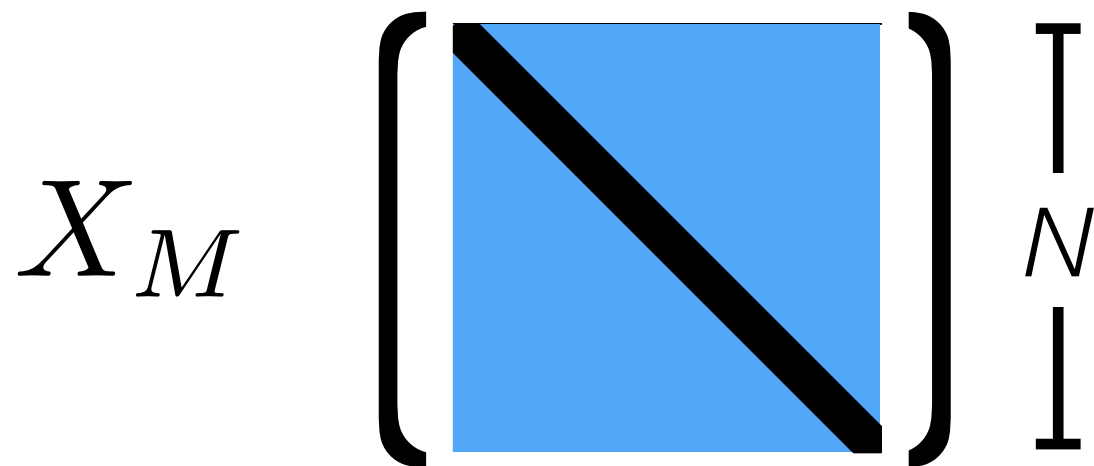
- 10D SUGRA at low temperature
- Dimensionful coupling, easy scale setting!

BFSS Conjecture:
This theory $\equiv$ M theory!

BFSS Cartoon

Witten hep-th/9510135

$$L = \frac{1}{2g_{YM}^2} \text{Tr} \left\{ (D_t X_M)^2 + i\bar{\psi}^\alpha D_t \gamma_{\alpha\beta}^{10} \psi^\beta + \bar{\psi}^\alpha \gamma_{\alpha\beta}^M [X_M, \psi^\beta] + [X_M, X_{M'}]^2 \right\}$$



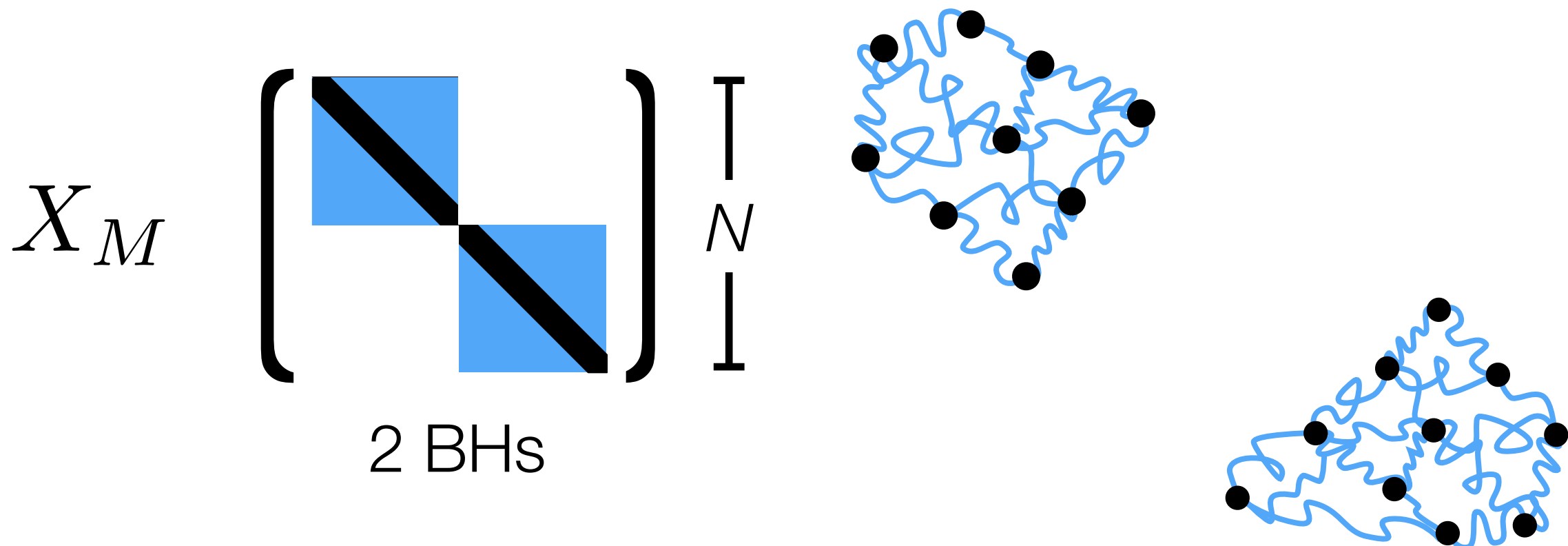
coordinates
couplings

One big bunch $\sim$ black 0-brane $\sim$ BH

BFSS Cartoon

Witten hep-th/9510135

$$L = \frac{1}{2g_{YM}^2} \text{Tr} \left\{ (D_t X_M)^2 + i\bar{\psi}^\alpha D_t \gamma_{\alpha\beta}^{10} \psi^\beta + \bar{\psi}^\alpha \gamma_{\alpha\beta}^M [X_M, \psi^\beta] + [X_M, X_{M'}]^2 \right\}$$

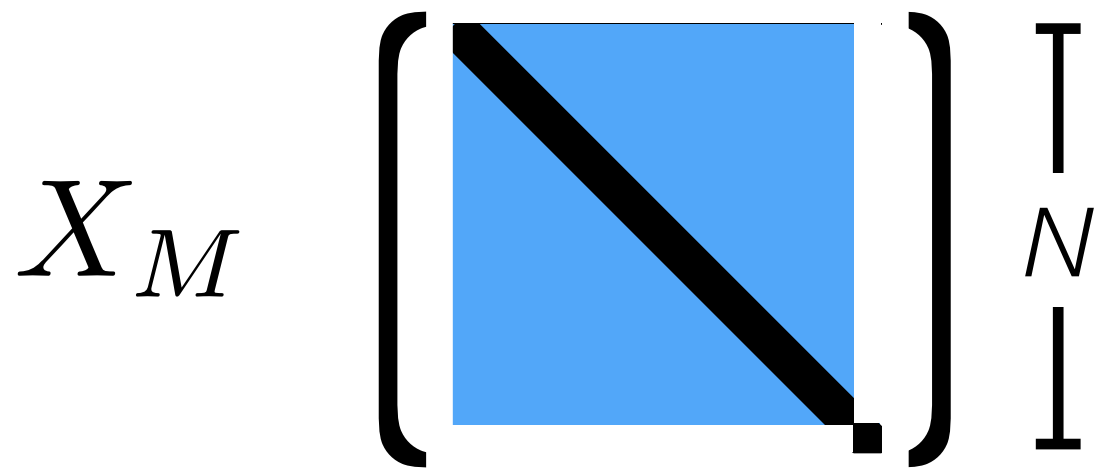


At large N BFSS is a 2nd quantized theory!

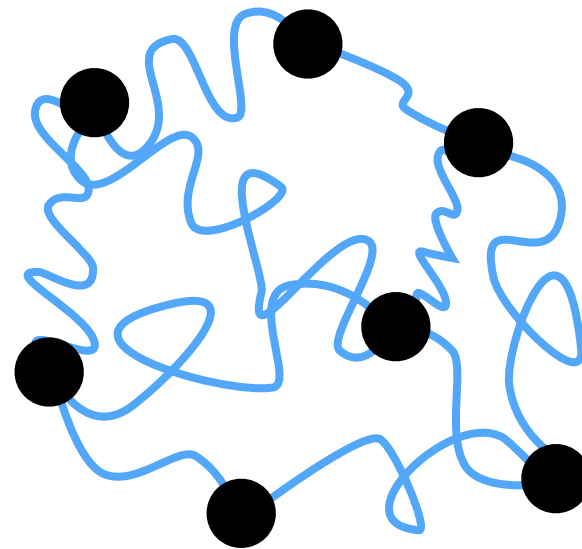
BFSS Cartoon

Witten hep-th/9510135

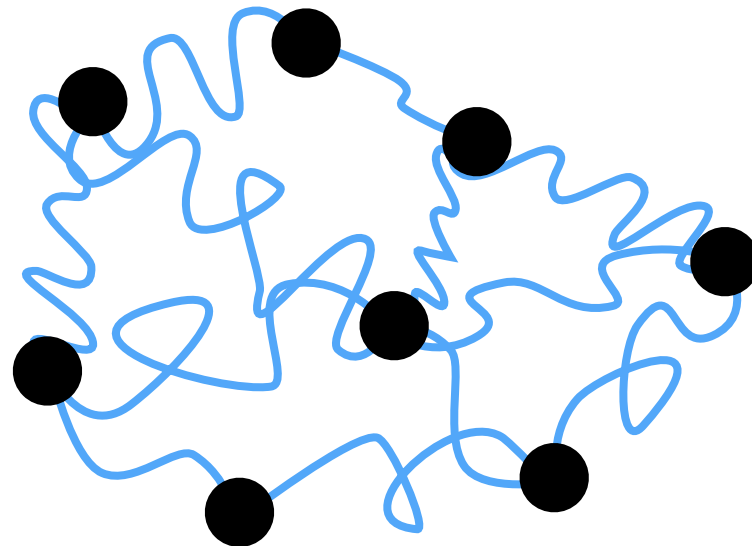
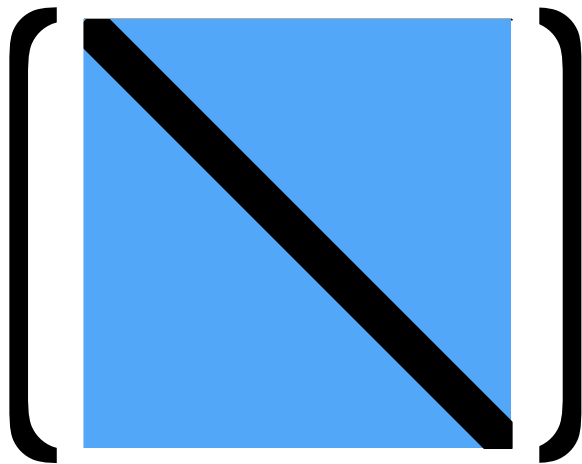
$$L = \frac{1}{2g_{YM}^2} \text{Tr} \left\{ (D_t X_M)^2 + i\bar{\psi}^\alpha D_t \gamma_{\alpha\beta}^{10} \psi^\beta + \bar{\psi}^\alpha \gamma_{\alpha\beta}^M [X_M, \psi^\beta] + [X_M, X_{M'}]^2 \right\}$$



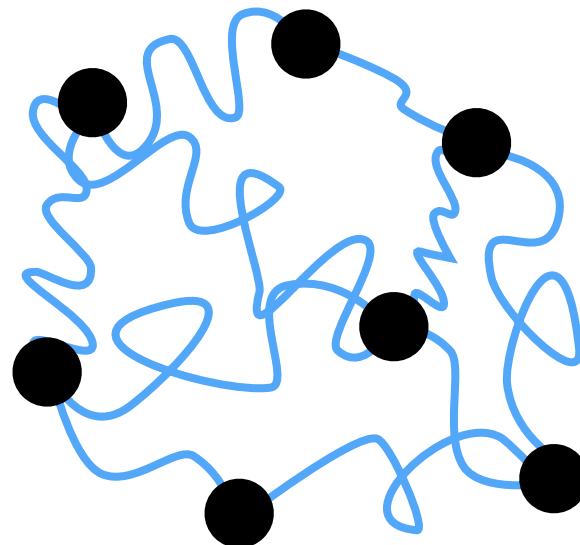
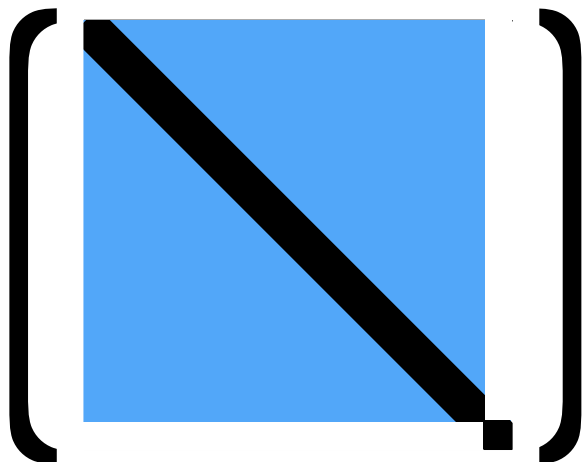
BH + D0 radiation



Monte Carlo Study of Metastable State?



$$\text{DOF} \sim N^2$$
$$t_{\text{recurrence}} \sim e^{+N^2}$$



$$\tau \sim e^{+N}$$

$$\text{DOF} \sim (N-1)^2$$
$$+ \# \log(V)$$



Possible Observables

Likely an incomplete list!

Fast Scrambling arXiv:1512.00019 Gur-Ari, Hanada, Shenker $t_{\text{scramble}} \sim \log N$

SUGRA 0707.4454 Anagnostopoulos et al.
0803.4273 Catterall+Wiseman
1503.08499 Kadoh, Kamata
1506.01366 Filev, O'Connor

Finite N 0811.3102 Hanada, Hyakutake, Nishimura, Takeuchi
1311.5603 1603.00538 Hanada, Hyakutake, Ishiki, Nishimura
1606.04948 1606.04951 MCSMC

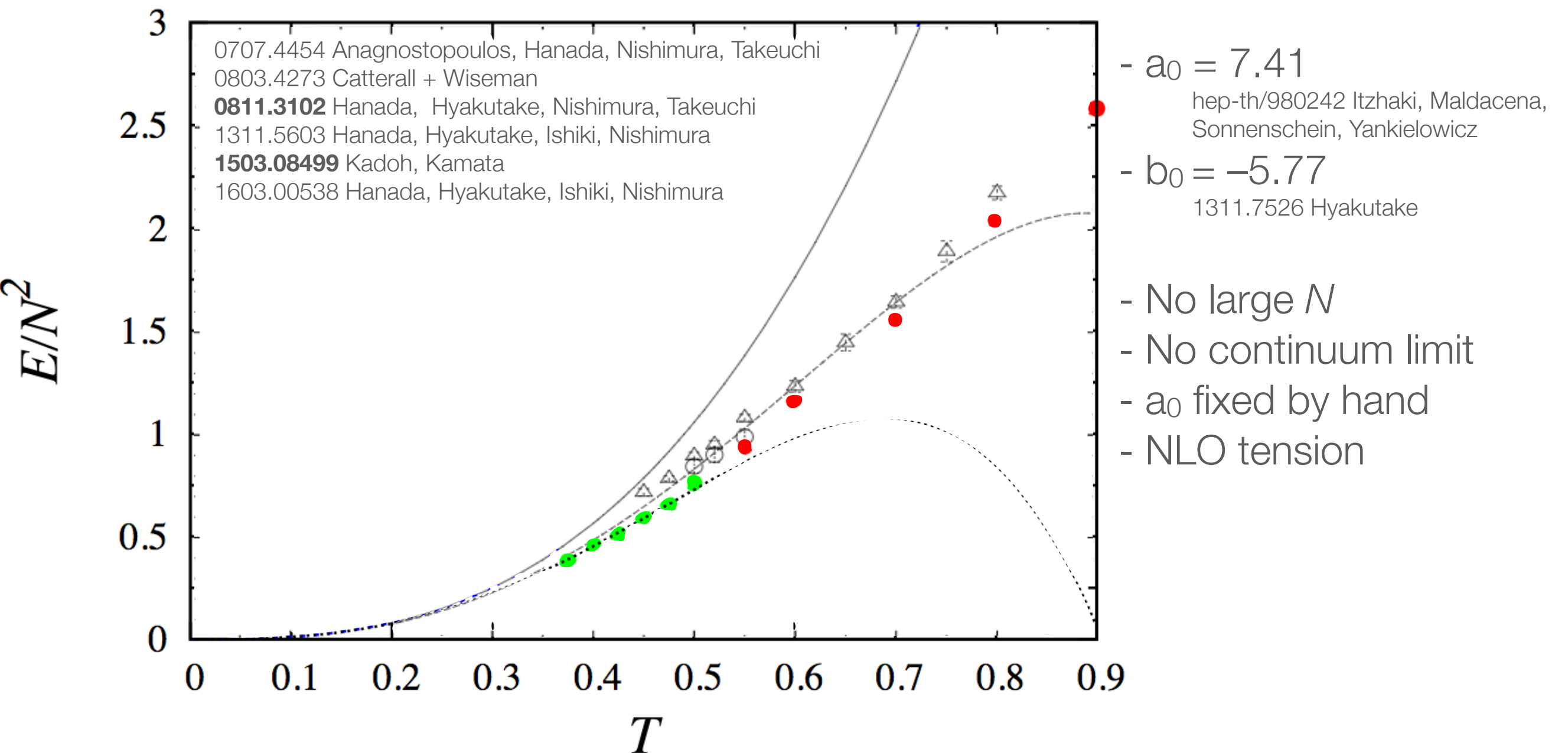
Polyakov loop 0811.2081 Hanada, Miwa, Nishimura, Takeuchi

2-point functions 1108.5153 Hanada, Nishimura, Sekino, Yoneya, 2009, 2011

Force 1709.01932 Rinaldi, Berkowitz, Hanada, Maltz, Vranas

Test: BH Internal Energy

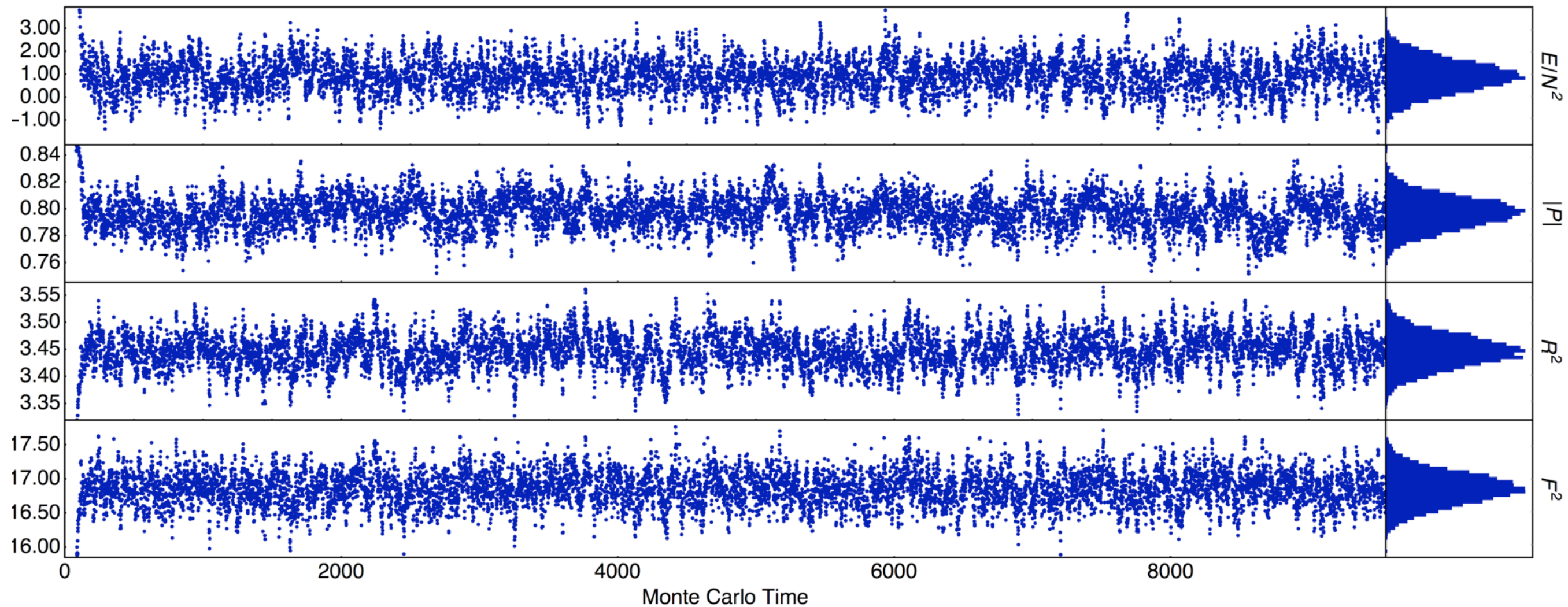
$$E/N^2 = \frac{(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \dots)}{N^0} + \frac{(b_0 T^{0.4} + b_1 T^{2.2} + \dots)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$



Example Monte Carlo History

MCSMC 1606.04948 1606.04951

$T=0.5$ $N=24$ $L=32$

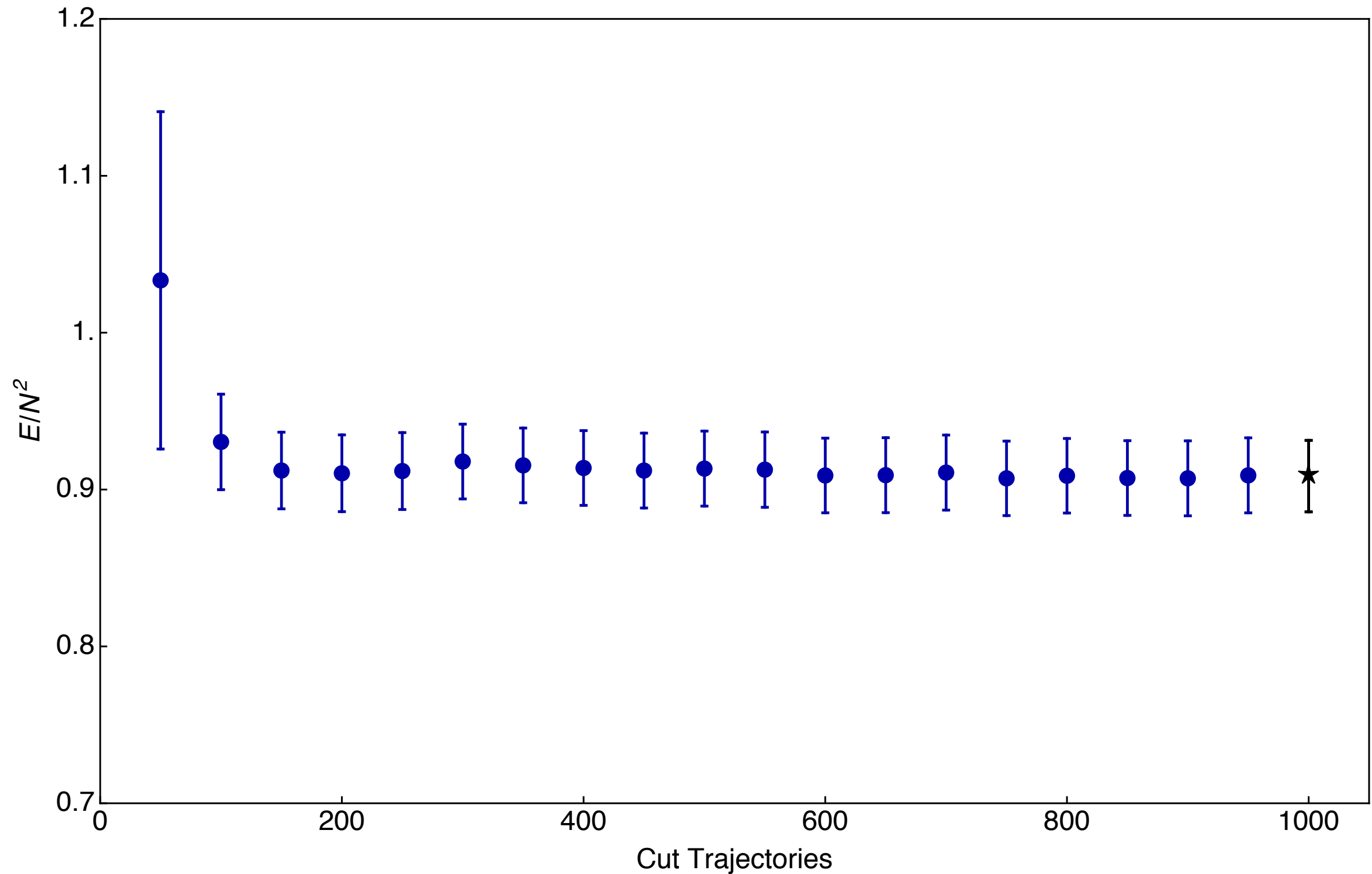


Long correlations can be seen in each observable.

Thermalization Cut

MCSMC 1606.04948 1606.04951

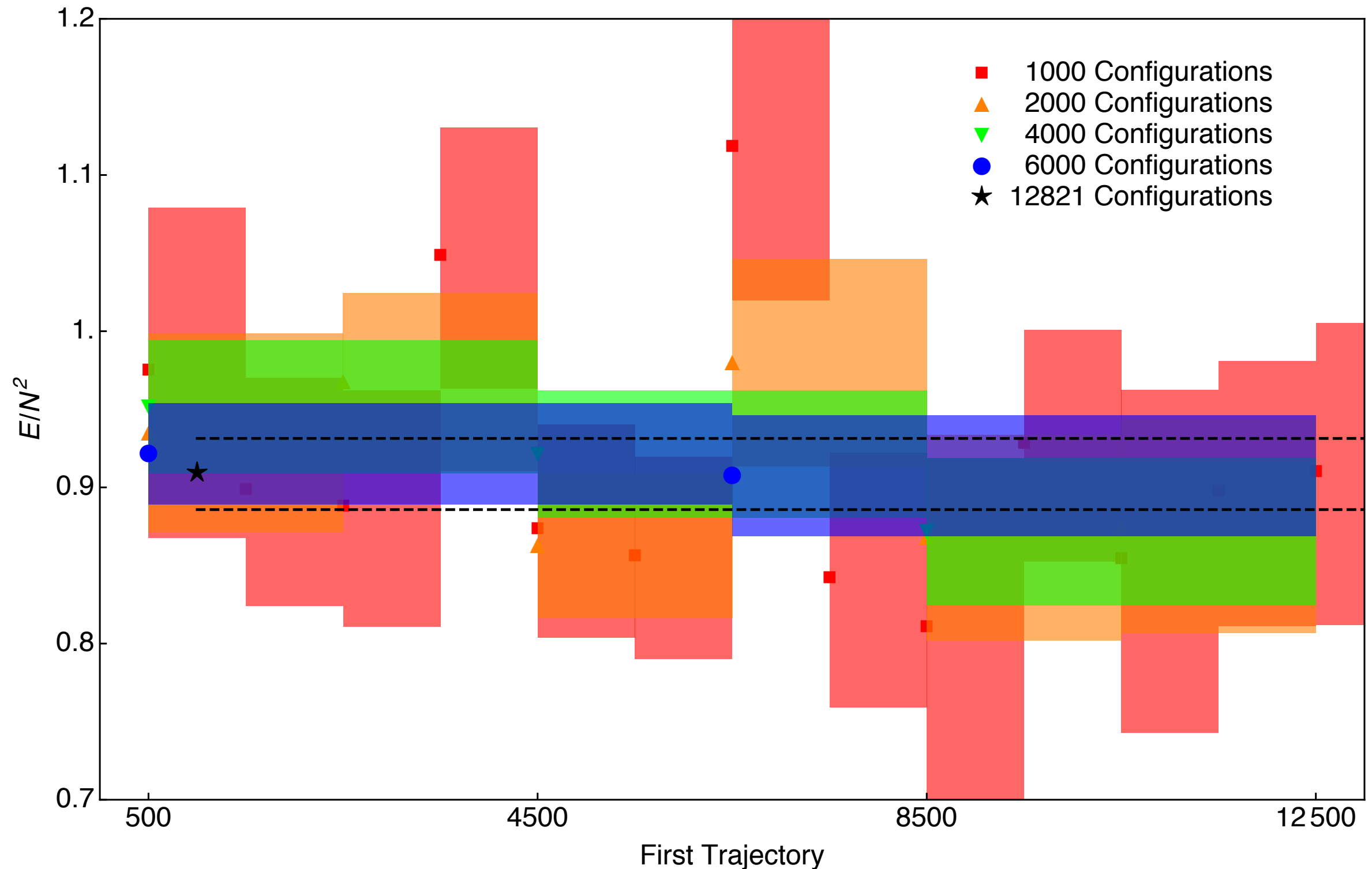
$T=0.5$ $N=16$ $L=32$



Statistical Stability

MCSMC 1606.04948 1606.04951

$T=0.5$ $N=16$ $L=32$



Compute!



Ensembles

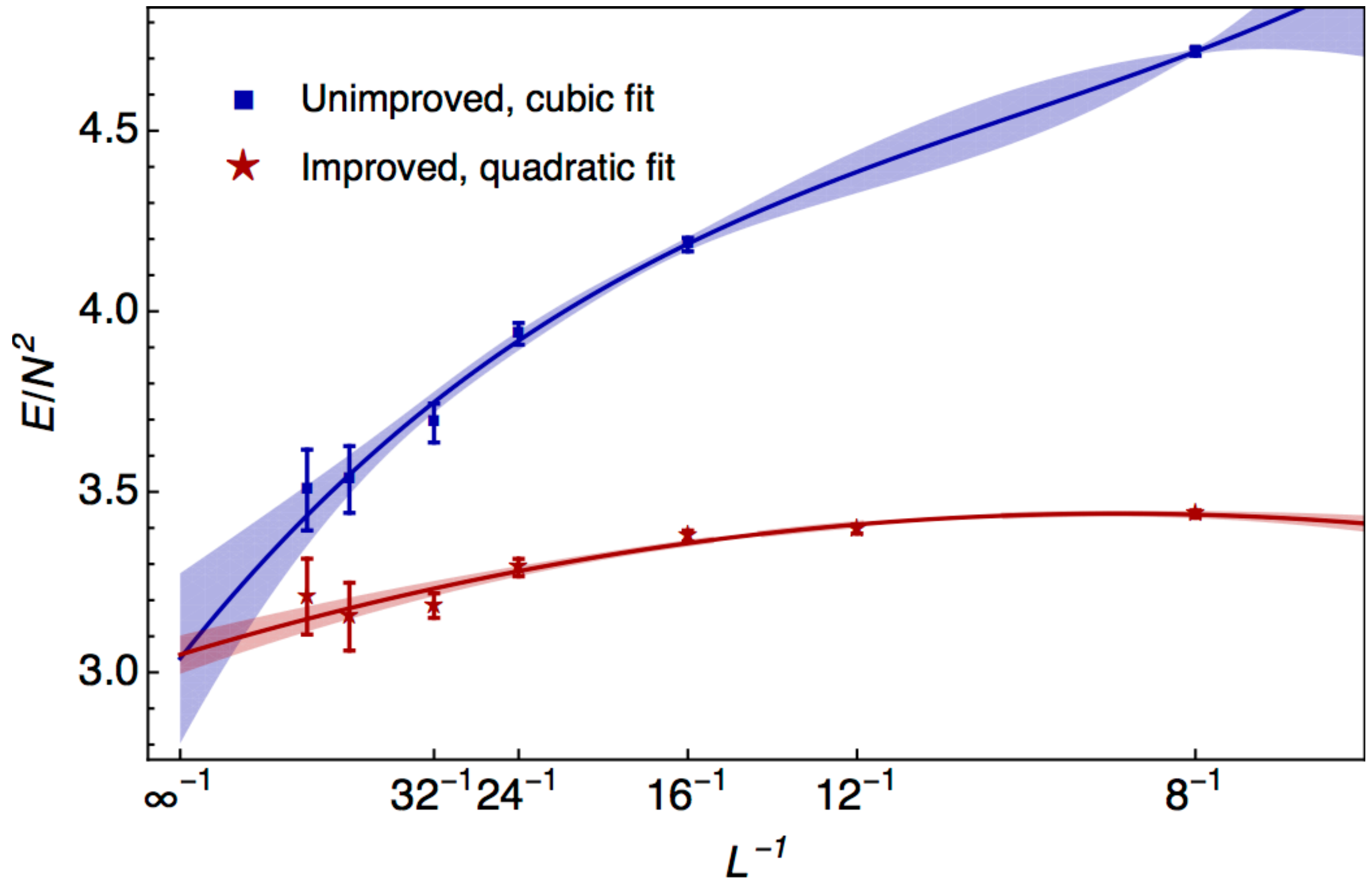
<i>T</i>	<i>N</i>	<i>L</i>	action	<i>N</i> _{cfg}	<i>E</i> / <i>N</i> ²	<i>P</i>	<i>R</i> ²	<i>F</i> ²
0.4	24	16	improved	15935	0.827±0.005	0.72770±0.00035	3.2504±0.0015	14.530±0.002
		24	improved	2321	0.719±0.031	0.72888±0.00129	3.3459±0.0039	15.627±0.011
		32	improved	6625	0.657±0.027	0.72721±0.00116	3.4110±0.0020	16.319±0.008
		32	8	improved	3057	0.903±0.009	0.74150±0.00348	13.846±0.006
	32	12	improved	2491	0.907±0.010	0.72754±0.00089	3.1663±0.0024	13.651±0.005
		16	improved	8242	0.835±0.007	0.72732±0.00054	3.2387±0.0012	14.518±0.003
		24	improved	1331	0.692±0.052	0.72919±0.00453	3.3414±0.0025	15.635±0.012
		32	improved	1888	0.629±0.029	0.72849±0.00142	3.4016±0.0018	16.311±0.008
0.5	16	8	improved	21101	1.229±0.004	0.78847±0.00031	3.1104±0.0026	13.068±0.003
		12	improved	17201	1.140±0.007	0.79566±0.00032	3.2304±0.0014	14.374±0.003
		16	improved	17933	1.081±0.009	0.79599±0.00035	3.3086±0.0012	15.207±0.004
		32	improved	15101	0.907±0.020	0.79689±0.00049	3.4747±0.0017	16.897±0.006
	24	8	improved	20951	1.243±0.004	0.78964±0.00028	3.0776±0.0007	13.038±0.002
		16	improved	19765	1.092±0.006	0.79718±0.00020	3.2883±0.0005	15.194±0.002
		24	improved	14957	0.979±0.010	0.79741±0.00029	3.3898±0.0006	16.240±0.003
		32	improved	10469	0.941±0.024	0.79727±0.00051	3.4457±0.0012	16.851±0.007
	32	8	improved	16253	1.248±0.003	0.78995±0.00020	3.0712±0.0006	13.032±0.002
		12	improved	3569	1.155±0.010	0.79600±0.00049	3.2012±0.0010	14.357±0.004
		16	improved	7885	1.093±0.009	0.79730±0.00034	3.2830±0.0007	15.196±0.003
		24	improved	2873	0.946±0.047	0.79852±0.00123	3.3815±0.0032	16.223±0.012
		32	improved	5469	0.955±0.023	0.79833±0.00044	3.4386±0.0011	16.841±0.006
0.6	16	8	improved	27221	1.560±0.005	0.83423±0.00018	3.1410±0.0006	13.728±0.002
		12	improved	19051	1.475±0.007	0.84077±0.00021	3.2708±0.0008	15.001±0.003
		16	improved	18141	1.432±0.010	0.84156±0.00023	3.3477±0.0010	15.790±0.004
		24	improved	8977	1.339±0.021	0.84184±0.00034	3.4410±0.0020	16.754±0.008

T	N	L	action	N_{cfg}	E/N^2	$ P $	R^2	F^2	
0.6	16	32	improved	18677	1.267±0.021	0.84181±0.00028	3.4951±0.0014	17.327±0.006	
		24	8	improved	23971	1.569±0.004	0.83474±0.00017	3.1290±0.0005	13.731±0.002
	32	12	improved	19171	1.481±0.007	0.84083±0.00018	3.2602±0.0007	15.012±0.003	
		16	improved	19961	1.429±0.008	0.84205±0.00018	3.3349±0.0006	15.790±0.003	
		24	improved	25249	1.346±0.009	0.84176±0.00015	3.4262±0.0006	16.753±0.003	
		32	improved	12577	1.276±0.025	0.84212±0.00030	3.4780±0.0012	17.309±0.007	
		8	improved	19017	1.575±0.005	0.83539±0.00024	3.1248±0.0007	13.731±0.003	
		16	improved	10071	1.442±0.009	0.84182±0.00022	3.3306±0.0006	15.787±0.004	
0.7	16	8	improved	30641	1.959±0.005	0.86564±0.00013	3.1941±0.0006	14.377±0.003	
		12	improved	20051	1.885±0.008	0.87096±0.00014	3.3145±0.0008	15.579±0.004	
		16	improved	20187	1.843±0.011	0.87181±0.00015	3.3891±0.0009	16.333±0.005	
		24	improved	10605	1.763±0.022	0.87126±0.00024	3.4702±0.0018	17.214±0.009	
		32	unimproved	21633	2.344±0.031	0.86981±0.00017	3.5681±0.0015	18.615±0.007	
			improved	19921	1.672±0.023	0.87191±0.00021	3.5193±0.0014	17.739±0.007	
	24	8	improved	20701	1.968±0.005	0.86574±0.00014	3.1854±0.0005	14.385±0.003	
		12	improved	19997	1.893±0.008	0.87095±0.00013	3.3088±0.0007	15.601±0.004	
		16	improved	21451	1.849±0.007	0.87203±0.00012	3.3789±0.0006	16.338±0.004	
		24	improved	28925	1.755±0.011	0.87126±0.00012	3.4634±0.0007	17.237±0.004	
		32	improved	16135	1.682±0.025	0.87186±0.00019	3.5069±0.0012	17.726±0.008	
		32	8	improved	18989	1.966±0.006	0.86612±0.00017	3.1837±0.0007	14.394±0.004
			16	improved	10849	1.850±0.010	0.87187±0.00015	3.3771±0.0008	16.341±0.004
		0.8	16	8	unimproved	19281	3.674±0.009	0.89048±0.00013	3.5758±0.0028
	improved			19171	2.400±0.008	0.88790±0.00012	3.2468±0.0007	14.994±0.004	
12			improved	22001	2.356±0.009	0.89220±0.00011	3.3601±0.0008	16.138±0.005	
	16		unimproved	24081	3.283±0.016	0.89113±0.00011	3.5305±0.0011	18.305±0.006	
			improved	21421	2.309±0.012	0.89318±0.00011	3.4295±0.0010	16.856±0.005	
24	unimproved		21591	3.000±0.025	0.89069±0.00012	3.5588±0.0012	18.663±0.007		
			improved	17521	2.214±0.019	0.89292±0.00014	3.5070±0.0013	17.687±0.008	
32	unimproved		14157	2.710±0.044	0.89119±0.00017	3.5803±0.0016	18.880±0.010		

<i>T</i>	<i>N</i>	<i>L</i>	action	<i>N</i> _{cfg}	<i>E</i> / <i>N</i> ²	<i>P</i>	<i>R</i> ²	<i>F</i> ²
0.8	16	32	improved	20187	2.107±0.024	0.89261±0.00015	3.5452±0.0014	18.131±0.009
		24	8	improved	22151	2.417±0.006	0.88779±0.00011	3.2416±0.0006
			12	improved	18175	2.346±0.009	0.89207±0.00011	3.3553±0.0008
		16	improved	20721	2.303±0.010	0.89317±0.00009	3.4232±0.0007	16.868±0.004
	32	24	improved	9153	2.216±0.020	0.89239±0.00015	3.4968±0.0012	17.681±0.008
		32	improved	13867	2.176±0.028	0.89287±0.00015	3.5366±0.0015	18.134±0.009
		8	improved	19213	2.424±0.007	0.88809±0.00014	3.2394±0.0008	15.012±0.004
			16	improved	13495	2.340±0.017	0.89302±0.00018	3.4216±0.0014
0.9	16	8	unimproved	20411	4.221±0.010	0.90519±0.00009	3.5113±0.0012	17.926±0.005
			improved	20401	2.895±0.009	0.90433±0.00009	3.3002±0.0008	15.599±0.004
		12	improved	20501	2.863±0.011	0.90783±0.00009	3.4048±0.0009	16.679±0.005
		16	unimproved	37701	3.710±0.014	0.90689±0.00007	3.5475±0.0007	18.635±0.005
	24		improved	21461	2.796±0.014	0.90856±0.00009	3.4717±0.0010	17.377±0.006
		32	unimproved	21851	3.450±0.028	0.90639±0.00010	3.5807±0.0013	19.001±0.008
			improved	17955	2.716±0.021	0.90826±0.00011	3.5400±0.0014	18.139±0.009
		32	unimproved	12221	3.243±0.049	0.90720±0.00013	3.6037±0.0019	19.209±0.012
			improved	14601	2.673±0.031	0.90849±0.00013	3.5750±0.0017	18.545±0.011
	32	8	improved	24331	2.931±0.006	0.90413±0.00008	3.2958±0.0006	15.613±0.004
		12	improved	17557	2.872±0.010	0.90785±0.00009	3.4025±0.0009	16.707±0.006
		16	improved	24441	2.822±0.010	0.90861±0.00007	3.4658±0.0007	17.384±0.005
		24	improved	8917	2.761±0.023	0.90774±0.00013	3.5349±0.0014	18.156±0.009
	32	32	improved	16709	2.719±0.028	0.90824±0.00010	3.5663±0.0014	18.538±0.009
		8	improved	18695	2.931±0.008	0.90441±0.00011	3.2948±0.0009	15.620±0.005
			16	improved	12061	2.835±0.019	0.90836±0.00016	3.4643±0.0016
		16	unimproved	21291	4.719±0.011	0.91705±0.00007	3.5139±0.0010	18.245±0.006
			improved	20641	3.439±0.010	0.91672±0.00008	3.3515±0.0009	16.185±0.005
		12	improved	20751	3.397±0.012	0.91968±0.00008	3.4485±0.0010	17.217±0.006
			unimproved	25379	4.185±0.019	0.91907±0.00007	3.5769±0.0010	19.034±0.007
		16	improved	21641	3.378±0.015	0.92033±0.00008	3.5115±0.0011	17.876±0.007

T	N	L	action	N_{cfg}	E/N^2	$ P $	R^2	F^2
1.0	16	24	unimproved	23391	3.937±0.030	0.91855±0.00008	3.6099±0.0013	19.376±0.009
			improved	17469	3.290±0.024	0.91979±0.00009	3.5755±0.0017	18.600±0.011
		32	unimproved	13503	3.690±0.054	0.91895±0.00012	3.6323±0.0019	19.582±0.013
			improved	15555	3.185±0.034	0.91985±0.00011	3.6068±0.0018	18.971±0.013
	48	unimproved	12026	3.534±0.092	0.92064±0.00014	3.6592±0.0028	19.810±0.021	
			improved	5772	3.154±0.094	0.92101±0.00025	3.6431±0.0042	19.401±0.027
		64	unimproved	15024	3.505±0.112	0.92051±0.00015	3.6808±0.0029	19.999±0.020
			improved	7280	3.210±0.105	0.92070±0.00021	3.6752±0.0042	19.715±0.030
	24	8	improved	22605	3.467±0.007	0.91663±0.00007	3.3490±0.0007	16.208±0.004
		12	improved	19817	3.413±0.011	0.91956±0.00007	3.4478±0.0009	17.251±0.006
		16	improved	20655	3.363±0.013	0.92041±0.00007	3.5084±0.0009	17.901±0.006
		24	improved	21725	3.283±0.019	0.91964±0.00008	3.5705±0.0011	18.609±0.008
32	8	improved	19383	3.268±0.028	0.91997±0.00009	3.5993±0.0014	18.964±0.010	
	16	improved	17881	3.469±0.008	0.91684±0.00009	3.3482±0.0009	16.216±0.006	
	16	improved	14915	3.386±0.019	0.92018±0.00011	3.5078±0.0013	17.902±0.009	

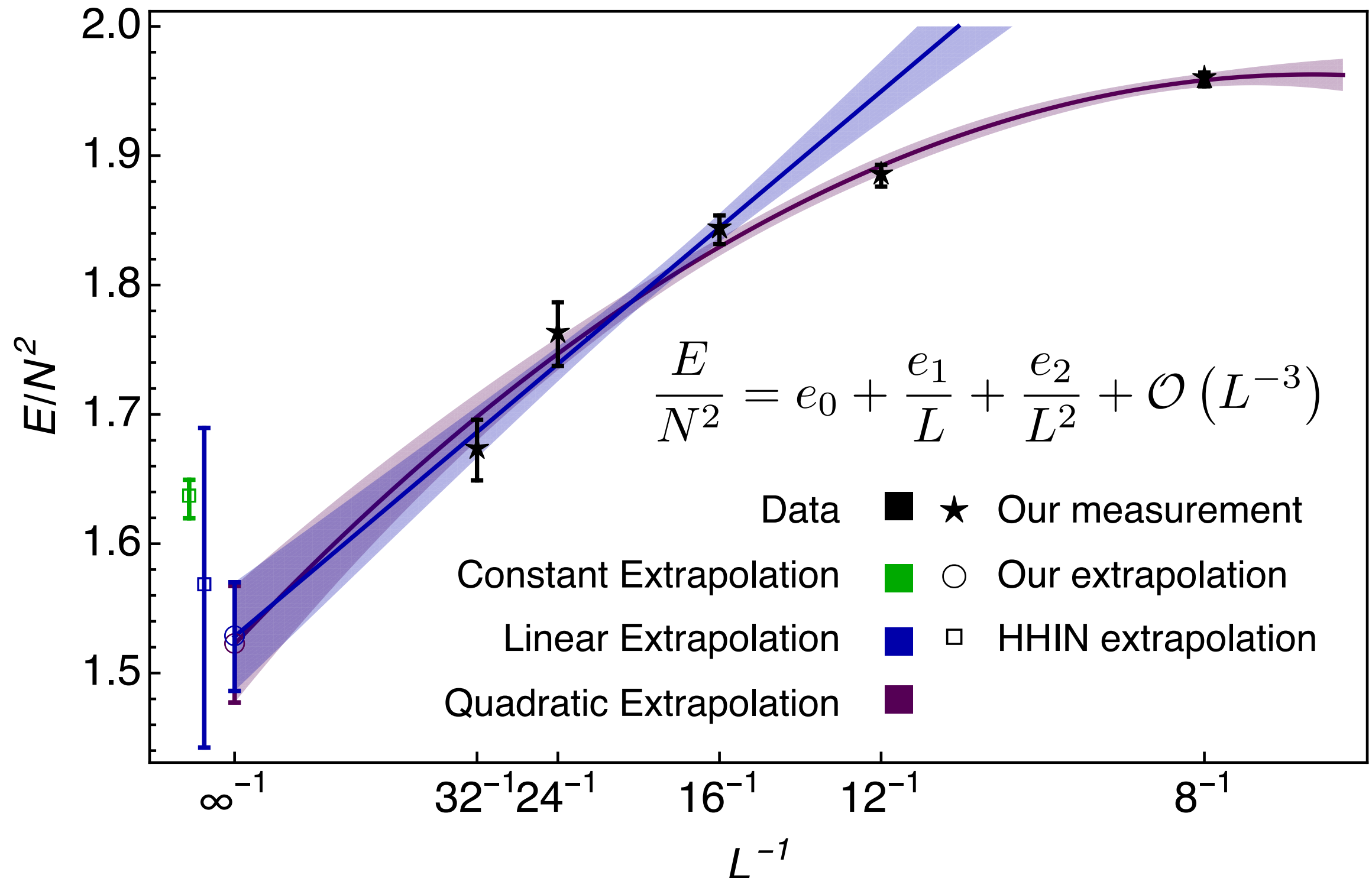
Derivative Improvement



Fixed- N Continuum Extrapolation

MCSMC 1606.04948 1606.04951

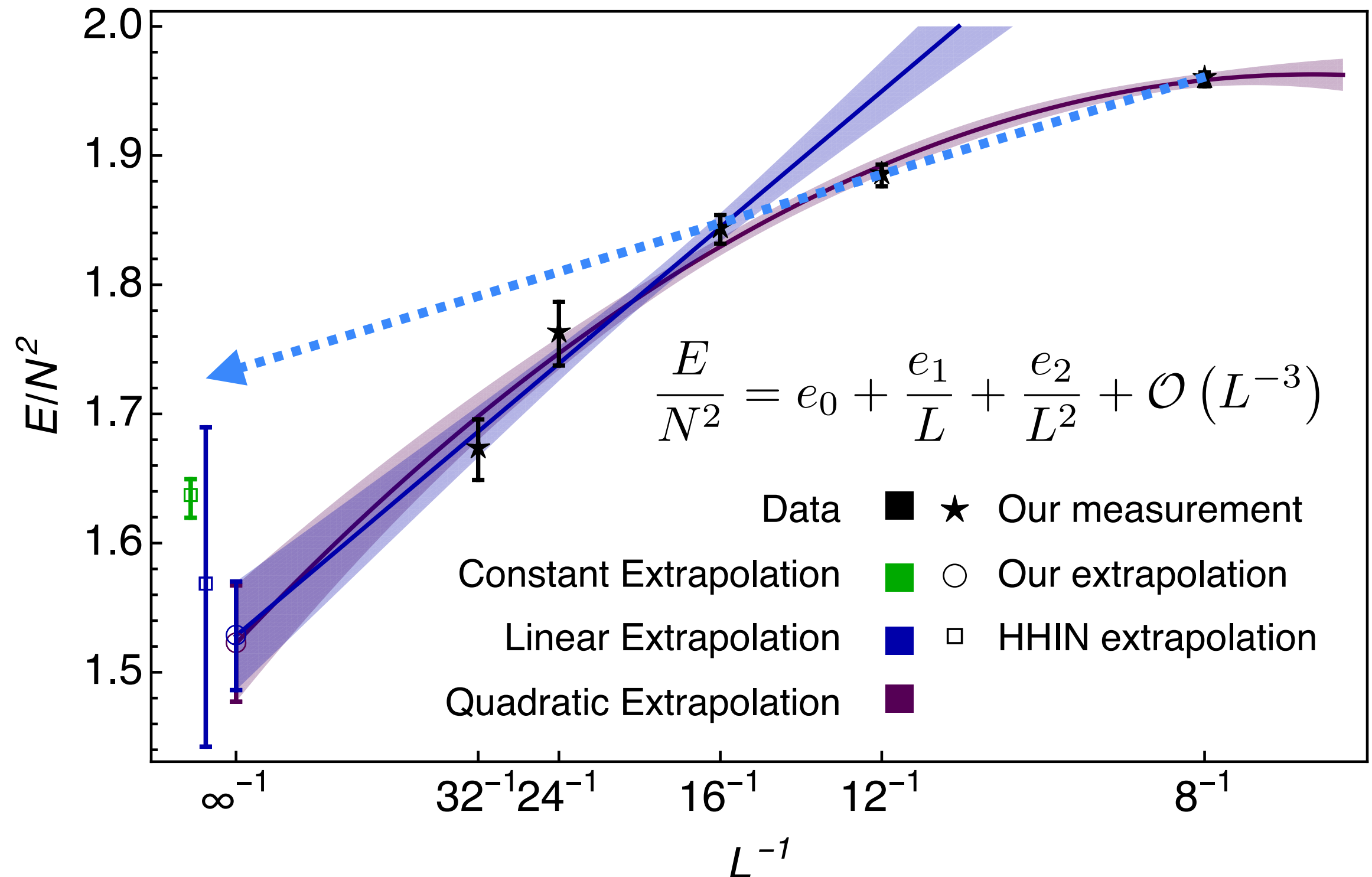
$T=0.7$ $N=16$



Fixed- N Continuum Extrapolation

MCSMC 1606.04948 1606.04951

$T=0.7$ $N=16$



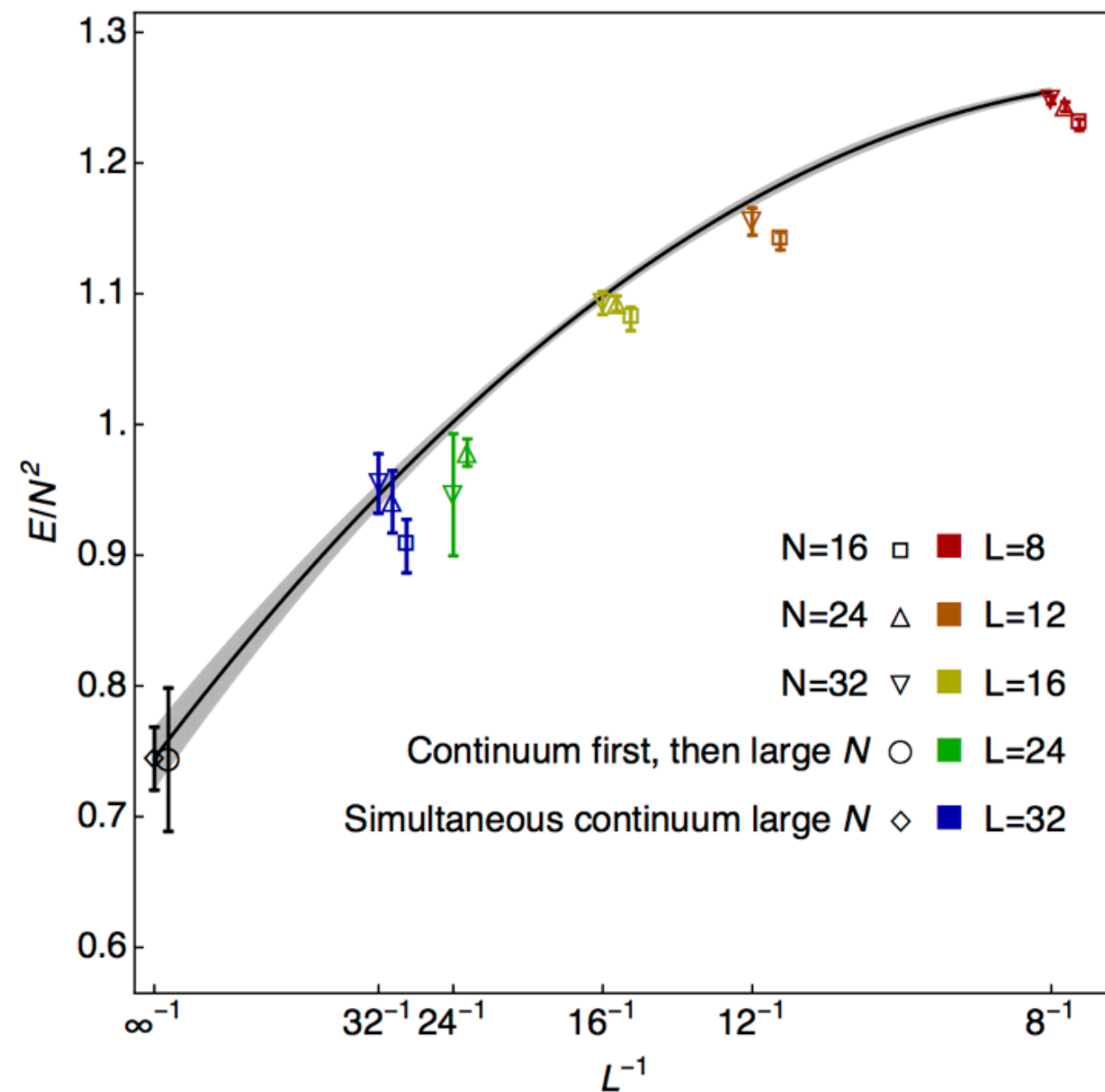
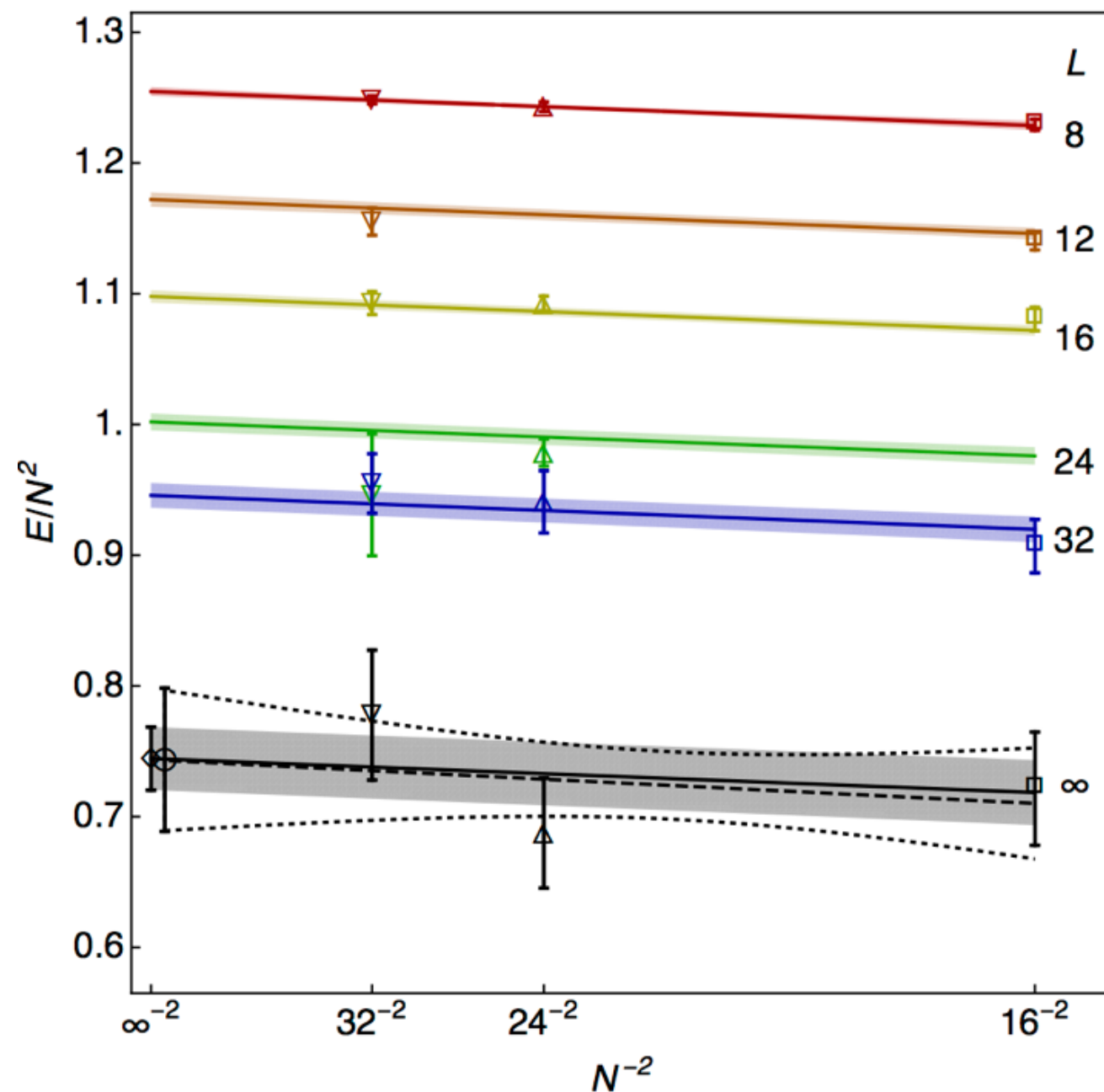
Simultaneous Continuum + Large- N Extrapolation

MCSMC 1606.04948 1606.04951

$$\frac{E}{N^2} = \sum_{ij \geq 0} \frac{e_{ij}}{N^{2i} L^j}$$

$e_{00}, e_{01}, e_{02}, e_{10}$

$T=0.5$



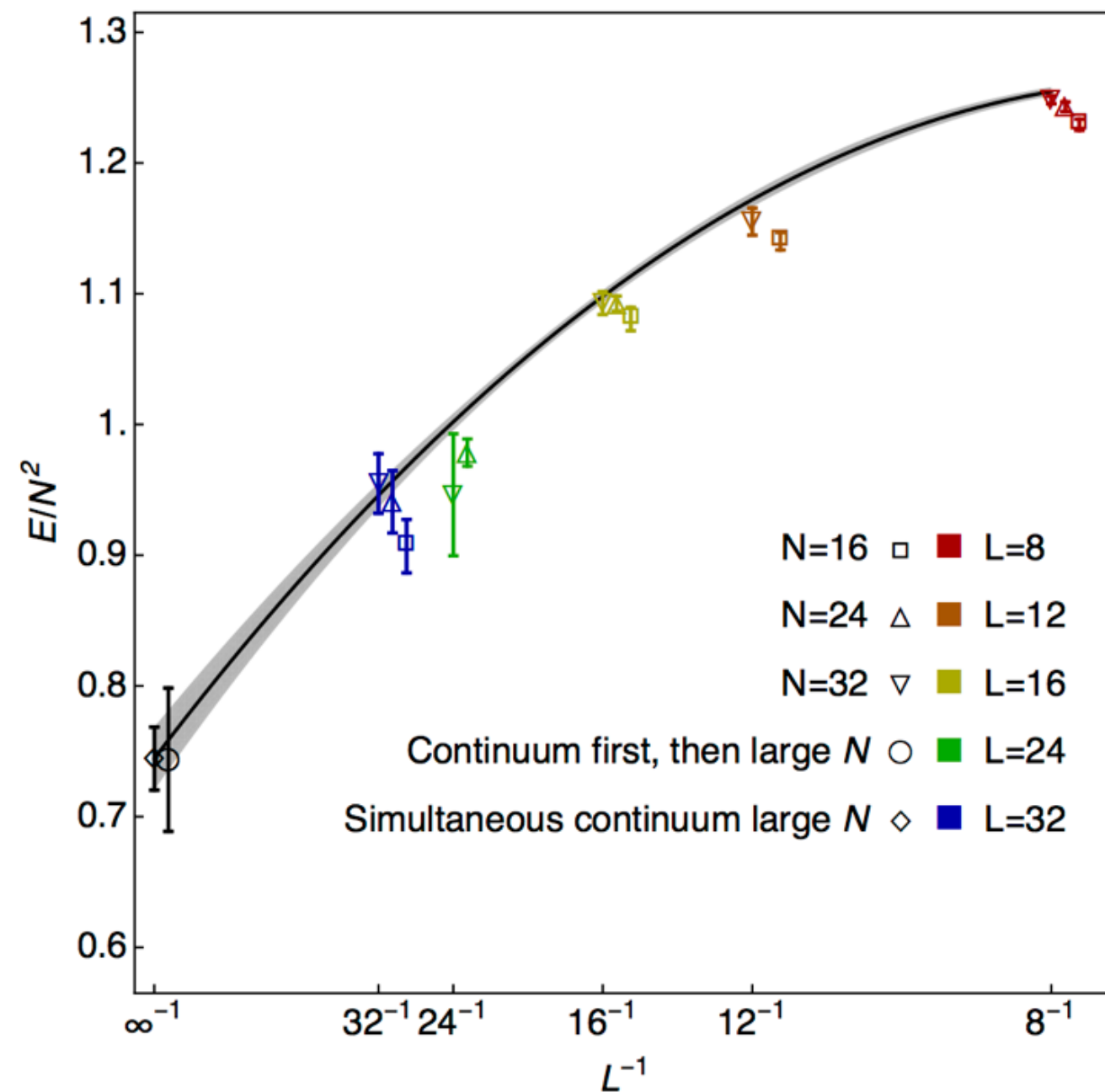
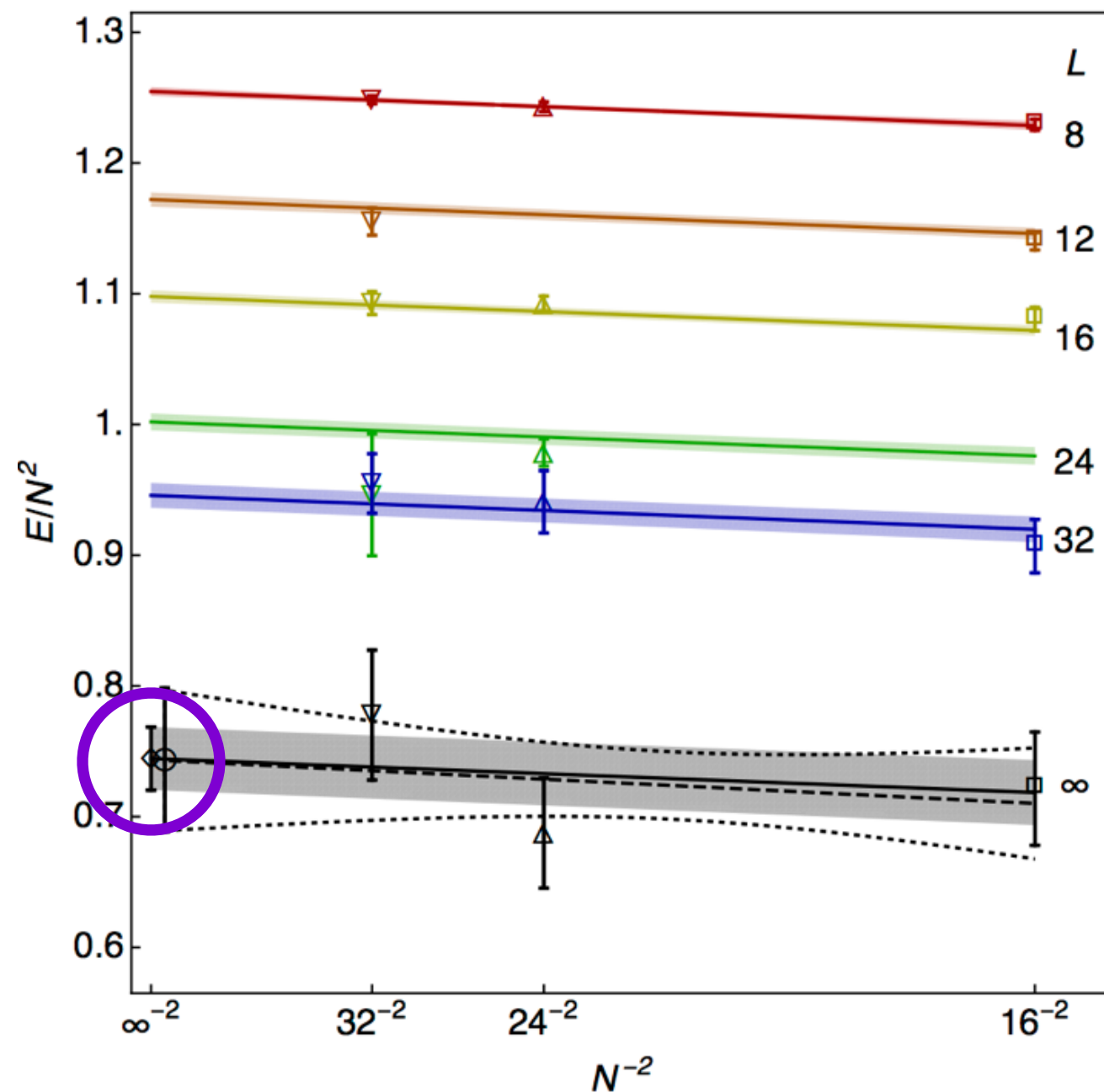
Simultaneous Continuum + Large- N Extrapolation

MCSMC 1606.04948 1606.04951

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e_{00} e_{01}, e_{02}, e_{10}

$T=0.5$



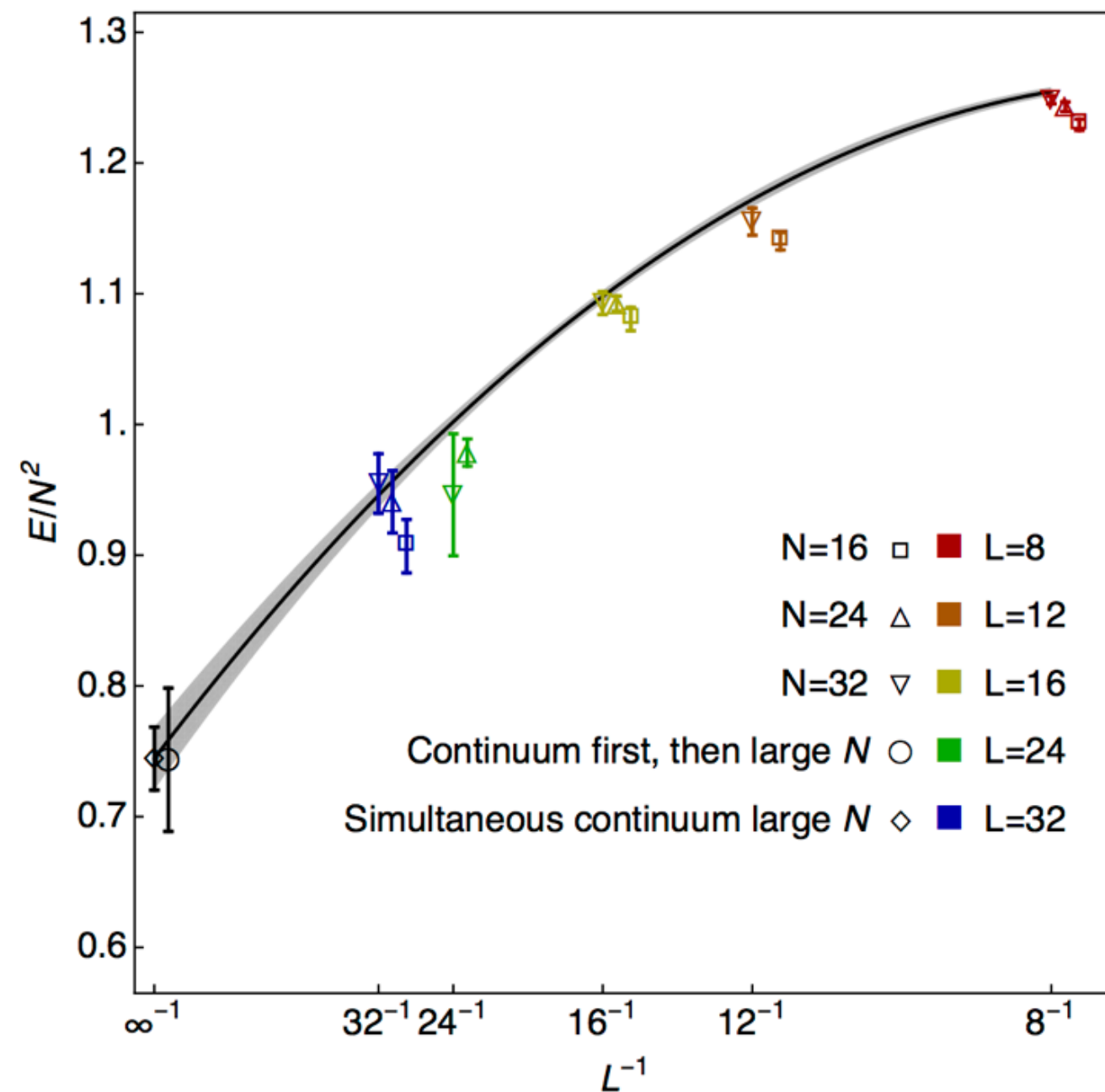
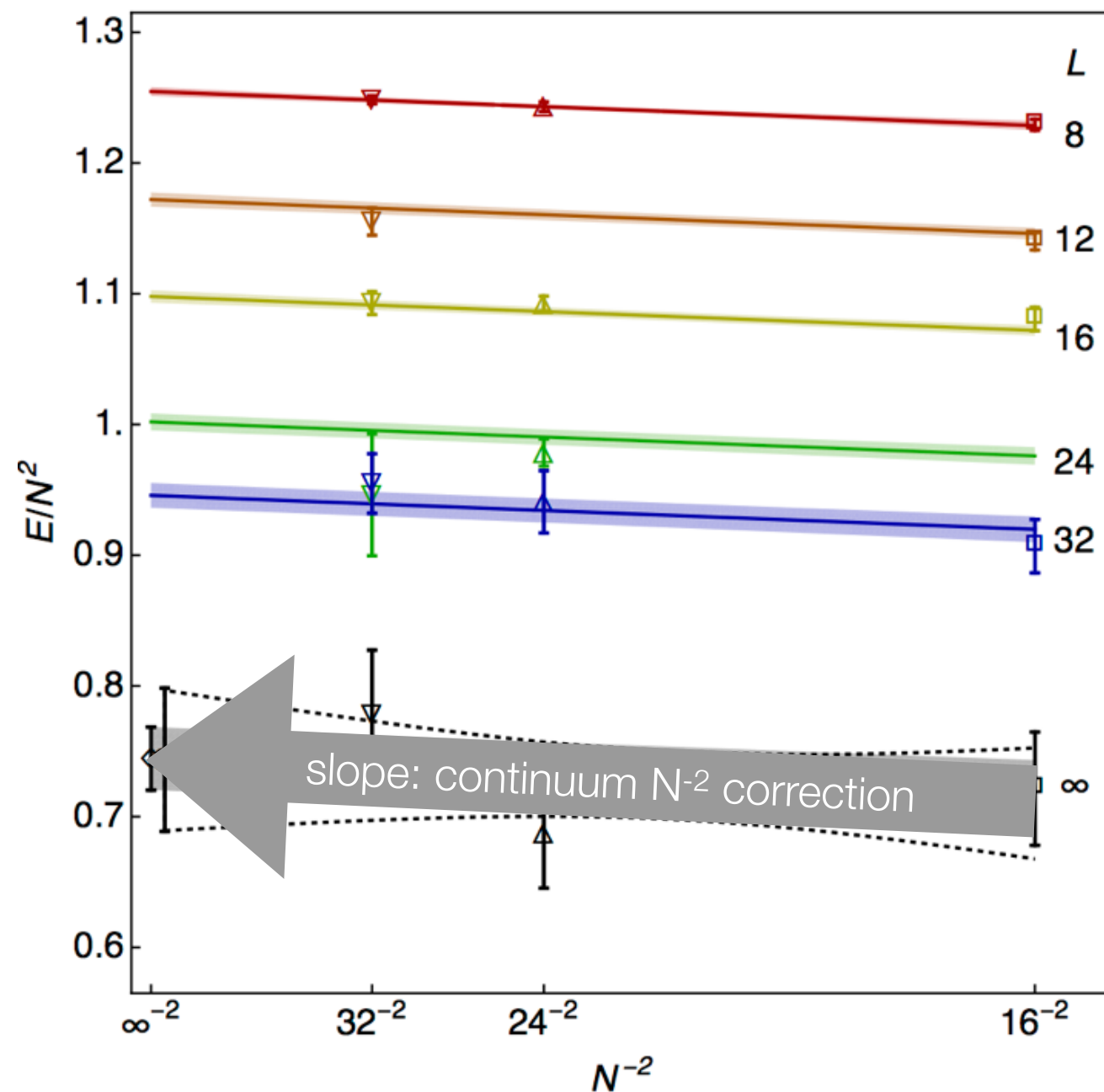
Simultaneous Continuum + Large- N Extrapolation

MCSMC 1606.04948 1606.04951

$$\frac{E}{N^2} = \sum_{ij \geq 0} \frac{e_{ij}}{N^{2i} L^j}$$

$e_{00}, e_{01}, e_{02}, e_{10}$

$T=0.5$



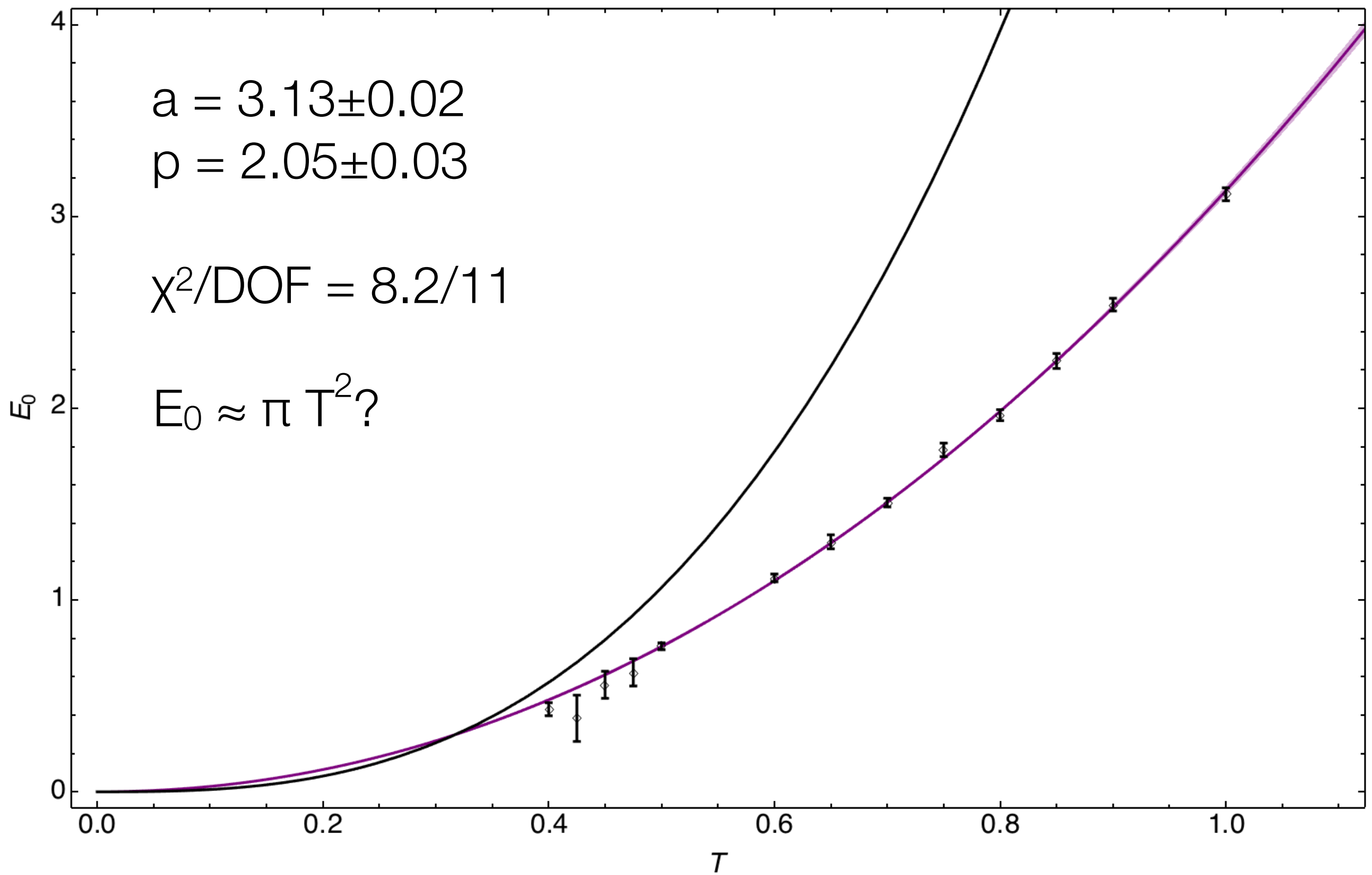
No stringy input: $a T^p$?

$$a = 3.13 \pm 0.02$$

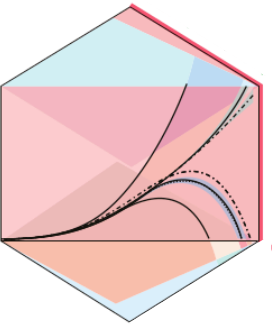
$$p = 2.05 \pm 0.03$$

$$\chi^2/\text{DOF} = 8.2/11$$

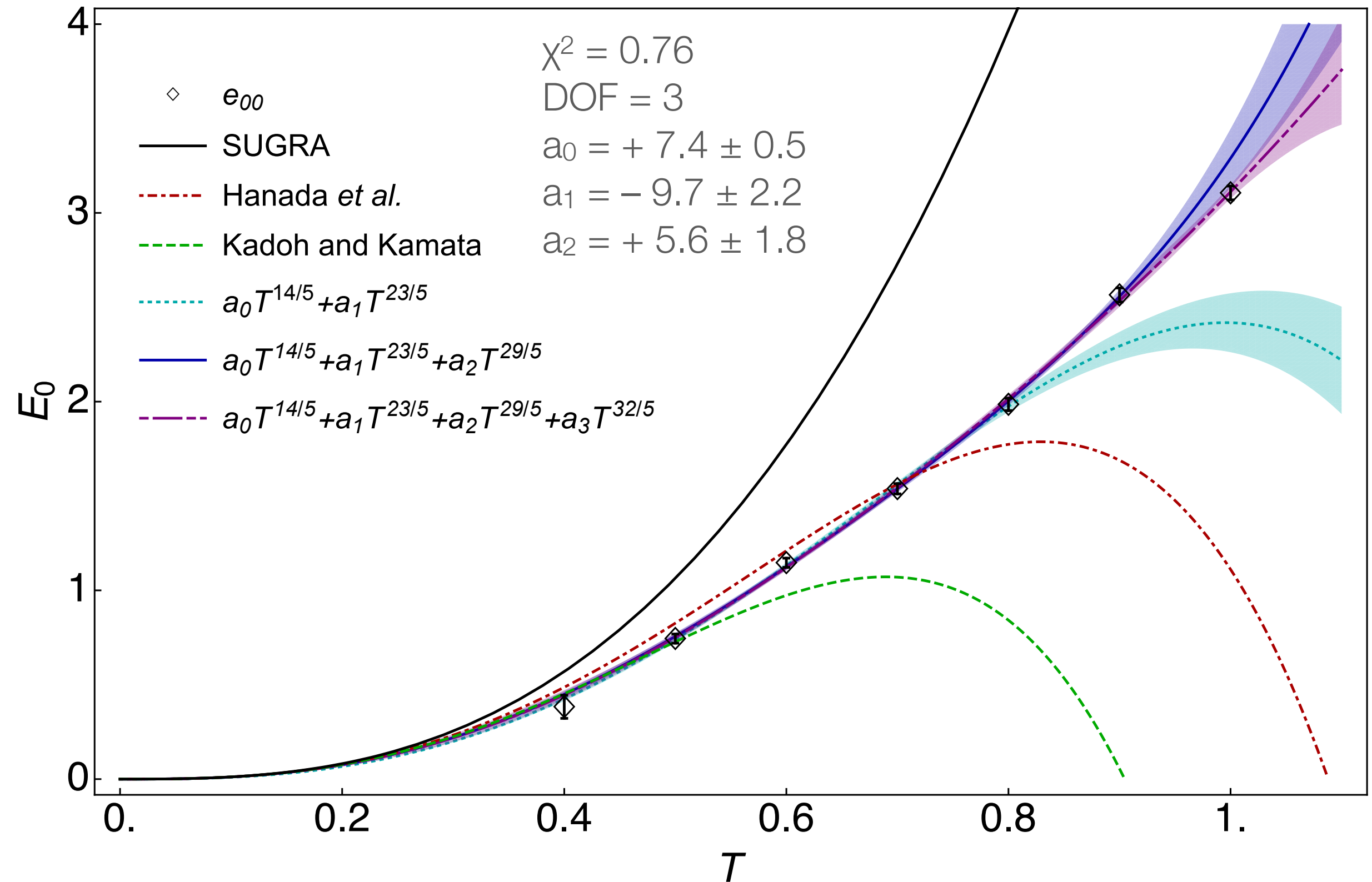
$$E_0 \approx \pi T^2?$$



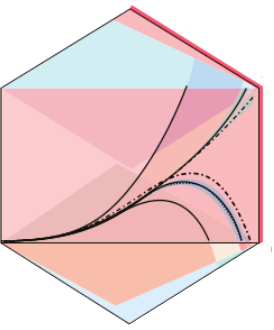
$$E/N^2 = N^0 \left(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \dots \right) + \mathcal{O} \left(N^{-2} \right)$$



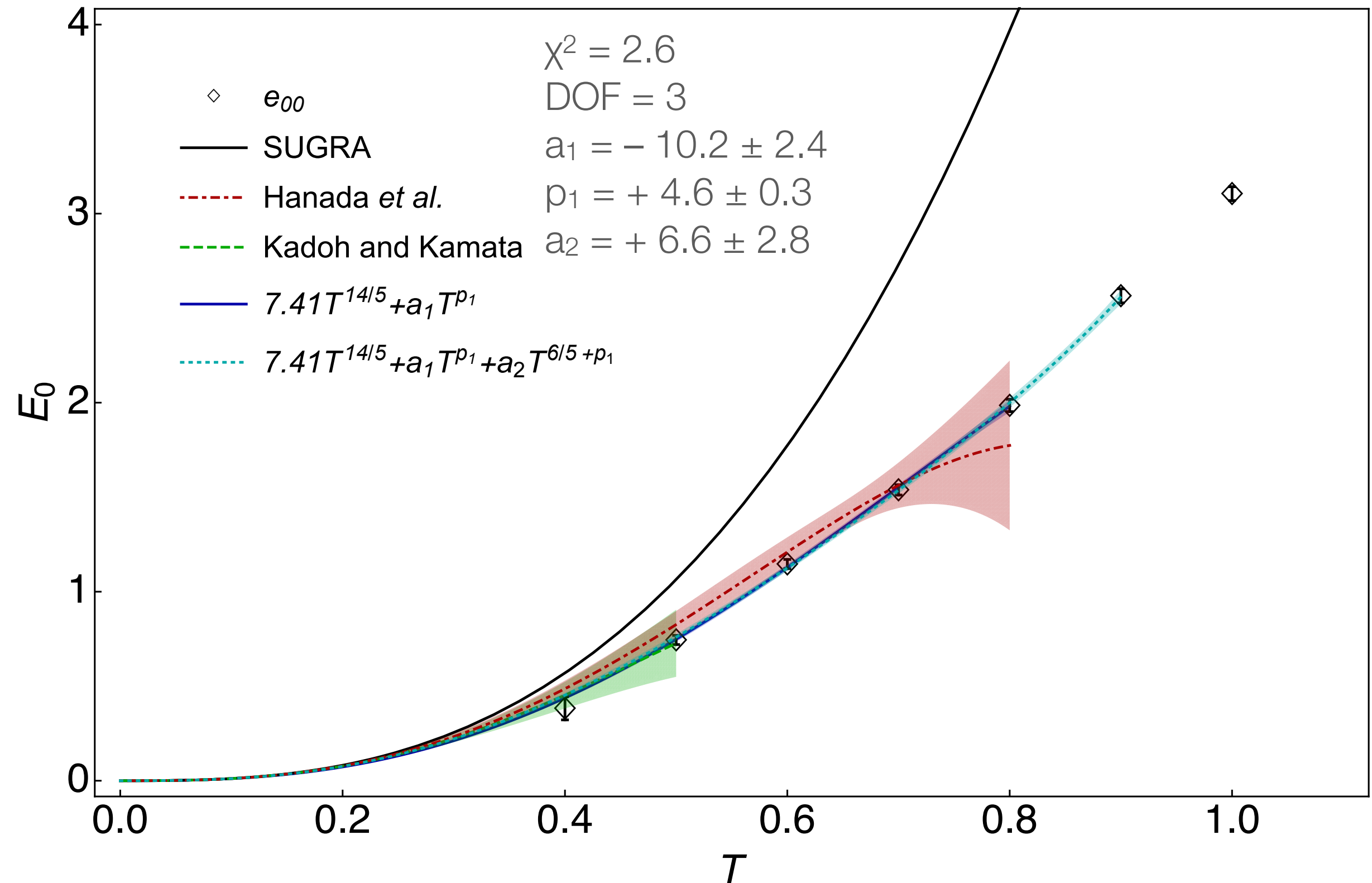
MCSMC 1606.04948 1606.04951



$$E/N^2 = N^0 \left(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \dots \right) + \mathcal{O} \left(N^{-2} \right)$$



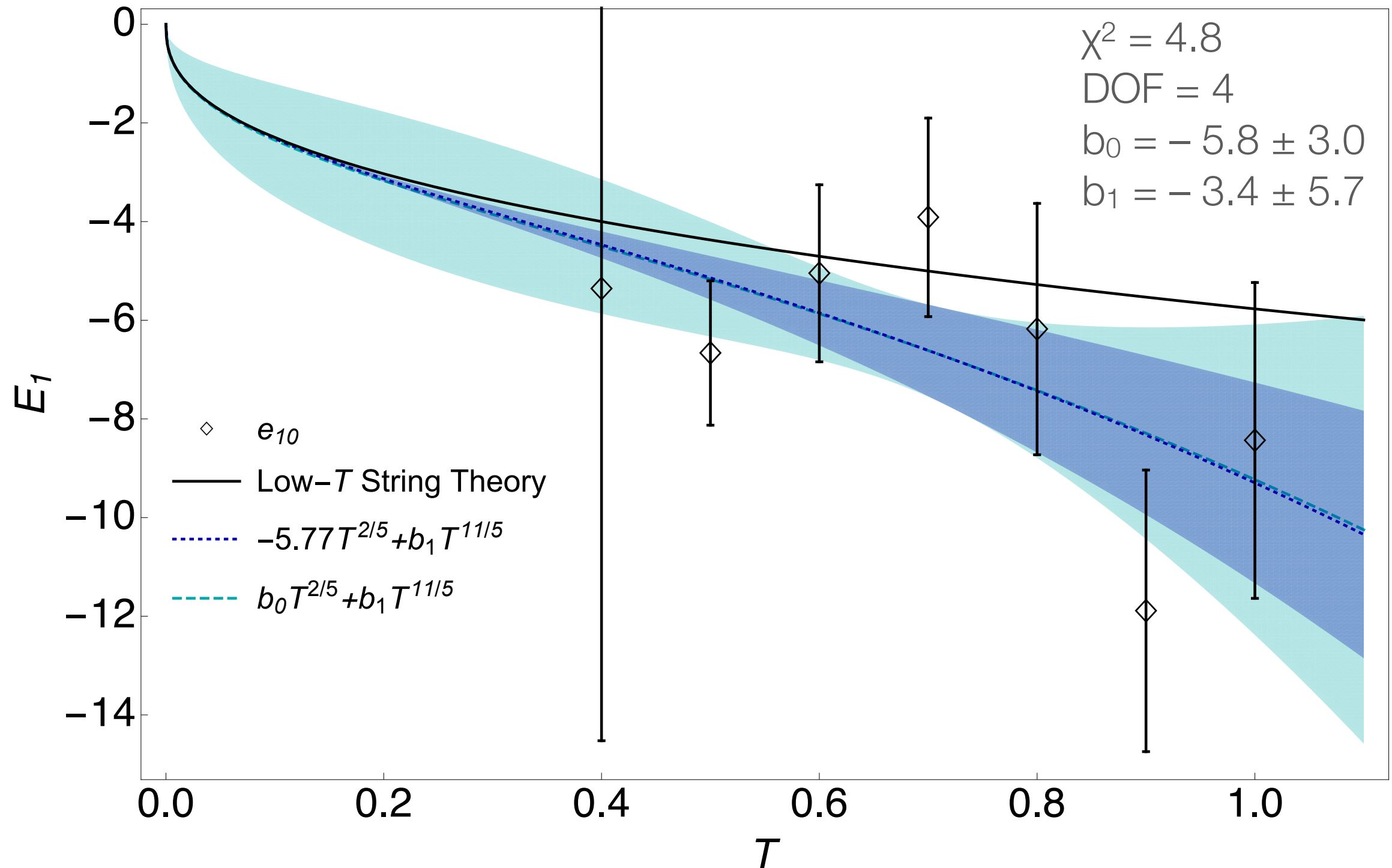
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$$\mathcal{O}(N^{-2}) = N^{-2} (b_0 T^{0.4} + b_1 T^{2.2} + \dots)$$

← slope: continuum N^{-2} correction

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Summary

Quantum Gravity



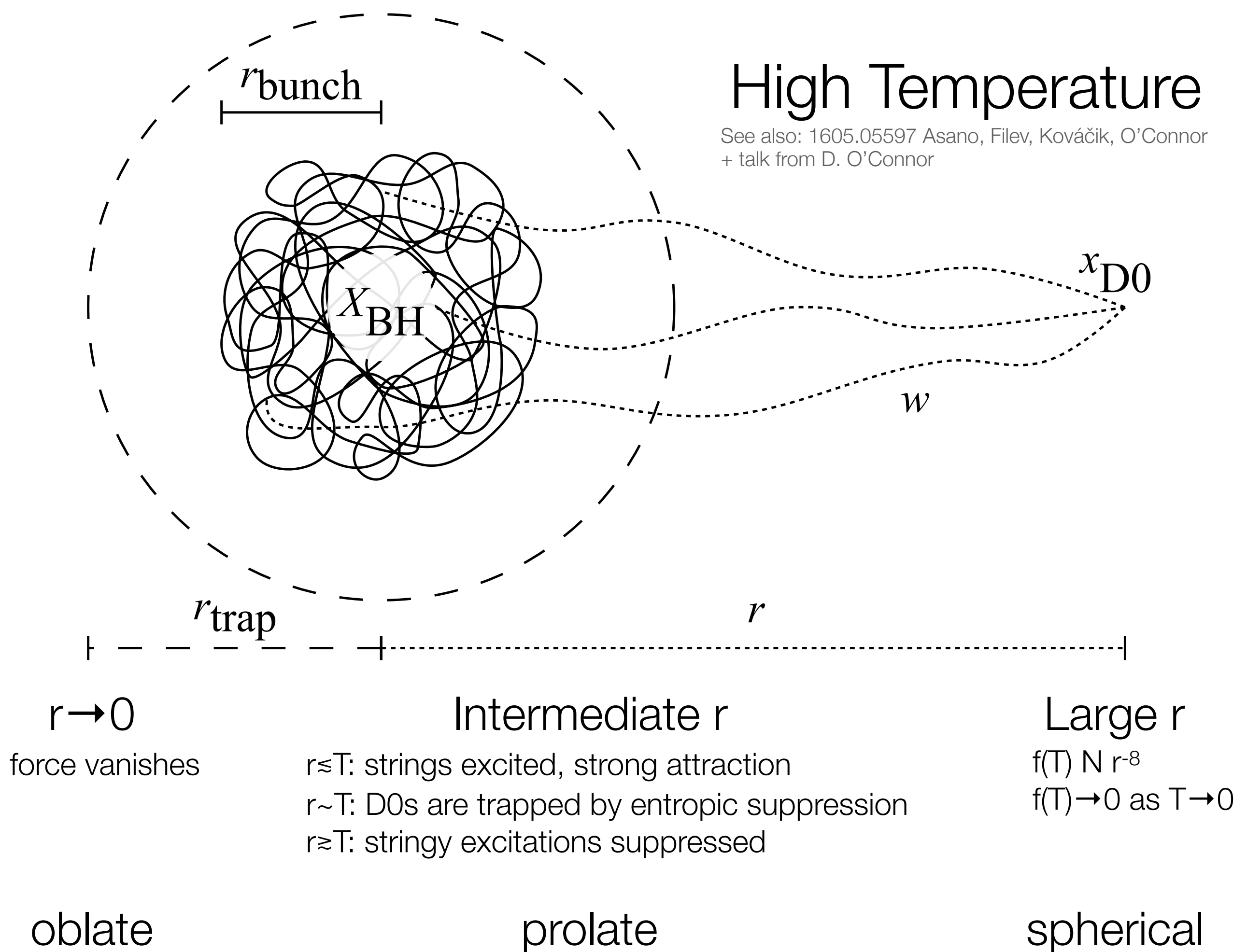
Gauge Theory

- 0+1D BFSS reproduces known 10D SUGRA result
- Nontrivial checks of gauge / gravity duality
- Predictions about (quantum!) stringy corrections.

Backup Slides

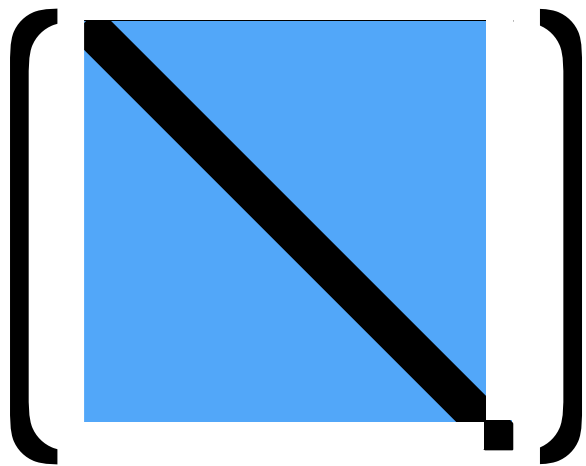
High Temperature

See also: 1605.05597 Asano, Filev, Kováčik, O'Connor
+ talk from D. O'Connor



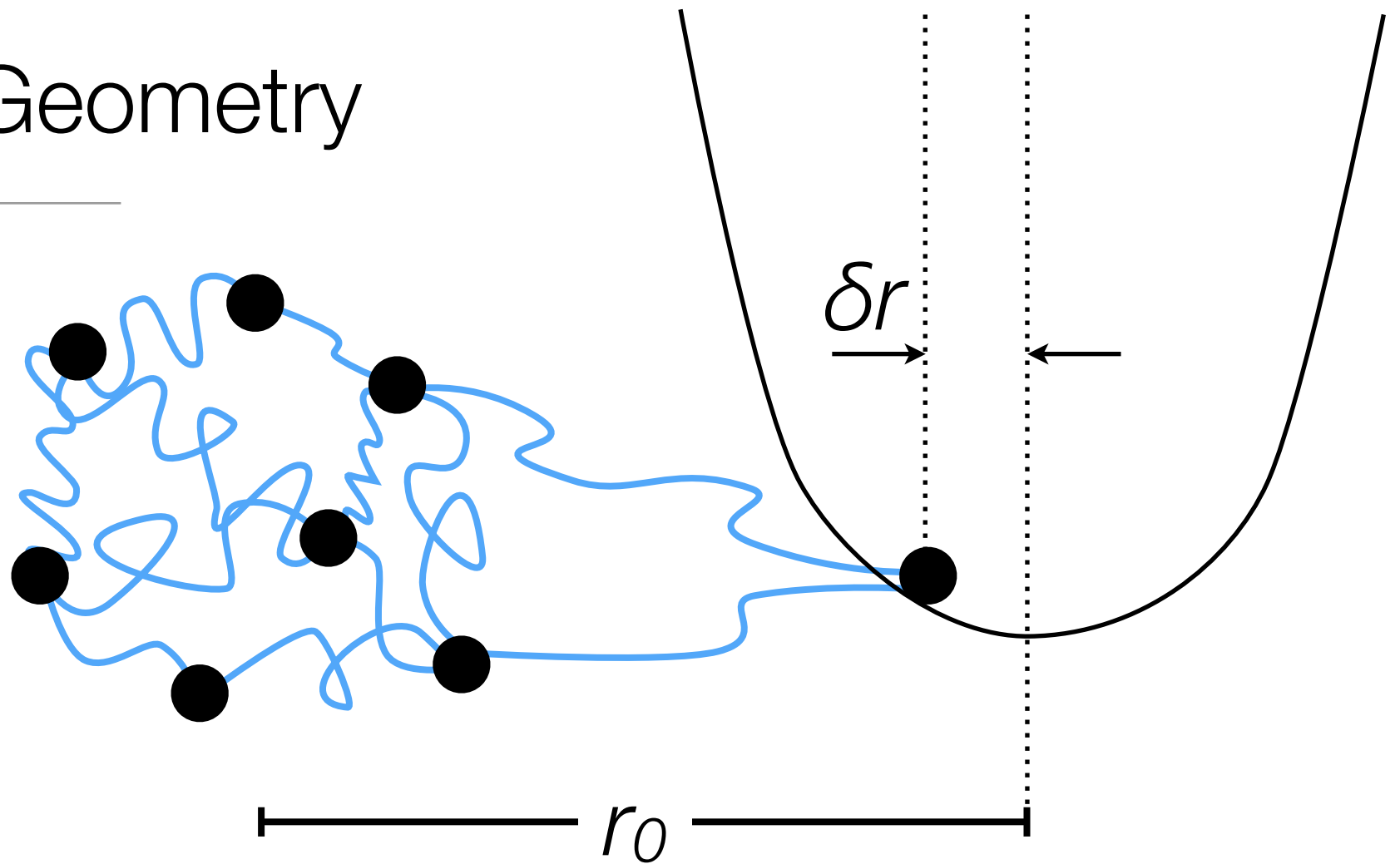
Force / Emergent Geometry

MCSMC 1709.01932



BH + probe D0

$$X^M = \begin{pmatrix} X_{\text{BH}}^M & w^M \\ w^{\dagger M} & x_{\text{D0}}^M \end{pmatrix}$$



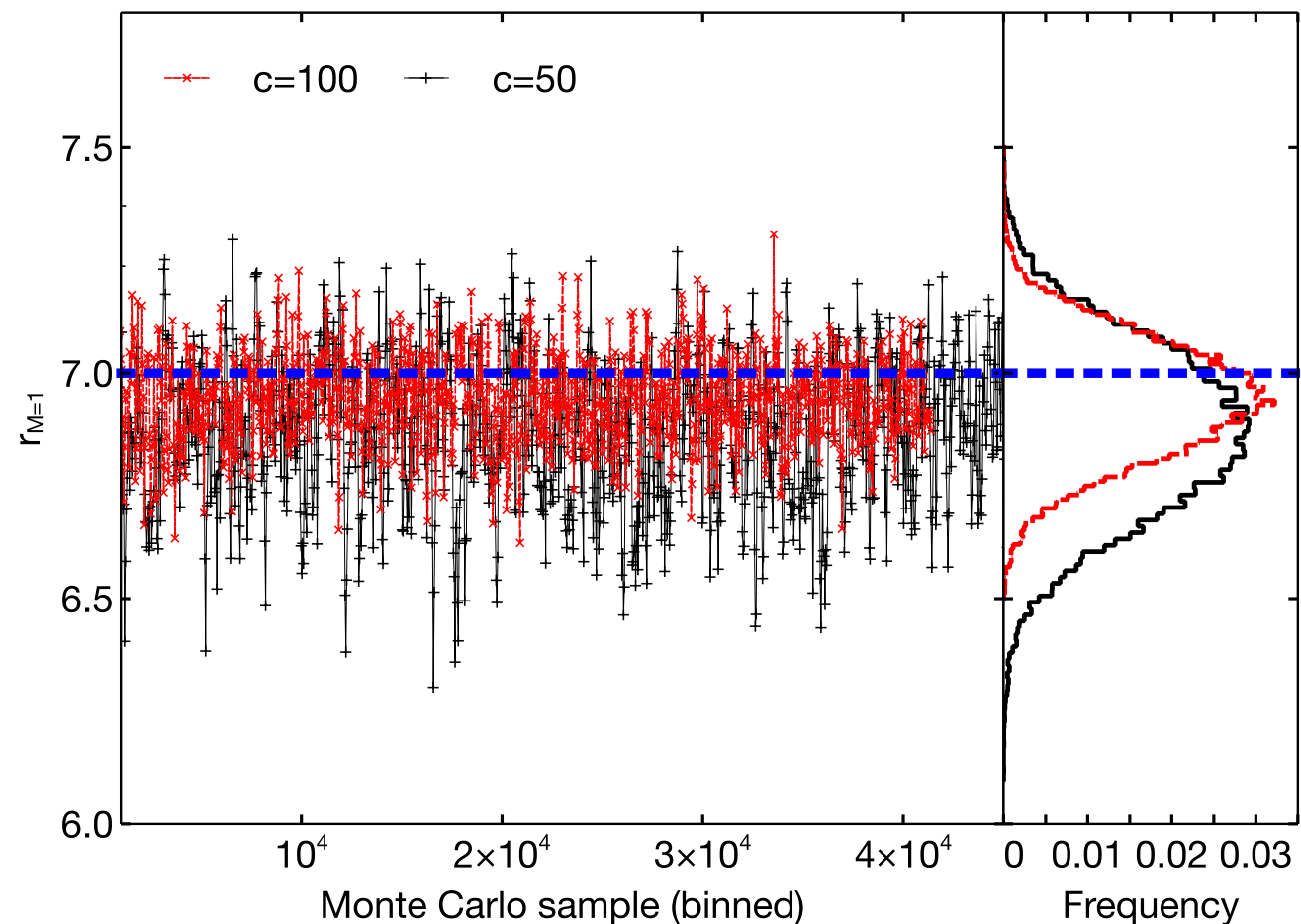
$$\Delta L = -c \left\{ \underbrace{\left(\frac{\text{Tr} X_{\text{BH}}^1}{N-1} - x_{\text{D0}}^1 - r_0 \right)^2}_{\delta r} + \sum_{M=2}^9 \left(\frac{\text{Tr} X_{\text{BH}}^M}{N-1} - x_{\text{D0}}^M \right)^2 \right\} - c' |w_1|^2$$

δr

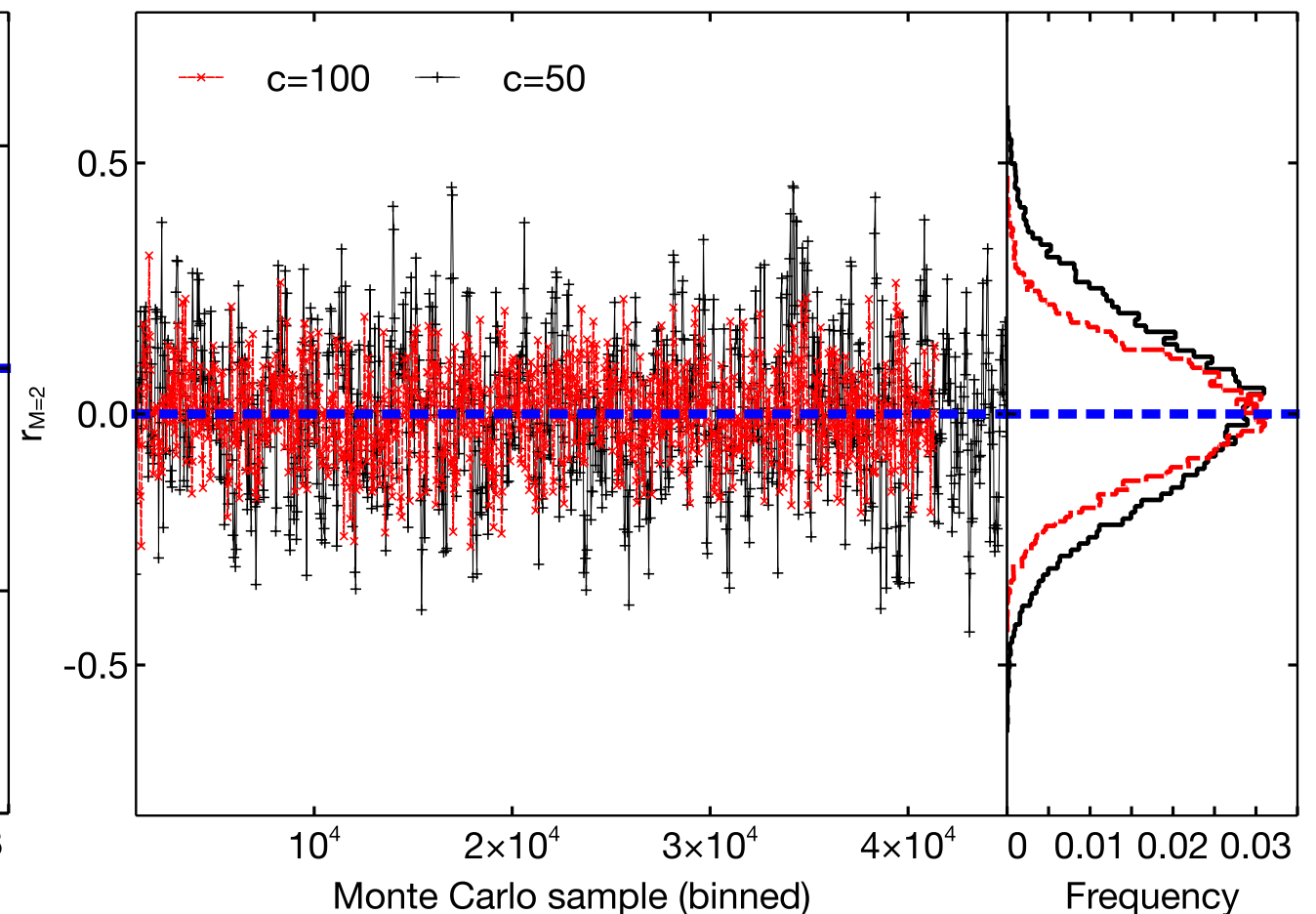
$$F(N, L, r_0) = 2 c \langle \delta r \rangle$$

$\langle \delta r \rangle$ in different directions

$$r_0=7 \quad T=3.0 \quad N=8 \quad L=8$$



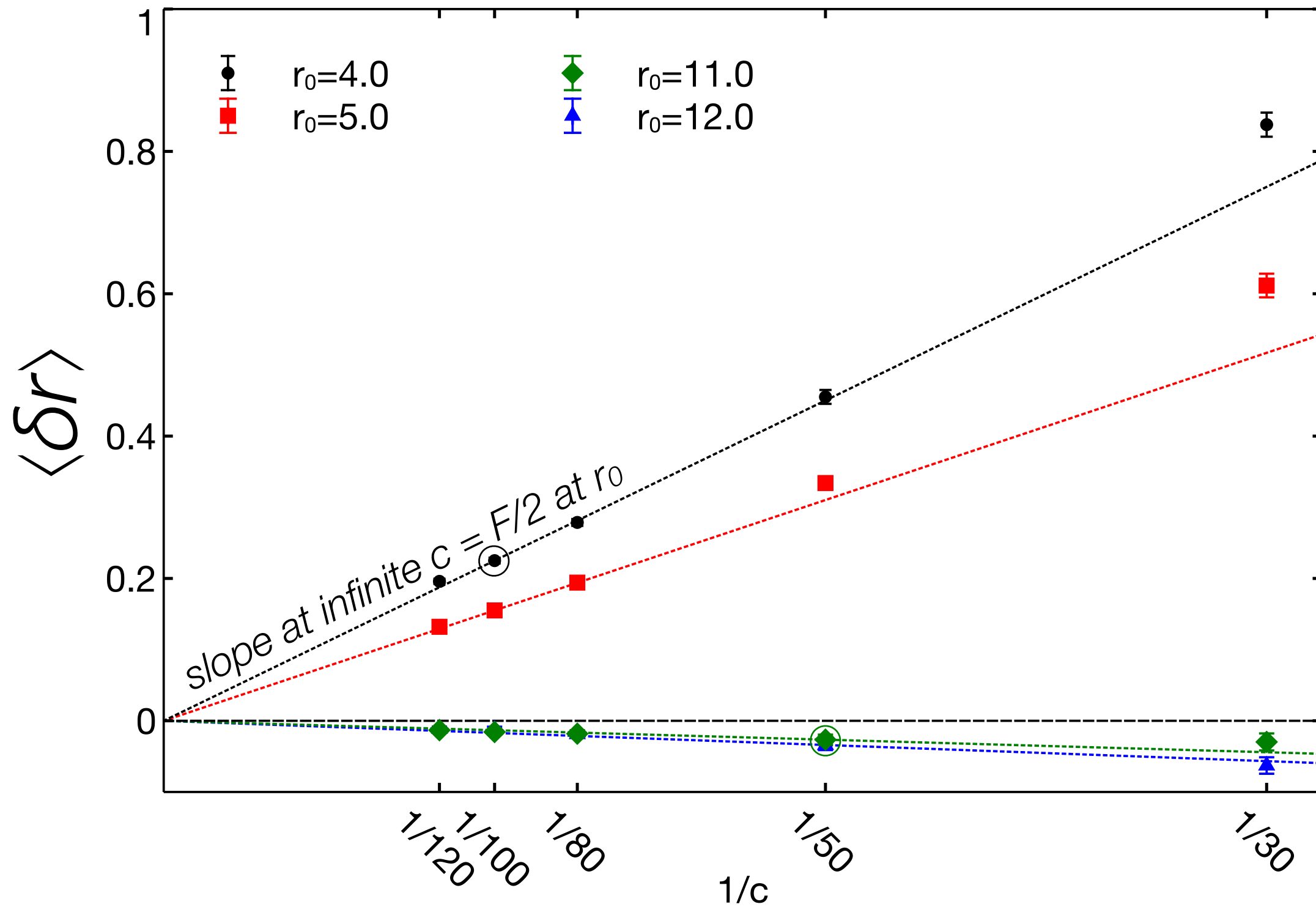
probe displacement direction



perpendicular direction

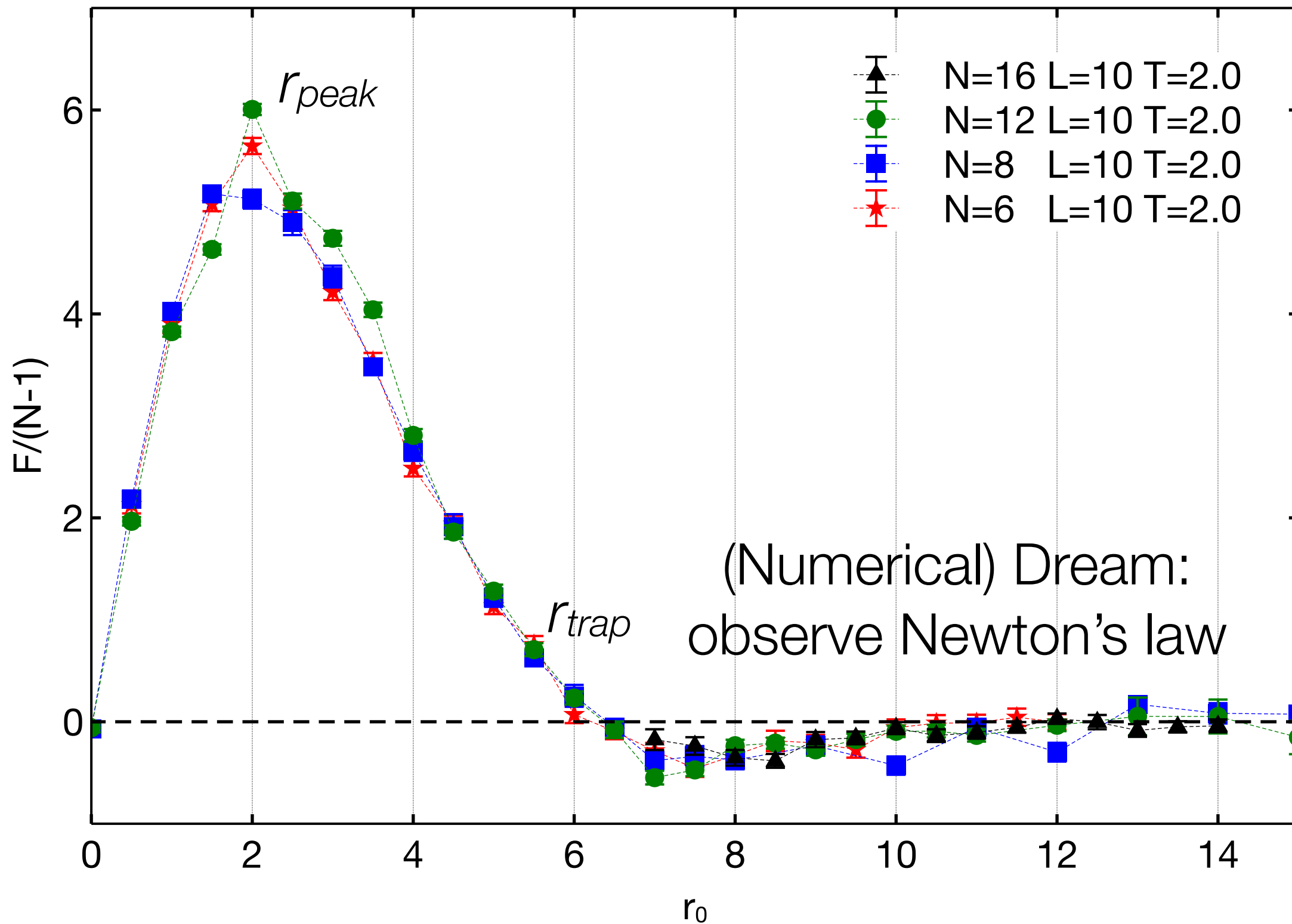
$$\langle \delta r \rangle = F(N, L, r_0) / 2 c$$

$T=3.0 \ N=8 \ L=8$



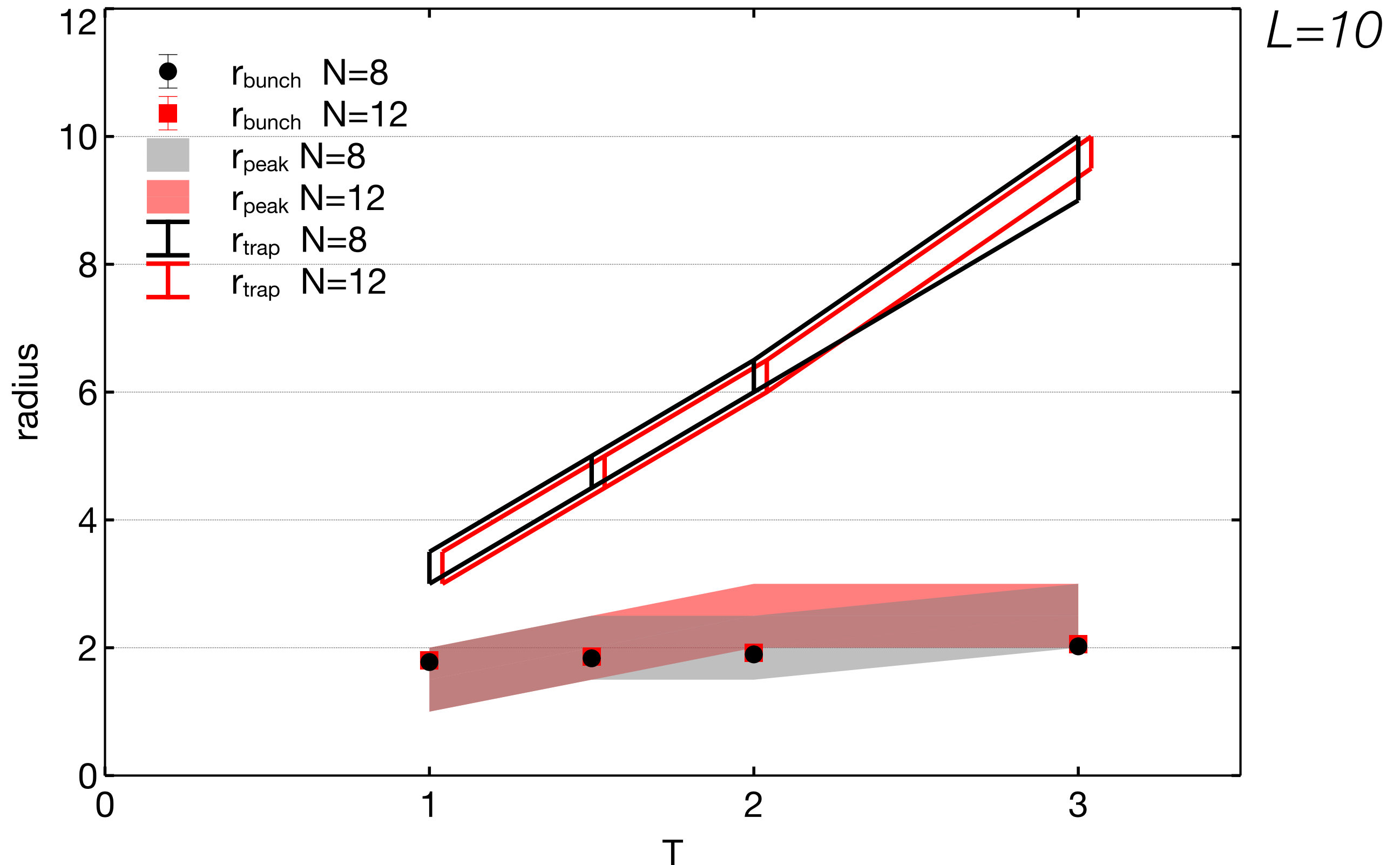
Force on probe D0 brane

MCSMC 1709.01932



Length Scales

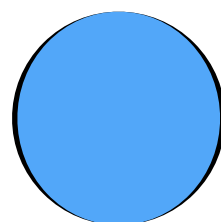
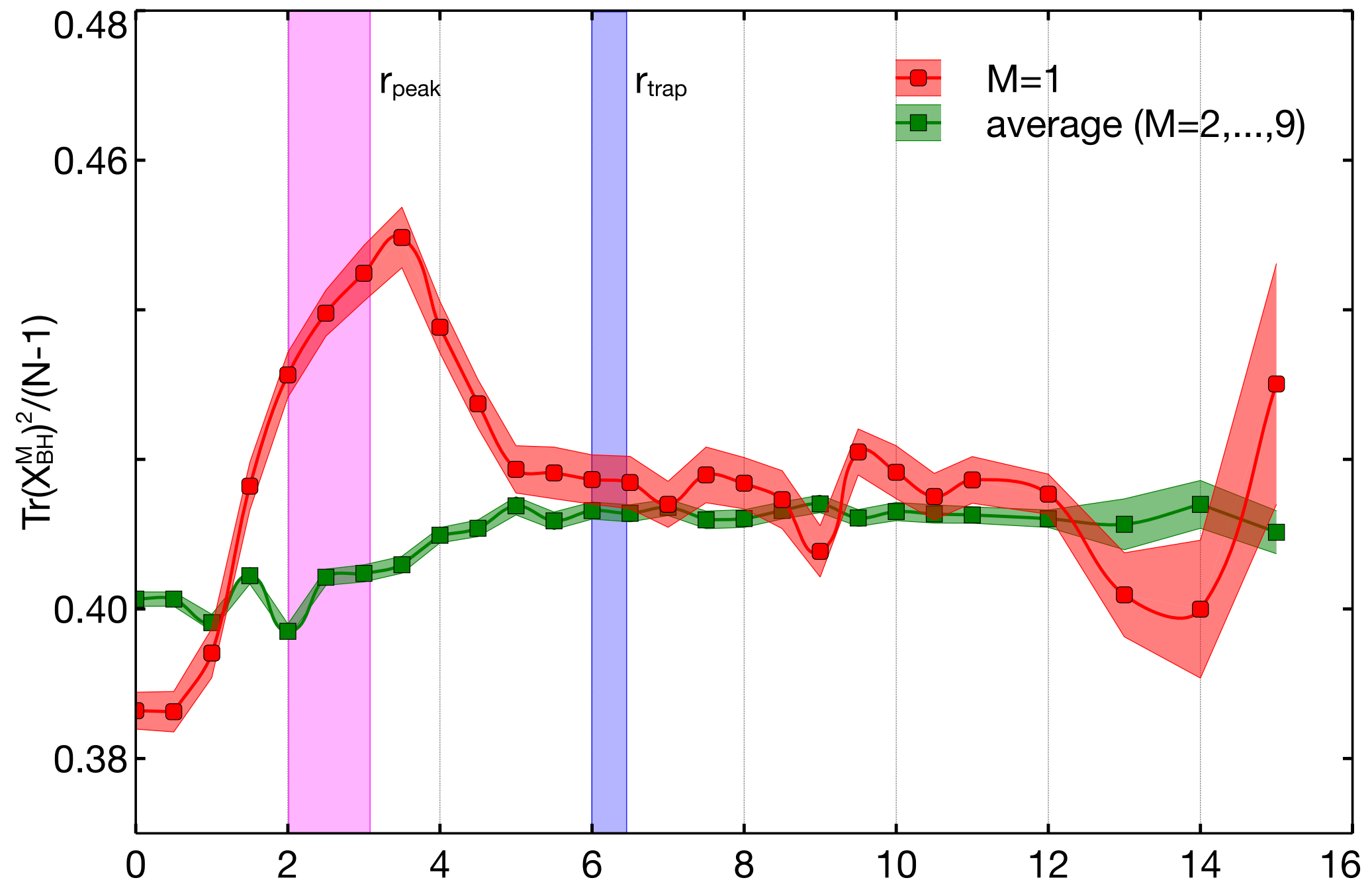
MCSMC 1709.01932



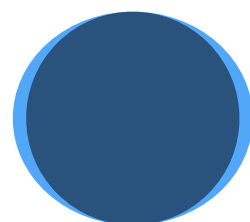
Bunch Deformation

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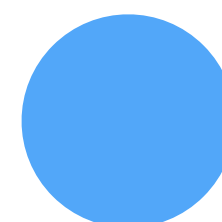
$T=2.0$ $N=12$ $L=10$



oblate



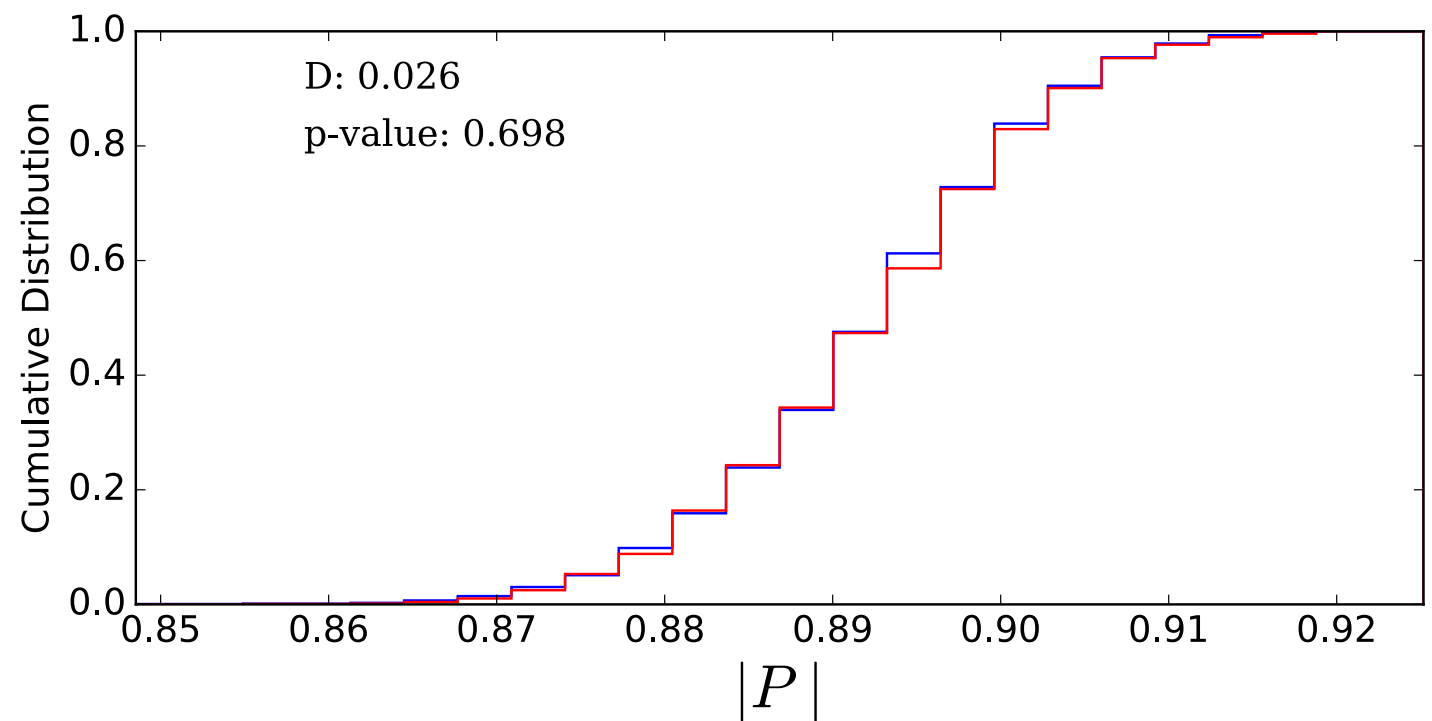
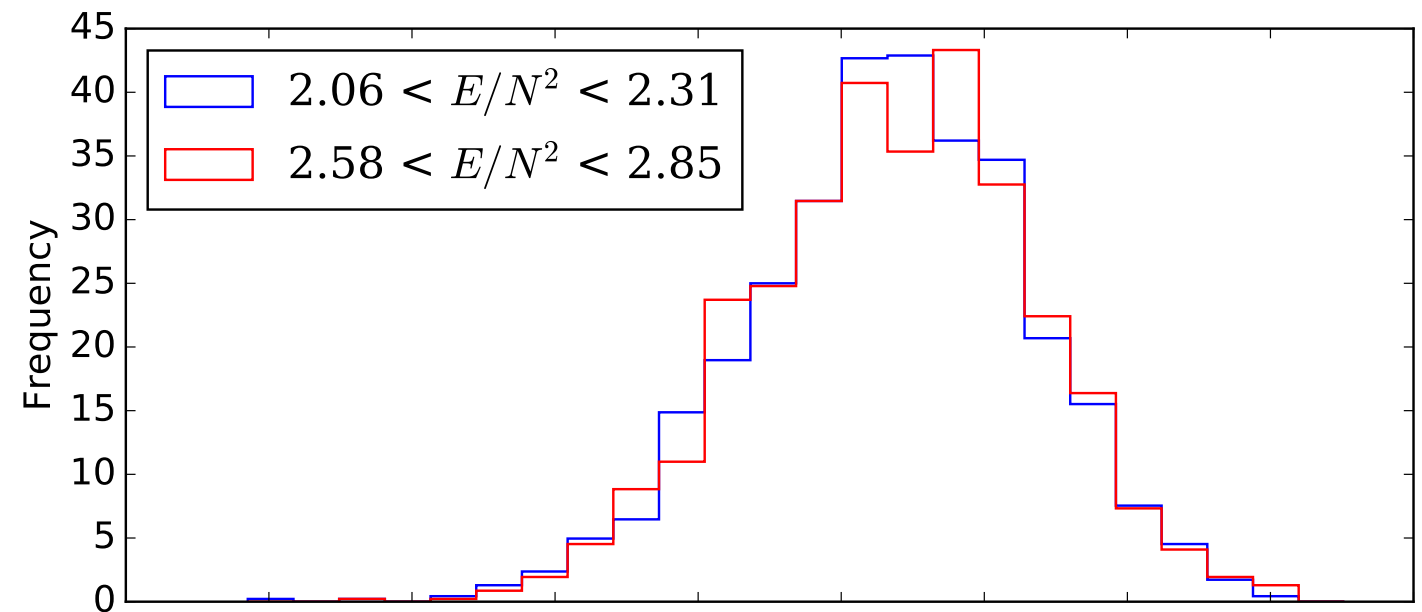
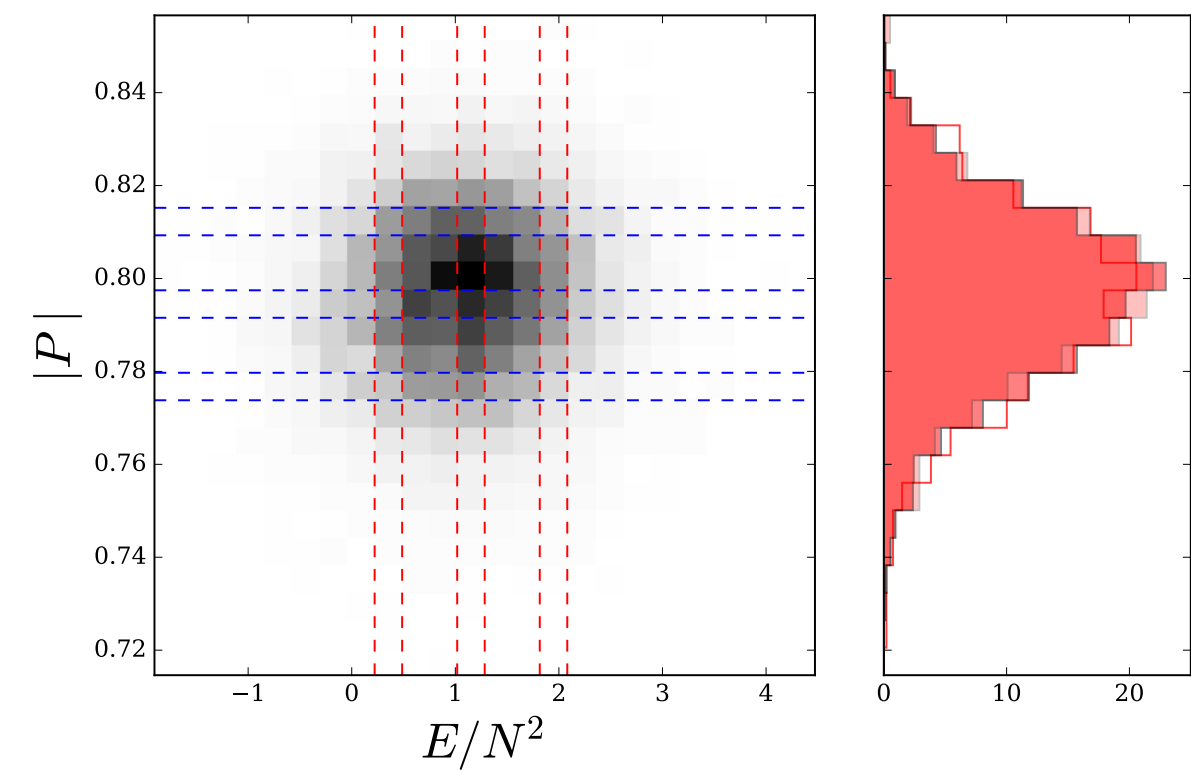
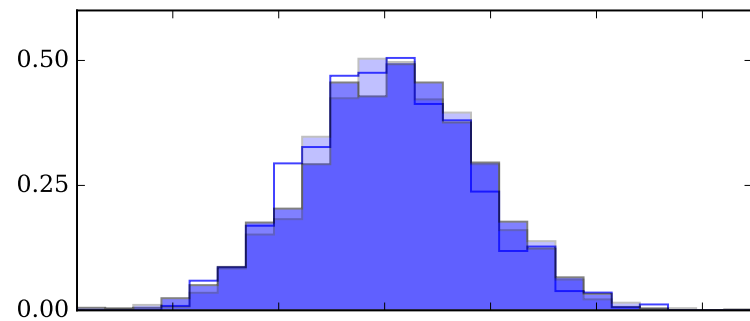
prolate



spherical

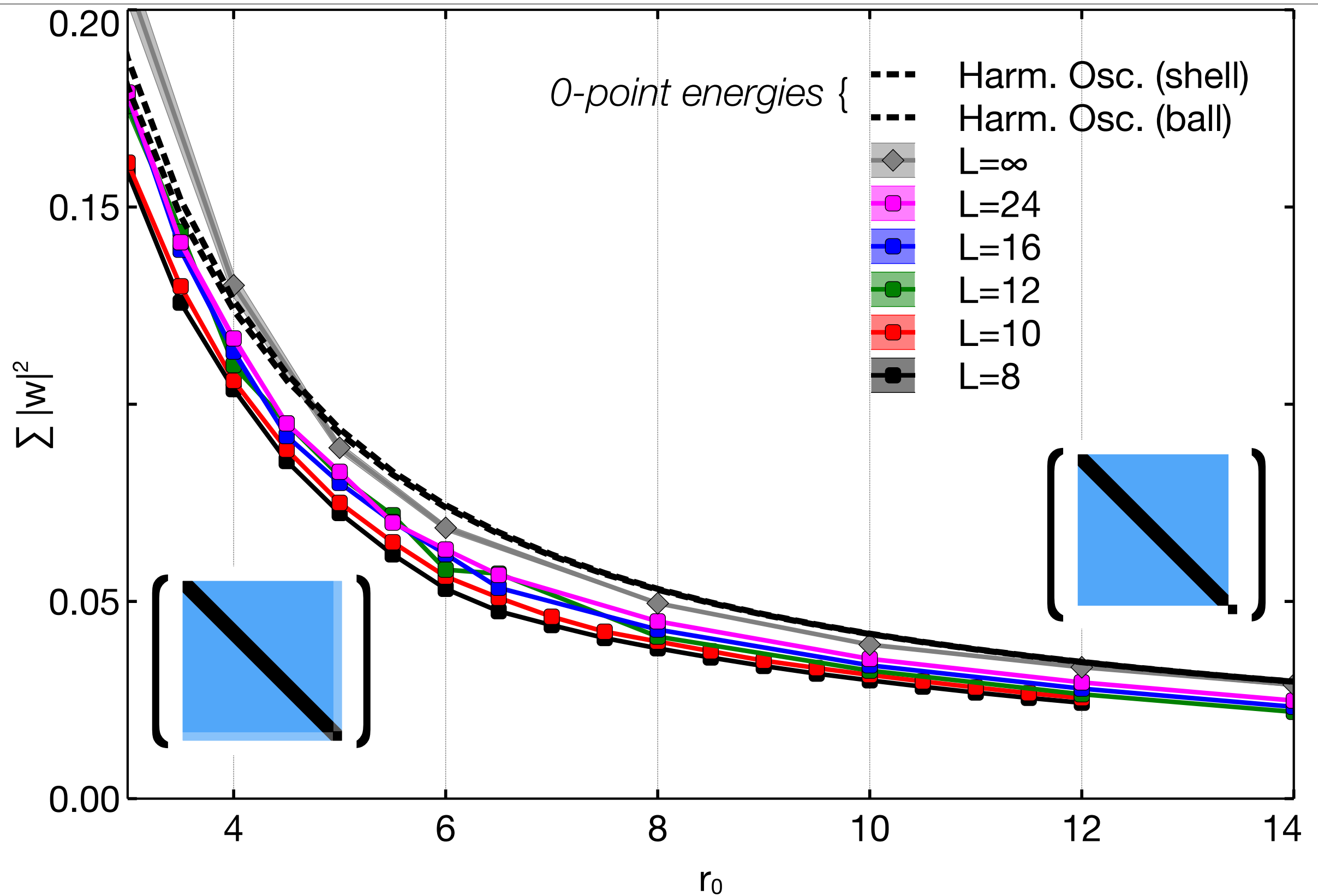
Backup² Slides

Phase Quench



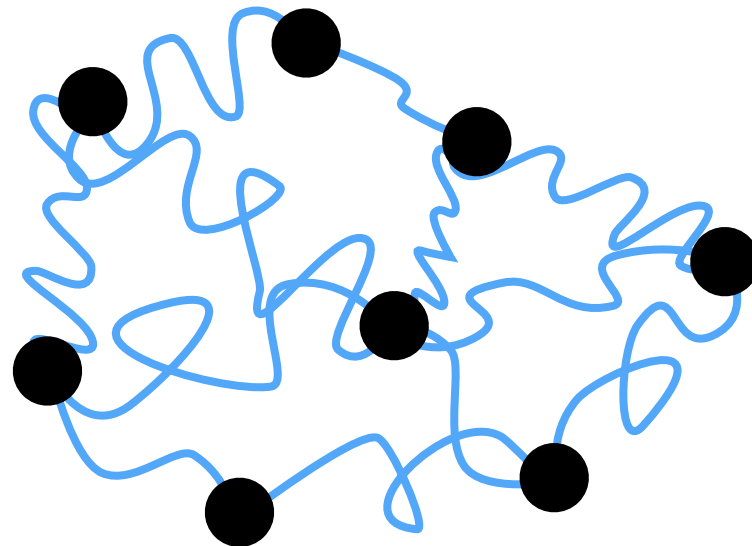
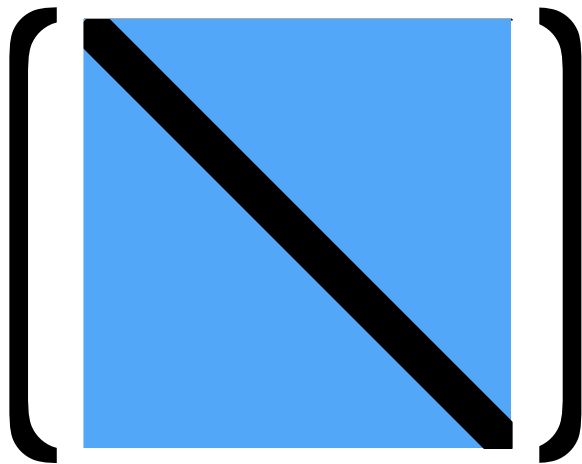
Off-diagonal Elements

$T=2.0$ $N=12$

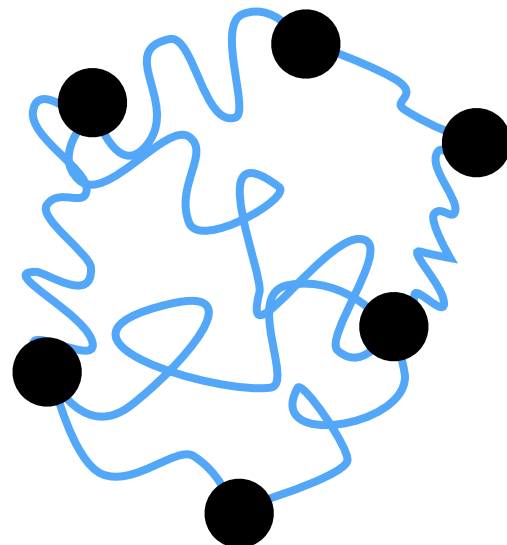
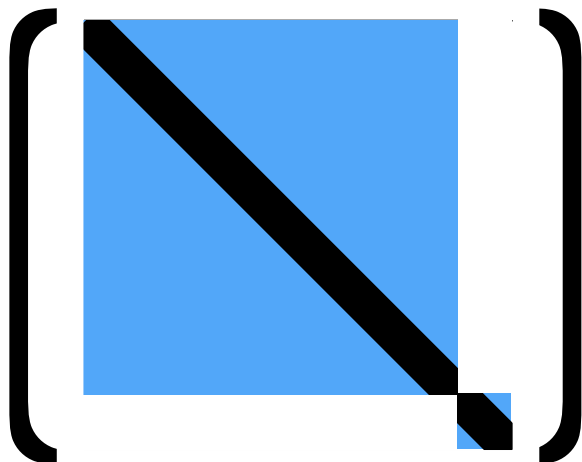


Emission of “clumps” is suppressed

Berkowitz, Hanada, Maltz - arXiv:1602.01473



$$\text{DOF} \sim N^2$$
$$t_{\text{recurrence}} \sim e^{+N^2}$$



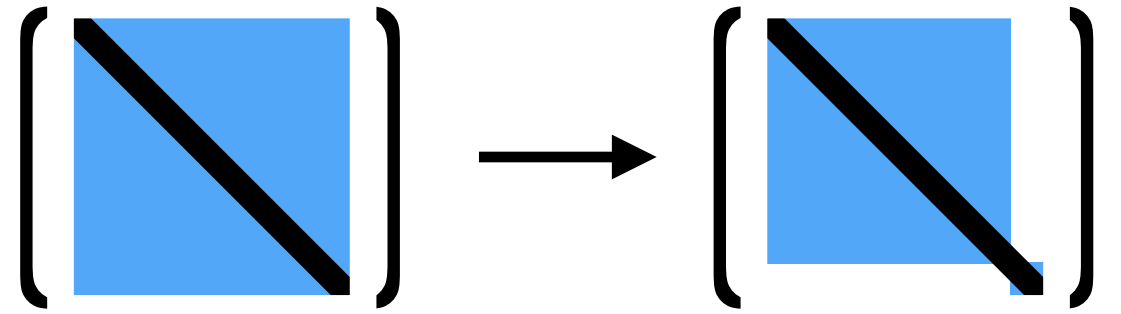
$$\tau \sim e^{+2N}$$

$$\text{DOF} \sim (N-2)^2$$
$$(+ \# \log(V))$$



Negative Specific Heat

1602.01473 Berkowitz, Hanada, Maltz



Classically

$$E \sim cN^2T$$

(Virial: $c=6$)

$$cN^2T = c(N-1)^2T' + E_{D0}$$

$$T' = \left(\frac{N}{N-1} \right)^2 T - \frac{E_{D0}}{cN^2}$$
$$\approx \left(1 + \frac{2}{N} \right) T$$

Quantum Mechanically

$$E = a_0 \lambda^{-3/5} T^{14/5} N^2$$

(SUGRA: $a_0=7.41$)

$$\lambda' = \frac{N-1}{N} \lambda$$

$$T'^{14/5} = \frac{\lambda^{-3/5} N^2}{\lambda'^{-3/5} (N-1)^2} T^{14/5} - \frac{E_{D0}}{a_0 \lambda' (N-1)^2}$$

$$T' \approx \left(1 + \frac{1}{2N} \right) T$$

$$T' > T!$$

1602.01473 Berkowitz, Hanada, Maltz

1602.01473 Berkowitz, Hanada, Maltz

Evaporation:

High T $\lambda_{\text{max}} \sim T^{1/4}$ $KS \propto N^2 T^{1/4} \rightarrow (N-1)^2 \underbrace{(T + \Delta T)^{1/4}}$

$$\Delta T \approx \frac{2}{N} T \rightarrow \left(N^2 - \frac{3N}{2} \right) T^{1/4}$$

Low T $\lambda_{\max} \sim T$ $KS \propto N^2 T \rightarrow (N-1)^2 (T + \Delta T)$

$$\Delta T \approx \frac{1}{2N} T \rightarrow \left(N^2 - \frac{3N}{2} \right) T$$

Lyapunov exponent grows but KS entropy falls!

Mergers: KS grows dramatically

bigger black holes are faster scramblers