Probing the BFSS dual geometry

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Nonperturbative and Numerical Approaches to Quantum Gravity, String Theory and Holography

International Centre for Theoretical Sciences

Bengaluru

V. Filev and D.O'C. [1506.01366,1512.02536,1710.02565]

Y. Asano, V. Filev, S. Kováčik and D.O'C. [1605.05597, 1612.09281].

- **i)** Introduction
- **ii)** Supermembranes and gauge/gravity duality.
- **iii)** D4-brabe probes
- **iv)** Lattice formulation
- **v)** Results

Membrane Actions

Nambu Gota—the simplest

$S_{NG} = \int_{\mathcal{M}} d^{p+1}x\sqrt{-detG} \qquad G_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N g_{MN}$

Higher form gauge field on the world volume

$$S_{p-form} = -\int_{\mathcal{M}} \frac{1}{(p+1)!}\epsilon^{\mu_1\dots\mu_{p+1}} C_{\mu_1\dots\mu_{p+1}}$$

$$C_{\mu_1\dots\mu_{p+1}} = \partial_{\mu_1} X^{M_1} \dots \partial_{\mu_{p+1}} X^{M_{p+1}} C_{M_1\dots M_{p+1}}$$

We can add

- an anti-symmetric part to $G_{\mu\nu}$ to get a Dirac-Born-Infeld action.
- extrinsic curvature terms.

Supersymmetric S_{NG} exist only in 4, 5, 7 and 11 dim-spacetime.

The BFSS model

$$S_{SMembrane} = \int \sqrt{-G} - \int C + \quad \text{Fermionic terms}$$

The susy version only exists in $4, 5, 7$ and 11 spacetime dimensions.

BFFS Model — The supersymmetric membrane à la Hoppe

$$\mathrm{H} = \mathrm{Tr}(\tfrac{1}{2}\sum_{a=1}^{9} P^a P^a - \tfrac{1}{4}\sum_{a,b=1}^{9}[X^a, X^b][X^a, X^b] + \Theta^T \gamma^a [X^a, \Theta])$$

The model is claimed to be a non-perturbative 2nd quantized formulation of M-theory.

It also describes a system of N interacting D0 branes.

Note the flat directions.

The BMN or PWMM

The supermembrane on the maximally supersymmetric plane wave spacetime

$$ds^2 = -2dx^+dx^- + dx^a dx^a + dx^i dx^i - dx^+ dx^+ ((\frac{\mu}{6})^2 (x^i)^2 + (\frac{\mu}{3})^2 (x^a)^2)$$

with

$$dC = \mu dx^1 \wedge dx^2 \wedge dX^3 \wedge dx^+$$

so that $F_{123+} = \mu$. This leads to the additional contribution to the Hamiltonian

$$\Delta \mathrm{H}_\mu = \frac{N}{2} \mathbf{Tr} \Big((\frac{\mu}{6})^2 (X^a)^2 + (\frac{\mu}{3})^2 (X^i)^2 + \frac{2\mu}{3} i\epsilon_{ijk} X^i X^j X^k + \frac{\mu}{4} \Theta^T \gamma^{123} \Theta \Big)$$

Finite Temperature Model

The partition function and Energy of the model at finite temperature is

$$Z = Tr_{Phys}(\mathrm{e}^{-\beta\mathcal{H}}) \qquad \text{and} \qquad E = \frac{Tr_{Phys}(\mathcal{H}\mathrm{e}^{-\beta\mathcal{H}})}{Z} = \langle\mathcal{H}\rangle$$

The 16 fermionic matrices $\Theta_\alpha = \Theta_{\alpha A} t^A$ are quantised as

$$\{\Theta_{\alpha A}, \Theta_{\beta B}\} = 2\delta_{\alpha\beta}\delta_{AB}$$

The $\Theta_{\alpha A}$ are $2^{8(N^2-1)}$ and the Fermionic Hilbert space is

$$\mathcal{H}^F = \mathcal{H}_{256} \otimes \cdots \otimes \mathcal{H}_{256}$$

with $\mathcal{H}_{256} = \mathbf{44} \oplus \mathbf{84} \oplus \mathbf{128}$ suggestive of
the graviton (**44**), anti-symmetric tensor (**84**) and gravitino (**128**)
of $11 - d$ SUGRA.

For an attempt to find the ground state see: J. Hoppe et al
arXiv:0809.5270

Lagrangian formulation

.

The BFSS matrix model is also the dimensional reduction of ten dimensional supersymmetric Yang-Mills theory down to one dimension:

$$
\begin{aligned}
S_M \quad &= \frac{1}{g^2}\int dt\,\mathrm{Tr}\left\{\frac{1}{2}(\mathcal{D}_0 X^i)^2 + \frac{1}{4}[X^i, X^j]^2 \right. \\
&\qquad \left. -\frac{i}{2}\Psi^T C_{10}\,\Gamma^0 D_0\Psi + \frac{1}{2}\Psi^T C_{10}\,\Gamma^i[X^i,\Psi]\right\} \ ,
\end{aligned}
$$

where Ψ is a thirty two component Majorana–Weyl spinor, Γ^μ are ten dimensional gamma matrices and C_{10} is the charge conjugation matrix satisfying $C_{10}\Gamma^\mu C_{10}^{-1} = -\Gamma^{\mu\,T}$.

$$\Delta S_\mu = \frac{N}{2}\int_0^\beta d\tau \mathbf{Tr}\Big((\frac{\mu}{6})^2 (X^a)^2 + (\frac{\mu}{3})^2 (X^i)^2 + \frac{2\mu}{3} i\epsilon_{ijk} X^i X^j X^k + \frac{\mu}{4}\Psi^T \gamma^{123}\Psi \Big)$$

The gravity dual and its geometry

Gauge/gravity duality predicts that the strong coupling regime of the theory is described by II_A supergravity, which lifts to 11-dimensional supergravity.

The bosonic action for eleven-dimensional supergravity is given by

$$S_{11D} = \frac{1}{2\kappa_{11}^2} \int [\sqrt{-g}R - \frac{1}{2}F_4 \wedge *F_4 - \frac{1}{6}A_3 \wedge F_4 \wedge F_4]$$

where $2\kappa_{11}^2 = 16\pi G_N^{11} = \frac{(2\pi l_p)^9}{2\pi}$.

The relevant solution to eleven dimensional supergravity for the dual geometry to the BFSS model corresponds to N coincident $D0$ branes in the IIA theory. It is given by

$$ds^2 = -H^{-1}dt^2 + dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

with $A_3 = 0$
The one-form is given by $C = H^{-1} - 1$ and $H = 1 + \frac{\alpha_0 N}{r^7}$ where $\alpha_0 = (2\pi)^2 14\pi g_s l_s^7$.

Including temperature

The idea is to include a **black hole** in the gravitational system.

The Hawking termperature provides the temperature of the system.

Hawking radiation

We expect difficulties at low temperatures, as the system should Hawking radiate. It is argued that this is related to the flat directions and the propensity of the system to leak into these regions.

The black hole geometry

$$ds_{11}^2 = -H^{-1}Fdt^2 + F^{-1}dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

Set $U = r/\alpha'$ and we are interested in $\alpha' \to \infty$
$H(U) = \frac{240\pi^5\lambda}{U^7}$ and the black hole time dilation factor
$F(U) = 1 - \frac{U_0^7}{U^7}$ with $U_0 = 240\pi^5\alpha'^5\lambda$. The temperature

$$\frac{T}{\lambda^{1/3}} = \frac{1}{4\pi\lambda^{1/3}} H^{-1/2} F'(U_0) = \frac{7}{2^4 15^{1/2} \pi^{7/2}} (\frac{U_0}{\lambda^{1/3}})^{5/2}.$$

From black hole entropy we obtain the prediction for the Energy

$$S = \frac{A}{4G_N} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{9/2} \Longrightarrow \frac{E}{\lambda N^2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{14/5}$$

The observables that we focus on

$$
\begin{aligned}
E/N^2 &= \left\langle \frac{1}{N\beta} \int_0^\beta dt \, \mathrm{Tr} \left(-\frac{3}{4}[X^i, X^j]^2 + \frac{3}{4} \Psi^T C_{10} \gamma^i [X^i, \Psi] \right) \right\rangle , \\
\langle R^2 \rangle &= \left\langle \frac{1}{N\beta} \int_0^\beta dt \, \mathrm{Tr} \left(X^i \right)^2 \right\rangle , \\
< |P| > &= \langle | \frac{1}{N} \mathrm{Tr} U \, | \rangle , \\
U &\equiv \mathcal{P} \exp \left(i \int_0^\beta dt \, A_0(t) \right) .
\end{aligned}
$$

Lattice discretisation

For *Spin*(9) the simplest representation of the γ-matrices has $C = \mathbf{1}$ and all γ^a are symmetric. In this representation the BFSS fermionic action is

$$\int_0^\beta \frac{1}{2} Tr(\psi^T D_\tau \psi - \psi \gamma^i [X^i, \psi])$$

1st order formulation

For the bosons

$$D_\tau \to \frac{\mathrm{e}^{aD_\tau} - 1}{a}$$

while for fermions

$$D_\tau \to \frac{\mathrm{e}^{aD_\tau} - 1}{a} \equiv \frac{\mathrm{e}^{aD_\tau} - \mathrm{e}^{-aD_\tau}}{2a} := K \quad \text{by antisymmetry.}$$

This has fermion doublers.

A simple Wilson term is symmetric.

Avoiding fermion doublers

1st order Lattice Dirac, with Wilson term

$$[D_\tau]_{Lat} = K + ar_1 \Sigma \Delta_{Wilson} \qquad \Sigma = \Sigma^\dagger = -\Sigma^T, \Sigma^2 = \mathbf{1}$$

$$K = \frac{e^{aD_\tau} - e^{-aD}}{2a} \qquad \Delta_{Wilson} = \frac{2 - e^{aD_\tau} - e^{-aD_\tau}}{a^2}$$

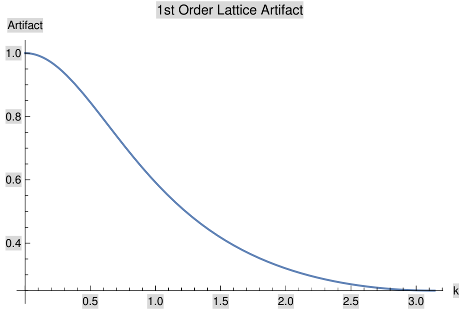
Options for Σ

$$\Sigma = \sum_{ij} i a_{ij} \gamma^{ij} + \sum_{ijk} i b_{ijk} \gamma^{ijk}, \quad \text{e.g.} \quad \Sigma = i\gamma^{89} = \sigma^2 \otimes \mathbf{1_8}$$

Lattice Artifact

Inverse of Lattice Dirac

$$[D_\tau]^{-1}_{Lat} = \frac{-K + ar_1\Sigma\Delta}{\Delta - \frac{a^2\Delta^2}{4} + r_1^2 a^2 \Delta^2} \sim \frac{1}{D_\tau} + ar_1\Sigma + \dots$$



The artifact leads to some negative and some positive masses or order a^2 the scalars. This is especially problematic for the BMN (mass deformed model).

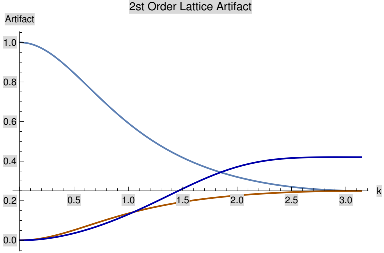
2nd order form

1st order Lattice Dirac, with Wilson term

$$[D_\tau]_{Lat} = K_2 + \Sigma(ar_1\Delta + a^3 r_{2F}\Delta^2)$$

$$K_2 = (1-r)(\mathrm{e}^{aD_\tau} = \mathrm{e}^{-aD_\tau}) + r(\mathrm{e}^{2aD_\tau} - \mathrm{e}^{-2aD_\tau})$$

Note with $r = -1/3$ $K_2 = D_\tau - \frac{a^5}{30} D_\tau^5 + \cdots$

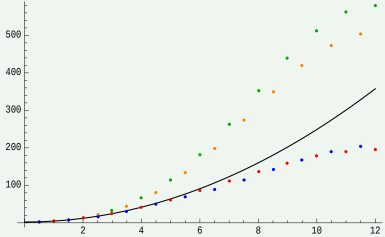


lightblue $\equiv$ 1st order; brown(Hanada) $\equiv r = -1, r_1 = 0, r_{2F} = 1/4$;
blue $\equiv r = -1/3, r_1 = 0, r_{2F} = 0.1488$

Optimising the free dispersion relation

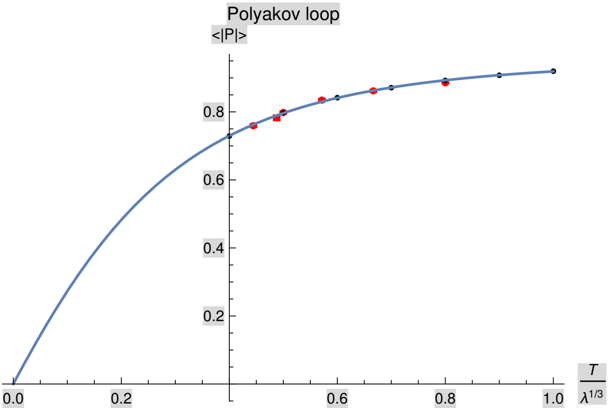
We choose r_{2F} to maximise the number of Matsubara frequencies matched in the dispersion relation. The BMN model has a mass deformation proportional to $i\gamma^{123}$ so choosing $\Sigma = i\gamma^{789}$ anti-commutes with this. We implement the 3-γ prescription.

Comparison of dispersion relations



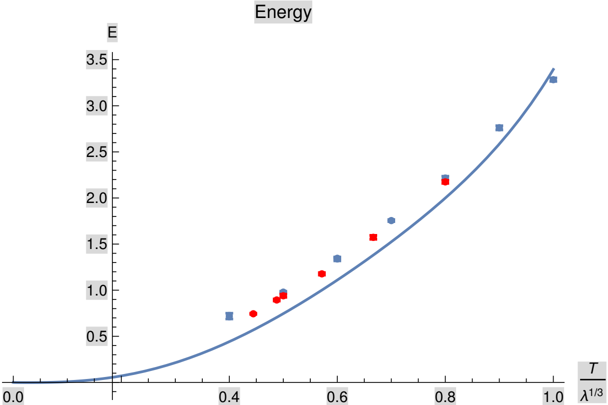
upper curves (Hanada) $r = -1$, $r_{2F} = 1/4$, lower $r = -1/3$, $r_{2F} = 0.1488$.

The average Polyakov loop $\langle |P| \rangle$.



Berkowitz et al [arXiv:1606.04951] blue
our data the red points.
The continuous curve is a fit to Berkowitz et al.
Data $N = 16$ $L = 24$.

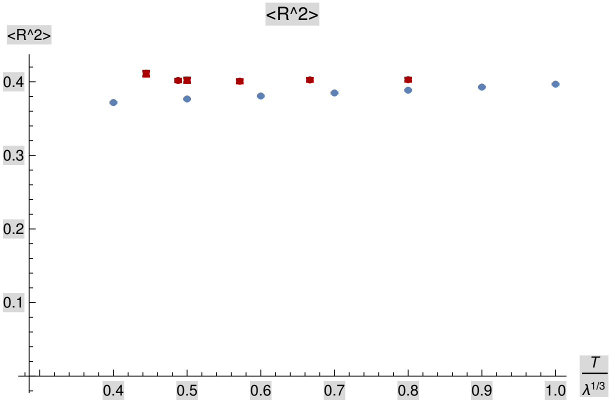
The Internal Energy $\langle H \rangle$.



Berkowitz et al [arXiv:1606.04951] blue
our data ($N = 16$ $L = 24$) the red points.
The continuous curve from Berkowits et al is

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{14}{5}} - (10.0 \pm 0.4)\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{23}{5}} + (5.8 \pm 0.5)\, T^{\frac{29}{5}} + \dots$$

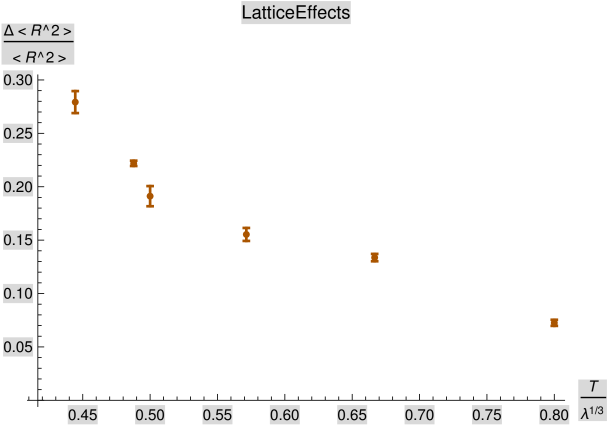
$\langle R^2 \rangle$.



Berkowitz et al [arXiv:1606.04951] blue
our data the red points.
Data $N = 16$ $L = 24$.

Lattice effect

$$\frac{\Delta < R^2 >}{R^2} = \frac{\sum_{i=7}^{9} < R_i^2 > /3 - \sum_{i=1}^{6} < R_i^2 >}{\sum_{i=1}^{6} < R_i^2 >}$$



.

Data $N = 16$ $L = 24$.

Probing the dual geometry.

We use the $p = 0$ version of the $Dp - D(p+4)$ setup. We add D_4 branes which bring new fundamental matter fields to the Hamiltonian. In IIA string theory this describes a $D0 - D4$ system.

The model was first discussed by M. Berkooz and M. R. Douglas, "Five-branes in M(atrix) theory," [hep-th/9610236]
and M. Van Raamsdonk, "Open dielectric branes,"
[hep-th/0112081].

Add new bosonic and fermionic degrees of freedom Φ_α as two complex $N \times N_f$ matrices

$$\mathcal{S}_{\Phi}^{\mathrm{E}} = \frac{1}{g^2}\int_0^\beta d\tau\,\mathrm{tr}\Big(D_\tau\bar{\Phi}^\rho D_\tau\Phi_\rho - \bar{\Phi}^\alpha(\sigma^A)_\alpha^{\ \beta}\, J^A_{ab}\,[X^a, X^b]\Phi_\beta$$

$$+\bar{\Phi}^\rho(X^i)^2\Phi_\rho - \frac{1}{2}\bar{\Phi}^\alpha\Phi_\beta\bar{\Phi}^\beta\Phi_\alpha + \bar{\Phi}^\alpha\Phi_\alpha\bar{\Phi}^\beta\Phi_\beta\Big)\ .$$

J^A and K^A are the $SO(3)$ generators

$$J^A_{ab} = \frac{1}{2}(L_{A4})_{ab} + \frac{1}{4}\varepsilon^{ABC}(L_{BC})_{ab}\ ,\ K^A_{ab} = -\frac{1}{2}(L_{A4})_{ab} + \frac{1}{4}\varepsilon^{ABC}(L_{BC})_{ab}$$

equivalent to the

$SO(4)$ generators $\quad (L_{ab})_{cd} = i(\delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd})$

$$
\begin{aligned}
S_\chi &= \frac{1}{g^2}\int \mathrm{tr}\Big(i\chi^\dagger D_\tau \chi + \bar{\chi}\gamma^a X^a \chi \\
&\qquad + \sqrt{2}\, i\, \varepsilon_{\alpha\beta}\, \bar{\chi}\lambda_\alpha \Phi_\beta - \sqrt{2}\, i\, \varepsilon_{\alpha\beta}\, \bar{\Phi}^\alpha \bar{\lambda}_\beta \chi \Big) \ .
\end{aligned}
$$

where $\lambda_\alpha = P_\alpha^\iota \psi_\iota$.

The full model is

$$
S_{BD} = S_{BFSS} + S_\Phi + S_\chi \, .
$$

It is a dimensional reduction of $\mathcal{N} = 1$, $d = 6$ Susy.
The lattice discretisation is again delicate but it works!

The D4-brane as a probe of the geometry.

The dual adds N_f D4 probe branes. In the probe approximation $N_f \ll N_c$, their dynamics is governed by the Dirac-Born-Infeld action:

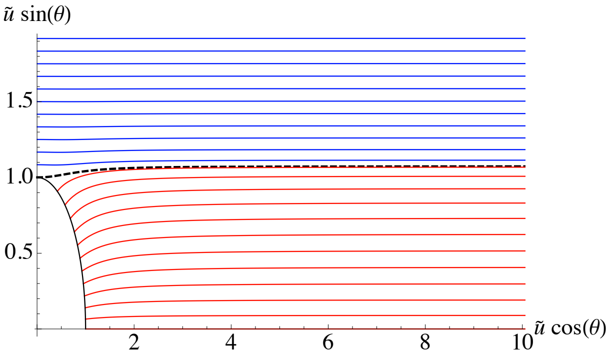
$$S_{\mathrm{DBI}} = -\frac{N_f}{(2\pi)^4\,\alpha'^{5/2}\,g_s}\int d^4\xi\, e^{-\Phi}\sqrt{-\mathrm{det}||G_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta}||}\,,$$

where $G_{\alpha\beta}$ is the induced metric and $F_{\alpha\beta}$ is the $U(1)$ gauge field of the D4-brane. For us $F_{\alpha\beta} = 0$.

$$d\Omega_8^2 = d\theta^2 + \cos^2\theta\, d\Omega_3^2 + \sin^2\theta\, d\Omega_4^2$$

and taking a D4-brane embedding extended along: t, u, Ω_3 with a non-trivial profile $\theta(u)$.

Embeddings



$$\tilde{u} \sin\theta = m + \frac{\tilde{c}}{\tilde{u}^2} + \ldots .$$

The Bosonic model (and ADHM Data)

$$S_{\text{bos}} = N \int_0^\beta d\tau \left[\mathrm{Tr} \left(\frac{1}{2} D_\tau X^a D_\tau X^a + \frac{1}{2} D_\tau \bar{X}^{\rho\dot{\rho}} D_\tau X_{\rho\dot{\rho}} - \frac{1}{4}[X^a, X^b]^2 + \frac{1}{2}[X^a, \bar{X}^{\rho\dot{\rho}}][X^a, X_{\rho\dot{\rho}}] \right) + \mathrm{tr} \left(D_\tau \bar{\Phi}^\rho D_\tau \Phi_\rho + \bar{\Phi}^\rho (X^a - m^a)^2 \Phi_\rho \right) + \frac{1}{2} \mathrm{Tr} \sum_{A=1}^{3} \mathcal{D}^A \mathcal{D}^A \right].$$

where $X^a = 0$ with

$$\mathcal{D}^A = \sigma^{A\,\sigma}_{\rho} \left(\frac{1}{2}[\bar{X}^{\rho\dot{\rho}}, X_{\sigma\dot{\rho}}] - \Phi_\sigma \bar{\Phi}^\rho \right) = 0$$

specify ADHM data for Yang-Mills instantons on $\mathbb{R}^4$.

Results for the flavoured bosonic model

With $N_f << N$ the fundamental fields act as probes on the adjoint background. The $SO(9)$ symmetry has been broken to $SO(5) \times SO(4)$. In the low temperature phase the system is well described by a gaussian model with three masses $m_A^t = 1.964 \pm 0.003$, $m_A^l = 2.001 \pm 0.003$ and $m_f = 1.463 \pm 0.001$, the adjoint longitudinal and transverse masses and the mass of the fundamental fields respectively.
These agree well very with theoretical estimates based on a $1/d$ espansion.
Yuhma Asano, Veselin G. Filev, Samuel Kováčik and D. O'C. arXiv 1605.05597

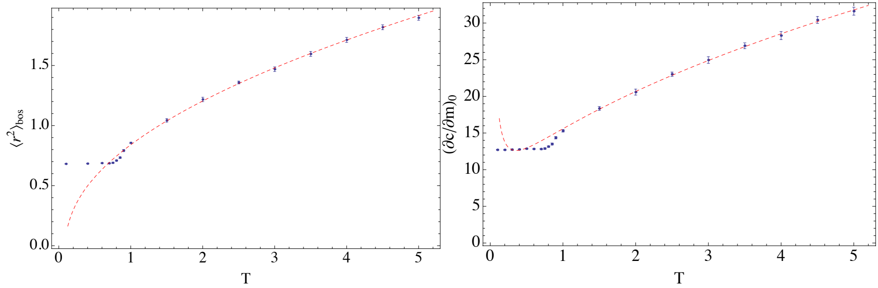
Two new observables

$$r^2 = \frac{1}{\beta N_f} \int_0^\beta d\tau \ \mathrm{tr}\, \bar{\Phi}^\rho \Phi_\rho$$

and the condensate defined as

$$c^a(m) = \frac{\partial}{\partial m^a} \left(-\frac{1}{N\beta} \log Z \right)$$

$N_f = 1$ and $N = 10$:



With $X^a \to X^a - m^a$ and for $m^a = 0$ we can look at the condensate susceptibility:

$$\left(\frac{\partial c}{\partial m}\right)_0 = \frac{2}{\beta}\int_0^\beta d\tau \, \mathrm{tr}\, \bar{\Phi}^\rho \Phi_\rho - \frac{N}{5\beta}\left(\int_0^\beta d\tau \, \mathrm{tr}\, 2\bar{\Phi}^\rho X^a \Phi_\rho\right)^2 .$$

The condensate and the dual prediction

Parametrising the unit S^8 in the metric dual geometry as

$$d\Omega_8^2 = d\theta^2 + \cos^2\theta\, d\Omega_3^2 + \sin^2\theta\, d\Omega_4^2$$

and taking a D4–brane embedding extended along t, u, Ω_3 with a non-trivial profile $\theta(u)$, we obtain (after Wick rotation)

$$S^E_{\rm DBI} = \frac{N_f\,\beta}{8\,\pi^2\,\alpha'^{5/2}\,g_s}\int du\, u^3 \cos^3\theta(u)\,\sqrt{1+u^2\,f(u)\,\theta'(u)^2}\ .$$

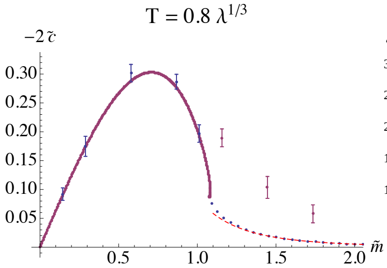
Extremising gives the embedding. The AdS/CFT for large u gives

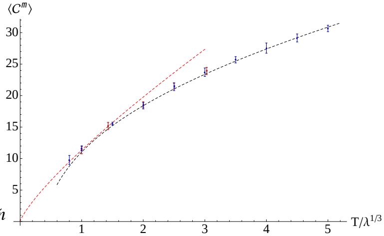
$$\sin\theta = \frac{\tilde m}{\tilde u} + \frac{\tilde c}{\tilde u^3} + \ldots \qquad \tilde u = u/u_0.$$

$\tilde m$ and $\tilde c$ are proportional to the bare mass and condensate. We find the mass susceptibility at zero bare mass

$$\langle \mathcal{C}^m \rangle = 14^{1/5} 15^{2/5} \pi^{9/5} \frac{\csc(\pi/7)\,\Gamma(3/7)\,\Gamma(5/7)}{\Gamma(1/7)^2\,\Gamma(2/7)\,\Gamma(4/7)} N_f\, N_c \left(\frac{T}{\lambda^{1/3}}\right)^{4/5}$$

Measurements and comparison with prediction

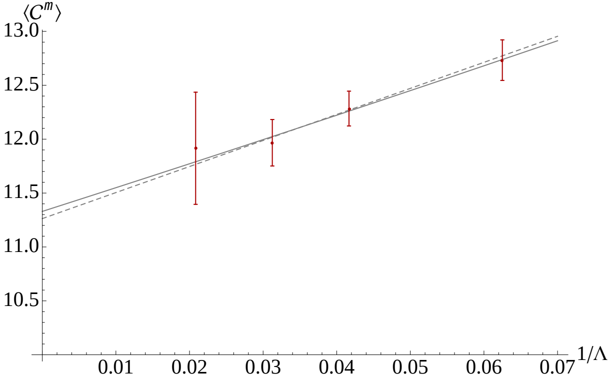




V. Filev and D. O'C. arXiv 1512.02536.

The data overlaps surprisingly well with the gravity prediction in the region where the $D4$ brane ends in the black hole.

Extrapolation to the continuum



The red bars correspond to measurements at $\Lambda = 16$, 24, 32 and 48 for $T = \lambda^{1/3}$ and $N = 10$. The dashed gray line represents the linear extrapolation, and the solid gray curve corresponds to the quadratic extrapolation. The gravity prediction is $\langle \mathcal{C}^m \rangle|_{T=\lambda^{1/3}} = 11.36$.

Conclusions

- The Berkooz Douglas model provides a non-trivial test of gauge/gravity duality.
- The strong coupling condensate agrees well with the prediction from a $D4$-brane propagating on the dual supergravity background.
- The $D4$-brane probes the geometry in a non-trivial fashion.
- We are only at the beginning!

Thank you for your attention!