An effective theory for warm QCD

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Outline

The grand-daddy of sign problems

The path integral for a quantum system in real time has extreme phase cancellations. Direct simulation impossible.

Lattice techniques: Wick rotation to Euclidean time. VEVs of operators or equilibrium thermal expectations obtained by Monte Carlo.

Continuation to real time requires care. Extraction of transport coeffients at finite temperature require prior knowledge of spectral function. Circular problem?

Urgently needed to make sense of Beam Energy Scan at RHIC: is the QCD critical point in reach of experiments?

Which transport coefficients?

The most promising signals are checks of critical fluctuations. Longest range correlations at the QCD critical point are baryonic: divergence in the baryon number susceptibility.

ILGTI 2005, 2008, 2011, 2013. RBRC 2017. Many others

Flavoured transport coefficients come from correlation functions which excite quarks in the medium. Transport coefficients: long distance slow physics, $p \ll T$. Can one use EFT?

Effective theories

$T \ll T_{co}$: pion gas; $T \gg T_{co}$: weak coupling QCD.
What about $T \simeq T_{co}$?

Outline

Restrict scope to make progress

1. Require only flavoured currents; no external gluons. Maybe possible to subsume all gluon effects into the couplings of an EFT? Needs check: subject of this talk
2. Try to describe low momentum n-point functions in some temperature range around T_0. Try to understand $p \ll T_0$, in both Euclidean and Minkowski theories: so screening and off-equilibrium physics.
3. But an EFT cannot give the full dynamics: high momenta are truncated so full EOS may be out of reach. Some parts can be obtained. In particular should be able to obtain universal soft parts determined by the symmetries. (Otherwise failure of universality)
4. Symmetries include chiral $SU_V(2)\times SU_A(2)$; valid at zero quark masses. How large can quark masses become before the theory becomes invalid? $N_f = 3$ later.

The effective Lagrangian up to dimension 6

$$L = d^3 T_0 \overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi \longleftarrow \text{pole/screening mass}$$

The effective Lagrangian up to dimension 6

$$
\begin{aligned}
L &= d^3\,T_0\overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi \\
&\quad + \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\tau^a\psi)^2\right] \longleftarrow \text{NJL type terms}
\end{aligned}
$$

No dimension 5 terms: chiral symmetry.

The effective Lagrangian up to dimension 6

$$
\begin{aligned}
L &= d^3 T_0 \overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi \\
&+ \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\tau^a\psi)^2\right] \\
&+ \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\psi)^2\right]
\end{aligned}
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& + \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\tau^a\psi)^2\right] \\
& + \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2
\end{aligned}
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&+ \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 \\
&+ \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2
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&+ \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 \\
&+ \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2 \\
&+ \frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
&+ \frac{d^{68}}{T_0^2}\left[(\overline{\psi}i\gamma_i\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\gamma_i\tau^a\psi)^2\right]
\end{aligned}
$$

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The effective Lagrangian up to dimension 6

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\begin{aligned}
L \;=\;& d^3 T_0 \overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi \\
&+ \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\tau^a\psi)^2\right] \\
&+ \frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\psi)^2\right] + \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 \\
&+ \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2 \\
&+ \frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
&+ \frac{d^{68}}{T_0^2}\left[(\overline{\psi}i\gamma_i\tau^a\psi)^2 + (\overline{\psi}i\gamma_5\gamma_i\tau^a\psi)^2\right] \\
&+ \frac{d^{69}}{T_0^2}\left[(\overline{\psi}iS_{i4}\psi)^2 + (\overline{\psi}S_{ij}\tau^a\psi)^2\right] + \frac{d^{60}}{T_0^2}\left[(\overline{\psi}iS_{i4}\tau^a\psi)^2 + (\overline{\psi}S_{ij}\psi)^2\right]
\end{aligned}
$$

No dimension 5 terms: chiral symmetry.

The mean field approximation

Mean field approximations: a field operator is replaced by a condensate. Then doing the Fierz transformations, obtain a single combination of couplings

$$\lambda = (\mathcal{N} + 2)d^{61} - 2d^{62} - d^{63} + d^{64} + d^{65} - d^{66} + d^{69} - d^{60}.$$

where $\mathcal{N} = 4N_c N_f$.

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where $\mathcal{N} = 4N_c N_f$.
Using this, MFT same as in ordinary NJL.

$$L = -\mathcal{N}\left(\frac{T_0^2}{4\lambda}\right)\Sigma^2 + (\Sigma + d^3 T_0)\,\overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\partial\!\!\!/_i\psi$$

where $\Sigma = 2\lambda\langle\overline{\psi}\psi\rangle/T_0^2$. In the chiral limit ($d^3 = 0$) critical point T_c. We find $\lambda = 12(d^4)^3$ using the choice of convention $T_0 = T_c$..
Input parameter T_0 can be changed, couplings then change.

Counting couplings

10 couplings collapse to a single combination λ.

λ used to fix T_c — dimensional transmutation. Easy to work in the convention $T_0 = T_c$.

But T_c is unknown. Also d^3 and d^4. Total of 3 unknowns. Strategy: use location of the cross over temperature T_{co} (at finite quark mass), and two measurements of correlation functions to fix all parameters.

Match to lattice data. Then T_{co} is fixed by finding the peak of the chiral susceptibility. Also, gauge invariant long-distance correlators on the lattice involve pion quantum numbers. So the matching must include a description of fluctuations.

Neat: if $T_0 < 1/a$ then continuum effective theory possible from cutoff lattice computations. Caveats later.

Outline

Fluctuations

Replace MFT ansatz by Hubbard-Stratanovich transformation

$$\overline{\psi}_\alpha \psi_\beta \simeq \langle \overline{\psi}\psi \rangle U_{\alpha\beta} \qquad \text{where} \qquad U = \exp\left(\frac{i\pi\gamma_5}{f}\right).$$

The pion fields, π, are fluctuations around the MFT. Introduce this into the MFT quadratic Lagrangian, integrate over the fermions, and find the EFT for pions. Result in the form

$$L_f = c^2 T_0^2 \pi^2 + \frac{1}{2}(\partial_0 \pi)^2 + \frac{c^4}{2}(\nabla\pi)^2 + c^{41}\pi^4 + \cdots$$

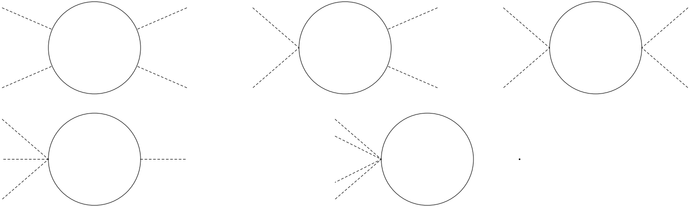
Tadpole and cubic term π^3 vanish.

Similar pion theory used before, using intuition from hydrodynamics, but π^4 term missed. **Son, Stephanov** Our contribution: full EFT expansion, microscopic derivation using fermion EFT (relations between f, c^2, c^4 and c^{41}), matching to lattice computations using 2-point functions.

Fermion loops

2-point functions

4-point functions



Loops in dimensional regularization (introduces regularization scale M). No power corrections in m/M, logs in m/M under control.

Chiral symmetry

At 1-loop order finite temperature GMOR:

$$f^2 m_\pi^2 = -m_0 \langle \overline{\psi}\psi \rangle$$

Similar results also earlier: **Klevansky (review)**. Self-consistency check, since loop corrections fix all parameters.

In the chiral limit c^{41} must vanish **Weinberg**. Explicit computation gives

$$c^{41} = -\frac{m_\pi^2}{3f^2} + \text{subleading terms} \propto m_0.$$

up to one-loop order. Small but measurable.

Outline

Lattice and current correlators

Tune T_0 to match the peak of the chiral susceptibility to T_{co} measured on the lattice. After this screening correlators of PS and AV are enough to fix d^3 and d^4. If $1/a \gg T_0$ then EFT of continuum limit from measurements at finite lattice spacing!

Technical difficulty (1): lattice has power corrections *ma* which may not have been subtracted. This will be necessary for improvement of matching. For now we neglect this; new computations under way.

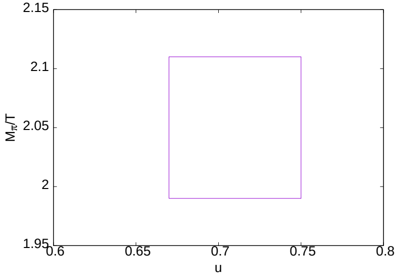
Technical difficulty (2): different quantities extracted from lattice measurements have co-variance. Not reported; so we fit as if they were independent. This may be improved in future.

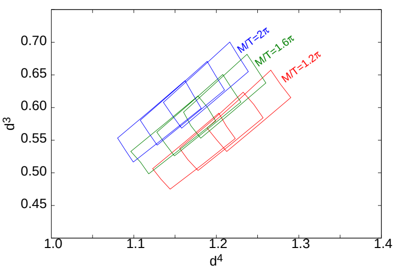
Data taken from **Brandt et al, 1406.5602**

Fermion parameters

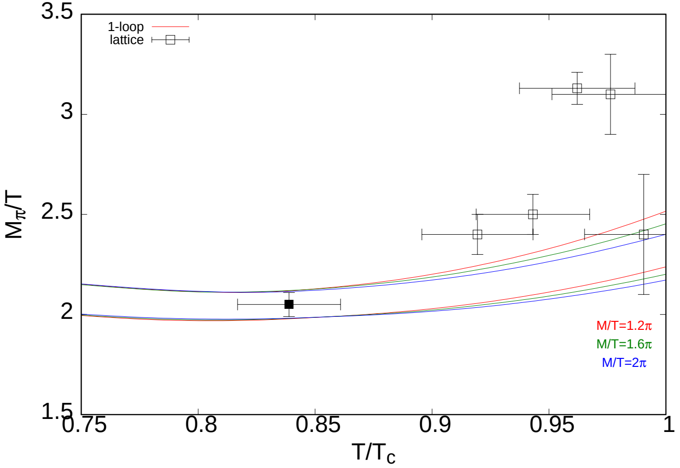
$$T_{co} = (1.24 \pm 0.03)\ T_0 \qquad \text{implies} \qquad T_c = 170 \pm 6 \text{ MeV}.$$

Agrees with chiral extrapolation of **Philipsen et al (2 flavours)**
Other parameters:



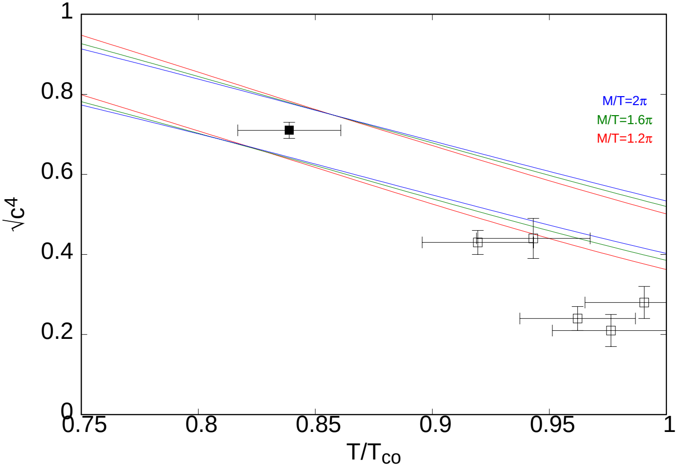


Predictions



Numerical approximations in prediction: λ fitted in the chiral limit; T_0 without power corrections; d^3, d^4 and λ assumed to be independent of T.

Predictions



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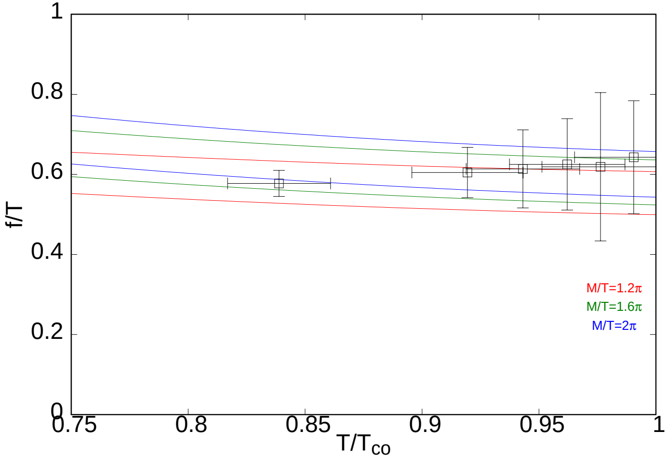

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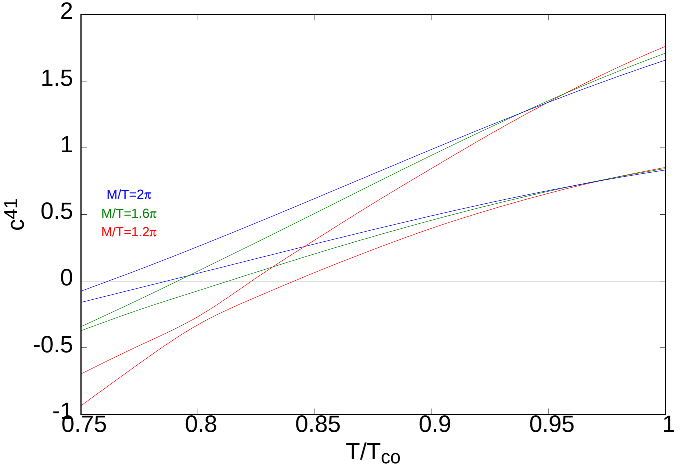

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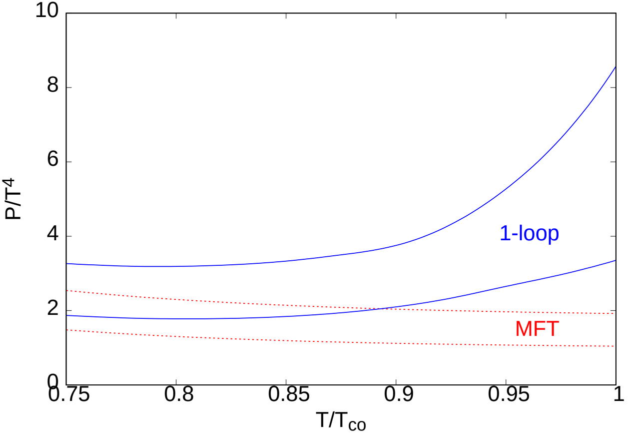
Pressure



E/T^4 less well controlled than P/T^4 due to extra $E(p)$ inside the integral: more UV dominated. Rise in P/T^4 robust, due to Jacobian factor $(c^4)^{-3/2}$.

Outline

Summary

1. Fermion EFT written down, complete up to dimension 6. Mean-field theory same as NJL MFT. One new prediction: curvature of critical line. Systematic improvements possible.
2. Derived a theory of pion fluctuations: second EFT. Finite T GMOR only a one-loop result. Weinberg power-counting may be violated.
3. Can be matched easily to lattice computations. Results are encouraging. Predictions of f, M_π, speed of sound, pion self-coupling. Gluons can be neglected!
4. In the chiral limit clear need to go beyond dimension 4 terms in the pion EFT, for example for the computation of critical indices.
5. To come: use of the EFT for transport computation. Extension to finite μ, and $N_f = 3$.