Nonperturbative Study of Heavy Quarkonia in Quark Gluon Plasma

Saumen Datta

Tata Institute of Fundamental Research, Mumbai

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QGP

- At large T, strongly interacting matter exists in a deconfined, chirally symmetric state ("Quark-gluon plasma", or QGP).
- At zero net baryon density, the transition is a crossover at temperature is $\sim$ 150-170 MeV, depending on the order parameter.
- For SU(3) without quarks, it is a first order transition at $T \sim$ 260 MeV.
- Realization in lab: relativistic heavy ion collision experiments.
- $T_{\rm freezeout} \sim$ 8 fm in RHIC, 10 fm in LHC

Role of heavy quarks

- Quarks like b, c: $M_Q \gg \Lambda_{\rm QCD}$
- Quarks approximately non-relativistic in bound states
- Typical heavy ion collision expt: $M_Q \gg T$ so heavy quarks generated only in initial hard collisions
- Works as a probe of the evolving medium
- $\bar{Q}Q$ bound states (quarkonia): dissolution will indicate deconfining medium

T. Matsui & H. Satz, Phys. Lett. B178, 416 ('86)

- Different dissociation temp. for diff. states: "thermometer"

F. Karsch and H. Satz, Z. Phys. C51, 209 (1991)
S. Digal *et al.*, Phys. Rev. D 64, 094015 (2001)

In-medium behavior

How to calculate interaction of quarkonia with medium?

Take a meson decay through a certain current, e.g., $J/\psi \to \mu^+\mu^-$

$$\begin{aligned}
\mathcal{M} &= e^2\ \bar{u}(p)\gamma_\mu v(-p)\ \frac{1}{q^2}\ \langle H|J_\mu|0\rangle \\
\Sigma(\omega) &= \frac{\alpha^2}{3\pi^3 q^2}\ n_B(q^0)\rho^{\mu}_{V,\mu}(\omega) \\
\rho_H(\omega,\vec{p}) &= \int d^4x\ e^{iq.x}\ \langle [J_H(\vec{x},t), J_H(\vec{0},0)]\rangle_T
\end{aligned}$$

Relation to Euclidean (Matsubara) correlator

$$G(\tau,T) = \int_0^\infty \frac{d\omega}{\pi}\ \rho(\omega,T)\ \frac{\cosh \omega(\tau - \frac{1}{2T})}{\sinh \frac{\omega}{2T}}$$

$\rho(\omega)$ from lattice

- QCD in non-perturbative regime: Lattice QCD
- Direct inversion of this equation to extract $\rho(\omega)$ not possible: numerically unstable.
- Analysis requires incorporation of any prior knowledge, e.g., high ω behavior.

  Asakawa, Nakahara & Hatsuda, Prog. Part. Nucl. Phys. 46 (2001) 459.
- Also restrict the search to a suitable singular space.

  R. K. Bryan, Eur. Biophys. J. 18 ('90) 165
- Quantitative information about change of spectral function with temperature very difficult to extract.

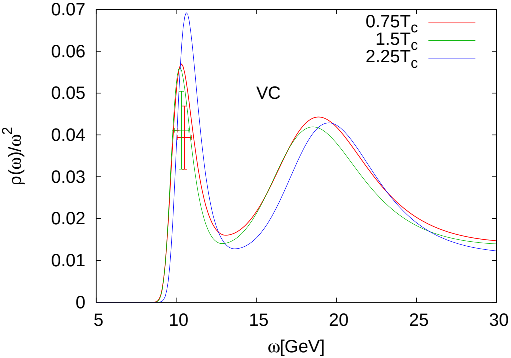
Bottomonia on lattice

- Bottomonia on lattice: large discretization error ($m_b a \sim 1$)
- Gluonic plasma: can use sufficiently fine lattices, $a^{-1} > 10$ GeV, and nonperturbatively $\mathcal{O}(a)$ improved bottom action.
- For thermal decay of bottomonia, thermal quarks not expected to be important.

D. Kharzeev & H. Satz, Phys. Lett. B334 (1994) 155

- Useful to compare the finite temperature correlator with correlator reconstructed from spectral function at low temperatures,
$G_{rec}(t; T) = \int \frac{d\omega}{\pi} \frac{\cosh\, \omega(t - 1/2T)}{\sinh\, \omega/2T} \rho(\omega, T' \approx 0)$

Υ at 1.5 T_c



No dramatic change on crossing T_c

ρ using Bayesian analysis: indicative of very small change in $\Upsilon(1S)$ upto quite high temperatures.

Point correlator not very sensitive to width.

Bottomonia using NRQCD

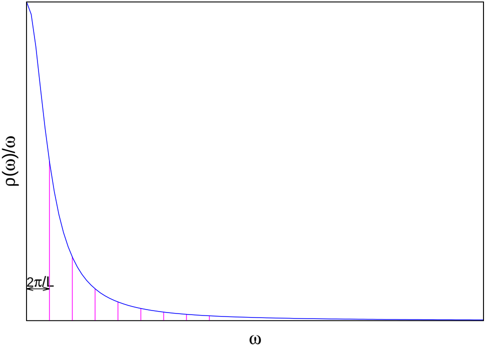
Since $m_b \gg$ any other scale of interest, can integrate it out to go to NRQCD action.

$$\mathcal{L}_{NR} = \theta^\dagger \left[i\partial_0 - gA_0 + \frac{D^2}{2M} \right] \theta + \phi^\dagger [...] \phi$$
$$+ \frac{1}{2M} \left[\theta^\dagger \sigma.gB\theta + \phi^\dagger \sigma.gB\phi \right] + \frac{i}{2M} \left[\theta^\dagger \sigma.gE\phi + h.c. \right] + ...$$

- Study using lattice NRQCD: $\Upsilon(1S)$ and $\eta_b(1S)$ survive till $> 2T_c$
  However, difficult to get reliable result for width.
  G. Aarts, et al., JHEP 11 (2011), 103; JHEP 12 (2013) 064.
  S. Kim, P. Petreczky and A. Rothkopf, PRD 91 (2015) 054511
- The NRQCD correlator does not include the diffusion peak.

"Diffusion" part in lattice correlator

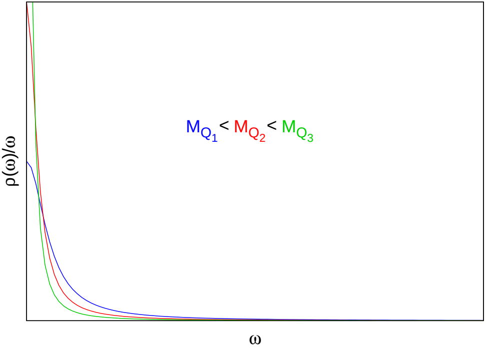
$$\rho_V(\omega) \underset{low\ \omega}{\sim} \chi_{00} D \frac{\omega\eta^2}{\eta^2+\omega^2}, \qquad \eta \sim \frac{1}{M}$$



For extraction of diffusion part: $\frac{2\pi}{L} \ll \eta$
For $DT \sim 1/\pi$: $\quad LT \gg 2M/T \sim 7$ for charm at 1.5 T_c

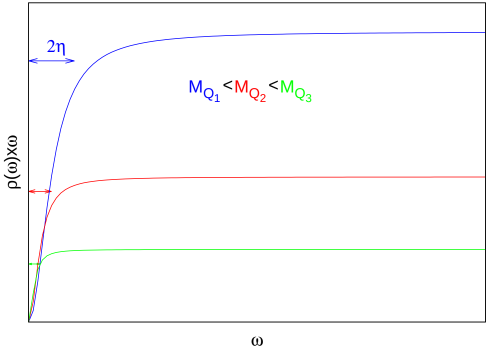
Extracting the diffusion part

$$\frac{1}{\chi_{00}} \frac{\rho_V(\omega)}{\omega} \quad \underset{low\ \omega}{\sim} \quad D \frac{\eta^2}{\eta^2 + \omega^2}, \qquad \eta \sim \frac{1}{M}$$



Extracting the diffusion part

$\frac{1}{\chi_{00}} \quad \frac{\rho(\omega)}{\omega} \quad x\omega^2$



$\Rightarrow$ spectral function for $\frac{dJ_i}{dt}$

$$\omega^2 \rho_V(\omega) \;=\; \int dt\; e^{i\omega(t-t')} \int dx\; \frac{1}{2}\langle[\dot{J}_i(\mathbf{x},t), \dot{J}_i(\mathbf{0}], t')\rangle$$

Force-force correlator

$$M^2 \times : \qquad \int dt \; e^{i\omega(t-t')} \int dx \; \frac{1}{2} \langle [M\dot{J}_i(\mathbf{x},t), M\dot{J}_i(\mathbf{0},t')] \rangle$$

From NRQCD Lagrangian, $M\dot{\vec{J}}_i \;=\; \phi^\dagger g\vec{E}\phi - \theta^\dagger g\vec{E}\theta + \mathcal{O}(\frac{1}{M})$
In the static limit,

$$G_E(\tau) = \frac{\mathrm{Re\ Tr}\; U(\beta,\tau_1)\vec{E}(\tau_1)U(\tau_1,\tau_2)\vec{E}(\tau_2)U(\tau_2,0)}{\mathrm{Re\ Tr}\; U(\beta,0)}$$

Caron-Huot, Laine & Moore, JHEP (2009)

G_E has a finite renormalization: calculated to 1-loop order.

Christensen & Laine, PLB (2016)

We expect $\lim_{\omega\to 0} \frac{\rho_E(\omega)}{\omega}$ to have finite static limit.

Langevin description of heavy quark in plasma

Svetitsky '88; Moore & Teaney '05; Rapp & van Hees '05; Mustafa '05

For thermal heavy quark, $M \gg T, \quad p \sim \sqrt{MT}$
Takes $\mathcal{O}(M/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
Scattering with thermal quarks: uncorrelated momentum kicks

$$\frac{dp_i}{dt} = \xi_i(t) - \eta p_i, \qquad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

$$\langle p^2 \rangle = 3MT \rightarrow \eta = \frac{\kappa}{2MT} \qquad \langle x_i(t) x_j(t) \rangle = 2Dt\delta_{ij} \rightarrow D = \frac{2T^2}{\kappa}$$

Static limit of ρ_F: color electric field correlator; Fluctuation dissipation theorem $\Rrightarrow$

$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)$$

Caron-Huot, Laine & Moore (2009)

- Calculated in HTL PT. Diffusion part $\propto \omega$ at small ω
However, very small κ/T^3, even negative at small temperatures.

Burnier, Laine, Langelage, Meher (2010)

- Calculated on lattice: $\frac{\kappa}{T^3} \sim 3$, rather flat, for $T \lesssim 1.5\,T_c$

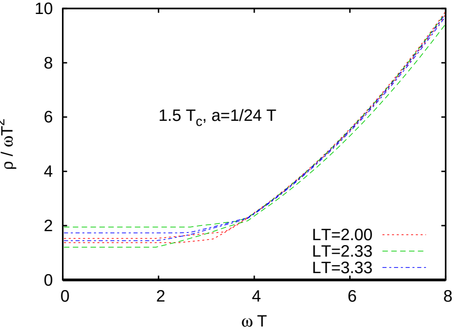
Banerjee, Datta, Gavai, Majumdar (2012);
Francis, Kaczmarek, Laine, Neuhaus & Ohno (2015)

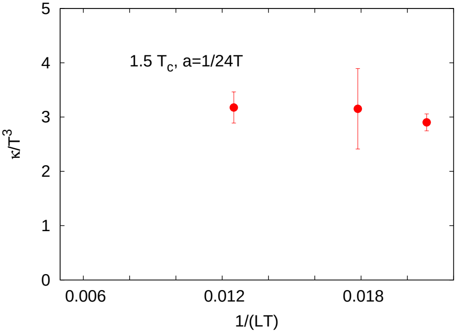
- κ from force-force correlator calculated in $\mathcal{N} = 4$ SYM, giving $\frac{\kappa}{T^3} = \pi\sqrt{\lambda_{tH}}$.

Casalderrey-Solana & Teaney (2008)

- Our ongoing study: finite volume and a effect, and approach to higher temperature.

Finite volume effect

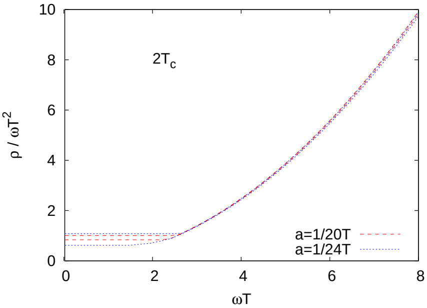


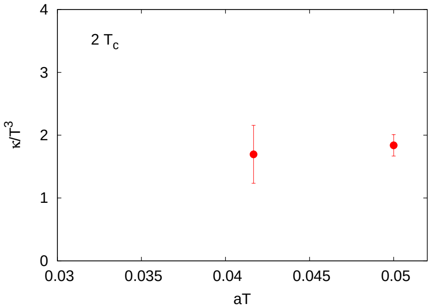


The correlator fitted to form $\rho_0 = (c_1\omega, c_2\omega^3)$ with IR and UV parts matched smoothly

also test a correction, $\rho_0 \left(1 + c' \sin \pi \frac{x}{1+x}\right)$, $\qquad x = \log(1 + \frac{\omega}{\pi T})$.

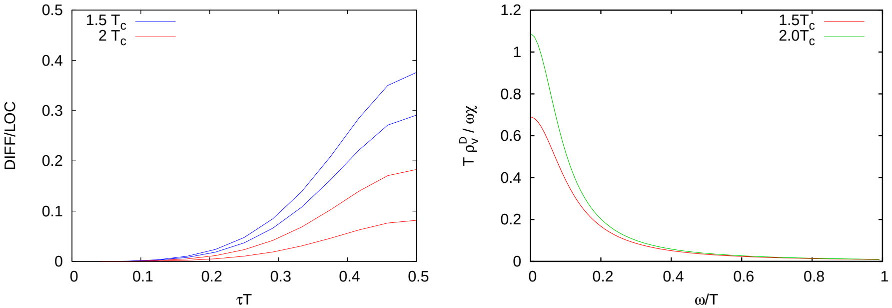
Cutoff effect at 2 T_c





There is an indication of κ decreasing with temperature. Need to go to higher temperatures to check the trend.

Diffusion contribution



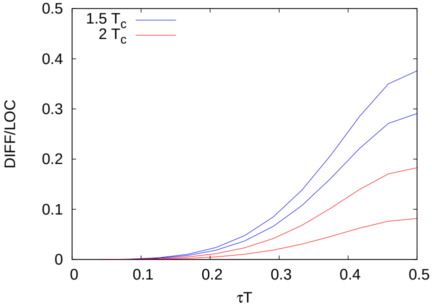
System NR: can use Einstein relations

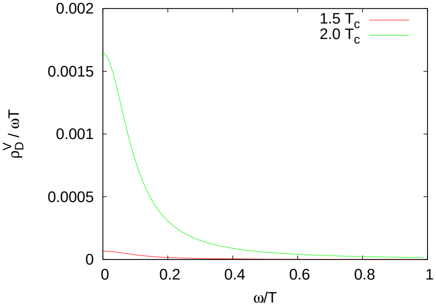
$$D = \frac{2T^2}{\kappa} \qquad \eta = \frac{\kappa}{2MT}$$

to get the diffusive peak in vector current correlator:

$$\rho_{VV} = \chi_{00} \, D \frac{\omega \eta^2}{\omega^2 + \eta^2}$$

Diffusion contribution





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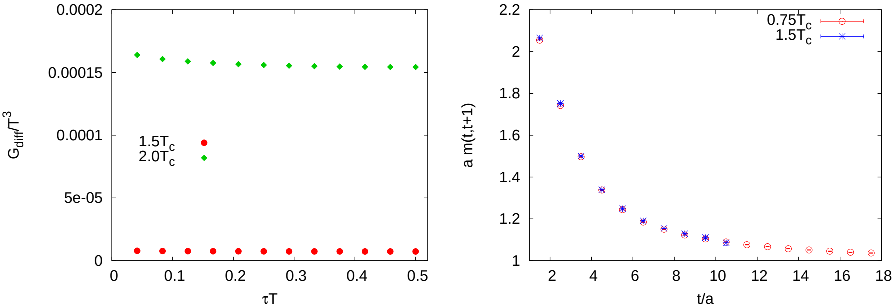
Diffusive part to correlator



Diffusive part gives flat contribution to the vector correlator. Once this part is removed, the Υ correlator shows very low change up to 1.5 T_c, indicating very small width at this temperature.

Summary

- Understanding of finite temperature vector quarkonia correlator of great interest for phenomenology of quark-gluon plasma.
- A study of bottomonia correlator using Bayesian analysis indicates that the Υ peak survives in plasma till temperatures in excess of 2 T_c.
- The diffusion peak, and thermal width of Υ, difficult to extract from the correlator.
- Diffusive part can be understood well using nonrelativistic field theory.
- Almost all modification of the vector correlator can be explained by the modification of diffusive part, indicating very small thermal modification of Υ.

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