Supersymmetry on the lattice and the light bound states of four dimensional $\mathcal{N} = 1$ supersymmetric Yang-Mills theory

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1. Supersymmetry on the lattice
2. Supersymmetric Yang-Mills theory on the lattice
3. The bound state spectrum of SU(3) supersymmetric Yang-Mills theory
4. SU(2) supersymmetric Yang-Mills theory at finite temperatures and compactifications

in collaboration with S. Ali, H. Gerber, P. Giudice, S. Kuberski, C. Lopez, G. Münster, I. Montvay, S. Piemonte, P. Scior

Why study SUSY on the lattice?

1. BSM physics: Supersymmetric particle physics requires breaking terms based on an unknown non-perturbative mechanism.
   $\Rightarrow$ need to understand non-perturbative SUSY
2. Supersymmetry is a general beautiful theoretical concept: (Extended) SUSY simplifies theoretical analysis and leads to new non-perturbative approaches.
   $\Rightarrow$ need to bridge the gap between “beauty” and **"reality"**

Lattice simulations of SUSY theories

Lattice simulations would be the ideal method to investigate non-perturbative sector of SUSY theories . . .

Theory: → next part

- Can we define a lattice SUSY?
- Can we control SUSY breaking?

Practical Simulations: → example SYM

- SUSY theories have nice properties, but require to rework numerical methods

. . . but are challenging from theoretical and practical point of view.

[G.B., S. Catterall, arXiv:1603.04478]

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SUSY breaking and the Leibniz rule on the lattice

Like Nielsen-Ninomiya theorem: locality contradicts with SUSY

On the lattice:

There is no Leibniz rule for a discrete derivative operator. The action can only be invariant with a non-local derivative and non-local product rule. [GB],[Kato,Sakamoto,So],[Nicolai,Dondi]

Further problems:

- fermonic doubling problem, Wilson mass term
- gauge fields represented as link variables

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"The lattice is the only valid non-perturbative definition of a QFT and it can not be combined with SUSY. Therefore SUSY can not exist!" (Lattice theorist)

General solution by generalized Ginsparg-Wilson relation?

"Mrs. RG, the good physics teacher…"
(Peter Hasenfratz)

Symmetry in the continuum ($S[(1+\varepsilon\tilde{M})\varphi] = S[\varphi]$) implies relation for lattice action S_L:

Generalized Ginsparg-Wilson relation

$$M^{ij}_{nm}\phi^j_m \frac{\delta S_L}{\delta \phi^i_n} = (M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S_L}{\delta \phi^j_m}\frac{\delta S_L}{\delta \phi^i_n} - \frac{\delta^2 S_L}{\delta \phi^j_m \delta \phi^i_n}\right)$$

$$\Phi[\tilde{M}\varphi] = M_{nm}\Phi_m[\varphi]$$

Still open problem how to find solutions. [GB, Bruckmann, Pawlowski]

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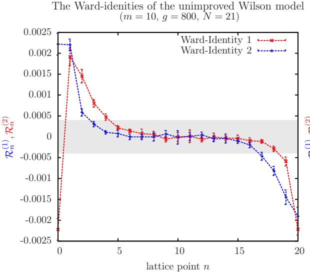
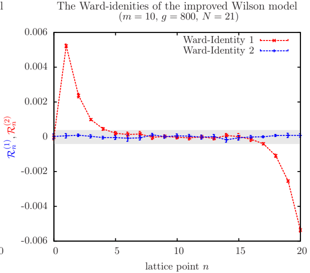
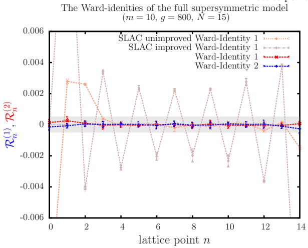
...but we still don't completely understand her lesson.

Sketch of solutions







- only model dependent solutions
- partial realization of extended SUSY
- non-local actions
- otherwise: fine tuning.

Super Yang-Mills theory

Supersymmetric Yang-Mills theory:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right]$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- λ Majorana fermion in the adjoint representation
- SUSY transformations: $\delta A_\mu = -2i\bar{\lambda}\gamma_\mu\varepsilon$, $\delta\lambda = -\sigma_{\mu\nu}F_{\mu\nu}\varepsilon$

Why study supersymmetric Yang-Mills theory on the lattice ?

1. extension of the standard model
   - gauge part of SUSY models
   - understand non-perturbative sector: check effective actions etc.
2. controlled confinement [Ünsal,Yaffe, Poppitz] :
   - compactified SYM: continuity expected
   - small R regime: semiclassical confinement
3. connection to QCD [Armoni,Shifman]:
   - orientifold planar equivalence: SYM $\leftrightarrow$ QCD
   - Remnants of SYM in QCD ?
   - comparison with one flavor QCD

Supersymmetric Yang-Mills theory: Symmetries

SUSY

- gluino mass term $m_g \Rightarrow$ soft SUSY breaking

$U_R(1)$ symmetry, "chiral symmetry": $\lambda \to e^{-i\theta\gamma_5}\lambda$

- $U_R(1)$ anomaly: $\theta = \frac{k\pi}{N_c}$, $U_R(1) \to \mathbb{Z}_{2N_c}$
- $U_R(1)$ spontaneous breaking: $\mathbb{Z}_{2N_c} \stackrel{\langle\bar{\lambda}\lambda\rangle \neq 0}{\to} \mathbb{Z}_2$

Supersymmetric Yang-Mills theory on the lattice

Lattice action:

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- Wilson fermions:

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right] + \text{clover}$$

  gauge invariant transport: $T_\mu \lambda(x) = V_\mu \lambda(x+\hat{\mu})$;

  $$\kappa = \frac{1}{2(m_g + 4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$ of SU(2), SU(3)

Lattice SYM: Symmetries

Wilson fermions:

- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

fine tuning:

- add counterterms to action
- tune coefficients to obtain signal of restored symmetry

special case of SYM:

- tuning of m_g enough to recover chiral symmetry [1]
- same tuning enough to recover supersymmetry [2]

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

Recovering symmetry

Fine-tuning:

chiral limit = SUSY limit $+O(a)$, obtained at critical $\kappa(m_g)$

- no fine tuning with Ginsparg-Wilson fermions (overlap/domainwall) fermions[3]; but too expensive

practical determination of critical κ:

- limit of zero mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

- cross checked with SUSY Ward identities

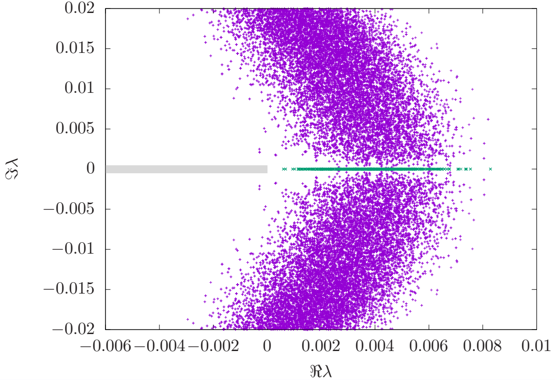
[3] [Fleming, Kogut, Vranas, Phys. Rev. D 64 (2001)], [Endres, Phys. Rev. D 79 (2009)], [JLQCD, PoS Lattice 2011]

The sign problem in supersymmetric Yang-Mills

Majorana fermions:

$$\int \mathcal{D}\lambda e^{-\frac{1}{2}\int \bar{\lambda} D\lambda} = \text{Pf}(CD) = (-1)^n \sqrt{\det D}$$

n = number of degenerate real negative eigenvalue pairs



no sign problem
@ current
parameters

Low energy effective theory

- confinement: colourless bound states
- symmetries + confinement → low energy effective theory
- glueballs, gluino-glueballs, gluinoballs (mesons)
- build from chiral multiplet type

	multiplet[1]	**multiplet**[2]
scalar	meson $a-f_0$	glueball 0^{++}
pseudoscalar	meson $a-\eta'$	glueball 0^{-+}
fermion	gluino-glue	gluino-glue

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]

[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

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Supersymmetry

Particles must have same mass.

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[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

Bound states in supersymmetric Yang-Mills theory

- like in YM and QCD: glueball bound states of gluons
- meson states (like flavour singlet mesons in QCD)
  - a–f_0 ($\bar{\lambda}\lambda$), a–η' ($\bar{\lambda}\gamma_5\lambda$)
  - on the lattice: disconnected contributions
- gluino-glue spin-1/2 state
  - Specific feature of adjoint representation: colourless mixed fermion-glue states.

$$\sum_{\mu,\nu} \sigma_{\mu\nu} \mathrm{tr}\left[F^{\mu\nu}\lambda\right]$$

Quite challenging to get good signal for the correlators of these operators. Mass determined from exponential decay of the correlator.

The status of the project

Advanced methods of lattice QCD required:

- disconnected contributions [LATTICE2011]
- eigenvalue measurements [GB,Wuilloud]
- variational methods (including mixing of glueball and meson operators) [LATTICE2017]

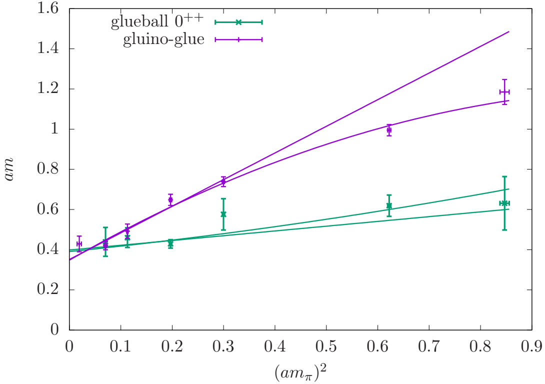
SU(2) SYM:

- multiplet formation found in the continuum limit of SU(2) SYM [JHEP 1603, 080 (2016)]

SU(3) SYM:

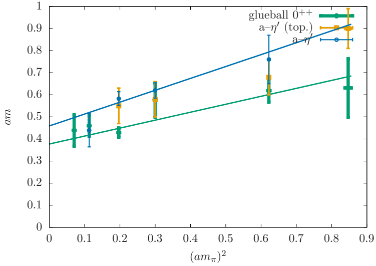
- adjoint representation much more demanding than fundamental one (limited to small lattice sizes)
- first SU(3) simulations [LATTICE99,LATTICE2016,LATTICE2017]
- results presented here: [arXiv:1801.08062]

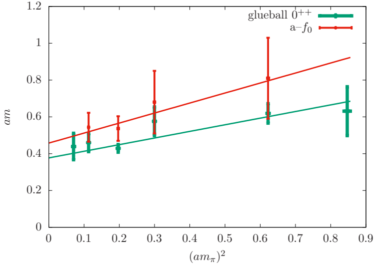
The fermion-boson mass degeneracy



- gluino-glue and glueballs become degenerate in the chiral limit (lattice size $16^3 \times 32$, $\beta = 5.5$)

The mesonic states and complete multiplet





Masses in units of r_0:

gluino-glue	glueball 0^{++}	a–η'	a–f_0
2.83(44)	3.22(95)	3.70(71)	3.69(63)

Towards the continuum limit

Challenges:

- lattice artefacts important: need fine lattices

Systematic uncertainties:

- finite volume effects checked
- topological freezing checked

preliminary results in units of $w_0^{0.2}$:

β	gluino-glue	glueball 0^{++}	volume	topology
5.4	0.90(13)	0.6240(59)	●	●
5.5	0.743(77)	0.84(20)	●	●
5.6	0.673(66)	0.60(15)	○	□

SU(2) supersymmetric Yang-Mills theory at finite temperature

Deconfinement:

- above $T_c^{\mathrm{deconf.}}$ plasma of gluons and gluinos
- Order parameter: Polyakov loop

Chiral phase transitions:

- above T_c^{chiral} fermion condensate melts and chiral symmetry gets restored
- order parameter: $\langle\bar{\lambda}\lambda\rangle$

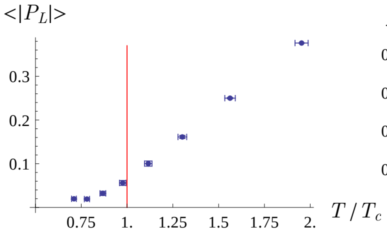
In QCD:

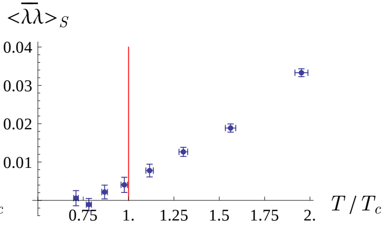
- quarks add screening effects
- explicit chiral symmetry breaking

$\rightarrow$ both transitions become crossover

In SYM: two independent transitions (at $m_g = 0$)

Lattice results SYM at finite T



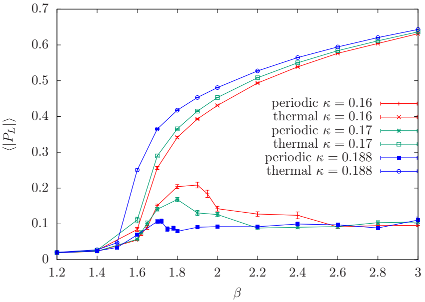


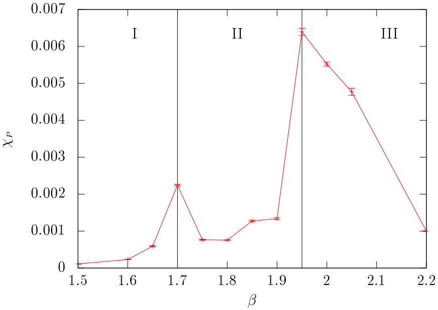
- second order deconfinement transition

$$\frac{T_c(\mathrm{SYM})}{T_c(\mathrm{pure\ Yang\text{-}Mills})} = 0.826(18).$$

- coincidence of deconfinement and chiral transition
  $T_c^{\mathrm{chiral}} = T_c^{\mathrm{deconf.}}$ (within current precision) [JHEP 1411 (2014) 049]

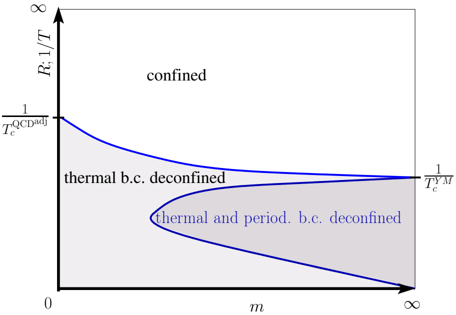
Compactified SYM with periodic boundary conditions





- fermion boundary conditions: thermal $\rightarrow$ periodic
- at small m (large κ) no signal of deconfinement
- intermediate masses: two phase transitions (deconfinement + reconfinement) [GB,Piemonte]

Phase diagram at finite temperature/compactification



- change of boundary conditions in compact direction $Z(\beta_B) \to \tilde{Z}(\beta_B)$ (Witten index)
- Witten index can not have β_B dependence: states can only be lifted pairwise $\Rightarrow$ continuity in SYM

Conclusions

- simulation of supersymmetric theories on the lattice is still in some aspects an open theoretical problem

Supersymmetric Yang-Mills theory:

- theoretical problem is solvable, practical challenges
- SUSY breaking is under control and formation of chiral multiplet observed for the gauge groups SU(2) and SU(3)
- interesting non-perturbative physics like the phase diagram can be investigated on the lattice
- further aspects of the spectrum are currently investigated: mixing of glueballs and gluinoballs, excited states, further bound states