SU(N) gauge theories beyond large N: universalities of spectral ratios

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Outline

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The 't Hooft's large-N limit

The Conjecture

In the limit $N \to \infty$ and $g \to 0$ with $\lambda = g^2 N$ fixed, physical quantities in SU(N) gauge theories can be expressed as functions of $1/N$ (if N_f fermions in the fundamental representations are present) or $1/N^2$ (in the Yang-Mills case), with a finite large-N limit and a convergent series expansion about that limit down to some $N = N^\star$

Relevance

- Explanation of observed QCD features (OZI rules, stability of particles, . . .)
- Potential for analytic calculations
- Connection with gauge-string dualities

Support

Large-N extrapolation of lattice results

[For a review, see B. Lucini and M. Panero, Phys. Rept. 526 (2013) 93]

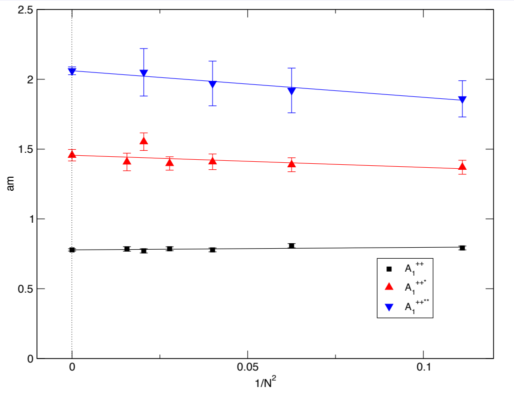
Large-N limit on the lattice

The lattice approach allows us to go beyond perturbative and diagrammatic arguments. In SU(N) YM, for a given observable

1. Continuum extrapolation
   - Determine its value at fixed a and N
   - Extrapolate to the continuum limit
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$
2. Fixed lattice spacing
   - Choose a in such a way that its value in physical units is common to the various N
   - Determine the value of the observable for that a at any N
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$

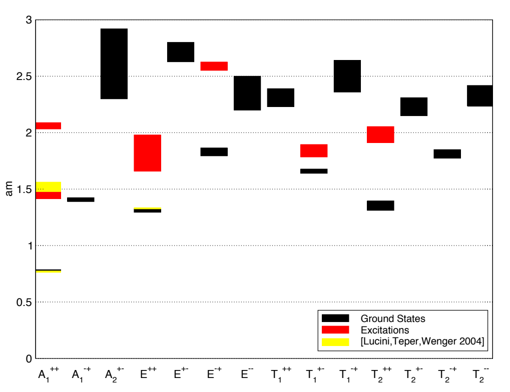
Study performed for various observables both at zero and finite temperature for $2 \leq N \leq 8$ (and $N = 17$!)

The A^{++} glueball channel



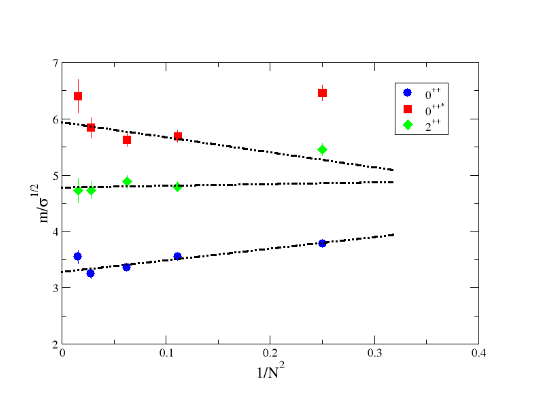
Lattice spacing fixed by requiring $aT_c = 1/6$
[B. Lucini, A. Rago and E. Rinaldi, JHEP 1008 (2010) 119]

The glueball spectrum at $aT_c = 1/6$



[B. Lucini, A. Rago and E. Rinaldi, JHEP 1008 (2010) 119]

Large-N extrapolation of glueball masses



[B. Lucini, M. Teper and U. Wenger, JHEP 0406 (2004) 012]

Glueball masses at $N = \infty$

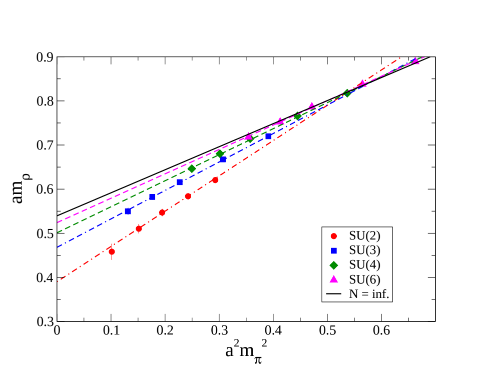
0^{++} $$\frac{m}{\sqrt{\sigma}} = 3.28(8) + \frac{2.1(1.1)}{N^2}$$

0^{++*} $$\frac{m}{\sqrt{\sigma}} = 5.93(17) - \frac{2.7(2.0)}{N^2}$$

2^{++} $$\frac{m}{\sqrt{\sigma}} = 4.78(14) + \frac{0.3(1.7)}{N^2}$$

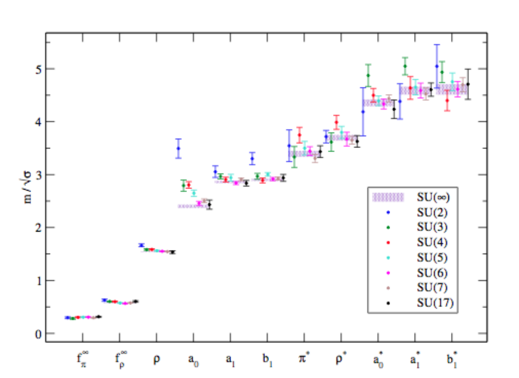
Accurate $N = \infty$ value, small $\mathcal{O}(1/N^2)$ correction

m_ρ vs. m_π^2 at $N = \infty$



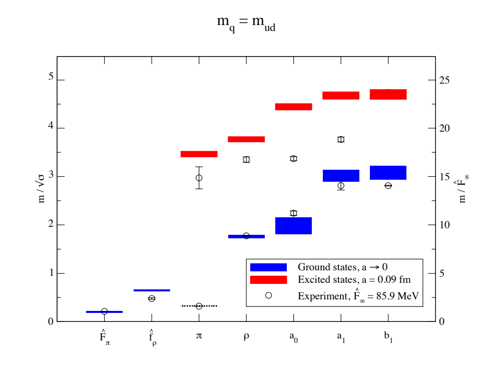
[L. Del Debbio, B. Lucini, A. Patella and C. Pica, JHEP 0803 (2008) 062]
[see also G. Bali and F. Bursa, JHEP 0809 (2008) 110]

The meson spectrum approaching $N = \infty$



[G. Bali *et al*,, JHEP 1306 (2013) 071]

Comparison with QCD



$\sqrt{\sigma}$ fixed from the condition $\hat{F}_\infty = 85.9$ MeV, m_{ud} from $m_\pi = 138$ MeV [G. Bali *et al*,, JHEP 1306 (2013) 071]

What do we learn?

- Lowest-lying mesons broadly compatible with QCD, excitations off by 20% (however, excitations less controlled in our calculation)
- The calculated large-N masses $m_\rho = 753(14)\ \mathrm{MeV}$ and of the $m_\phi = 981(44)\ \mathrm{MeV}$ are remarkably close to their experimental values $m_\rho = 775$ and $m_\phi = 1019\ \mathrm{MeV}$
- Observed degeneracy of (ρ, a_0) and $(a_1, \pi^\star)$ (predicted by χPT)

Outline

Casimir scaling of the string tension and of the scalar glueball mass

The conjecture

In a Yang-Mills theory, the ratio $m^2_{0^{++}}/\sigma$ is proportional to the ratio of the quadratic Casimir of the adjoint over that of the fundamental representation of the gauge group, with the proportionality constant η being **universal** (i.e. dependent only on the dimensionality):

$$\frac{m^2_{0^{++}}}{\sigma} = \eta(0^{++})\frac{C_2(A)}{C_2(F)}$$

Relevance

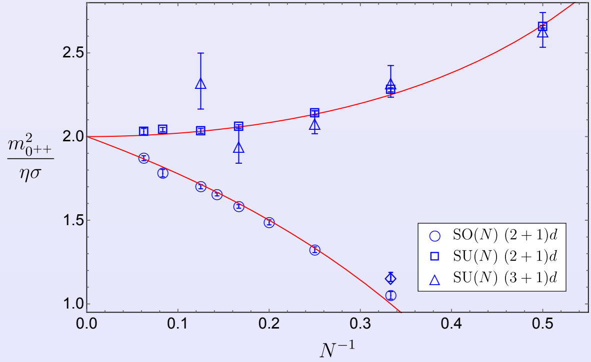
Mechanisms of colour confinement

Support

- Lattice data on glueball masses
- Semi-analytic arguments

[Hong, Lee, Lucini, Piai and Vadacchino, Phys. Lett. B775 (2017) 89, arXiv:1705.00286]

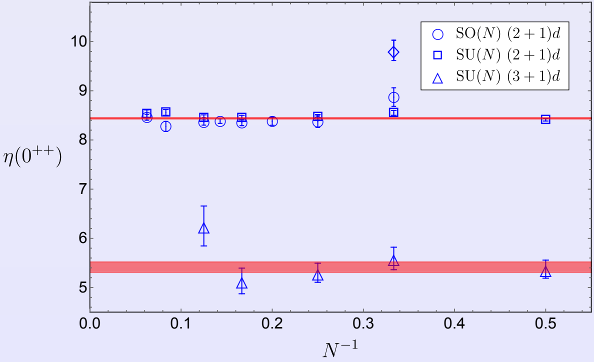
Testing with lattice results



SU(N) data in $d = 3 + 1$ from Lucini, Teper, Wenger, hep-lat/0404008
SU(N) data in $d = 2 + 1$ from Athenodorou, Lau, Teper, arXiv:1504.08126
SO(N) data in $d = 2 + 1$ from Lau, Teper, arXiv:1701.06941

(Also Sp(4) result in $d = 2 + 1$ from Bennett *et al.*, arXiv:1712.04220)

Universality of η

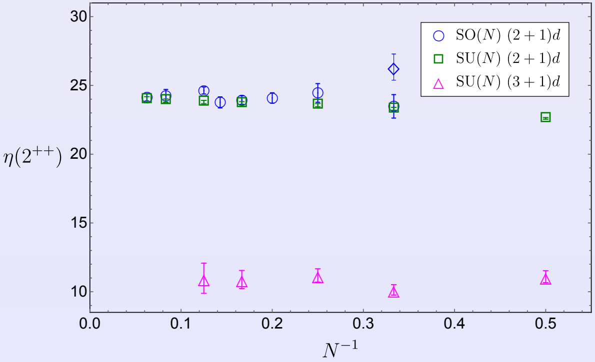


$$\eta(0^{++}) \equiv \frac{m^2_{0^{++}}}{\sigma} \cdot \frac{C_2(F)}{C_2(A)} = \begin{cases} 5.41(12)\,, \ \chi^2 \simeq 1 \text{ for } d = 3+1 \\ 8.440(14)(76)\,, \ \chi^2 \simeq 1.9 \text{ for } d = 2+1 \end{cases}$$

Some (semi-)analytic derivations

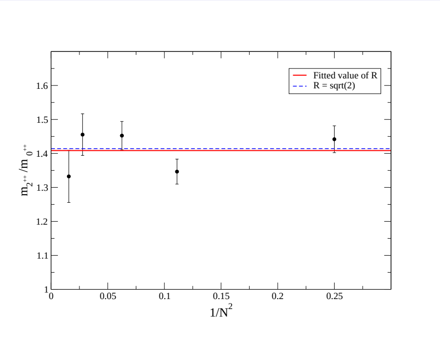
- Ground-state calculation in $d = 2 + 1$ (Leigh, Minik, Yelnikov, 2007)
- Constituent gluon model (Buisseret, Bicudo, . . .)
- Bethe-Salpeter equation (Hong *et al.*, 2017)
- Glueball as a dilaton (Hong *et al.*, 2017)
- Sum rules (Hong *et al.*, 2017)
- Coulomb gauge Hamiltonian (Greensite, private communication, 2017)

Casimir scaling of $m^2_{2^{++}}/\sigma$?



Data are inconclusive: bad constant ansatz for $d = 2 + 1$, plausible constant ansatz for $d = 3 + 1$

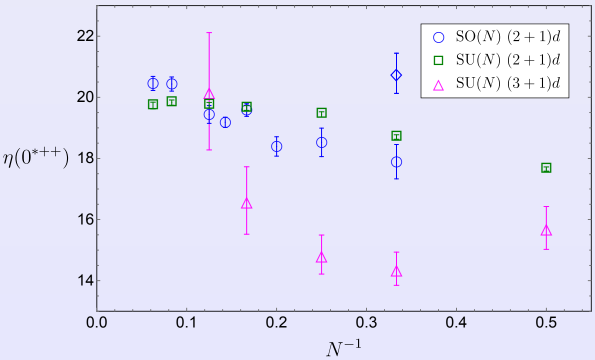
Universality of $m_{2^{++}}/m_{0^{++}}$?



Fit result: $m_{2^{++}}/m_{0^{++}} = 1.409(26)$, $\chi^2 \simeq 1.5$

In $d = 3 + 1$, is $m_{2^{++}}/m_{0^{++}} = \sqrt{2}$ independently of the gauge group?

(No) Casimir scaling of $m^2_{0^{++*}}/\sigma$



No clear sign of universality in either d=2+1 or d=3+1

Outline

Mass anomalous dimension and glueballs

The conjecture

In an asymptotically free SU(N) gauge theory with N_f fermions in the fundamental or a two index irreducible representation of the gauge group, the anomalous dimension of the fermion condensate is a monotonically increasing function of the glueball mass ratio $m_{2^{++}}/m_{0^{++}}$

Relevance

Studies of nearly-conformal and conformal gauge theories in relation to novel strongly interacting dynamics beyond the standard model

Support

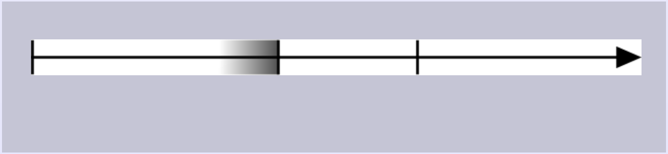
- Gauge-string duality
- Lattice calculations

[A. Athenodorou *et al.*, JHEP 1606 (2016) 114, arXiv:1605.04258]

Phases of a gauge theory

At fixed N a critical number of flavours N_f^{cu} exists above which asymptotic freedom is lost

Banks and Zaks conjectured that an N_f^{lu} exists such that a non-trivial infrared fixed point appears for $N_f^{lu} \leq N_f \leq N_f^{cu}$ (conformal window)



At fixed fermion representation N_f^{lu} depends on the number of flavours

Near the BZ point naive scaling arguments can not be applied and walking can arise

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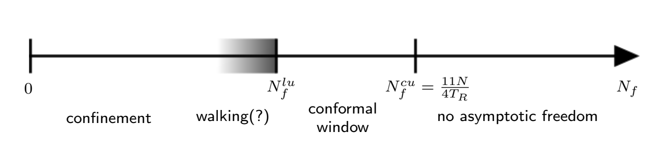

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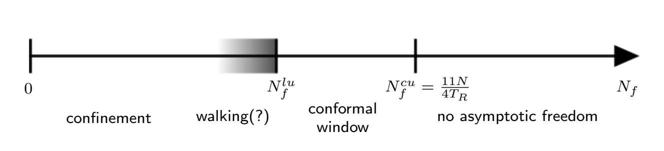

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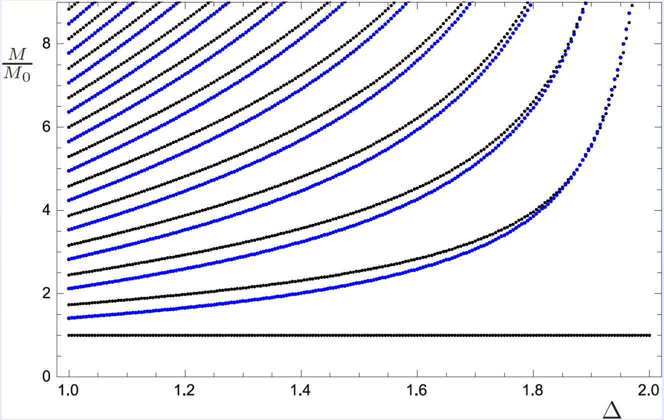
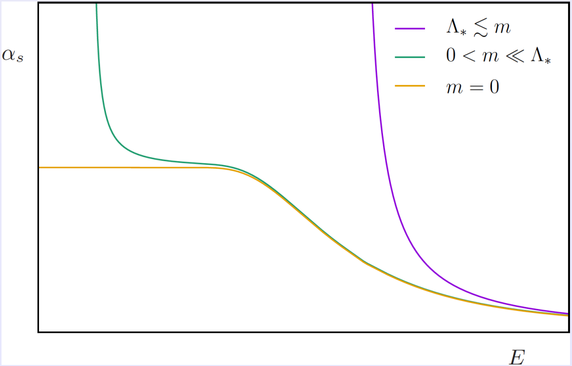
Glueball masses and anomalous dimension in the GPPZ model



Mass of the 2^{++} glueball monotonically increases with $\Delta = 1 + \gamma^\star$

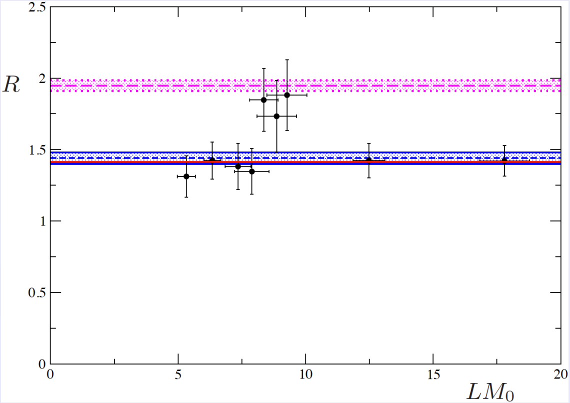
Near-conformality and lattice calculations



Strategy

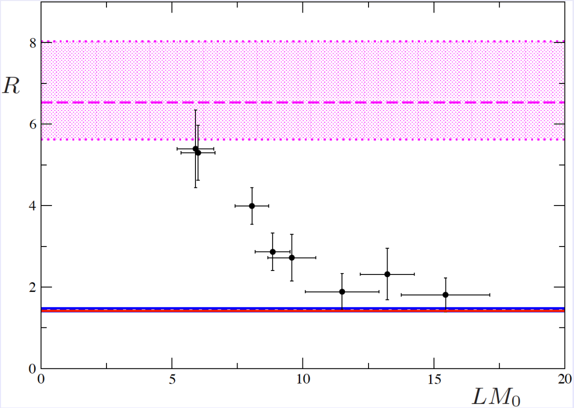
Compute $R = m_{2^{++}}/m_{0^{++}}$ as a function of the box size measured in units of $m_{0^{++}}$

Results for SU(2) with two adjoint flavours



Reference value $\gamma^\star = 0.371(20)$
[value from Del Debbio *et al.*, arXiv:1512.08242]

Results for SU(2) with one adjoint flavour



Reference value $\gamma^\star = 0.925(25)$
[value from Athenodorou *at al.*, arXiv:1412.5994]

Outline

Conclusions

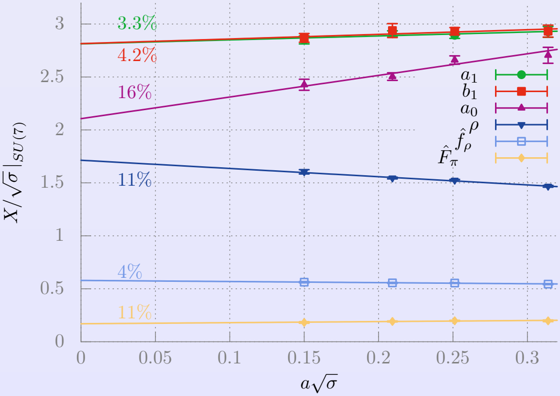
Numerical results obtained in the framework of lattice gauge theories suggest that

- A large-N limit *à la* 't Hooft can be constructed in SU(N) gauge theories, with finite-N observables being close (in the sense of a perturbative expansion in $1/N$ or $1/N^2$) to the latter all the way down to the physical case of SU(3)
- In Yang-Mills gauge theories, the ratio of the scalar glueball mass squared over the string tension is proportional to a ratio of quadratic Casimirs, with the proportionality constant being universal (i.e. depending only on the dimension)
- The ratio of the tensor and scalar glueball is $\sqrt{2}$
- In theories with fermions in the fundamental or a two-index representation, the ratio of the tensor over the scalar glueball is a universal monotonically increasing function of the chiral condensate anomalous dimension

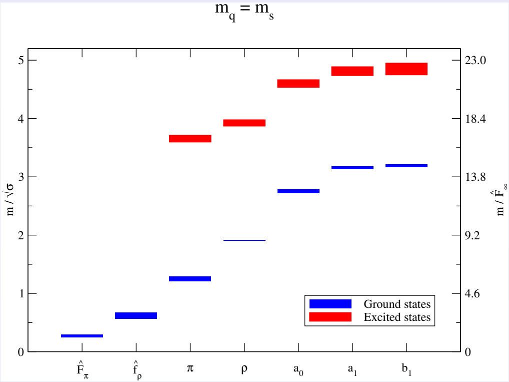
Future directions include

- to further explore numerically those conjectures
- to obtain more information from analytic arguments
- to understand the mechanisms that determine those universalities
- to identify potential violations

Continuum meson spectrum – SU(7)

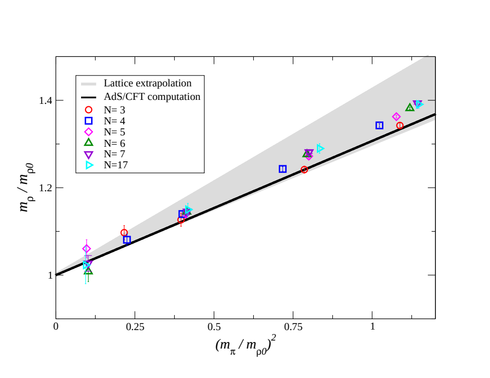


The strange meson spectrum



m_s fixed from $m_\pi(m_s) = (m^2_{K^\pm} + m^2_{k^0} - m^2_{\pi^\pm})^{1/2} = 686.9\ \mathrm{MeV}$

Meson masses from gauge-string duality



Ads/CFT data from Erdmenger *et al.*, Eur.Phys.J. A35 (2008) 81-133 [arXiv:0711.4467]

Relevant quadratic Casimir ratios

$$\frac{C_2(A)}{C_2(F)} = \begin{cases} \frac{2N^2}{N^2-1} & \text{for } \mathrm{SU(N)} \\ \frac{2(N-2)}{N-1} & \text{for } \mathrm{SO(N)} \\ \frac{4(N+1)}{2N+1} & \text{for } \mathrm{Sp(2N)} \end{cases}$$