Twisted matrix models: Recent results and future prospects

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Twisted matrix models

- Class of matrix models which, in the large N limit, are claimed to be equivalent to large N lattice field theories theories at infinite volume.
- Their equivalence at all values of the lattice coupling implies their equivalence in the continuum limit also.
- A common feature of these theories is the presence of integer fluxes in their formulation.
- These theories can be considered in some cases regularized versions of non-commutative field theories.

TEK model (*GA Okawa 1983*)

This is the first of these models and the best studied. The partition function is this

$$Z = \prod_{\mu} \left(\int dV_{\mu} \right) e^{-S_{\mathrm{TEK}}}$$

with

$$S_{\mathrm{TEK}} = -\frac{N}{\lambda_L} \sum_{\mu,\nu} z_{\mu\nu} \mathrm{Tr}(V_{\mu} V_{\nu} V_{\mu}^{\dagger} V_{\nu}^{\dagger}) + \mathrm{h.c.}$$

and $z_{\mu\nu} = z_{\nu\mu}^{*} = e^{2\pi i n_{\mu\nu}/N}$, where $n_{\mu\nu} \in (Z/NZ)^d$

- SU(N) Lattice gauge theory with twisted boundary conditions in a lattice of size $L^d = 1$. Obviously bigger sizes $L > 1$ can also be considered.

The claim is that this matrix model with an adequate choice of the twist tensor $n_{\mu\nu}$ is equivalent to large N gauge theories at infinite volume.

Choice of twist

- ♣ The non-perturbative proof of equivalence (*Eguchi Kawai 1982*) demands that the model remains invariant under a sufficiently large subgroup of the center symmetry group $\mathbb{Z}_N^d$
$V_\mu \longrightarrow z_\mu V_\mu$
- ♠ Our analysis in perturbation theory (*GA Okawa 1983*) demands that twist tensor is orthogonal and irreducible.

  In 4D a simple symmetric example: $N = \hat{L}^2$ and $|n_{\mu\nu}| = k\hat{L}$ and k coprime with $\hat{L}$.

QUESTION

How to choose the flux k to ensure center symmetry and also to minimize finite N corrections?

Finite N corrections in Perturbation Theory

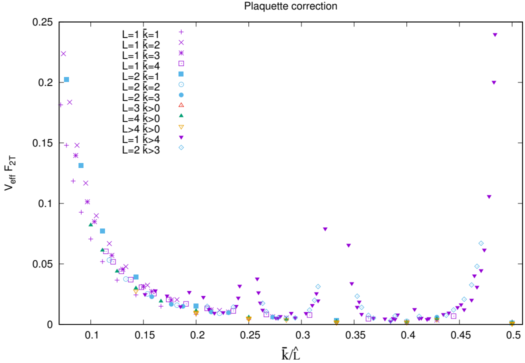
Perturbation theory gives information on the effect of finite N. Illustrated by the dependence of $R \times T$ Wilson loops on L, N and k.

- Part of the corrections amount to finite size effects on a lattice of size $L_{\rm eff}^4$ (with $L_{\rm eff} = L\sqrt{N}$).
  Thus loop sizes $R \ll L_{\rm eff}$. This determines the range of $\sqrt{N}$ values to be those of standard lattice calculations:
  $\sqrt{N} = 17, \ldots, 29, 37$.
- The second correction comes from inefficient suppression of **non-planar contributions**.
  The choice of k and $\bar{k}$ can be very important
  ($\bar{k}k = 1 \bmod N$).

Perturbative calculation of Wilson loops to order λ^2 *(Garcia Perez, AGA, Okawa 2017)*: Non-planar diagram contribution goes as $\delta W_2(R \times R) \sim (\frac{R}{L_{\rm eff}})^4 c(k, N, L)$

Non Planar contribution to order λ^2

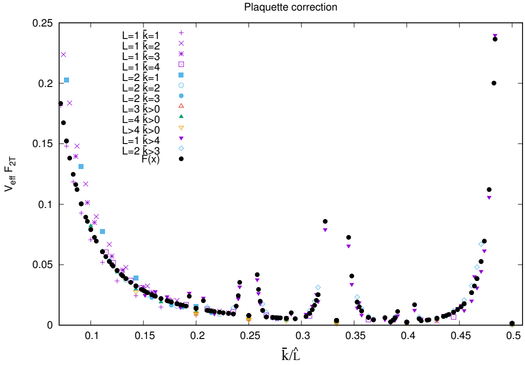
Plot of $c(k, N, L)$, the coefficient in $1/L_{\rm eff}^4$, as function of $\bar{k}/\sqrt{N}$



Strong dependence on $\bar{k}/\sqrt{N}$

Non Planar contribution to order λ^2

The coefficient is mostly controlled by $Z_{\min} = \min_n n||n\bar{k}/\sqrt{N}||$
Plot of $c(k, N, L)$ compared with $F(\bar{k}/N) = c_0/Z_{\min}^{2.5}$ with $c_0 = 0.00025$



Slight dependence on c_0

Results at intermediate λ

- Signs of center symmetry breaking observed (*Ishikawa-Okawa, Vairinhos-Teper, Azeyanagi et al, Bietenholz et al*)
- All problematic cases correspond to $k/\hat{L},\ \bar{k}/\hat{L} < 0.1$ (*GA, Okawa 2010*).
  No signs of symmetry breaking observed whenever $k/\hat{L},\ \bar{k}/\hat{L} > 0.1$ for $N \leq 37^2 = 1369$
- Direct test of equivalence (*AGA, Okawa 2014*)

Testing the equivalence

- Direct test on the lattice (*AGA, Okawa 2014*):
  a) $R \times R$ Wilson loops ($L^4 = 16^4$ lattice with PBC, $b = 1/\lambda_L$). Extrapolated from $N = [8, 16]$
  b) TEK $L = 1^4$ lattice $N = 1369$

	R=1 b=0.36	R=4 b=0.36	R=1 b=0.37	R=4 b=0.37
a)	0.558012(12)	0.005851(26)	0.578978(17)	0.009966(45)
b)	0.558019(11)	0.005861(13)	0.578959(5)	0.009926(3)

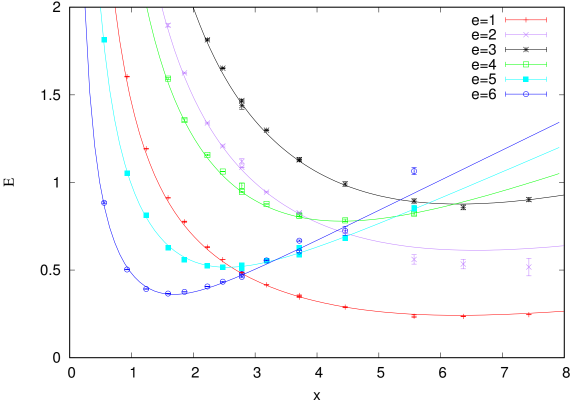
- Direct test in the continuum limit (*AGA, Okawa 2013*)

String tension from Wilson loops:	$\Lambda_{\overline{MS}}/\sqrt{\sigma}$
N=3,5,6,8 $L^4 = 32^4$ PBC extrapolated	**0.515(3)**
TEK L=1 N=841	**0.513(6)**

Lessons from SU(N) gauge theory on $T_2 \times \mathbb{R}$

- This simpler system serves a testing ground of the interplay between L, N and $n_{12} = k$.
- The main observables are the ground state energies in each electric flux sector $E(\vec{e}, k, N, L, \lambda) = \lambda \mathcal{E}(\vec{e}, k, N, x)$ with $x = \lambda N L/(4\pi)$.
- For some values of k and N there is a condensation of some fluxes at intermediate values of k:**Tachyonic instabilities**
- By Morita duality this system is a NC field theory on the non-commutative torus of size LN and NC dimensionless parameter $\bar{\theta} = \bar{k}/N$
- Our lattice study (*Garcia Perez, GA, Koren, Okawa*) exploring $N \in [5, 89]$, $L \in [1, 28]$ and $x \in [0.2, 8]$ shows that $\mathcal{E}^2$ described by a function $G_{\rm pert} + G_{\rm string} \sim (\frac{a+bx+cx^2+dx^4}{x^2})$

Size dependence of energies $N = 17$ $k = 3$



Implications (*Chamizo GA 2017*)

The minimum of $\min_{\vec{e}} \mathcal{E}(\vec{e})$ is a function of $Z_{\rm min}(k,N)$ (as before with $\sqrt{N} \to N$)

- If $Z_{\rm min}(k,N) > 0.1$ there are no tachyonic instabilities.
- For almost all N there exist k which avoids instabilities (*Huang 2015*)
- Removing the *almost* is equivalent to an unproven mathematical conjecture (*Zaremba 1974*)
- The optimal choice (maximal $Z_{\rm min}$) is given by the Fibonacci numbers: $N = F_p$ and $k = \bar{k} = F_{p-2}$
- Defining the NC theory on the NC torus as limit of ordinary theories without tachyonic instabilities (*Alvarez-Gaumé Barbón 2001*) is only possible for irrationals belonging to Cantor-type set (zero measure, positive Haussdorff dimension).

What can be done?

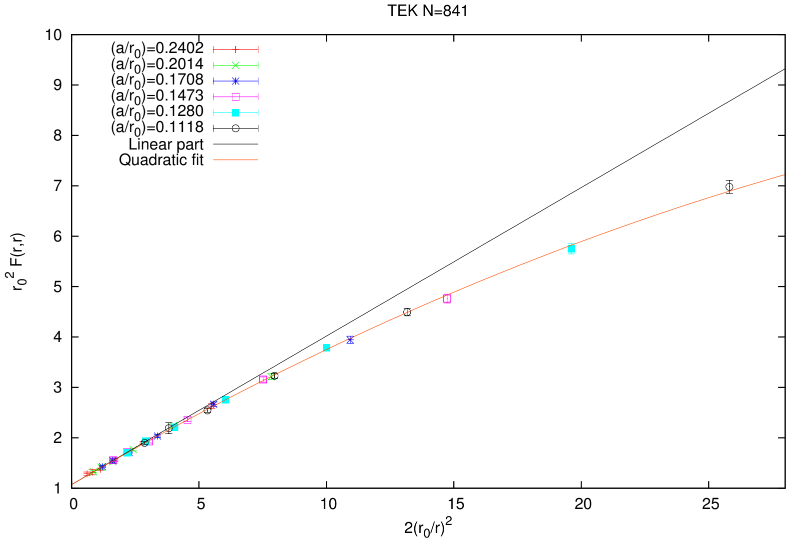
Alternative to extrapolations to compute physical quantities of the large N gauge theory. Apart from string tension

- A continuum observable for Wilson loops

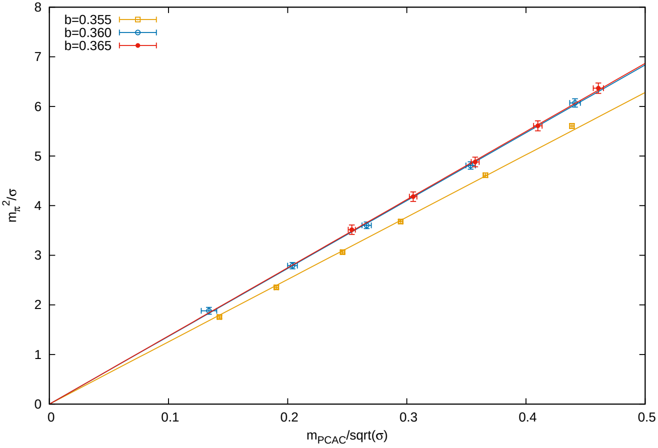
$$F(r) = -\frac{\partial \log W(r,t)}{\partial r \partial t}\Big|_{t=r}$$

- The Meson spectrum and decay constants. The method has been tested and continuum limit results are under way (*Garcia Perez, GA, Okawa 2016, 2018?*)
- The gradient flow running coupling (*Garcia Perez, GA, Keegan, Okawa 2015*)
- Others: Yang-Mills at finite temperature (*Das and Kogut*), Chiral condensate, Glueball spectrum(?)
- Large N Yang-Mills theory in 2 and 3 dimensions is been studied: Mesons and Glueballs!!
- Non-perturbative behaviour of Non-commutative Field theories

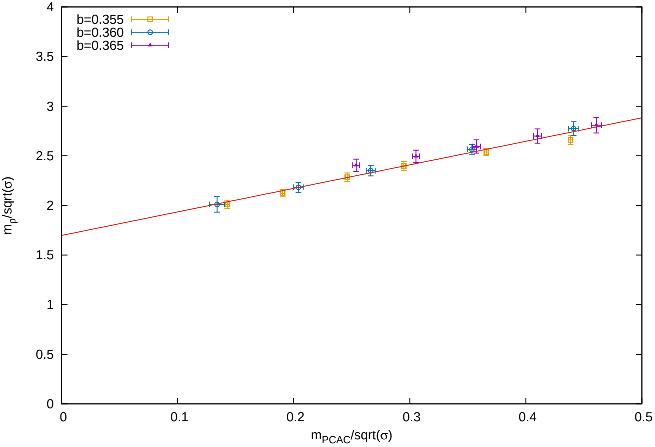
Wilson loop observable



Pion mass and Chiral symmetry

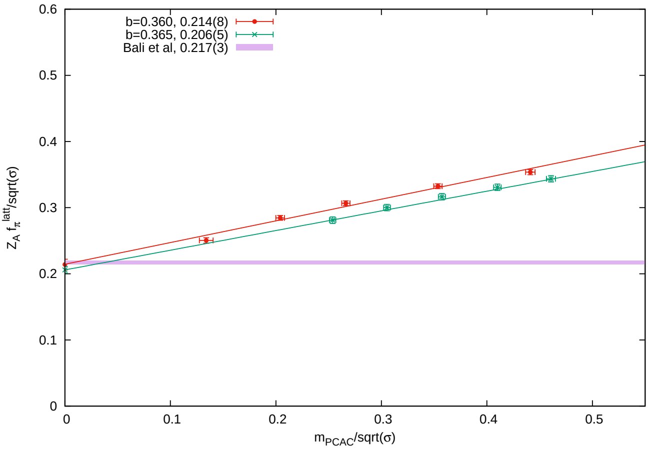


Rho mass



b=0.355
b=0.360
b=0.365
4
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
m_ρ/sqrt(σ)
m_{PCAC}/sqrt(σ)

F_π



b=0.360, 0.214(8)
b=0.365, 0.206(5)
Bali et al, 0.217(3)
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
Z_A f_π^latt/sqrt(σ)
m_PCAC/sqrt(σ)

Other twisted matrix models

- ♣ Yang-Mills theory in d dimensions with N_f fermions in the adjoint rep (*GA Okawa 2013*)
  - Mass anomalous dimension for $N_f = 2$: // $\gamma_* = \mathbf{0.269 \pm 0.002 \pm 0.05}$ (*Garcia Perez, GA, Keegan, Okawa 2015*)
  - Fundamental Meson spectrum

  $\mathcal{N} = 1$ **SUSY Yang-Mills?**

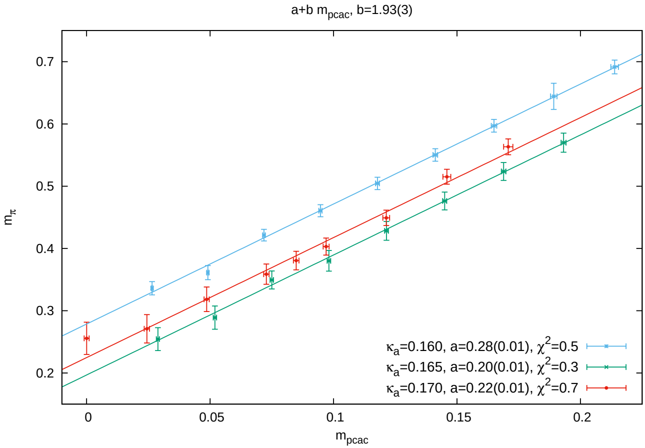
- ♠ Yang-Mills theory in d dimensions in the Veneziano limit for certain values of N_f/N

- ♣ Non-gauge matrix models in the large N limit such as principal chiral model in d dimensions.

$$\partial_\mu \Phi \longrightarrow \Gamma_\mu \Phi \Gamma_\mu^\dagger$$

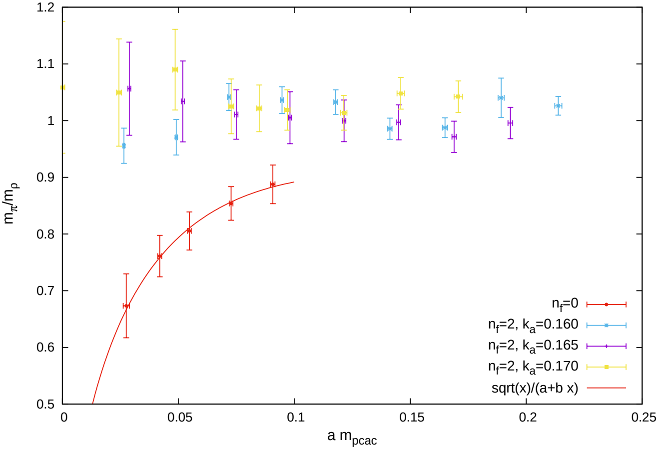
with $\Gamma_\mu \Gamma_\nu = e^{2\pi i/N} \Gamma_\nu \Gamma_\mu$

Pion mass



a+b m_pcac, b=1.93(3)
m_π
m_pcac
0.7
0.6
0.5
0.4
0.3
0.2
0
0.05
0.1
0.15
0.2
κ_a=0.160, a=0.28(0.01), χ^2=0.5
κ_a=0.165, a=0.20(0.01), χ^2=0.3
κ_a=0.170, a=0.22(0.01), χ^2=0.7

Mass Ratio m_ρ/m_π



Conclusions

- Twisted matrix models provide a useful instrument to study of large N gauge and non-gauge theories in the large N limit. They provide an alternative to extrapolations for pure gauge systems. Their advantage is crutial in theories with dynamical fermions. To minimize finite N errors it is convenient to choose fluxes having large values of $Z_{\rm min}$.