Recent results from lattice supersymmetry in $2 \leq \mathrm{d} < 4$ dimensions

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arXiv: 1800.00012 and work in progress with
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Outline

- Motivation and possibilities
- Two dimensional $\mathcal{N} = (2,2)$ SYM –susy breaking ?
- Holographic connection via three dimensional SYM (16 S.C)

Why lattice supersymmetry (SUSY) ?

Discretization on the lattice furnishes gauge-invariant regularization of gauge theories and provides non-perturbative insights into

- Gauge/gravity (AdS/CFT) duality - potential non-perturbative definition of string theory
- Finite N regime and large N limit of supersymmetric theories.
- Confinement, phase transitions, symmetry breaking and conformal field theories.

Lattice SUSY : Problem and resolution

Problem

Supersymmetry generalizes Poincaré symmetry by adding spinorial generators Q and $\bar{Q}$ to translations, rotations, boosts

The algebra includes $Q\bar{Q} + \bar{Q}Q = 2\sigma^{\mu} P_{\mu}$,

P_{μ} generates infinitesimal translations, which don't exist on the lattice. Supersymmetry explicitly broken at the classical level.

Solution

Preserve a subset of SUSY algebra exactly on the lattice. Possible for theories with $Q \geq 2^{D}$. For ex : $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM). Methods are based on orbifold construction and topological twisting. I will focus on the latter in this talk.

Lattices in various dimensions

THEORY	R-SYMMETRY	LATTICE CONSTRUCTION ?
$d = 2, \mathcal{Q} = 4$	$SO(2) \bigotimes U(1)$	✓
$d = 2, \mathcal{Q} = 8$	$SO(4) \bigotimes SU(2)$	✓
$d = 2, \mathcal{Q} = 16$	$SO(8)$	✓
$d = 3, \mathcal{Q} = 4$	$U(1)$	
$d = 3, \mathcal{Q} = 8$	$SO(3) \bigotimes SU(2)$	✓
$d = 3, \mathcal{Q} = 16$	$SO(7)$	✓
$d = 4, \mathcal{Q} = 4$	$U(1)$	
$d = 4, \mathcal{Q} = 8$	$SO(2) \bigotimes SU(2)$	
$d = 4, \mathcal{Q} = 16$	$SO(6)$	✓

$\mathcal{N} = (2,2)$ SYM in d=2

The action of continuum $\mathcal{N} = (2,2)$ SYM takes the following $\mathcal{Q}$-exact form after topological twisting

$$S = \frac{N}{2\lambda}\mathcal{Q}\int d^2x\,\Lambda,$$

where

$$\Lambda = \mathrm{Tr}\,\left(\chi_{ab}\mathcal{F}_{ab} + \eta[\overline{\mathcal{D}}_a, \mathcal{D}_b] - \frac{1}{2}\eta d\right),$$

and $\lambda = g^2 N$ is the 't Hooft coupling.

The nilpotent supersymmetry transformations associated with the scalar supercharge $\mathcal{Q}$ are given by

$$
\begin{aligned}
\mathcal{Q}\,\mathcal{A}_a &= \psi_a, \\
\mathcal{Q}\,\psi_a &= 0, \\
\mathcal{Q}\,\overline{\mathcal{A}}_a &= 0, \\
\mathcal{Q}\,\chi_{ab} &= -\overline{\mathcal{F}}_{ab}, \\
\mathcal{Q}\,\eta &= d, \\
\mathcal{Q}\,d &= 0.
\end{aligned}
$$

The four degrees of freedom appearing in this theory are just the twisted fermions $(\eta, \psi_a, \chi_{ab})$ and complexified gauge field $\mathcal{A}_a$. The complexified field is constructed from the usual gauge field A_a and the two scalars B_a present in the untwisted theory: $\mathcal{A}_a = A_a + iB_a$. The twisted theory is naturally written in terms of the complexified covariant derivatives

$$\mathcal{D}_a = \partial_a + \mathcal{A}_a, \qquad \overline{\mathcal{D}}_a = \partial_a + \overline{\mathcal{A}}_a, \tag{1}$$

and complexified field strengths

$$\mathcal{F}_{ab} = [\mathcal{D}_a, \mathcal{D}_b], \qquad \overline{\mathcal{F}}_{ab} = [\overline{\mathcal{D}}_a, \overline{\mathcal{D}}_b]. \tag{2}$$

The action can be written as, $\mathrm{S} = \mathrm{S}_B + S_F$,where the bosonic action is

$$S_B = \frac{N}{2\lambda} \sum_{\mathbf{n}} \mathrm{Tr} \Big(-\overline{\mathcal{F}}_{ab}(\mathbf{n}) \mathcal{F}_{ab}(\mathbf{n}) + \frac{1}{2} \Big(\overline{\mathcal{D}}_a^{(-)} \mathcal{U}_a(\mathbf{n}) \Big)^2 \Big),$$

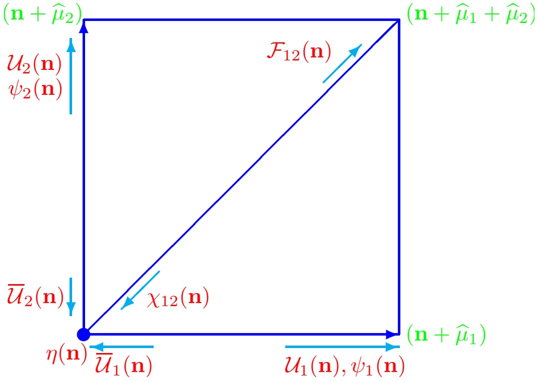
and the fermionic piece

$$S_F = \frac{N}{2\lambda} \sum_{\mathbf{n}} \mathrm{Tr} \Big(-\chi_{ab}(\mathbf{n}) \mathcal{D}_{[a}^{(+)} \psi_{b]}(\mathbf{n}) - \eta(\mathbf{n}) \overline{\mathcal{D}}_a^{(-)} \psi_a(\mathbf{n}) \Big).$$

Also an additional mass term (breaks $\mathcal{Q}$ supersymmetry)

$$S_{\mathrm{soft}} = \frac{N}{2\lambda} \mu^2 \sum_{\mathbf{n},\ a} \mathrm{Tr} \left(\overline{\mathcal{U}}_a(\mathbf{n}) \mathcal{U}_a(\mathbf{n}) - \mathbb{I}_N \right)^2,$$

Fields on the lattice



Extrapolations

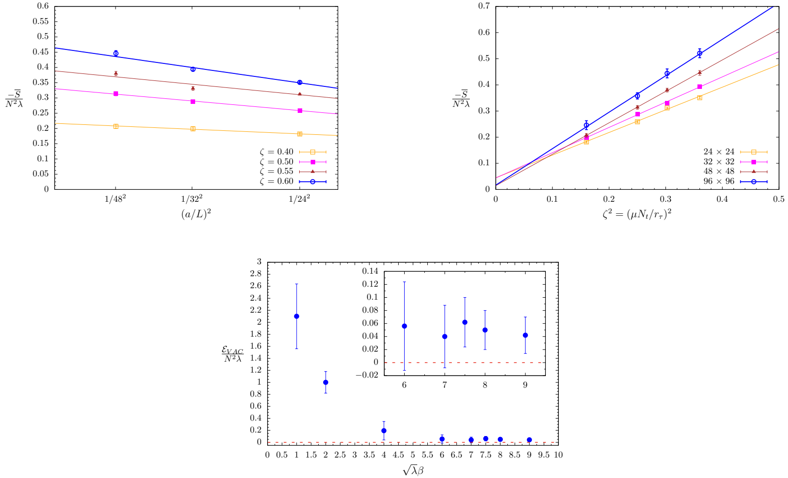


Figure: Left : $\lim_{a\to 0}$, Right : $\lim_{\mu^2\to 0}$, Bottom : $\lim_{\beta\to\infty}$

Supersymmetry breaking

- Calculate the ground state energy density in the limit $\beta \to \infty$.
- Need to use small mass term μ to control flat directions, which we extrapolate to zero after doing continuum extrapolation ($a \to 0$).
- Upper bound on energy density $\frac{\mathcal{E}_{\text{VAC}}}{N^2\lambda} = 0.05(2)$, statistically consistent with zero.

[Similar study done earlier by Kanamori, Sugino and Suzuki based on A-twist Sugino's action]

Applications to holography - gauge/gravity

Original AdS/CFT correspondence

4D $\mathcal{N} = 4$ U(N) super-Yang-Mills theory associated with N D3-branes, is dual to Type IIB string theory on $AdS_5 \times S_5$ in the large N limit.

More general holographic dualities in lower dimensions

Maximally supersymmetric YM in $p+1$ dimensions dual to Dp-branes

At low temperatures, and in the decoupling limit : dual description in terms of black holes in Type II A/B supergravity

Decoupling limit: $N \to \infty$ and $t = T/\lambda^{\frac{1}{3-p}} \ll 1$

Maximal SYM for p < 3

- Dimensionally reduce lattice $\mathcal{N} = 4$ SYM along (3-p) spatial directions.
- Dimensional reduction : $A_4^* \rightarrow$ A$_{p+1}^*$ giving a skewed torus with $\gamma = -1/(p+1)$ ($\gamma = \cos\,\theta$).
- 't Hooft coupling (λ) is dimensionful in p ¡ 3 dimensions and we construct a dimensionless coupling given by $\hat{\lambda} = r_{\text{eff}} = \lambda_p \beta^{3-p}$, where $\beta = 1/T$.
- No phase transition (single de-confined phase) in 1-d QM case, richer structure for p = 1,2.

Regime of valid supergravity (SUGRA) description

To have a valid SUGRA description, we need :

- Radius of curvature should be large in units of α'. This implies $r_{\text{eff}} \gg 1$.
- String coupling should be small.

We can combine both requirements to get a constraint on the effective dimensionless coupling we can probe for a well-defined SUGRA description (p < 3)

$$1 \ll \lambda_p \beta^{3-p} \ll N^{\frac{10-2p}{7-p}}$$

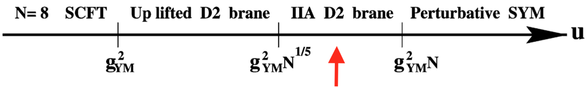
Various dimensions - progress report

- p=0 : [Hanada, Nishimura and Takeuchi in 0706.1647 + Catterall & Wiseman, 0706.3518]
- p=1 : [See talk by David Schaich and Daisuke Kadoh]
- p=2 : This talk [Preliminary work]
- . . . .
- p = 3 : Thermodynamics of $\mathcal{N} = 4$ SYM. Statement : Can we understand $f(\lambda) \ni$, $f(0) = 1$ and $f(\infty) = 3/4$?

Move to p=2 : Maximal SYM in (2+1)-dimensions

't Hooft coupling has dimensions of energy. Construct $r_{\text{eff}} = \lambda\beta = 1/t$ as dimensionless coupling. Type IIA SUGRA description is valid when the energy scale, $u = r/\alpha'$ (defined as fixed expectation value of a scalar) is in the range shown below :

O. Aharony et al. / Physics Reports 323 (2000) 183–386



This translates to the condition (for our dimensionless coupling) as,

$$1 \ll r_{\text{eff}} \ll N^{\frac{6}{5}}$$

Divergence of thermal partition function - I

First discussed by [Kabat, Lifshitz and Lowe, hep-th/9910001, hep-th/0105171], the thermal SYM partition function has divergence.

It was shown that the thermal Euclidean partition function can be schematically written as [Catterall & Wiseman, hep-th/0909.4947] ,

$$I \sim kN \log(f(\zeta)) + N^2 I_{\text{finite}}$$

So technically, one can avoid the issue of divergence if $N \to \infty$ (another need for large N) because the finite contribution dominates. For the N we can access in our numerical simulations, we need to do more !

Use a mass term (μ) related to ζ in our lattice action to restrict the moduli space and then extract the finite piece carefully and compare to the thermodynamics of Dp-branes.

Thermodynamics of D2-branes

For a uniform Dp-brane (p $<$ 3) , we have a prediction for free energy density which is [Itzhaki et al., hep-th/9802042, Harmark and Obers, hep-th/0407094],

$$\mathcal{F} = -k_p N^2 \lambda^{\frac{1+p}{3-p}} t^{\frac{14-2p}{5-p}}$$

where, k can be read off the table in the above reference.

p	0	1	2	3	4
k_p	$(2^{21}3^2 5^7 7^{-19}\pi^{14})^{1/5}$	$2^4 3^{-4}\pi^{5/2}$	$(2^{13}3^5 5^{-13}\pi^8)^{1/3}$	$2^{-3}\pi^2$	$2^5 3^{-7}\pi^2$

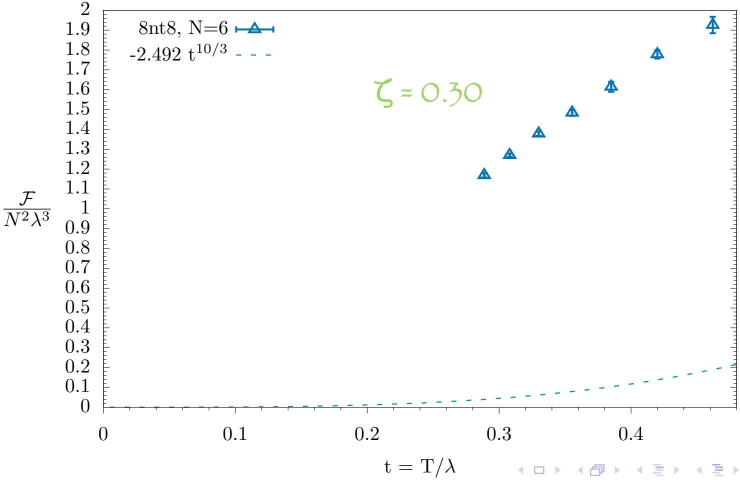
For our case of $i.e$ p $=$ 2, we get :

$$\mathcal{F} = -2.492\ N^2 \lambda^3 t^{\frac{10}{3}}$$

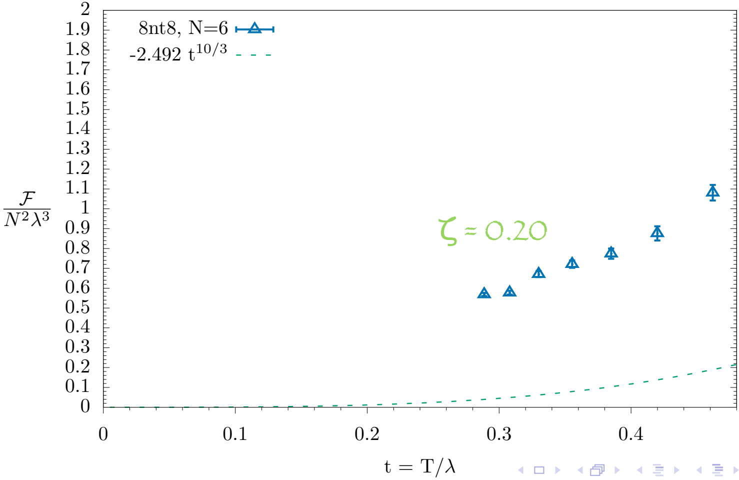
Lattice simulations

- We focus on calculating the free energy density for the SYM theory on the lattice restricting to uniform D2 phase.
- Choose temperatures $t \ll 1$ and large N for multiple lattices.
- Computational cost scales as $\sim N^{7/2}$, so we restrict to $N_{\text{maximum}} = 8$ on $8^3, 10^3$ and 12^3 lattices.
- We need to use small mass regulator ζ (discussed before), which we extrapolate to zero as $\zeta^2 \to 0$.
- Publicly available lattice code for arbitrary N : github.com/daschaich/susy

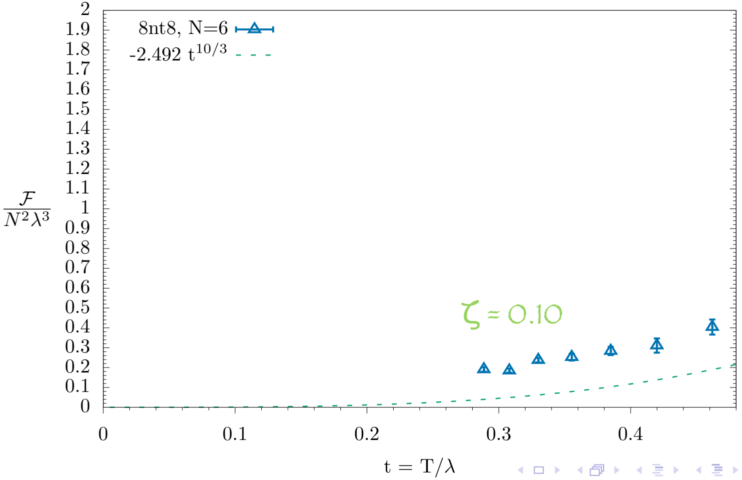
Preliminary numerical results



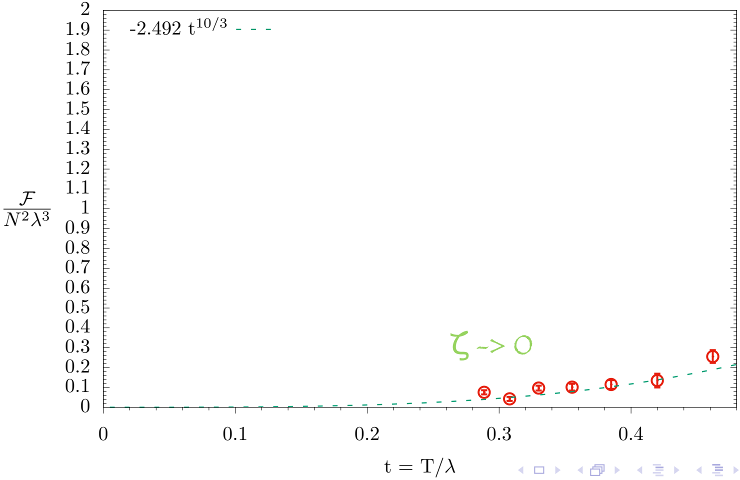
Preliminary numerical results



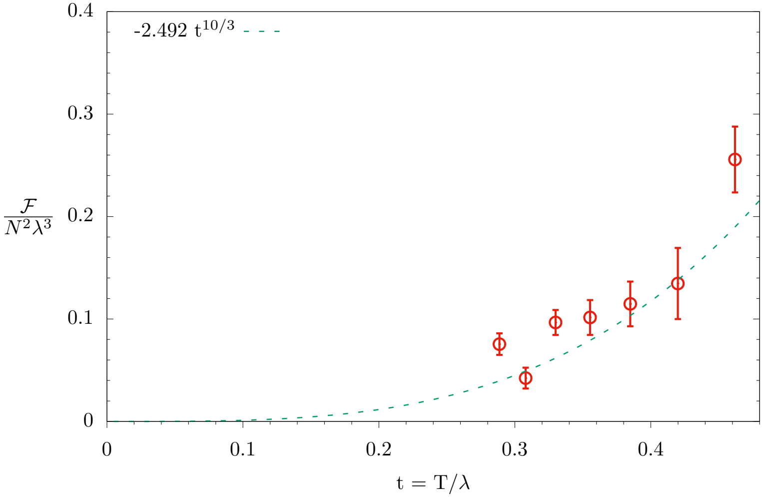
Preliminary numerical results

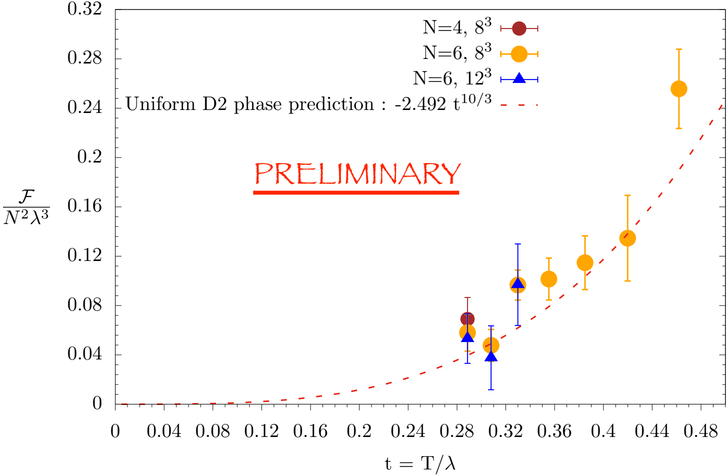


Preliminary numerical results ($8 \times 8 \times 8, N = 6$)



Preliminary numerical results ($8 \times 8 \times 8, N = 6$)




N=4, 8^3
N=6, 8^3
N=6, 12^3
Uniform D2 phase prediction : -2.492 $t^{10/3}$
PRELIMINARY
$\frac{\mathcal{F}}{N^2\lambda^3}$
0.32
0.28
0.24
0.2
0.16
0.12
0.08
0.04
0
0 0.04 0.08 0.12 0.16 0.2 0.24 0.28 0.32 0.36 0.4 0.44 0.48
$t = T/\lambda$

Wilson loops

The quark-anti-quark potential calculated for this theory goes as [Maldacena, hep-th/9803002]

$$E \sim \frac{(g_{YM}^2 N)^{1/3}}{L^{2/3}} \sim \frac{(\alpha r_\tau)^{1/3}}{L}$$

This is only valid for $\alpha\lambda\beta \gg 1$, choosing $\alpha \sim \mathcal{O}(1)$ implies that $\lambda\beta \gg 1$. Calculated only when the size of the loop is big [not perturbative] !

Thank you !

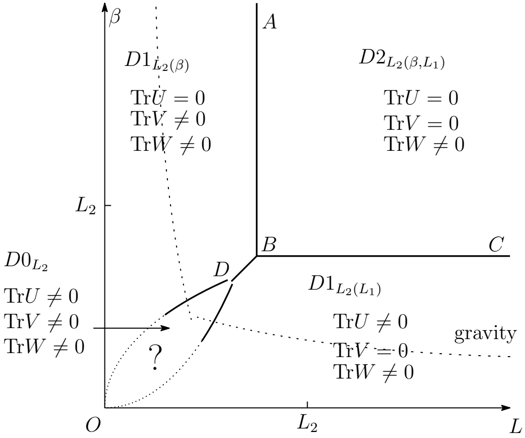
Thank you !

Funding and computing resources

Backup 1

Lower-dimensional sixteen supercharge SYM with **apbc** has no sign problem.

Backup 2



β
A
$D1_{L_2(\beta)}$
$\mathrm{Tr}U = 0$
$\mathrm{Tr}V \neq 0$
$\mathrm{Tr}W \neq 0$
$D2_{L_2(\beta,L_1)}$
$\mathrm{Tr}U = 0$
$\mathrm{Tr}V = 0$
$\mathrm{Tr}W \neq 0$
L_2
B
C
$D0_{L_2}$
$\mathrm{Tr}U \neq 0$
$\mathrm{Tr}V \neq 0$
$\mathrm{Tr}W \neq 0$
D
$D1_{L_2(L_1)}$
$\mathrm{Tr}U \neq 0$
$\mathrm{Tr}V = 0$
$\mathrm{Tr}W \neq 0$
gravity
?
O
L_2
L