Lattice Supersymmetry II

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Quick recap

- Certain SUSY theories can be formulated in terms of twisted variables which facilitates discretization
  - One (or more) nilpotent supersymmetry $\mathcal{Q}$ appears
  - Action is $\mathcal{Q}$-exact ..
  - $\mathcal{Q}$ remains exact on lattice
- Twisted fermion action = Kähler-Dirac action. Fermions as Grassman integer spin objects. Assigned to links of lattice. No doubling.
- Bosons appear as complex gauge links
- Requirements of $\mathcal{Q}$-symmetry, gauge invariance and no doubling place severe constraints on structure of lattice theory ...

Lattice Kähler-Dirac = reduced staggered fermions

- Introduce a lattice with half spacing – now all link fields are site fields on fine lattice.
- Forward/backward difference operators become symmetric differences on fine lattice. Free KD action reduces to free staggered action – famous staggered fermion phases now arise from antisymmetry of derivatives in KD equation.

Staggered U(1) symmetry: rotate $\psi_{1,2} \to e^{\alpha}\ \psi_{1,2}$ and $\eta, \chi_{12} \to e^{-\alpha}\ \eta, \chi_{12}$.

Note: KD/taggered fermions exhibit no doubling related to lattice artifacts but they do describe more than 1 fermion. Indeed describe precisely correct number of fermions in target theory with extended SUSY

General features of lattice action

- Fermions are in algebra of $U(N)$ are antisymmetric objects living on links.
- Bosons are complex link fields valued in algebra of $GL(N, C)$.
- Hence measure $D\mathcal{U}D\overline{\mathcal{U}}$ flat. But still $U(N)$ gauge invariant !
- S^2 permutation symmetry. Plus $U(1)^2$ center symmetry $\mathcal{U}_\mu(x) \to Z_\mu \mathcal{U}_\mu(x)$.
- However: derivatives arise from $\mathcal{U}_\mu = I + \mathcal{A}_\mu + \ldots$. Corresponds to giving a vev to imaginary part of $\mathcal{U}_\mu$. Breaks center. Picks pt on moduli space.
- Requires a soft $\mathcal{Q}$-breaking term eg.

$$\delta S = \mu^2 \sum_x \text{tr} \left(\mathcal{U}_\mu(x)\overline{\mathcal{U}}_\mu(x) - I\right)^2$$

Exact $\mathcal{Q}$ at $\mu = 0$ ensures $\mathcal{Q}$-breaking terms vanish as $\mu \to 0$.

Topological Observables

Any $\mathcal{Q}$-invariant observable $\mathcal{O}$ can be computed exactly in the semi-classical limit

$$\frac{\partial}{\partial t} < \mathcal{O} >= \frac{1}{Z} \int D\Phi \mathcal{O} \mathcal{Q} \Lambda e^{t\mathcal{Q}\Lambda}$$

Clearly $\mathcal{O}\mathcal{Q}\Lambda = \mathcal{Q}\left(\mathcal{O}\Lambda\right)$ so that

$$\frac{\partial}{\partial t} < \mathcal{O}(\mathrm{t}) >= 0$$

This implies the observable can be computed exactly at one loop in the $t = 1/g^2$ parameter.

One trivial class of topological observable are $\mathcal{O} = \mathcal{Q}\Sigma$ - Ward identities of the SYM theory

Example: partition function

Matsuura:

Let $\mathcal{O} = 1$. Expand $\mathcal{U}$ around vacuum config.

$$\mathcal{U}_\mu = \mathcal{U}^0_\mu + \delta\mathcal{U}_\mu(x)$$

and $\mathcal{U}^0_\mu = I + b_\mu$ with b_μ is any constant diagonal matrix (flat directions). Fix gauge in usual way and introduce FP ghosts

$$Z(b_\mu)_{\rm 1loop} = \frac{\det \Box_{KD} \det \Box_{\rm ghosts}}{\det^2 \Box_{\rm bosons}}$$

and $\Box$ is the covariant discrete Laplacian in b_μ background field. Thus Z and hence effective potential $V(b_\mu)$ independent of b_μ at 1loop.

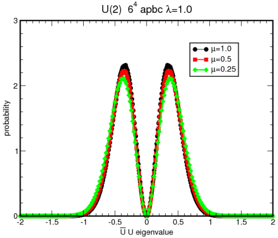
Key observation

Since Z is topological no $V(b_\mu)$ induced to all orders of perturbation theory!

Flat directions ...$[B_\mu, B_\nu] = 0$

Moduli space

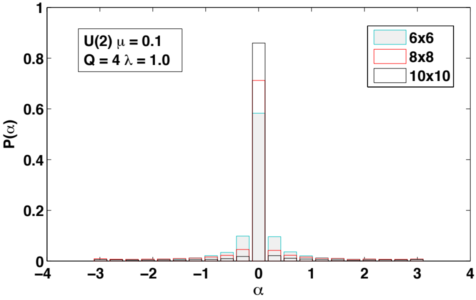
- To get correct naive continuum limit need to add a term to pick that particular point on the vacuum manifold.
- Just saw that formally at $\mu^2 = 0$ flat directions survive quantum correction. Is the path integral well defined as $\mu^2 \to 0$. Practical issue: are Monte Carlo simulations stable ?
- The limit seems harmless. Reason: fermions have exact 0 mode on flat directions. Suppresses those configurations even for $\mu^2 = 0$



Sign problem ?

To do Monte Carlo simulations we need $\det M_F$ real, positive. This is true in the continuum but is *not* true on lattice.

Nevertheless the fluctuations in the phase are small and go to zero as $L \to \infty$



At least for this theory no barrier to using numerical simulations to probe theory at strong coupling...

Summary so far ..

- Learned how to discretize a topologically twisted version of SYM to retain 1 exact supersymmetry.
- Novel lattice theories:
  - Fermions appear as integer spin objects assigned to links.
  - Bosons as complex link fields valued in *algebra*. Flat measure. Nevertheless gauge invariance is preserved.
  - Number of exact supercharges = number of site fermions. Need at least $2^{D/2}$ supercharges.
- Certain (topological) observables can be computed exactly at 1 loop in lattice.
- If no sign problem models can be simulated using standard Monte Carlo techniques drawn from lattice QCD.

Higher dimensions ?

Counting supercharges: Many interesting theories in $D < 4$

Let $n(\mathcal{Q})$ denote number supercharges

D=2

$n(\mathcal{Q}) = 4$ unique last time

Family of different lattices with $n(\mathcal{Q}) = 8, 16$

(Damgaard and Matsuura)

D=3

$n(\mathcal{Q}) = 8$ unique 3rd lecture

Family of theories with $n(\mathcal{Q}) = 16$

D=4

$n(\mathcal{Q}) = 16$ unique ($\mathcal{N} = 4$ SYM)

$\mathcal{N} = 4$ YM from twisted 5D theory

Continuum

- Start with $\mathcal{N} = 1$ in 10D
- Reduce to 5D: resultant theory has 5 scalars ϕ^I, $I = 1 \ldots 5$, 16 fermion dof, and 5 gauge fields A_a, $a = 1 \ldots 5$ and an $SO(5)$ R symmetry.

$$SO_{tw}(5) = \mathrm{diag}\left(SO_{\mathrm{euc}}(5) \times SO_R(5)\right)$$

Resultant action

- Bosonic dof: $\mathcal{A}_a = A_a + i\phi_a$, $a = 1 \ldots 5$.
- Fermionic dof: η, ψ_a, χ_{ab}.

$\mathcal{N} = 4$ theory obtained by further reduction along one direction

Twisted supersymmetry

As for $\mathcal{N} = 2$ twisted scalar supersymmetry $\mathcal{Q}$ acts as

$$\begin{aligned} &\mathcal{Q}\,\mathcal{A}_a = \psi_a && \mathcal{Q}\,\psi_a = 0 \\ &\mathcal{Q}\,\chi_{ab} = -\overline{\mathcal{F}}_{ab} && \mathcal{Q}\,\overline{\mathcal{A}}_a = 0 \\ &\mathcal{Q}\,\eta = d && \mathcal{Q}\,d = 0 \end{aligned}$$

$\nwarrow$ bosonic auxiliary field with e.o.m. $d = \overline{\mathcal{D}}_a \mathcal{A}_a$

- Scalars $\rightarrow$ vectors under twisted group. Combine with gauge fields.
- Two covariant derivatives $\mathcal{D} = \partial + \mathcal{A}$ and $\overline{\mathcal{D}} = \partial + \overline{\mathcal{A}}$. $\mathcal{F}_{ab} = [\mathcal{D}_a, \mathcal{D}_b]$
- The susy subalgebra $\mathcal{Q}^2 \cdot = 0$ is manifest

Twisted 5D action

$$S = \frac{N}{4\lambda}\mathcal{Q}\int_{M^4\times S^1} \mathrm{Tr}\left(\chi_{ab}\mathcal{F}_{ab} + \eta[\overline{\mathcal{D}}_a, \mathcal{D}_a] - \frac{1}{2}\eta d\right)$$
$$. \quad - \frac{N}{16\lambda}\int_{M^4\times S^1} \epsilon_{abcde}\, \mathrm{Tr}\,\chi_{ab}\overline{\mathcal{D}}_c\chi_{de}$$

$\mathcal{Q}^2 = 0$ and Bianchi guarantee supersymmetry
Note appearance of $\mathcal{Q}$-closed term

60 million dollar question

How do I construct 4D lattice theory from this ?

Proceeding naively ...

Assume a 5D hypercubic lattice. Assign complex link bosons as per $\mathcal{N}=2$ SYM in $D=2$

- The $\mathcal{Q}$-exact piece is lattice gauge invariant using the same prescription as before.
- But the $\mathcal{Q}$-closed term is not. Replacing derivatives by differences we can write

$$\sum_x \mathrm{Tr}\,\epsilon_{abcde}\chi_{ab}(x)\overline{\mathcal{D}}^{(-)}_c\chi_{de}(x-d-e)$$

where

$$\overline{\mathcal{D}}^{(-)}_c\chi_{de}(x)=\chi_{de}(x)\overline{\mathcal{U}}_c(x-c)-\overline{\mathcal{U}}_c(x+d+e-c)\chi_{de}(x-c)$$

Gauge invariance requires closed loop

$$\hat{a}+\hat{b}+\hat{c}+\hat{d}+\hat{e}=0 \qquad \text{basis vectors span } 4D!$$

Lattice fields and $\mathcal{Q}$

The lattice theory is gauge invariant if in fact it exists in 4D not 5D!

Structure same as for $\mathcal{N}=2$ in 2D

Bosons : $\mathcal{U}_a$, $a = 1\ldots5$ Fermions : $(\eta, \psi_a, \chi_{ab})$ assigned to links

Single nilpotent exact supercharge $\mathcal{Q}$ with $\mathcal{Q}\mathcal{U}_a = \psi_a$ etc. Lattice action:

$$S = \mathcal{Q}\sum \mathrm{Tr}\left(\chi_{ab}\mathcal{F}_{ab} + \frac{1}{2}\eta d + \eta\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a\right) + \mathcal{Q}_{\text{closed}}$$

Remarkably:

$\mathcal{Q}$-closed term remains supersymmetric since lattice $\mathcal{F}_{ab}$ satisfies exact Bianchi identity!

Bianchi ...

Remember:

$$\mathcal{F}_{ab}(x) = \mathcal{D}_a\mathcal{U}_b(x) = \mathcal{U}_a(x)\mathcal{U}_b(x+a) - \mathcal{U}_b(x)\mathcal{U}_a(x+b)$$

Automatically antisymmetric.
Additionally ... ($\overline{\mathcal{U}} \equiv \mathcal{U}^\dagger$)

$$\mathcal{F}_{ab}\overline{\mathcal{F}}_{ab} = 2\mathrm{Re}\left[\mathcal{U}_a(x)\mathcal{U}_b(x+a)\overline{\mathcal{U}}_a(x+b)\overline{\mathcal{U}}_b(x) + \mathcal{U}_a(x)\overline{\mathcal{U}}_a(x)\mathcal{U}_b(x)\overline{\mathcal{U}}(x)\right]$$

Remarkably satisfies an exact Bianchi

$$\epsilon_{abcde}\overline{\mathcal{D}}_c^{(-)}\mathcal{F}_{de}^\dagger(x-d-e) = \sum_{\text{pairs link paths}} \overline{\mathcal{U}}\,\overline{\mathcal{U}}\,\overline{\mathcal{U}} = 0$$

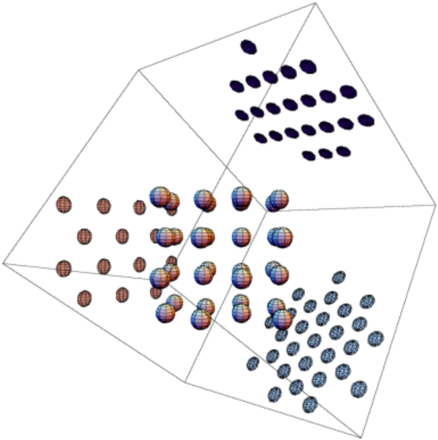
What is this 4D lattice ? ans: A_4^*

Need $\sum^5 \hat{\mu}_a = 0$. Maximize global symmetries of lattice theory if treat all five basis vectors symmetrically (S^5 symmetry)

—Start with hypercubic lattice in 5d momentum space

—**Symmetric** constraint $\sum_a \partial_a = 0$ projects to 4d momentum space

—Result is A_4 lattice $\longrightarrow$ dual A_4^* lattice in real space



Generalization of triangular lattice to 4D

More about the A_4^* lattice

Basis vectors

$$
\begin{aligned}
e_1 &= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_2 &= \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_3 &= \left(0, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_4 &= \left(0, 0, \frac{-3}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_5 &= \left(0, 0, 0, \frac{-4}{\sqrt{20}}\right)
\end{aligned}
$$

Position vector

$$R = a \sum_{\nu=1}^{4} n_\nu e_\nu$$

Lattice symmetry

- Clearly the lattice theory has S^5 permutation symmetry - analog of hypercubic rotational symmetry
- Remarkably the low lying irreps of S^5 match those of continuum twisted $SO(4)$

$$\begin{aligned}
\mathcal{U}_a &\rightarrow \mathcal{A}_\mu + \phi \\
\psi_a &\rightarrow \psi_\mu + \lambda \\
\mathbf{5} &\rightarrow \mathbf{4} + \mathbf{1} \\
\chi_{ab} &\rightarrow \chi_{\mu\nu} + \overline{\psi}_\mu \\
\mathbf{10} &\rightarrow \mathbf{6} + \mathbf{4}
\end{aligned}$$

Translating S^5 indices to $SO(4)$ indices

$$f_a = e^a_\mu f_\mu + \frac{1}{\sqrt{5}} f$$

(Kaplan and Unsal)

Symmetries and Counterterms

Lattice symmetries strongly constrain possible counterterms

Symmetries

- Gauge invariance
- $\mathcal{Q}$-symmetry.
- Point group symmetry S^5 and $U(1)^4$ center
- Exact fermionic shift symmetry $\eta \rightarrow \eta + \epsilon I$

Conclusion:

- S^5 PGS guarantees twisted $SO(4)'$ restored as $a \rightarrow 0$
- Power counting: only relevant ops correspond to 4 $\mathcal{Q}$-invariant terms already present in classical lattice action!

$$S = \mathcal{Q} \sum \alpha_1 \chi_{ab} F_{ab} + \alpha_2 \eta [\overline{\mathcal{D}}_a, \mathcal{D}_a] + \frac{\alpha_3}{2} \eta d + \alpha_4 L_{\text{closed}}$$

- As for $\mathcal{N} = 2$ $\mathcal{Q}$-symmetry ensures that $V_{\text{eff}}(\mathcal{A}^{\text{classical}}) = 0$ to all orders in p. theory ... no term $\mathcal{Q}(\eta \mathcal{U}_a \overline{\mathcal{U}}_a)$ term induced.

Computation of effective potential

- Classical vacua constant commuting complex matrices $\mathcal{U}_\mu$
- Expand to quadratic order about generic vacuum $\mathcal{U}_b(x) = I + \mathcal{A}_b^c + a_b(x)$. Integrate
- Bosons $\det^{-5}\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{D}_a^{(+)}\right)$
- Ghosts+Fermions: $\det\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{D}_a^{(+)}\right) + \left(Pf(M_F) \overset{Maple}{=} \det^4\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{D}_a^{(+)}\right)\right)$
- Thus $Z_{\rm pbc} = 1$ at 1-loop. $\mathcal{Q}$-exact structure – result good to all orders! Exact quantum moduli space

Summary

- Lattice $\mathcal{N} = 4$ is renormalizable. No new terms allowed by power counting.
- No lifting of flat directions.
- Four marginal couplings allowed $\alpha_i, \ i = 1 \ldots 4$. Require $\alpha_i = \alpha$ in continuum limit.

Can rescale the fermions to set 3 α's to 1.
Single marginal coupling remains
Potentially must be log tuned to take continuum limit:
Continuum limit:
Send $L \to \infty$ holding g^2 fixed

not like lattice QCD!

$\mathcal{Q}$ preserving real space RG (Giedt, SC)

Previous analysis implicitly assumes existence of RG that preserves $\mathcal{Q}$

One simple blocking exists:

$$\mathcal{U}_a^B(x') \stackrel{a'=2a}{=} \xi\,\mathcal{U}_a(x)\mathcal{U}_a(x+a)$$

$$\psi_a^B(x') \stackrel{a'=2a}{=} \xi\left(\psi_a(x)\mathcal{U}_a(x+a)+\mathcal{U}_a(x)\psi_a(x+a)\right)$$

$$\chi_{ab}^B(x') \stackrel{a'=2a}{=} \xi^2 \sum_{\text{paths P:x+2a+2b}\to\text{x}} f_P(\chi_{ab},\overline{\mathcal{U}},\overline{\mathcal{U}})$$

where ξ free parameter.
Taking

$$\mathcal{F}_{ab}^B = \mathcal{U}_a^B\mathcal{U}_b^B - \mathcal{U}_b^B\mathcal{U}_a^B$$

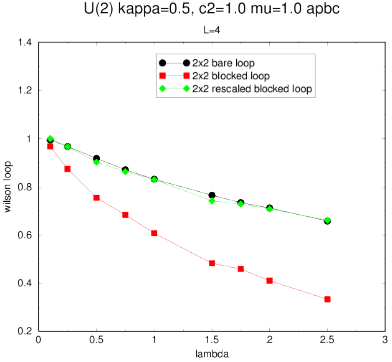
Preserves lattice $\mathcal{Q}$:

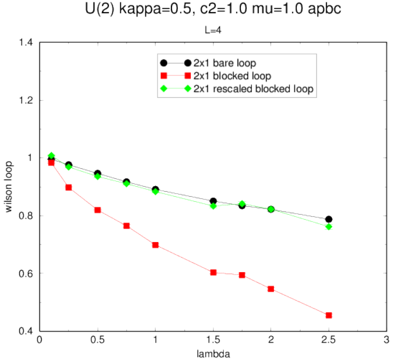
$$\mathcal{Q}\mathcal{U}^B = \psi^B \quad \mathcal{Q}\psi^B = 0 \quad \mathcal{Q}\chi^B = \overline{\mathcal{F}}^B \quad i.e\ \mathcal{Q}^2 = 0 \text{ on blocked fields}$$

Blocking

Determining ξ

Match eg. (1,1) and (2,2) Wilson loops on fine and blocked lattices





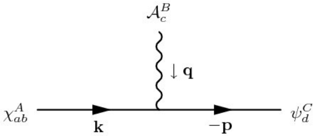
Single $\xi(g^2)$ serves to match all Wilson loops for weak coupling!

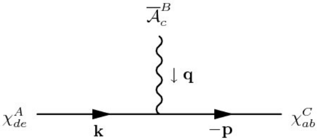
Going beyond symmetries – perturbation theory

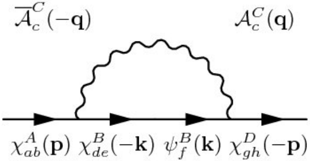
Lattice rules for A_4^* lattice (Feynman gauge):

- Boson propagator $< \overline{\mathcal{A}}_a^C(k)\mathcal{A}_b^D(-k) >= \frac{1}{\hat{k}^2}\delta_{ab}\delta^{CD}$ with $\hat{k}^2 = 4\sum_a \sin^2(k_a/2)$
- Fermion propagator $M_{\rm KD}^{-1}(k) = \frac{1}{\hat{k}^2}M_{\rm KD}(k)$ with $M(k)$ a 16×16 block matrix acting on $(\eta, \psi_a, \chi_{ab})$
- Vertices: $\psi\eta$, $\psi\chi$ and $\chi\chi$.
- Four one loop Feymann graphs needed to renormalize three fermion propagators. Yields 3 α's.
- One additional bosonic propagator for remaining α.

Example: chi-chi propagator







Self energies:

- $\Sigma_i(0) = 0; \quad \frac{\partial \Sigma_i}{\partial p} = A_i g^2 \ln \mu a + \text{finite} + \mathcal{O}(a)$
- Find universal $A_i = A$ all i.
- Implies $\sqrt{\alpha_i} = Z_i = 1 + \frac{A}{2} g^2 \ln \mu a + \ldots$ all α

Why so simple ?

- One loop lattice diagrams in 1-1 correspondence with continuum diagrams and have only log divergences.
- Divergences come from region near $pa \sim 0$ where lattice propagators and vertices approach continuum expressions
- Thus (divergent part of) 1-loop lattice diagram - same as continuum !
- In continuum twisted theory equivalent to usual - has full supersymmetry. Requires common wavefunction renormalization all fermions/bosons – log divergences must be same for all α_i ..
- Similar args indicate that $\boxed{\beta_{\rm lattice}(g) = 0}$ at 1-loop

Additional twisted supersymmetries

Construction preserves $\mathcal{Q}$ but breaks at $\mathcal{O}(a)$ other 15 SUSYs
$\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

These must be restored in any continuum limit

Discrete R symmetries

Action of eg. $\mathcal{Q}_a$ can be gotten by combining R_a and $\mathcal{Q}$.

$$\frac{\eta}{2} \to \psi_a \qquad \mathcal{D}_a \to \mathcal{D}_a$$

$$\psi_b \stackrel{b \neq a}{\to} -\chi_{ab} \qquad \mathcal{D}_b \stackrel{b \neq a}{\to} \overline{\mathcal{D}}_a$$

$$\chi_{bc} \to \frac{1}{2}\epsilon_{bcagh}\chi_{gh}$$

Testing for restoration of discrete R symmetries is much easier than measuring SUSY Ward identities ...

On lattice

Action of R_a:

$$\mathcal{U}_a \to \mathcal{U}_a$$

$$\mathcal{U}_b \to \left(\overline{\mathcal{U}}_b\right)^{-1}$$

Preserves gauge invariance ... Can test for restoration of R_a by comparing $W_{n\times m}(\mathcal{U})$ with $W_{n\times m}(R_a\mathcal{U})$

Notice that action of R_a does not commute with gauge transformations for fermions - $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ are never exact symmetries for $a \neq 0$

Taking the continuum limit

Keep g^2 fixed and send $L \to \infty$ while tuning α_2 to ensure $W_{n\times m}(\mathcal{U}) = W_{n\times m}(R_a\mathcal{U})$

What about the sign problem ?

In lattice gauge theory we compute operator expectation values

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\, \mathcal{O}\, e^{-S_B[\mathcal{U},\overline{\mathcal{U}}]}\, \text{pf}\, \mathcal{D}[\mathcal{U},\overline{\mathcal{U}}]$$

$\text{pf}\, \mathcal{D} = |\text{pf}\, \mathcal{D}| e^{i\alpha}$ can be complex for lattice $\mathcal{N} = 4$ SYM
$\longrightarrow$ Complicates interpretation of $\left[e^{-S_B}\, \text{pf}\, \mathcal{D}\right]$ as Boltzmann weight

Instead absorb $e^{i\alpha}$ into phase-quenched (pq) observables $\mathcal{O} e^{i\alpha}$
and reweight using $Z = \int e^{i\alpha}\, e^{-S_B}\, |\text{pf}\, \mathcal{D}| = \left\langle e^{i\alpha} \right\rangle_{pq}$

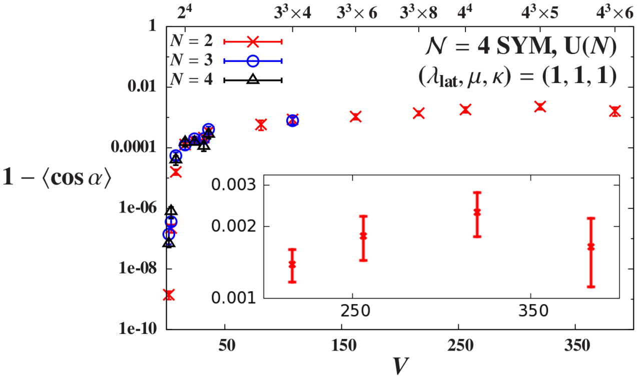
$$\langle \mathcal{O} \rangle_{pq} = \frac{1}{\mathcal{Z}_{pq}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\, \mathcal{O}\, e^{-S_B}\, |\text{pf}\, \mathcal{D}| \qquad \langle \mathcal{O} \rangle = \frac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{pq}}{\left\langle e^{i\alpha} \right\rangle_{pq}}$$

Sign problem: This breaks down if $\left\langle e^{i\alpha} \right\rangle_{pq}$ is consistent with zero

Pfaffian phase volume dependence

No indication of a sign problem at $\lambda_{\text{lat}} = 1$ with anti-periodic BCs

- Results from arXiv:1411.0166 using the unimproved action
- Fluctuations in pfaffian phase don't grow with the lattice volume
- Insensitive to number of colors $N = 2, 3, 4$



Why is sign problem so mild ..?

Some observations and puzzles

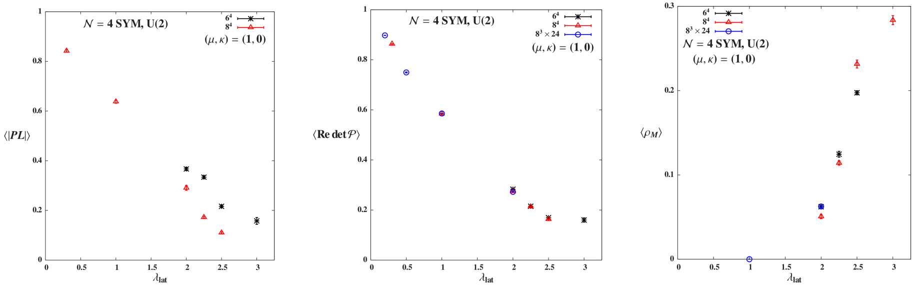
- No symmetry pairs eigenvalues of fermion operator. Generic Pf complex.
- Dynamics must be important in keeping system in region of config space where phase fluctuations are small. Phase vanishes if scalars are zero ...
- Partial products of eigenvalues $P(n) = \prod_{i=1}^{n}$ show a strong sign problem unless all eigenvalues included! Phase does not arise from low lying eigenvalues ...
- If use pbc see a sign problem. Nevertheless if we ignore topological vevs correct ..
- If use apbc very mild sign problem at $\lambda = \mathrm{O}(1)$.

This is not understood

Other problems for $\mathcal{N} = 4$ - monopole instabilities

Flat directions in U(1) gauge field sector can induce transition to confined phase at strong coupling

This lattice artifact is not present in continuum $\mathcal{N} = 4$ SYM



Around $\lambda_{\text{lat}} \approx 2\ldots$

Left: Polyakov loop falls towards zero

Center: Plaquette determinant falls towards zero

Right: Density of U(1) monopole world lines becomes non-zero

Supersymmetric lifting of the U(1) flat directions

arXiv:1505.03135 Schaich, SC

Better: modify e.o.m for auxiliary field d to add new moduli space condition $det\ P_{ab} = 1$

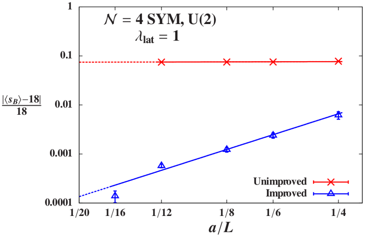
$$S = \frac{N}{2\lambda_{\text{lat}}}\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \quad \downarrow \quad -\frac{1}{2}\eta d\right) - \frac{N}{8\lambda_{\text{lat}}}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V$$

$$\eta\left(\overline{\mathcal{D}}_a\mathcal{U}_a + G\sum_{\mathcal{P}}\left[\det\mathcal{P} - 1\right]\mathbb{I}_N\right)$$

Scalar potential breaks $\mathcal{Q}$ softly. Ward identity restored as $1/L \to 0$

$\mathcal{O}(a)$ improved action

since $\mathcal{Q}$ forbids all dim-5 operators



Public code for lattice $\mathcal{N} = 4$ SYM

The lattice action is obviously very complicated

(the fermion operator involves $\gtrsim$100 gathers)

To reduce barriers to entry our parallel code is publicly developed at

github.com/daschaich/susy

Evolved from MILC code, presented in arXiv:1410.6971

In essence:

$$\mathrm{pf(M)} \to \det(M^\dagger M)^{\frac{1}{4}} = \int D\phi D\overline{\phi} e^{-\overline{\phi}\left(MM^\dagger\right)^{-\frac{1}{4}}\phi}$$

and

$$x^{-\frac{1}{4}} \sim \alpha_0 + \sum_{i=1}^{N} \frac{\alpha_i}{x+\beta_i} \quad \alpha_i, \beta_i \text{ determined by remez alg}$$

Summary of lattice $\mathcal{N} = 4$ YM

- Lattice action for $\mathcal{N} = 4$ YM is $\mathcal{Q}$ and $U(N)$ gauge invariant. S^5 lattice symmetry. $U(1)^4$ center symmetric, fermionic shift symmetry.
- Symmetry/power counting arguments reveal single marginal operator may need tuning to achieve a continuum limit with full SUSY.
- At one loop: no tuning necessary. Line of fixed points $\beta(g^2) = 0$ approached by $L \to \infty$.
- Practical issues: $SU(N)$ flat directions do not cause instabilities. $U(1)$ modes can be controlled with $\mathcal{Q}$-invariant truncation to $SL(N, C)$. Pfaffian phase observed to be small for $\lambda \leq 1$ and $T \neq 0$.

Can be used to investigate holography, QG, S-duality, and properties away planar limit (eg. anomalous dims non BPS ops).

Potential problems ..

Only just beginning to explore strong coupling regime
Do sign problems and lattice artifacts return there ?
If MC hard can we use other techniques eg strong coupling expansions ?