

Introduction to Wess-Zumino-Models and $\mathcal{N} = 1$ Super-Yang-Mills Theory on spacetime lattices

Andreas Wipf

Theoretisch-Physikalisches Institut
Friedrich-Schiller-University Jena



seit 1558

International Centre for Theoretical Sciences
ICTS-TIFR, January 29, 2018

in collaboration with:

Daniel August, Andre Sternbeck, Marc Steinhauser, Sebastian Uhlmann, Bjoern Wellegehausen, Christian Wozar, ...

- 1 A short introduction
- 2 Supersymmetric quantum mechanics
- 3 $\mathcal{N} = 1$ and $\mathcal{N} = 2$ Wess-Zumino-Models in two dimensions
- 4 Supersymmetric $\mathcal{N} = 1$ Yang-Mills theory in four dimensions

- **only extension** of Poincaré invariance $\{P_\mu, M_{\mu\nu}\}$
- $\mathcal{Q}_\alpha^i$ with $i = 1, \dots, \mathcal{N}$ fermionic charges

$$\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}^\beta\} = 2(\gamma^\mu)_\alpha{}^\beta P_\mu$$

$$[\mathcal{Q}_\alpha, P_\mu] = 0$$

$$[J_{\mu\nu}, \mathcal{Q}_\alpha] = (\Sigma_{\mu\nu})_\alpha{}^\beta \mathcal{Q}_\beta$$

- **Physics beyond Standard Model:**
hierarchy problem, gauge coupling unification
dark matter candidates, string theory, AdS/CFT
- insights into **strongly coupled gauge theories**
- susy inspired approaches/approximations (fermions)
- mathematical physics, integrable systems, ...

- $\mathcal{Q}_\alpha$ realized on local fields $\phi, \psi, A_\mu, \dots$
- conserved Noether-(super)currents and (super)charges

charge	$\mathcal{Q}$	$\mathcal{Q}_\alpha$	P^μ
current	J^μ	J^μ_α	$T^{\mu\nu}$
spin	1	3/2	2
source field	A_μ	ψ^α_μ	$g_{\mu\nu}$
particle	photon	gravitino	graviton

- **nonperturbative** ab initio definition of QFT
- access to susy breaking, phase diagrams, particle masses, ...
- check/confirm conjectured/existing results
 - effective low energy theory (e.g. $\mathcal{N} = 1$ SYM)
 - calculate phase transition lines
 - find particle spectrum
 - matching weak to strong coupling (extended SYM)
- important: treatment of **lattice fermions** (non-trivial fixed points!)
- results can be stepwise improved
- comparison with functional renormalization group, ...

- PIONEERING WORK:

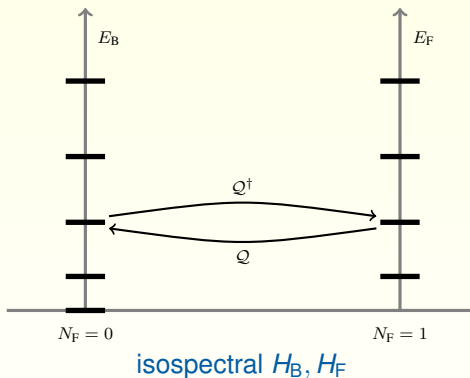
Dondi, Nicolai; Curcio, Veneziano; Elitzur, Schwimmer; Sakai, Sakamoto; Beccaria, Curci, D'Ambrosio, . . .

structural investigations, exact susy on lattice, $\mathcal{N} = (2, 2)$ Wess-Zumino in two dimensions, Nicolai mapping

- FURTHER RESULTS:

Bergner, Brower, Bonini, Catterall, Cohen, Damgaard, Farchiano, Feo, Giedt, Hanada, Harada, Joseph, Kanamori, Kaplan, Katz, Lemos, Montvay, Münster, Poppitz, Schaich, Sugino, Suzuki, Unsal, Wellegehausen, Wozar, . . .

susy breaking, precise numerical investigations: phases, symmetries, mass spectra; various fermion species: Wilson, overlap, SLAC; $\mathcal{Q}$ -exact formulations, susy-gauge theories in various dimensions and various $\mathcal{N}$



- $H = \{Q, Q^\dagger\}$, nilpotent $Q^2 = 0 \Rightarrow \mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_F$
- $Q, Q^\dagger : \text{eigenstate}_E \rightarrow \{0, \text{eigenstate}_E\}$

- particular realization of super-Hamiltonian $\mathcal{Q} = \sigma_+ \left(\frac{d}{d\phi} + P \right)$

$$H = -\frac{d^2}{d\phi^2} + P^2(\phi) - \sigma_3 P'(\phi) = \begin{pmatrix} H_B & 0 \\ 0 & H_F \end{pmatrix}$$

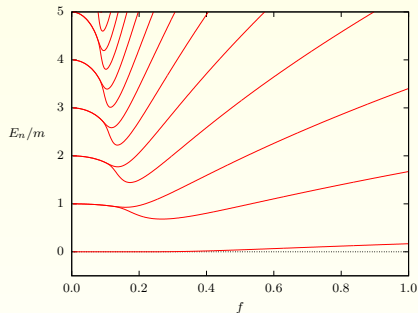
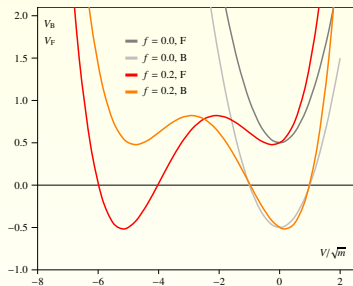
- **isospectral** up to possible zero-modes

Witten-Index: $\Delta = n_B - n_F = \text{tr}(e^{-\beta H_B} - e^{-\beta H_F})$

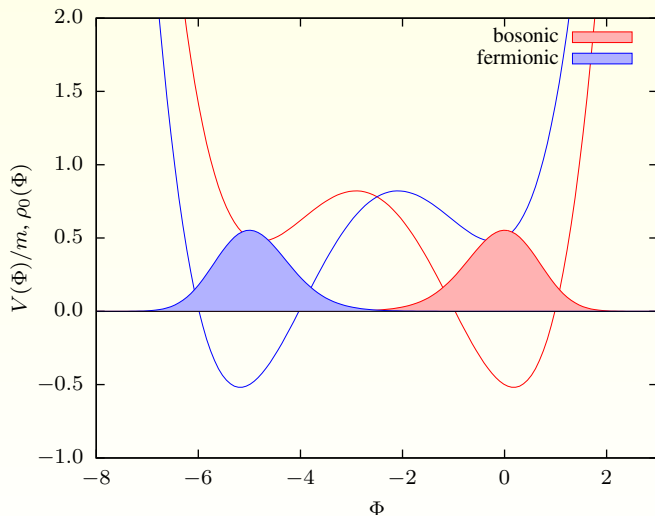
- $\Delta \neq 0 \Rightarrow$ susy unbroken, susy broken $\Rightarrow \Delta = 0$
- **susy anharmonic oscillator** with broken susy

$$P(\phi) = m\phi + h\phi^2 \Rightarrow \Delta = 0$$

- $f = h/m^{1.5}$ dimensionless coupling



- no ground state with zero energy
- every energy double degenerate
- $f \lesssim 0.1$: almost four-fold degeneracy (splitting $e^{-S/\hbar}$)
- dimensional parameters m and $\hbar$, m sets scale
- cp. Gerald Dunne last talk



- partition function and Witten index

$$\int \mathcal{D}\phi \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S[\phi, \psi, \bar{\psi}]} = \begin{cases} \mathcal{Z} & \text{anti-periodic bc} \\ \Delta & \text{periodic bc} \end{cases}$$

Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}P^2(\phi) + \bar{\psi}M(\phi)\psi, \quad M(\phi) = \partial + P'(\phi)$$

- supersymmetries

$$\begin{aligned} \delta^{(1)}\phi &= \bar{\varepsilon}\psi, & \delta^{(1)}\bar{\psi} &= -\bar{\varepsilon}(\dot{\phi} + P(\phi)), & \delta^{(1)}\psi &= 0 \\ \delta^{(2)}\phi &= \bar{\psi}\varepsilon, & \delta^{(2)}\psi &= (\dot{\phi} - P(\phi))\varepsilon, & \delta^{(2)}\bar{\psi} &= 0 \end{aligned}$$

- expectation values

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}\phi \mathcal{O}[\phi] \det M(\phi) e^{-S_B[\phi]}$$

- discretization, dimensionless lattice-fields and couplings

$$S = - \sum_{x,y} \frac{1}{2} \phi_x (\partial^2)_{xy} \phi_y + \frac{1}{2} \sum_x P(\phi_x)^2 + \sum_{x,y} \bar{\psi}_x (\partial_{xy} + P'(\phi_x) \delta_{xy}) \psi_y$$

- fermion determinant

$$\mathcal{Z} = \int \mathcal{D}\phi \det M[\phi] e^{-S_B[\phi]}, \quad M = \partial^{\text{slac}} + P'(\phi)$$

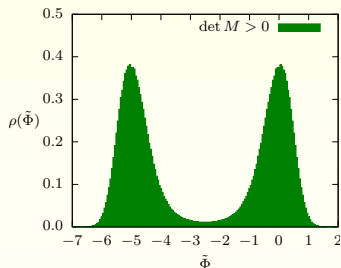
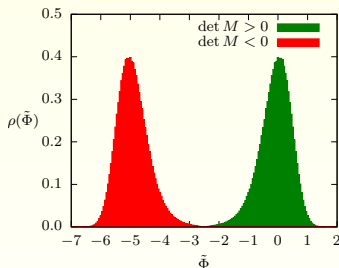
- (bosonic) expectation values

$$\langle \mathcal{O}[\phi] \rangle = \mathcal{Z}^{-1} \int \mathcal{D}\phi \mathcal{O}[\phi] \det M[\phi] e^{-S_B[\phi]}$$

- sign-quenched distribution

$$\rho[\phi] = e^{-S_B[\phi] + \ln |\det M[\phi]|}.$$

- sign quenched ensemble, 10^6 configurations, $m\beta = 4$, $f = 0.2$
- $\text{sign}(\det) \leftrightarrow \tilde{\phi} = \text{lattice average of field} \leftrightarrow \text{bc}$



- periodic (left, $N = 101$) and antiperiodic (rights, $N = 100$)

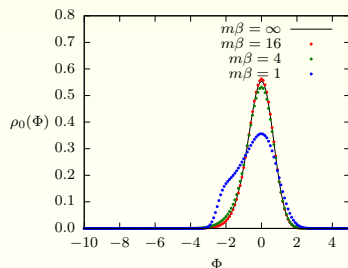
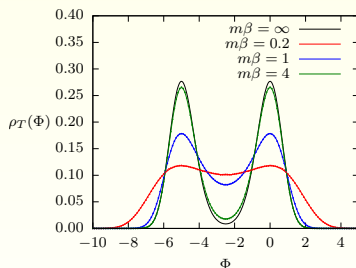
can show: for ∂^{slac} and even $P(\phi)$:

Wozar, AW

$$\det_{\text{ap}}(M[-\phi]) = \det_{\text{ap}}(M[\phi]), \quad \det_{\text{p}}(M[-\phi]) = -\det_{\text{p}}(M[\phi]) \Rightarrow \Delta = 0$$

- thermal probability distribution

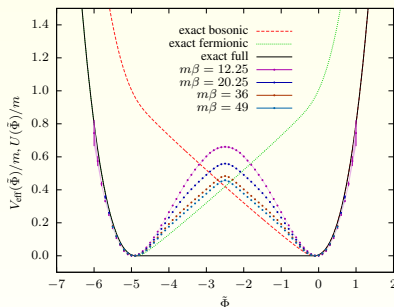
$$\rho_T(\phi) = Z^{-1} \sum_n e^{-E_n/T} |\langle \phi | \psi_n \rangle|^2$$



- exact results (lines) vs. simulation results (points) ($f = 0.2$)
- left: thermal distributions
- right: exact distribution for $|\psi_B\rangle$ and simulations with $\tilde{\phi} > -\frac{1}{2f}$.

- constraint effective potential (CEP)

$$U(\phi_0) = -\frac{1}{\beta} \ln \left(\int \mathcal{D}\phi \det M[\phi] e^{-S_B[\phi]} \delta(\tilde{\phi} - \phi_0) \right),$$



- exact V_{eff} for $T = 0$
exact: high precision numerics
- finite temperature:
CEP U from simulations
- not convex at low temperature
- $f = 0.2$, $N = 300$, $\approx 10^7$ conf

$$\min_{m\beta \rightarrow \infty} U(\phi_0) = \min \{ U_B(\phi_0), U_F(\phi_0) \}$$

- determination of energies (masses)

see C. Wozar, AW (2011)

on-shell formulation

$$S = \int d^2x \frac{1}{2} \left((\partial_\mu \phi)^2 + \bar{\psi} (\not{\partial} + P'(\phi)) \psi + P(\phi)^2 \right).$$

- minimal field content: real ϕ , Majorana ψ (real F)
- not a finite QFT, superpotential renormalized
- not obtained from dimensional reduction
- no $\mathcal{Q}$ -exact formulation, no Nicolai map
- $\Delta = 0$ possible $\Rightarrow$ susy may be broken
- **expect phase transition**: susy vs. $\mathbb{Z}_2$ (cp. FRG results)
- laboratory for **tensor networks**
- **most accurate**: fermion loop representation

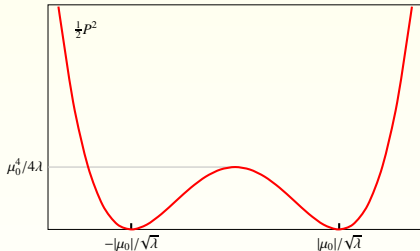
Witten

Wenger, Steinhauer

Beccaria, Campostini, Feo; Wozar, AW; Steinhauer, Wenger; Synatschke, Wipf, AW

- quadratic prepotential $\Rightarrow \Delta = 0$

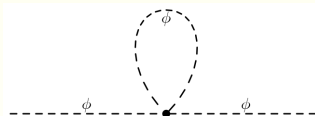
$$P(\phi) = \frac{1}{\sqrt{2\lambda}}(\mu_0^2 + \lambda\phi^2) \Rightarrow \frac{1}{2}P(\phi)^2 = \frac{\mu_0^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \text{const}$$



- $\mathbb{Z}_2$ -symmetric
- $\mathbb{Z}_2$ spontaneously broken?
- susy broken?
- $\mu_0^2 \ll 0$: system cannot tunnel
 $\Rightarrow$ susy not broken
- $\mu_0^2 > 0$: ground state energies lifted
 $\Rightarrow$ susy broken

FRG-analysis

- second order PT
- susy breaking $\Leftrightarrow \mathbb{Z}_2$ restoration
- massive susy phase for weak coupling
- broken phase with **goldstino and light scalar** ($m \propto 1/L$)
- continuum: **divergent diagram** (susy: fermion loop)



- λ not renormalized $\Rightarrow$ scale
- $\hat{\lambda} = \lambda a^2 \rightarrow 0$ determines a
- $\hat{\mu} = \mu a \rightarrow 0$
- $\hat{\mu}_0^2 = \hat{\mu}^2 - \# \hat{\lambda} A_{\hat{\mu}}$ counter term
- $A_{\hat{\mu}} = N^{-2} \sum (\hat{p}^2 + \hat{\mu}^2)^{-1}$
logarithmically divergent

$$\hat{V} = \frac{\hat{\mu}_0^2}{2} \phi^2 + \frac{\hat{\lambda}}{4} \phi^4$$

- fixed $\hat{\lambda} \rightarrow$ always 2nd order $\mathbb{Z}_2$ breaking PT at critical $\hat{\mu}_c^2$
- renormalized critical coupling

$$f_c = \left[\frac{\lambda}{\mu^2} \right]_{\text{crit}} = \lim_{\hat{\lambda} \rightarrow 0} \hat{f}_c, \quad \hat{f}_c = \frac{\hat{\lambda}}{\hat{\mu}_c^2}$$

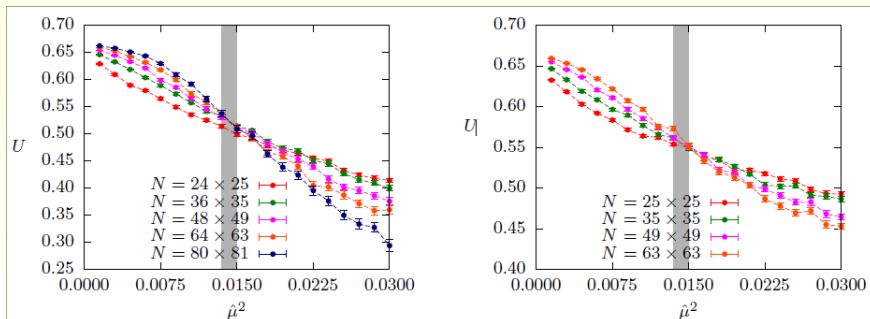
- algorithm:

- fix $\hat{\lambda} \Rightarrow$ determine $\hat{\mu}_c^2$ from Binder cummulant $\Rightarrow \hat{f}_c$
- extrapolate $\hat{f}_c \approx f_c + \alpha \hat{\lambda} + \beta \hat{\lambda} \log \hat{\lambda}$ to small $\hat{\lambda}$

Schaich, Loinaz

results for bosonic and susy model

model/method	f_c	authors
bosonic, MC Cluster (2009) on 1200^2	10.81(7)	Schaich, Loinaz.
bosonic, ferm. loop MC (2015)	11.10(2)	U. Wenger, privat comm.
bosonic, MC worm (2015) on 512^2	11.15(9)	Bosetti et al.
bosonic, matrix product states (2013)	11.064(20)	Milsted et al.
Hamiltonian truncation (2017):	11.04(12)	Miro, Rychkov, Vitale
bosonic, SLAC derivative on 256^2 :	10.92(13)	Wozar, AW
susy model (SLAC) on 80×81 :	21.1(1.1)	Wozar, AW



$$U = 1 - \frac{\langle \phi^4 \rangle}{3 \langle \phi^2 \rangle^2}$$

- Binder cumulant for thermal (left) and susy bc (right)
- periodic bc = anti-periodic bc
- $\mathbb{Z}_2$ breaking PT at $f > f_c \approx 21$; recall $\hat{\mu}_0^2 = \hat{\mu}^2(1 - \# \hat{f} A_{\hat{\mu}})$

Majorana fermions: $\mathcal{D}\psi$ yields Pfaffian

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}\psi e^{-S_B[\psi] - \psi^\dagger M[\phi] \psi} = \int \mathcal{D}\phi \text{Pf}(M[\phi]) e^{-S_B[\phi]}$$

- antisymmetric fermion matrix $M = \mathcal{C}(\not{\partial}^{\text{slac}} + P')$:

$$M = \begin{pmatrix} \partial_1^{\text{slac}} & \partial_0^{\text{slac}} - P' \\ \partial_0^{\text{slac}} + P' & -\partial_1^{\text{slac}} \end{pmatrix}.$$

- “square root” of determinant, $(\text{Pf } M)^2 = \det M$
- expansion and identities

$$\text{Pf}(M) = \frac{1}{2^N N!} \sum_{\sigma \in S_{2N}} \text{sign}(\sigma) \prod_{i=1}^N M_{\sigma_{2i-1}, \sigma_{2i}}$$

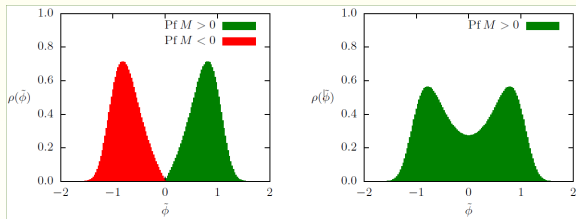
$$\text{Pf}(A) = (-1)^n \text{Pf}(A^t), \quad \text{Pf}(BAB^t) = \det(B) \text{Pf}(A)$$

- can show (transpose, $\mathcal{C}$): ϕ^{slac} and odd $P'(\phi) \Rightarrow$

$$\text{Pf}(M[-\phi]) = \begin{cases} +\text{Pf}(M[\phi]) & \text{thermal anti-periodic bc} \\ -\text{Pf}(M[\phi]) & \text{susy periodic bc} \end{cases}$$

- $\Delta = 0$ & continuums $\mathbb{Z}_2$ -symmetry for thermal bc

- left: 9×9 periodic bc
- right: 8×9 anti-periodic bc
- $\hat{f} = 100 > \hat{f}_c$
- $\hat{\lambda} = 0.1$
- 10^6 config.



- not true for Wilson-fermions!

sign of Pfaffian

$\mathbb{Z}_2$ -broken phase: $\tilde{\phi} \cdot \text{sign Pf}(M) > 0 \Rightarrow$ positive measure for thermal bc

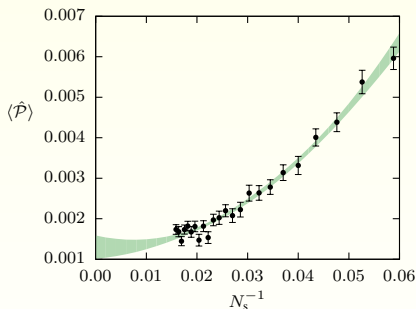
- $\mathbb{Z}_2$ -broken phase: system in **one ground state** $\Rightarrow$ fixed sign of Pf
physics insensitive to boundary conditions (Binder cummulant)
- approach phase transition from broken phase $f > f_c$
- to see susy-breaking: must control $T > 0$ and $a > 0$ effects
 $\Rightarrow$ careful extrapolations required!

Breaking of supersymmetry

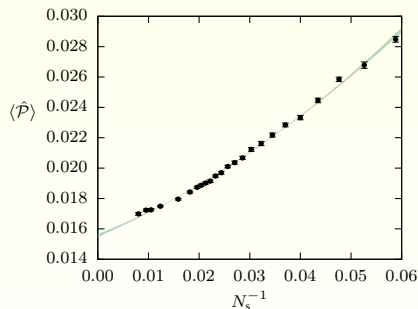
Ward identity $\langle \delta\psi \rangle = 0 \implies \langle \mathcal{P} \rangle = 0, \quad \mathcal{P} = \frac{1}{V} \sum P(\phi_x)$

- infinite volume extrapolations, $N_t = N_s + 1$

$$\langle \mathcal{P} \rangle = A + BN_s^{-1} + CN_s^{-2}$$

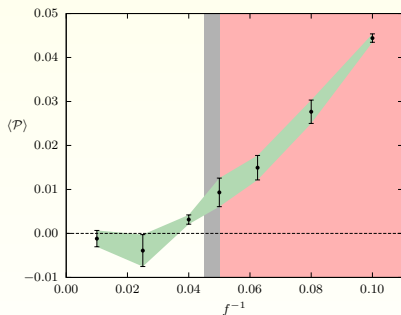


- $\mathbb{Z}_2$ broken phase $\hat{f} = 100$



- $\mathbb{Z}_2$ symmetric phase $\hat{f} = 10$

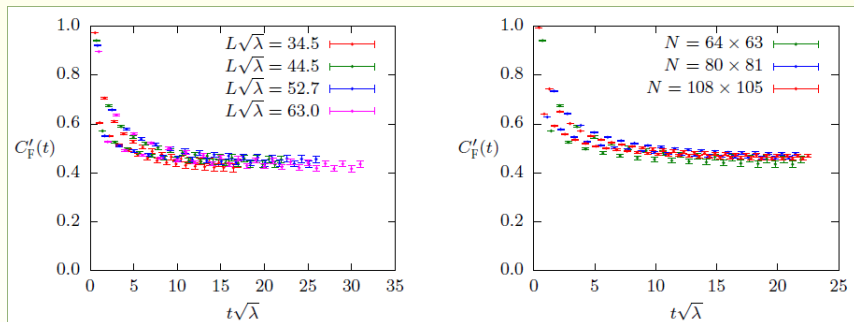
- for each $f \in \{10, 12.5, 16, 20, 25, 40, 100\}$:
extrapolate $\langle \mathcal{P} \rangle$ to its continuum value at $\hat{\lambda} \rightarrow 0$.



- $\mathbb{Z}_2$ and susy breaking at same f_c
- red: $\mathbb{Z}_2$ restored, susy broken
- gray: $\mathbb{Z}_2$ phase transition
- PT at $f_c \approx 21.1$

- Goldstino: massless mode in **susy broken phase**:

$$C_F'(t) = N_s^{-2} \sum_{\alpha, X, X'} \langle \bar{\psi}_{1,(t,X)} \psi_{1,(0,X')} - \bar{\psi}_{2,(t,X)} \psi_{2,(0,X')} \rangle$$

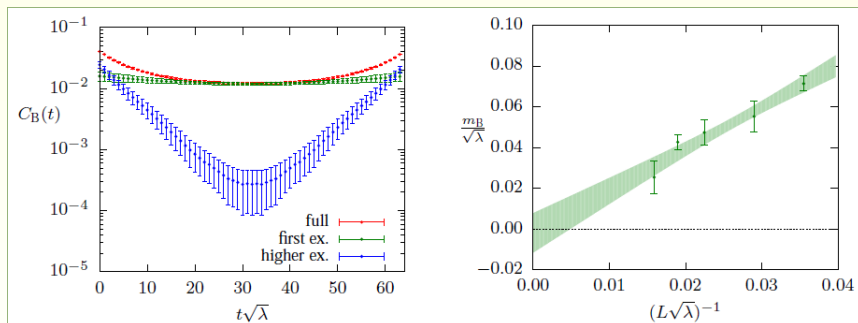


- left: different physical volumes
- right: fixed physical volume, different N
- **massless Goldstino**

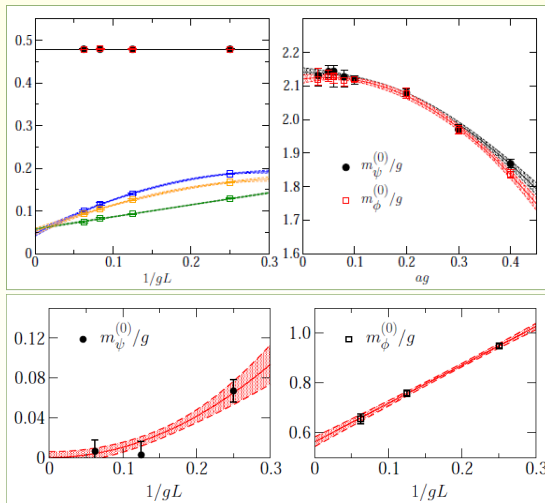
- FRG $\rightarrow$ massless boson

Gies, Synatschke, AW

$$C_B(t) = N_s^{-2} \sum_{x,x'} \langle \phi(t,x) \phi(0,x') \rangle$$



- bosonic states, first and second excited $\rightarrow$ massless boson
- discretization errors $<$ statistical errors ($64 \times 63, 80 \times 81, 108 \times 105$)



- left: infinite volume extrapolations
 m_{ϕ}, m_{ψ} in susy phase
 m_{ϕ} for $am = 0.3, 0.28, 0.02$ in susy broken phase
- right: $ag \rightarrow 0$ extrapolation in susy phase

-
- infinite volume extrapolation
 susy broken phase

K. Steinhauser, U. Wenger (2014)

$\mathcal{N} = 2$ WZ-model in two dimensions = dimensional reduction of $4d$ WZ-model

4 dimensions

Majorana spinor

two real scalar fields

2 dimensions

two Majorana spinors $\equiv$ one Dirac spinortwo real scalar fields $\equiv$ one complex field

- action

$$\mathcal{S}_{\text{cont}} = \int d^2x \left(2\bar{\partial}\bar{\varphi}\partial\varphi + \frac{1}{2}|W'(\varphi)|^2 + \bar{\psi}M\psi \right).$$

- fermion matrix

$$M = \gamma^z\partial + \gamma^{\bar{z}}\bar{\partial} + W''P_+ + \overline{W}''P_-.$$

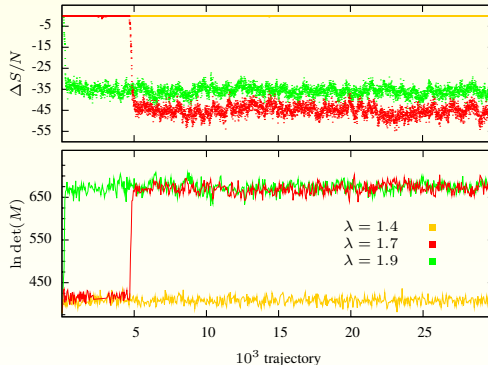
- holomorphic prepotential W not renormalized
- quantum corrections $\rightarrow$ Kähler metric $Z(\phi)(\bar{\partial}\bar{\varphi}\partial\varphi)$

- supersymmetries (ϵ complex)

$$\begin{aligned}\delta\varphi_x &= \bar{\epsilon}\psi_{1,x}, & \delta\bar{\psi}_{1,x} &= -\frac{1}{2}\bar{\xi}_x\bar{\epsilon}, & \delta\psi_{1,x} &= 0, \\ \delta\bar{\varphi}_x &= \bar{\epsilon}\psi_{2,x}, & \delta\bar{\psi}_{2,x} &= -\frac{1}{2}\bar{\xi}_x\bar{\epsilon}, & \delta\psi_{2,x} &= 0.\end{aligned}$$

- susy never broken, no Goldstino
- exist $\mathcal{Q}$ -exact formulation ($\mathcal{Q}^2 = 0$)
- equivalent to Nicolai map-construction (Parisi-Sourlas)
- no divergences (besides vacuum energy)
- detailed investigations of various discretization, fermions species
- measured: Ward identities, sign-problem, finite size effects, masses

- $\mathcal{Q}$ exact formulation may become instable for strong coupling
- $\mathcal{N} = (2, 2)$ WZ-model does:



- MC history of improvement term and fermion determinant $\hat{m} = 0.6$
- sudden jump into artifact phase with large gradients

Gauge theory with one supersymmetry

- $\mathcal{N} = 1$ vector multiplet $(A_\mu, \lambda = \lambda_c)$ in adjoint
- for gauge group $SU(3)$: **super-QCD**
- theory dynamical (not narrowed down by symmetries)
- on-shell **susy transformations**

$$\delta_\epsilon A_\mu = i\bar{\epsilon}\gamma_\mu\lambda, \quad \delta_\epsilon\lambda = i\Sigma_{\mu\nu}F^{\mu\nu}\epsilon$$

- field strength tensor $F_{\mu\nu}$
- inf. spin rotations $\Sigma_{\mu\nu} \equiv \frac{i}{4}[\gamma_\mu, \gamma_\nu]$
- inf. anti-commuting constant Majorana spinor ϵ

Continuum on-shell Lagrange density

$$\mathcal{L}_{\text{SYM}} = \text{tr} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda} \not{D} \lambda - \frac{m}{2} \bar{\lambda} \lambda \right) .$$

- relevant mass-term breaks susy softly
- needed for susy continuum limit
- fine-tuning to reach 'correct' continuum limit (vanishing gluino mass)
- expect **confinement** of color charge at low energies
- only color-neutral **bound states** admitted: Mesons, Glueballs, ...
- bound states arranged in super-multiplets
- **effective field theory** (symmetries plus anomaly matching)

- $a\text{-}f_0$ means adjoint f_0
- characterization of states J^{pc}
- multiplet of Veneziano and Yankielowicz

1 bosonic scalar	0^{++} gluinoball	$a\text{-}f_0 \sim \bar{\lambda}\lambda$
1 bosonic pseudoscalar	0^{-+} gluinoball	$a\text{-}\eta' \sim \bar{\lambda}\gamma_5\lambda$
Majorana-type spin $\frac{1}{2}$	gluino-glueball	$\chi \sim F_{\mu\nu}\Sigma^{\mu\nu}\lambda$

- multiplet of Later, Farrar, Gabadadze and Schwetz

1 bosonic scalar	0^{++} glueball	$0^{++} \sim F_{\mu\nu}F^{\mu\nu}$
1 bosonic pseudoscalar	0^{-+} glueball	$0^{-+} \sim \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$
1 Majorana-type spin $\frac{1}{2}$	gluino-glueball	$\chi \sim F_{\mu\nu}\Sigma^{\mu\nu}\lambda$

- $m = 0 \Rightarrow$ global chiral $U(1)_A$ symmetry

$$\lambda \mapsto e^{i\alpha\gamma_5} \lambda$$

- anomalous breaking $U(1)_A \Rightarrow \mathbb{Z}_{2N_c}$

$$\lambda \mapsto e^{i\alpha_n\gamma_5} \lambda \quad \text{with} \quad \alpha_n = \pi \frac{n}{N_c}, \quad n \in \{1, \dots, 2N_c\}$$

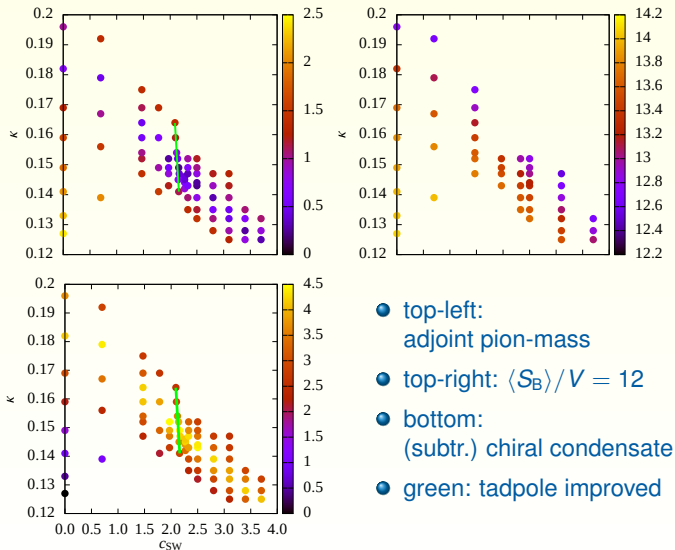
- further spontaneous breaking by gluino condensate $\langle \bar{\lambda}\lambda \rangle \neq 0$
- breaking pattern $\mathbb{Z}_{2N_c} \Rightarrow \mathbb{Z}_2$
- N_c physically equivalent vacua
- strongly interacting theory $\rightarrow$ lattice, FRG, ...
- no $\mathcal{Q}$ exact formulation
- conventional approach

- recover chiral symmetry in continuum $\rightarrow$ susy restoration
- **Ginsparg-Wilson fermions** $\rightarrow$ susy guaranteed in continuum
first benchmarks: gaugino condensate, static potential, spectrum of Dirac operator
Endress; Giedt, Brower, Catterall, Fleming, Vranas; JLQCD collaboration
- **Wilson fermions** $\rightarrow$ fine-tuning of m
mass-spectrum for SU(2), first results for SU(3), thermodynamics, Ward-Identities
SU(2), SU(3): DESY-Münster collaboration: Montvay, Münster, Bergner, ...
SU(3): Jena-group: Steinhäuser, Sternberg, Wellegehausen, AW

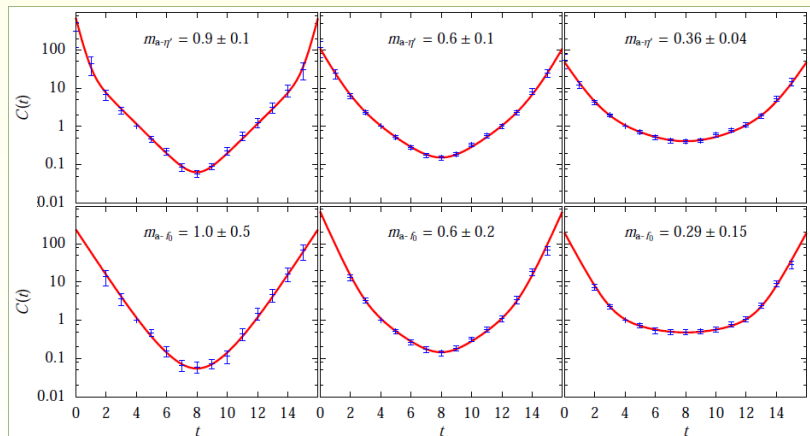
- find m with minimal mass of gluino and (adjoint) pion
- thermodynamic limit to avoid finite size effects
- continuum limit
- **reduce lattice artifacts**
 - Symanzik improved Lüscher-Weisz gauge action $O(a^2)$
 - Clover fermion operator

$$D_C(x, y) = D_W(x, y) - c_{SW} \frac{\kappa}{2} \Sigma_{\mu\nu} F^{\mu\nu} \delta_{x,y}$$

- proper choice for Sheikholeslami-Wohler coefficient $c_{SW} \Rightarrow O(a^2)$
- from 1-loop PT, Tadpole-improvement, Schrödinger functional, . . .
Jena group: **minimize** $m_{a-\pi}$



- $c_{SW} = 2.27$, $\beta = 5.2$, $\kappa \in \{0.142, 0.144, 1.45\}$
- on the way: larger lattices,

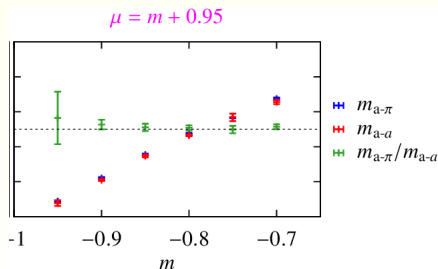
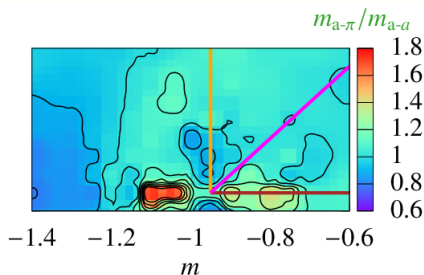


- $m_{a-\pi}$ too large: numerical experiments with 'twisted mass'

$$D_W^{\text{tw}}(x, y) = (4 + m + i\mu\gamma_5)\delta_{x,y} - \frac{1}{2} \sum_{\mu=\pm 1}^{\pm 4} (1 - \gamma_\mu) \mathcal{V}_\mu \delta_{x+\mu, y}$$

- along diagonal to critical point $(m_c, \mu_c) = (0.95, 0)$

$$\Rightarrow m_{a-\pi} \approx m_{a-a}$$



- much more about $\mathcal{N} = 1$ SYM: talk by G. Bergner

- susy-breaking under control in simple models
- $\mathbb{Z}_2$ breaking $\leftrightarrow$ susy breaking
- sign problem $\leftrightarrow$ order parameter
- Yukawa-type model: SLAC fermions useful (symmetries)
cp. recent results on 4– Fermi theories
- goldstino and massless boson in $\mathcal{N} = 1$ WZM
- deceneracy of masses in susy phase (high accuracy)
- $\mathcal{N} = 1$ SYM: optimization of Clover term
- numerical experiments with 'twisted' mass terms
- masses for SU(3): see DESY-Münster collaboration
- $\mathcal{N} = 1$ SYM in four dimensions $\rightarrow \mathcal{N} = (2, 2)$ in two dimensions
- convincings results for $(2, 2)$ (talk): moduli, masses, ...