Fidelity decay and entropy production in many-particle systems after random interaction quench

Sudip Kumar Haldar

Physical Research Laboratory

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  Physical Research Laboratory, Ahmedabad.
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Overview

Quench dynamics

What is it?

- **Suppose, initially the system is in the state $|i\rangle$.**
  At $t = 0$; $H_i \xrightarrow[\text{system parameter}]{\text{instantaneous change}} H_f$.
  Unitary time evolution under $H_f \rightarrow$ Quench dynamics.

Why important?

- **How equilibration is achieved in finite quantum many-body systems.**
- **To understand when and how such systems thermalize.**
- **Relations between quantum chaos and thermalization.**

Quantities of interest

- **Return probability or Fidelity decay $W_0(t)$.**
- **Entropy production with time $S(t)$.**

Embedded Random Matrix Ensembles

- **Isolated finite many-body systems $\rightarrow$ two-body interaction.**
- **The particles move in a mean-field.**
- **Suitable random matrix ensemble $\rightarrow$ Embedded Gaussian Orthogonal Ensemble EGOE(1+2).**

  $\{\hat{H}\} = \hat{h}(1) + \lambda\{\hat{V}(2)\}$

  $\{\hat{v}(2)\} \longrightarrow$ *GOE* **in two-particle space.**

  $\hat{h}(1) \longrightarrow$ **Fixed one body operator.**
- **Three chaos markers: λ_c, λ_F and λ_t.**

Embedded Gaussian Orthogonal Ensembles

EGOE(1+2)-s

- m **particles in Ω sp orbits each with spin $s = 1/2 \Rightarrow N = 2\Omega$.**
- **EGOE(1+2)-s: Embedded ensemble for one plus two body interaction with spin degrees of freedom for a system of fermions.**
- $\{\hat{H}\} = \hat{h}(1) + \lambda \hat{V}(2)$; $V(2) = \{V^{s=0}(2) + V^{s=1}(2)\}$

BEGOE(1+2)

- m **bosons in N sp states; $m > N$.**
- **BEGOE(1+2): Embedded ensemble for one plus two body interaction for a system of bosons.**
- $\{\hat{H}\} = \hat{h}(1) + \lambda\{\hat{V}(2)\}$

Fidelity decay or return probability

- $\Psi(t=0) = |k\rangle \xrightarrow[\textbf{Quench}]{\hat{V}(2)} \Psi(t) = |k(t)\rangle = \exp(-iHt)\;|k\rangle$.
  Eigenstate of $\hat{h}(1)$

  $$H = h(1) + \lambda V(2)$$

- $W_{k\to f}(t) = |\langle f|\exp[-iHt]|k\rangle|^2 = |A_{k\to f}(t)|^2$;

  $$\mathbf{A}_{k\to f}(t) = \sum_E C_k^E C_f^E \exp(-iEt)\ .$$

- **Return probability** $\Rightarrow$

  $$W_{k\to k}(t) = \left|\sum_E [C_k^E]^2 \exp(-iEt)\right|^2 = \left|\int F_k(E)\exp(-iEt)\;dE\right|^2.$$

  $F_k(E) = |C_k^E|^2 \rho(E) \to$ **strength function.**

  $\rho(E) \to$ **Density of states.**

EGOE formula

Gaussian and BW region

- $W_{k\to k}(t) \overset{\text{BW region}}{\longrightarrow} \exp[-\Gamma\, t]$; $\Gamma \to$ **width of spreading**
- $W_{k\to k}(t) \overset{\text{Gaussian region}}{\longrightarrow} \exp[-\sigma_k^2\, t^2]$; $\sigma_k^2 \to$ **spectral variance.**

BW to Gaussian intermediate region

- $$F_k(x:\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi}\,\sqrt{\nu}\,\Gamma\left(\frac{\nu}{2}\right)} \frac{dx}{\left(\frac{x^2}{\nu}+1\right)^{\frac{\nu+1}{2}}} \Longleftarrow \textbf{t-distribution.}$$

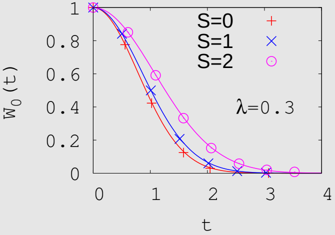
  $\alpha = (\nu+1)/2$; $(E - E_k) = \sqrt{\frac{\beta(\nu+1)}{2\nu}}\, x$; $\sigma_k^2 = \frac{\alpha}{2\alpha-3}\beta$; $\alpha > 3/2$

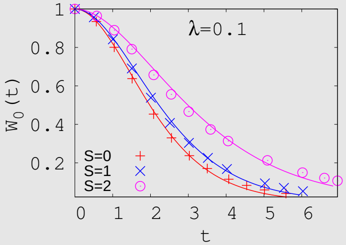
  D. Angom, S. Ghosh, and V.K.B. Kota, Phys. Rev. E 70, 016209 (2004).

- $$W_{k\to k}(t) \to \left| \frac{2^\nu (\sqrt{\nu})^\nu}{\Gamma(\nu)} \int_0^\infty dx[x(x+|t'|)]^{(\nu-1)/2} \exp[-\sqrt{\nu}\,(2x+|t'|)] \right|^2 ;\ t' = \sqrt{\frac{\beta(\nu+1)}{2\nu}}\, t\ .$$

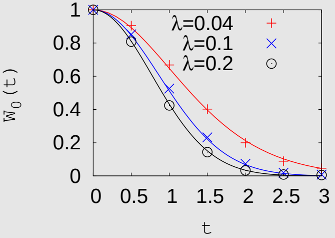
- **For** $\nu = 1$, $F_k(E) \longrightarrow \exp[-\Gamma\, t]$; $\beta = \Gamma^2/4$
  For $\nu \to \infty$, $F_k(E) \longrightarrow \exp[-\sigma_k^2\, t^2]$; $\sigma_k^2 = \beta/2$.

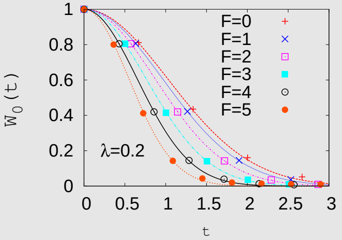
EGOE(1+2)-*s*





BEGOE(1+2)

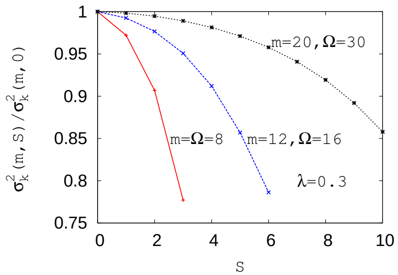




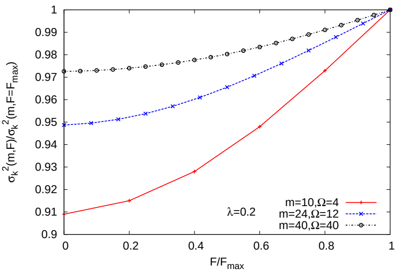
- $\overline{\sigma_k^2(m,S)} = \dfrac{\lambda^2\,\overline{\sigma^2_{V(2)}(m,S)}}{\overline{\sigma^2_H(m,S)}}$; $\overline{\sigma^2_H(m,S)} = \sigma^2_{h(1)}(m,S) + \lambda^2\overline{\sigma^2_{V(2)}(m,S)}$

M. Vyas, V.K.B. Kota and N.D. Chavda, Phys. Rev. E 81, 036212 (2010).

M. Vyas, V.K.B. Kota, N.D. Chavda and V. Potbhare, J. Phys. A: Math. Theor. 45, 265203 (2012).



EGOE(1+2)-s



BEGOE(1+2)-F

Entropy production with time and statistical relaxation

Definition

- $S(t) = -\sum_{f=0}^{d} W_f(t) \ln W_f(t)$ **.**
- $W_f(t) = \sum_E |C_0^E|^2 \, |C_f^E|^2 + 2 \sum_{E>E'} C_0^E C_f^E C_0^{E'} C_f^{E'} \; \cos(E-E')t$
  $= \mathbf{W}_f^{avg}(t) + W_f^{flu}(t)$.

Theoretical model

- **Assumptions: i)** $N_s \rightarrow$ **number of** f**'s with** $f \neq 0$ **that contribute**
  ii) The fluctuations in W_f **are small** $\longrightarrow$ W_f **can be replaced by** $\overline{W}$**.**
- $S(t) = -W_0(t) \ln W_0(t) - \sum_{r=1}^{N_s} \overline{W} \ln \overline{W}$; $\quad \overline{W} = \frac{1-W_0}{N_s}$
  = - $\mathbf{W}_0(t) \ln W_0(t) - [1 - W_0(t)] \ln(\frac{1-W_0(t)}{N_s})$ **.**

EGOE formula

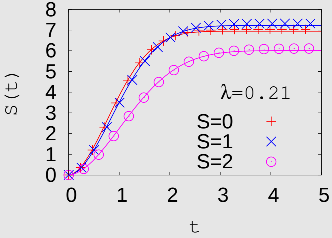
Gaussian region

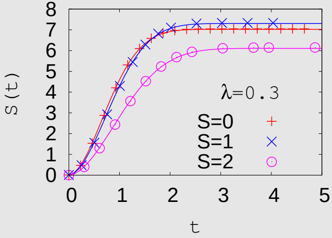
- $S(t) = \sigma_k^2 t^2 \exp(-\sigma_k^2 t^2) - \left[1 - \exp(-\sigma_k^2 t^2)\right] \ln\left(\frac{1-\exp(-\sigma_k^2 t^2)}{N_s}\right)$;
  $N_s = \langle \exp S(\infty) \rangle$.
- $N_s^{th} \sim \kappa \times \mathrm{NPC}_{\max} = \kappa \frac{\mathrm{d}}{3} \sqrt{1-\zeta^4}$; $\zeta^2 = 1 - \sigma_{\mathrm{k}}^2$. $\kappa = 2$
- $t_{sat} = \sqrt{\frac{\ln(1+N_s)}{1-\zeta^2}}$;
  $\lambda \to \infty$; $\zeta^2(m,S) \to 0 \implies t_{sat}^{min} \simeq \sqrt{\ln(\kappa\ \frac{d(m,S)}{3})}$
  $\lambda = \lambda_t$; $\zeta^2(m,S) = 0.5 \implies t_{th} \simeq \sqrt{2\ln(\kappa \frac{d}{6}\sqrt{3})}$.

BW region

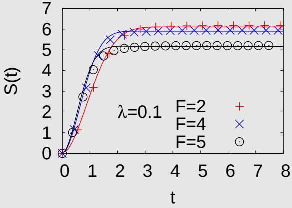
- $t_{sat} = \frac{\ln(1+N_s)}{\Gamma}$.

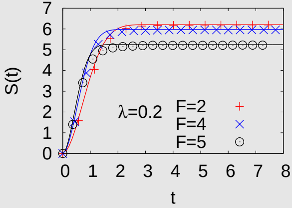
EGOE(1+2)-*s* examples





BEGOE(1+2)-*F* examples





t_{sat} in σ_{avg}^{-1} units for EGOE(1+2)-**s** examples.

λ	**S=0**		**S=1**		**S=2**	
	$\frac{\sigma_k^2}{\sigma_{avg}^2}$	t_{sat}	$\frac{\sigma_k^2}{\sigma_{avg}^2}$	t_{sat}	$\frac{\sigma_k^2}{\sigma_{avg}^2}$	t_{sat}
0.1	**0.21**	**8.3**	**0.18**	**8.8**	**0.11**	**8.9**
0.21	**0.6**	**3.32**	**0.499**	**3.73**	**0.32**	**4.21**
0.3	**0.81**	**2.7**	**0.68**	**3.2**	**0.44**	**3.65**

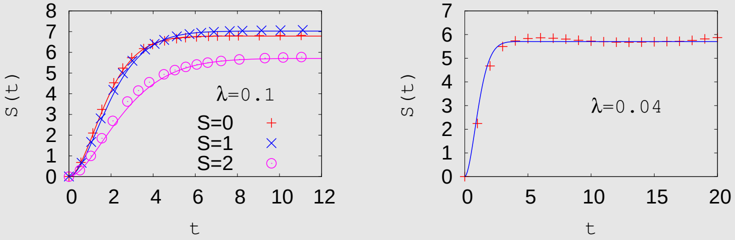
t_{sat} in σ_{avg}^{-1} unit for BEGOE(1+2)-F examples.

λ	**F=2**		**F=4**		**F=5**	
	$\frac{\sigma_k^2}{\sigma_{avg}^2}$	t_{sat}	$\frac{\sigma_k^2}{\sigma_{avg}^2}$	t_{sat}	$\frac{\sigma_k^2}{\sigma_{avg}^2}$	t_{sat}
0.1	**0.44**	**3.66**	**0.74**	**2.68**	**0.99**	**2.2**
0.2	**0.66**	**3.07**	**1.17**	**2.1**	**1.65**	**1.7**

BW to Gaussian transition region

- $N_s^{th} \sim \kappa \times \mathrm{NPC}_{\max} =$
  $\kappa \frac{d}{3}\left[\sqrt{\frac{2}{2\alpha-3}}\frac{\Gamma^2(\alpha)}{\Gamma^2(\alpha-\frac{1}{2})}\frac{1}{\zeta^2(1-\zeta^2)}\mathrm{U}\left(\frac{1}{2},\frac{3}{2}-2\alpha,\frac{(2\alpha-3)(1-\zeta^2)}{2\zeta^2}\right)\right]^{-1}$; **$\kappa = 2.5$**
- $t_{sat} = \frac{[\ln(1+N_s)]^{\frac{1}{2}(1+\frac{1}{\nu})}}{\sqrt{\frac{\beta}{2}}\left(1+\frac{1}{\nu}\right)^{3/2}}$.;

Numerical examples



(a) EGOE(1+2)-*s* (b) BEGOE(1+2)

Many-body results

1 of 10 156.38%

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Many-body entropies, correlations, and emergence of statistical relaxation in interaction quench dynamics of ultracold bosons

Axel U. J. Lode*
Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

Barnali Chakrabarti
Department of Physics, Presidency University, 86/1 College Street, Kolkata 700083, India

Venkata K. B. Kota
Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India
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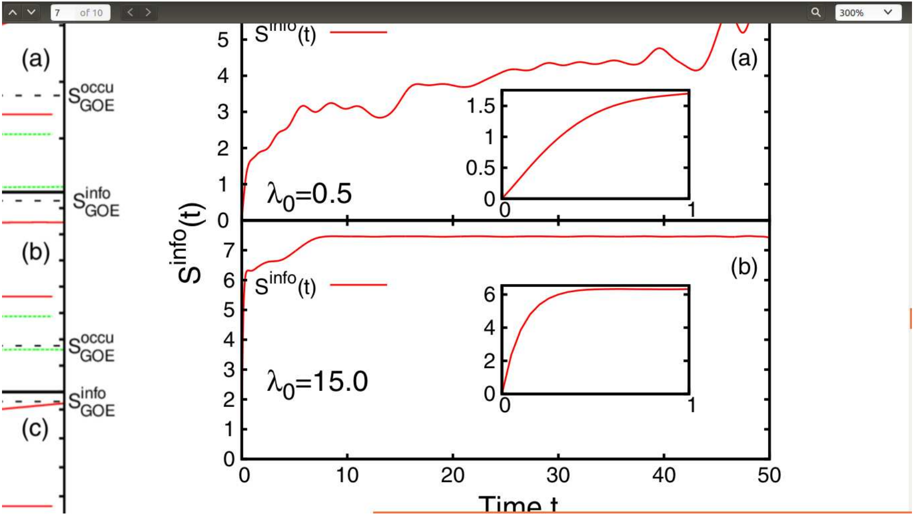
We study the quantum many-body dynamics and the entropy production triggered by an interaction quench in a system of $N = 10$ interacting identical bosons in an external one-dimensional harmonic trap. The multiconfigurational time-dependent Hartree method for bosons (MCTDHB) is used for solving the time-dependent Schrödinger equation at a high level of accuracy. We consider many-body entropy measures such as the Shannon information entropy, number of principal components, and occupation entropy that are computed from the time-dependent many-body basis set used in MCTDHB. These measures quantify relevant physical features such as irregular or chaotic dynamics, statistical relaxation, and thermalization. We monitor the entropy measures as a function of time and assess how they depend on the interaction strength. For larger interaction strength, the many-body information and occupation entropies approach the value predicted for the Gaussian orthogonal ensemble of random matrices. This implies statistical relaxation. The basis states of MCTDHB are explicitly time-dependent and optimized by the variational principle in a way that minimizes the number of significantly contributing ones. It is therefore a non-trivial fact that statistical relaxation prevails in MCTDHB computations. Moreover, we demonstrate a fundamental connection between the production of entropy, the buildup of correlations and loss of coherence in the system. Our findings imply that mean-field approaches such as the time-dependent Gross-Pitaevskii equation cannot capture statistical relaxation and thermalization because they neglect correlations. Since the coherence and correlations are experimentally accessible, their present connection to many-body entropies can be scrutinized to detect statistical

Many-body results



Summary

What achieved?

- **Using some approximations, EGOE(1+2) theory for the time evolution of entropy.**
- **Analytic formulae for t_{sat} and the saturation entropy derived.**
- **We observed significant spin dependence.**
- **An overall picture of relaxation of complex quantum systems in the absence of complete knowledge about it.**

What next?

- **Attempt to better understand the significance and magnitude of κ.**
- **Prethermalization using EGOE(1+2) formalism.**
- **Compare the formulas derived here with the results of realistic systems accessible to experiments$\rightarrow$ Quench dynamics of trapped Bose gas using MCTDHB.**

Thank you