Fast coarsening in strong phase separation

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Introduction

- A wide variety of systems show presence of ordering at low temperature.
- Starting from a high temperature disordered state, when such a system is quenched to a sufficiently low temperature, it undergoes coarsening
- If the dynamics conserves the order parameter, then in the long time limit, the system phase separates into macroscopic domains
- For most systems pure domains can be obtained only at zero temperature
- The coarsening towards a pure phase is associated with a very large time-scale

Strong phase separation

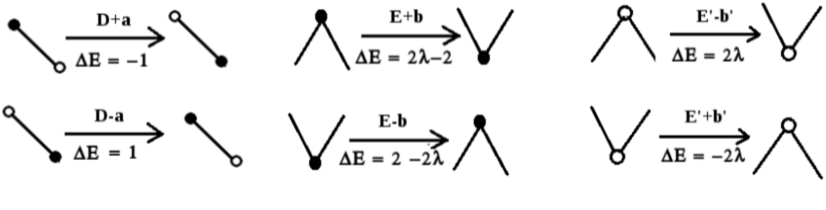
- Pure phase at all temperatures in the thermodynamic limit
- Long ranged interactions yield super-extensive energy which wins over extensive entropy at *all* finite temperatures
- Strong phase separation (SPS)
- *ABC* model [Evans *et al.*, 1998]
  Lahiri-Ramaswamy (LR) model [Lahiri et al., 1997, 2000]
- Existence of metastable states causes logarithmically slow coarsening
- Is it ever possible to reach a pure phase for sufficiently large L in a reasonable time-scale?

Fast coarsening to SPS

- We study a system that forms pure domains at all temperatures and coarsens over an algebraic time-scale $\sim L^2$, as opposed to exponentially large time-scale for conventional SPS
- Rich steady state dynamics where algebraic time-scale $\sim L^2$ and exponentially large time-scale $\sim e^L$, both are present
- Steady state is quintessentially nonequilibrium and carries a current $\sim 1/L$

Model description

- A system of hard-core particles sliding under gravity on a fluctuating surface in one dimension
- The local dynamics of the particles and the surface are coupled such that the particles tend to slide down along the local slope of the surface, and the surface gets pushed downward under the weight of the particles
- $1d$ lattice of length L in which the particles reside on the sites and the bonds, representing discrete surface elements, can have two possible orientations with slopes $\tau_{i+1/2} = \pm 1$



(a) (b) (c)

Detailed balance

- Condition for detailed balance

$$\frac{D-a}{D+a}=q, \qquad \frac{E-b}{E+b}=q^{2-2\lambda}, \qquad \frac{E'-b'}{E'+b'}=q^{2\lambda}$$

- Steady state shows Boltzmann measure $\sim \exp(-\beta\mathcal{H})$ with the Hamiltonian

$$\mathcal{H}=\sum_{i=1}^{L}(n_i-\lambda)h_i.$$

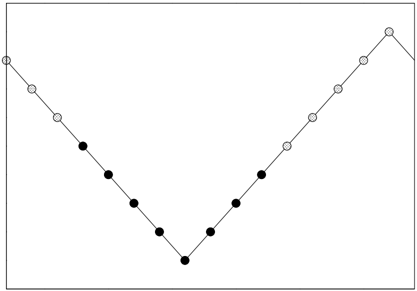
with $0\leq\lambda\leq 0.5$ and $q=e^{-2\beta}$

- Periodic boundary condition implies $\lambda=\rho$
- Although $\mathcal{H}$ is defined in terms of local height and local occupancy, the definition of the height field $h_i=\sum_{j=1}^{i-1}\tau_{j+1/2}$, generates long-ranged interactions between n_i and $\tau_{j+1/2}$

Conventional SPS for $\lambda \neq 0$

- Energy scales as L^2 and hence for any non-zero β, or equivalently, any $q < 1$, the system shows SPS
- The upslope and downslope surface bonds phase separate to form a single deep valley, where all the particles are present in a single cluster
- The parameter λ can be varied to obtain a family of Hamiltonians that characterizes the measure for the strongly phase separated steady state. The well-known LR model corresponds to $\lambda = 1/2$
- For any non-zero λ the qualitative behavior of the system remains the same as the LR model
- Presence of metastable states makes the relaxation process logarithmically slow

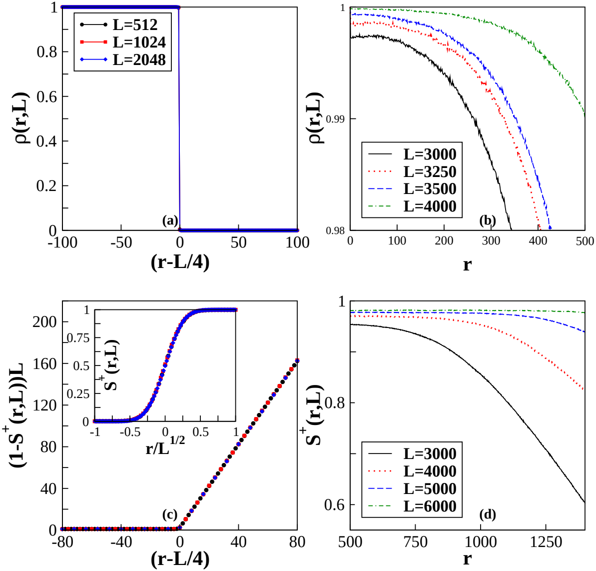
SPS for particles and surface



Fast relaxation in $\lambda = 0$ limit

- For $\lambda = 0$ the empty surface undergoes symmetric Edwards-Wilkinson type fluctuations
- The presence of the particles makes the transition between the (occupied) local hills and valleys asymmetric and pushes the surface down at a constant rate
- Detailed balance is violated for any finite ρ and one has a current-carrying steady state with SPS
- Most importantly, the relaxation towards this state takes place over an algebraic time-scale $\sim L^2$, unlike exponentially large time-scales for $\lambda \neq 0$

Static correlation functions in steady state



SPS for particles and surface

- Sharp domain wall between particles and holes
- As q increases, the boundary gets wider but pure phase is retrieved in the thermodynamic limit
- Surface correlations show crucial qualitative difference from conventional SPS
- The upslope and downslope surface bonds, lying under the particle cluster, phase separate completely to form a deep valley
- The domain boundary between these two pure phases lies at the deepest point of the valley: width of the domain boundary $\sim \sqrt{L}$

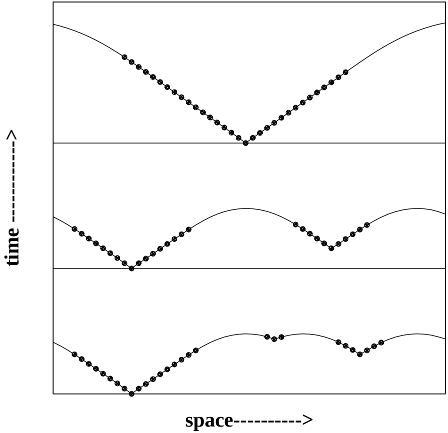
$1/L$ gradient in the empty part of the surface

- In the empty part of the surface no pure domain exists and $S^+(r, L)$ shows a linear gradient $\sim 1/L$
- System violates detailed balance and this gives rise to a downward velocity $\sim 1/L$ of the surface
- The linear gradient of $S^+(r, L)$ helps sustain the downward motion of the surface in the empty region, where the local dynamics is otherwise Edwards-Wilkinson type
- The empty part of the surface behaves like an open system which carries a current
- Pure domains of upslope and downslope bonds at the two ends act like two reservoirs with different chemical potentials that drives a current through the system

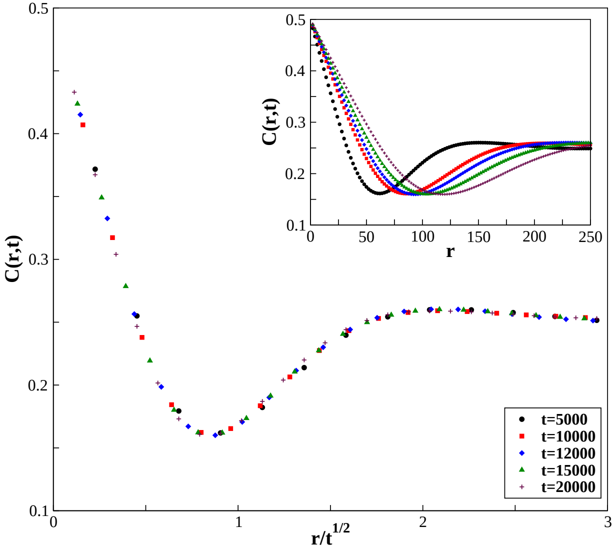
Algebraic coasening

- In the coarsening phase, small particle clusters are first formed in small valleys
- These valleys merge to form a deeper valley holding a large cluster of particles
- Two adjacent occupied valleys are separated from each other by an empty hill which must go away for merging
- Time-scale for this to happen $\sim q^{-2\lambda x}$ for a hill of size x
- For any $\lambda > 0$ the merging process becomes exponentially slow as x increases: metastable states
- For $\lambda = 0$ when the unoccupied hill can fluctuate without any energy cost, the merging process progresses diffusively with a time-scale $\sim x^2$

Time-evolution of a typical configuration



Coarsening length scale $\mathcal{L}(t) \sim \sqrt{t}$

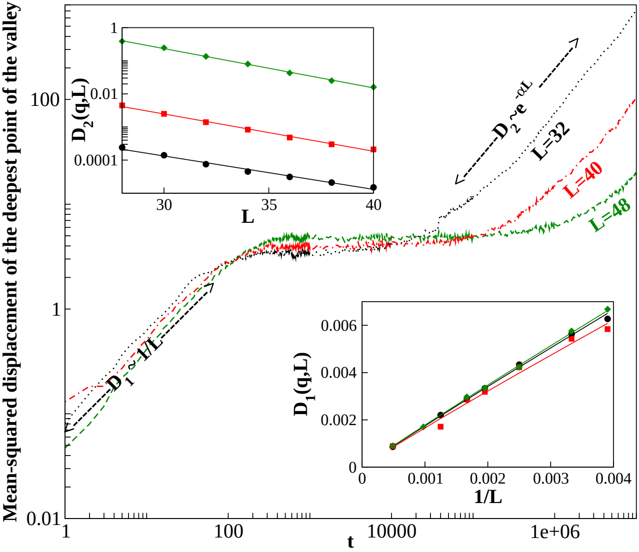


- Scaling of particle density correlation in the coarsening phase

Steady state dynamics

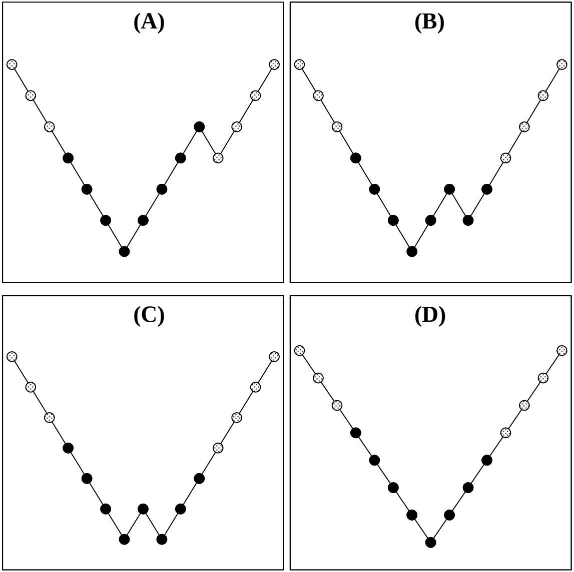
- Presence of an algebraic time-scale in the system gives rise to interesting dynamics in steady state
- Mean-squared displacement of the deepest point of the valley
- For small time $t \ll L^2$, the mean-squared displacement grows diffusively with a diffusion constant $D_1 \sim 1/L$
- For $t \sim L^2$, the mean-squared displacement reaches a plateau at a value $\sim L$
- Finally, at very large t, mean-squared displacement grows again with diffusion coeffecient $D_2 \sim e^{-\alpha L}$

Mean-squared displacement of the valley



Short time displacement

- Average distance of the nearest downslope bond from the right edge of the particle cluster $\sim \sqrt{L}$
- Therefore this downslope bond can diffuse through this distance over a time-scale $\sim L$, and form a local hill
- Local hill occupied by a particle and hence is unstable
- The downslope bond moves ballistically down the valley, over a time-scale $\sim L$, and reaches the deepest point causing a unit displacement of the deepest point towards right
- Process takes place over a time-scale $\sim L$ which explains the $D_1 \sim 1/L$ behavior for short times



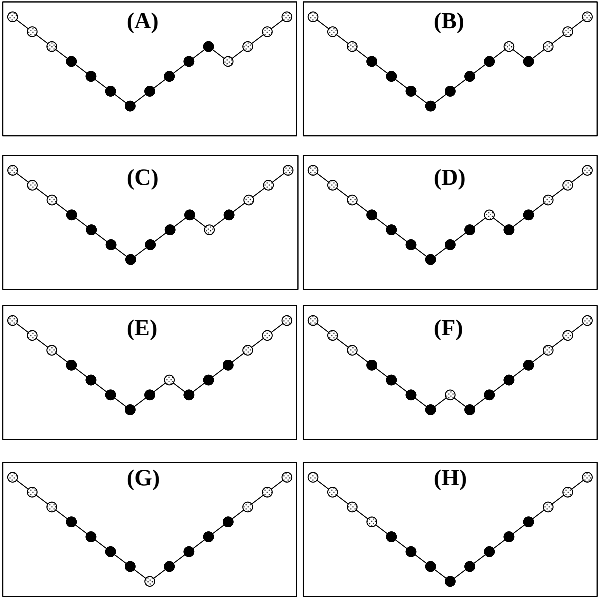
- No displacement for the particles, the deepest point of the valley slides back and forth beneath the particle cluster

Plateau region

- If the gravitational energy of the particles is defined as the sum of the height (measured from the deepest point of the valley) of the occupied sites
- Energy is minimum when the deepest point of the valley coincides with the center of mass of the particle cluster
- Any displacement from this position is associated with an energy cost
- For a distance Δ between the centre of mass and the deepest point, energy cost scales as Δ^2
- Thus the motion of the deepest point can roughly be described by an Ornstein-Uhlenbeck process
- The energy cost prevents the valley from taking large excursion in either direction
- Hence mean-squared displacement reaches a plateau at a value $\sim L$, consistent with the fluctuations expected with super-extensive energy

Displacement at very large t

- In the limit of very long time, the particle cluster will start moving around the system diffusively and the valley will naturally move along with it
- Very slow movement of a hole through the particle cluster
- When this hole reaches the bottom of the valley, the deepest point undergoes a displacement, along with the center of mass of the particle cluster
- The probability that an empty hill, starting from the domain boundary, survives till it reaches the bottom of the valley, is very low and decays exponentially with the domain size
- As a result, the diffusivity of the valley in this regime $\sim e^{-\alpha L}$
- Existence of an exponentially large time-scale breaks the translational invariance of the system but the relaxation time-scale is still algebraic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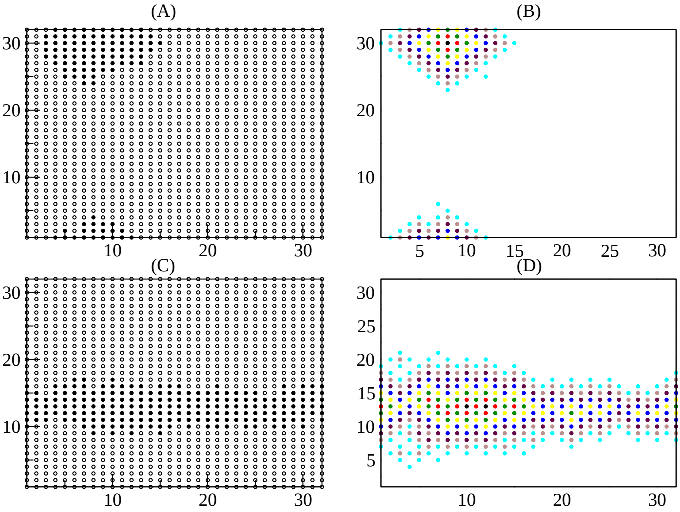
Conclusions

- A mechanism to speed up relaxation to a pure phase, earlier thought to be a logarithmically slow process
- The mechanism uses the fluctuations from a coexisting, nonequilibrium phase, and results in a much faster (power law) approach
- This simple idea can be applied to other systems as well, including higher dimensional systems which approach a pure phase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