Fluctuation-dominated Phase Ordering

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Work done with
Rajeev Kapri (IISER, Mohali)
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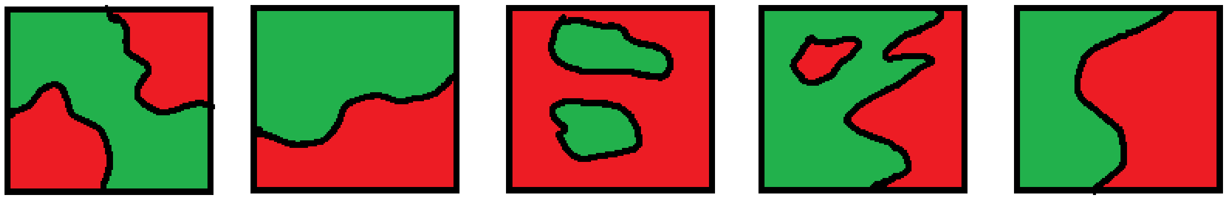
and earlier with
Dibyendu Das, Apoorva Nagar, Sakuntala Chatterjee,
Satya Majumdar, G. Manoj

Non-equilibrium Statistical Physics, ICTS-TIFR (October 2015)

Fluctuation-dominated Phase Ordering

Evolution to a state with

Long range order + Macroscopic fluctuations



Hallmarks

- Scaled 2-point correlation function → Cusp singularity (non-Porod)
- RMS fluctuations as large as mean order parameter

Examples

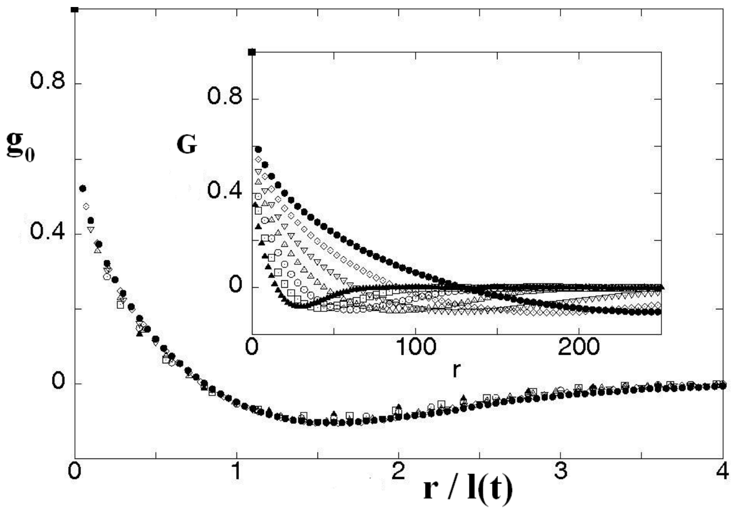
- Passive particles driven by a fluctuating force field
  (Driving field: Fluctuating surface, Burgers fluid, Cell membrane)
- Active nematics
- Granular gas with v-dependent restitution

Order parameter scaling

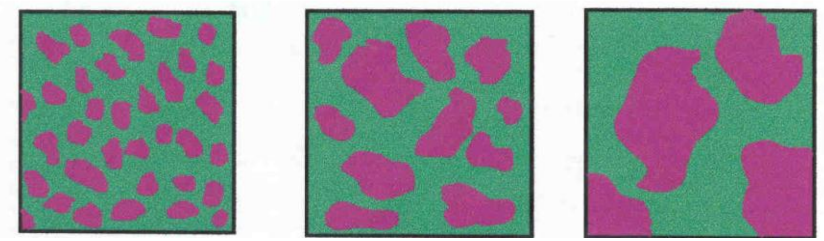
- All long-wavelength Fourier modes (m=1,2,3 …) required
- Scaling laws for static and dynamic correlations

Hallmarks of FDPO

Cusp Singularity in Scaled 2-point Correlation Function



Coarsening



$$G(r,t) \equiv \langle n(0,t)n(r,t)\rangle - \langle n\rangle\langle n\rangle$$

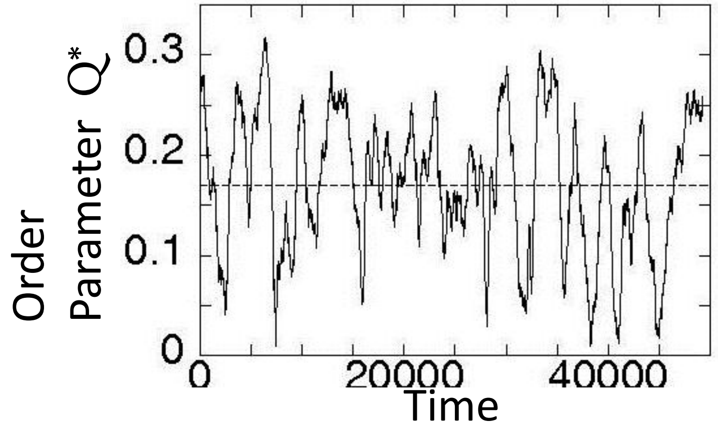
$$= \quad g\left(\frac{r}{l(t)}\right)$$

Domain size $l(t) \sim t^{1/z}$

Cusp: $g(y) \approx m_0{}^2 - cy^\alpha$ as $y \to 0$

(Porod Law: Linear fall → Fails to hold)

Giant Fluctuations in Steady State



Prob(Q*) remains broad as L→∞

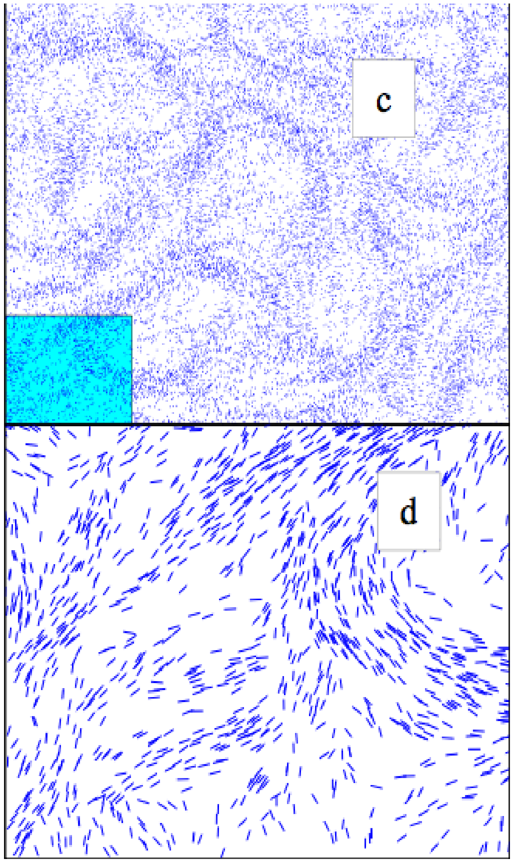
Active Nematics

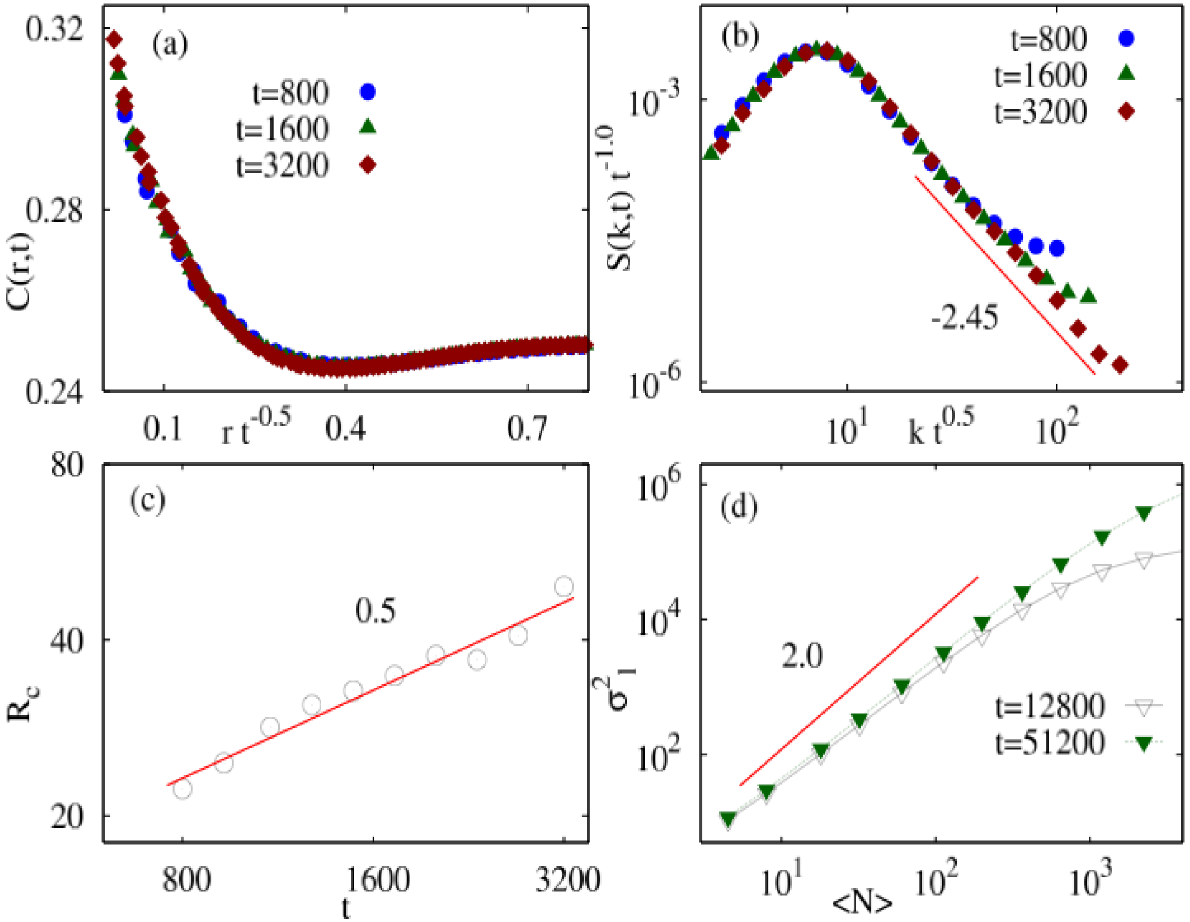
Model Movement of apolar rods depends on orientation

[H. Chate, F Ginelli, R. Montaigne (PRL, 2006)]

Analyze Correlation functions and fluctuations

[S. Dey, D. Das, R. Rajesh (PRL, 2012)]



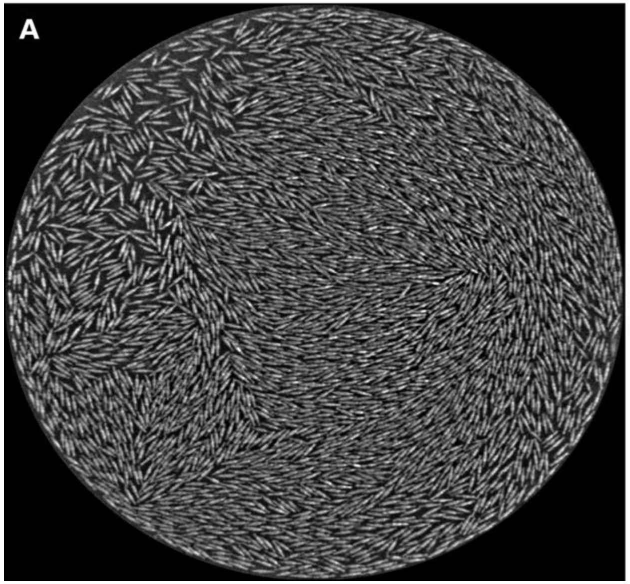


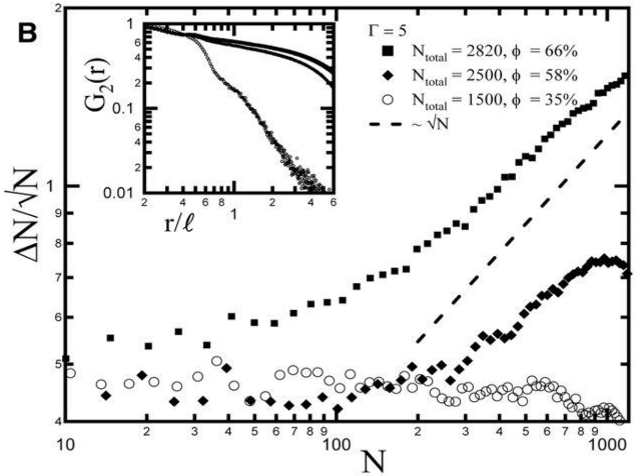
Cusp exponent $\cong 0.5$; Enters (sub-leading) into expression for σ^2

Experiment on Vibrated Rods

Observe

Giant number fluctuations [V. Narayan, N. Menon, S. Ramaswamy (Science, 2007)]



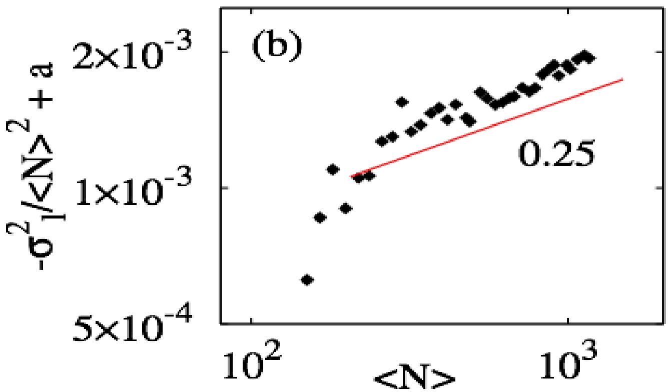


Extract

Correlation function contribution to fluctuations [S. Dey, D. Das, R. Rajesh (PRL, 2012)]

$$\frac{-\Delta N^2}{< N >^2} + m_0{}^2 \sim \frac{c}{l(t)^\alpha} < N >^{\alpha/d}$$

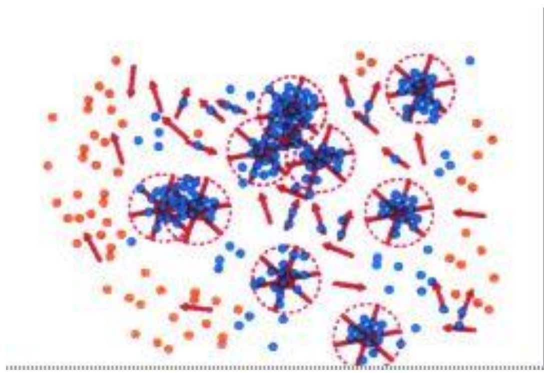
$$\frac{\alpha}{d} \cong \frac{0.5}{2} = 0.25$$

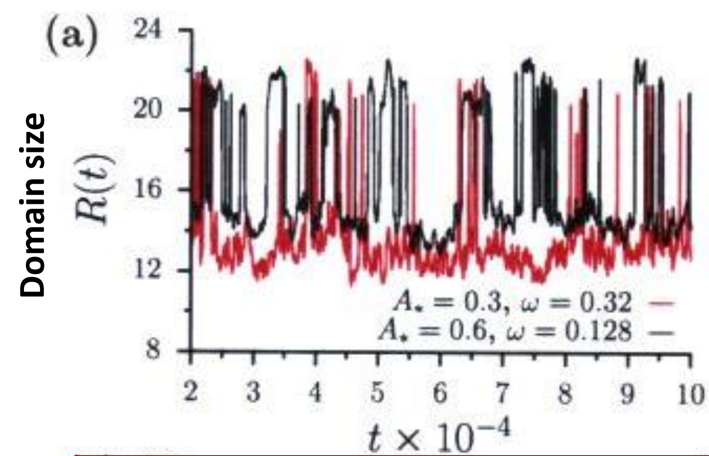


Molecular Clustering at Cell Surface

[A. Das, A. Polley, Madan Rao, cond-mat arXiv (2015)]

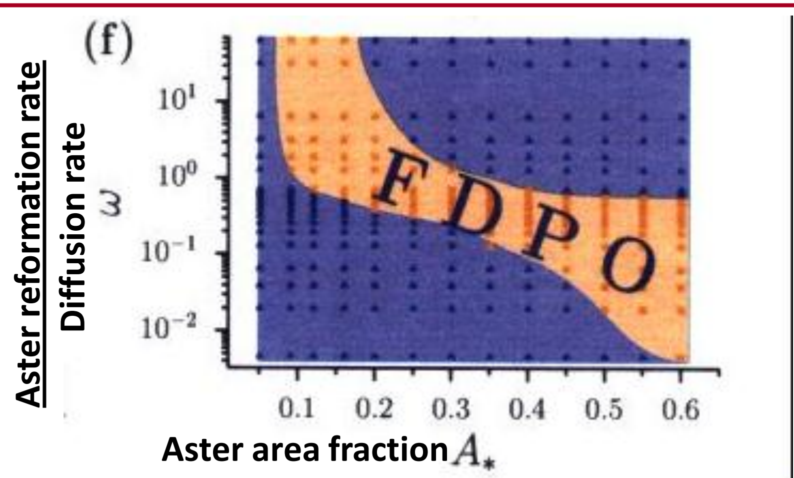
Phase segregation driven by activity:
Membrane stirred by actin activity → Clustering of advected membrane molecules

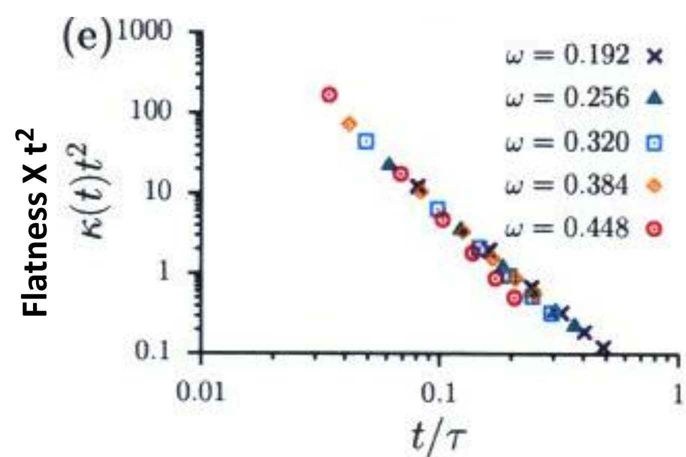




Find: Transition from micro-clustering to phase segregation with strong fluctuations

Intermittent domain sizes





Freely Cooling Granular Gases

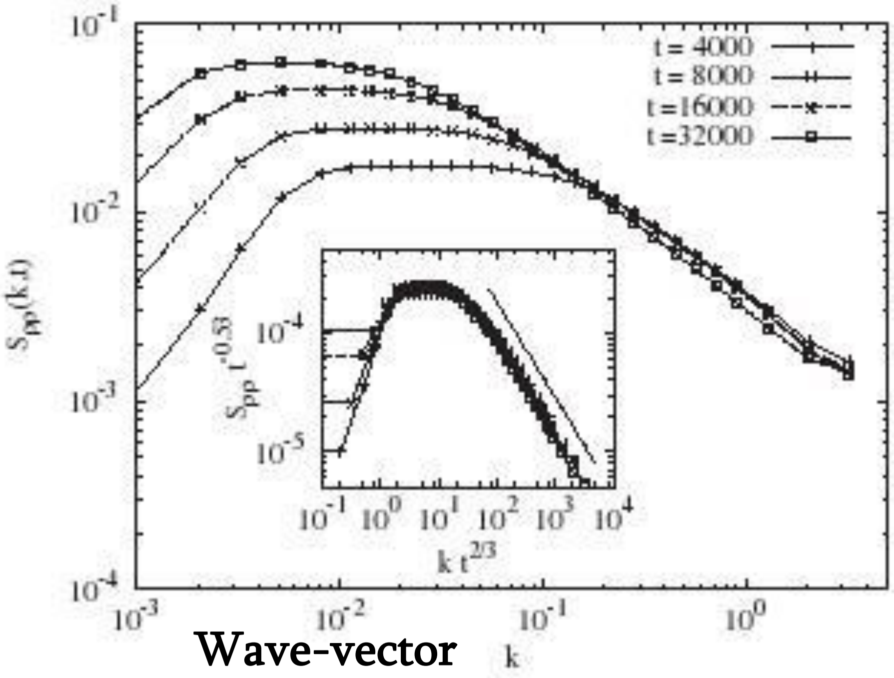
Inelastically colliding particles in 1-d

[M. Shinde, D. Das, R. Rajesh, Phys. Rev. Lett. (2007)]

Include velocity-dependent restitution

[C. V. Raman, Phys. Rev. (1918)]

Find: Density correlations show singular scaling properties



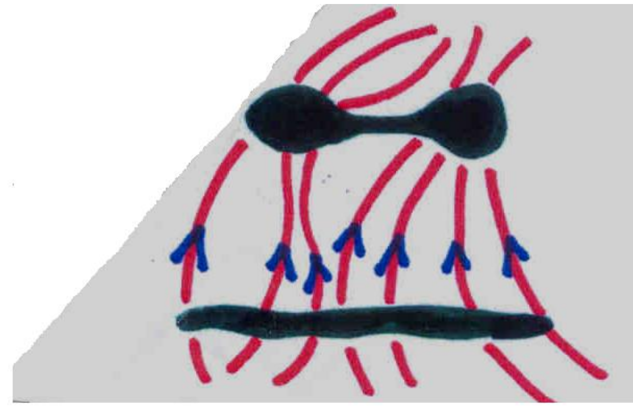
Passive Scalar Problem

One driven system drives another … but no back effect

Compressible fluid with noise

$$\frac{\partial v}{\partial t} + v.\nabla v = \mu\nabla^2 v + \nabla\vartheta$$

(Noisy Burgers Equation)



Advection of particles

$$\frac{dx_m}{dt} = av + \vartheta_m$$

$$\updownarrow$$

a v

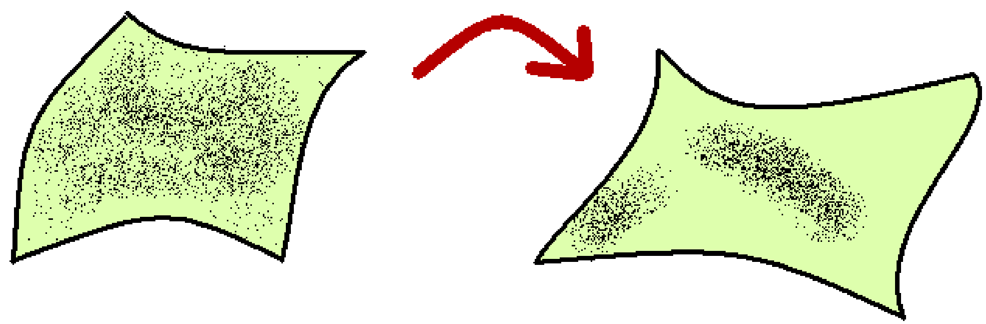
$\updownarrow$

-a∇h

Fluctuating surface

$$\frac{\partial h}{\partial t} = \mu\nabla^2 h + \tfrac{1}{2}(\nabla h)^2 + \vartheta$$

(Kardar, Parisi, Zhang Equation)



Sliding particles

$$\frac{dx_m}{dt} = -a\nabla h + \vartheta_m$$

Coupled Driven Systems

The Question of Stability

$$\frac{\partial \rho_\sigma}{\partial t} + \frac{\partial J_\sigma}{\partial x} = 0 \qquad J_\sigma = J_\sigma(\rho_\sigma, \rho_\tau)$$

$$\frac{\partial \rho_\tau}{\partial t} + \frac{\partial J_\tau}{\partial x} = 0 \qquad J_\tau = J_\tau(\rho_\sigma, \rho_\tau)$$

Define

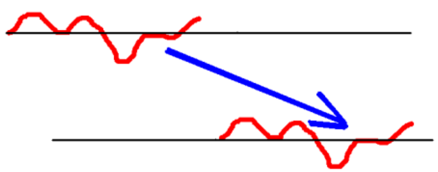
$$h_1(x,t) = \int^x dx' \delta\rho_\sigma(x',t); \qquad h_2(x,t) = \int^x dx' \delta\rho_\tau(x',t)$$

$$\frac{\partial h_1}{\partial t} = c_{11}\frac{\partial h_1}{\partial x} + c_{12}\frac{\partial h_2}{\partial x} + D_1 \frac{\partial^2 h_1}{\partial x^2}$$

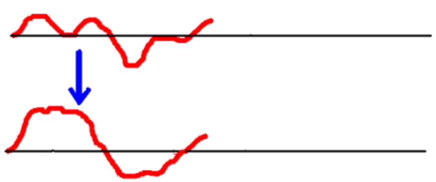
$$+\lambda_1 \left(\frac{\partial h_1}{\partial x}\right)^2 + \mu_1 \left(\frac{\partial h_2}{\partial x}\right)^2 + \nu_1 \left(\frac{\partial h_1}{\partial x}\right)\left(\frac{\partial h_2}{\partial x}\right) + \eta_1$$

+ Equation with $1 \leftrightarrow 2$.

Time evolution of fluctuations



Fluctuations may move as a wave



Fluctuations may grow

Linear terms determine stability

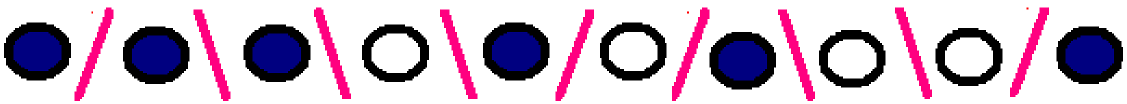
$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} (h_\pm) = c_\pm (h_\pm)$$

Eigenvalues

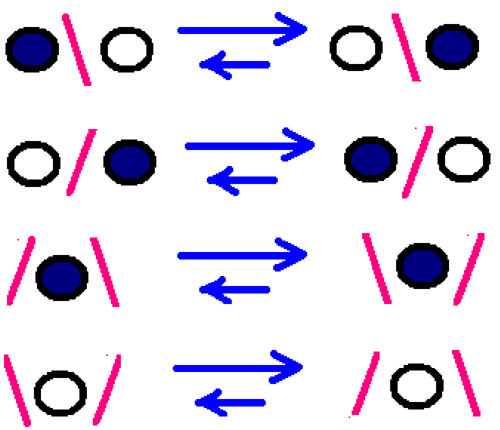
- c_+, c_- Real $\rightarrow$ Waves
- c_+, c_- Complex $\rightarrow$ Instability
- $c_{21} = 0 \rightarrow \delta\rho_\tau$ evolves autonomously

Two-species Lattice Model

Introduce two sets of discrete variables (particles and tilts)



Dynamical moves:



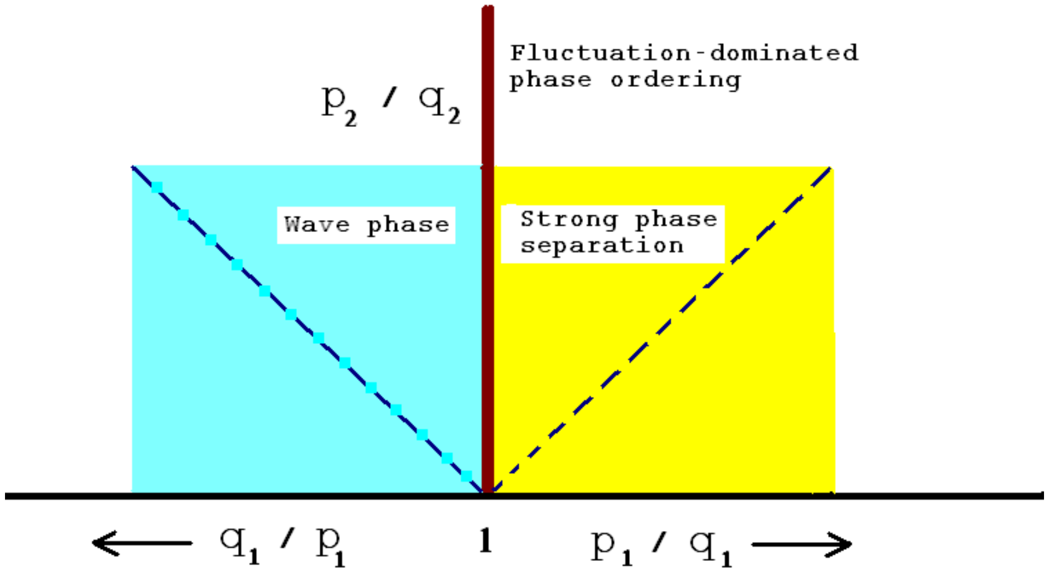
Rates: p_2 (forward); q_2 (backward)

Rates: p_1 (forward); q_1 (backward)

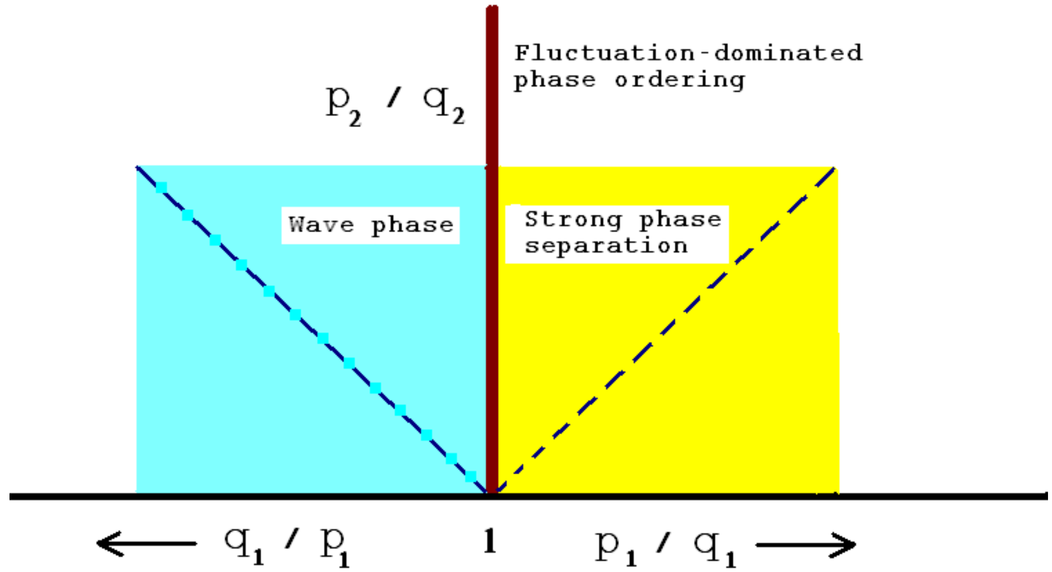
The nature of the steady state depends on p_1/q_1 and p_2/q_2

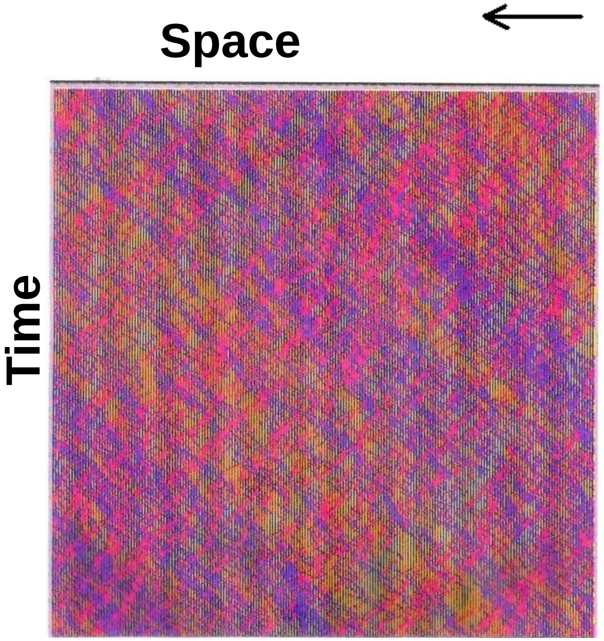
[R. Lahiri et al (1997, 2000)]

The Phase Diagram

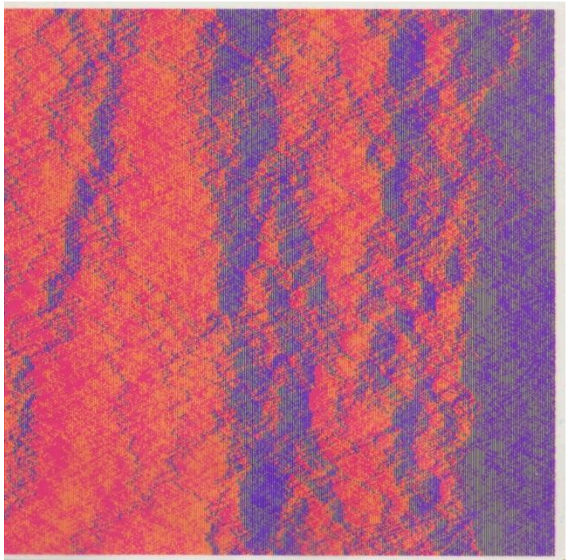


The Phase Diagram





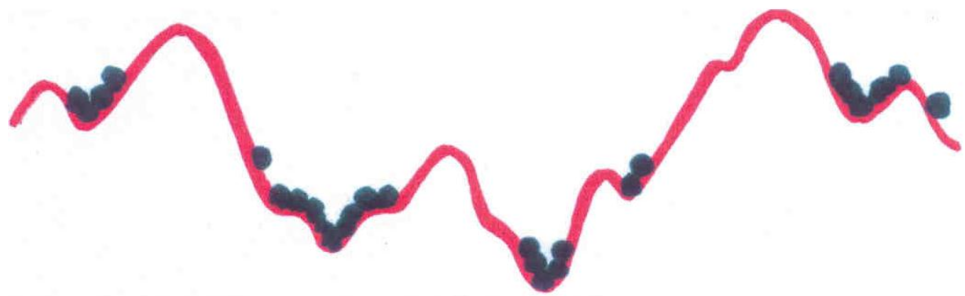
Wave phase



FDPO

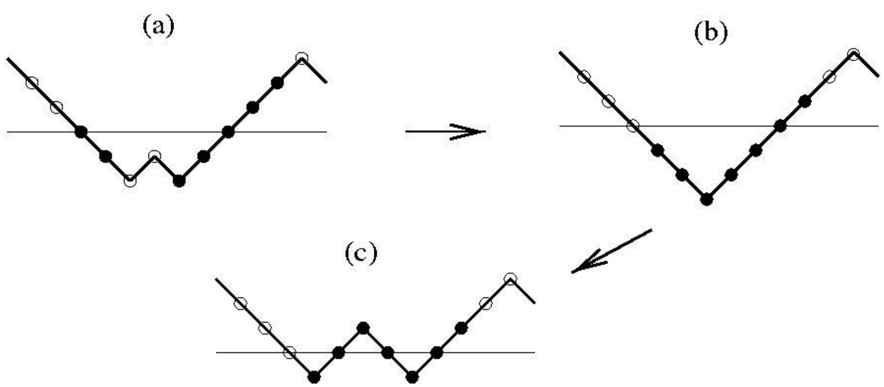
Strong Phase Separation

Passive Sliders on Fluctuating Surfaces



Particle motion: Biased random walks along local gradients

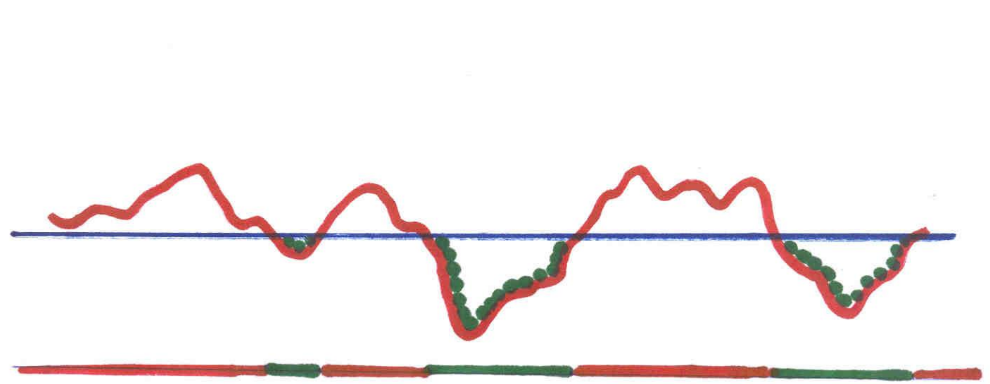
Surface evolution: Stochastic; unaffected by particles



Sliding + Surface fluctuations → Large-scale clustering

Caricature:

Depth Model of Surface



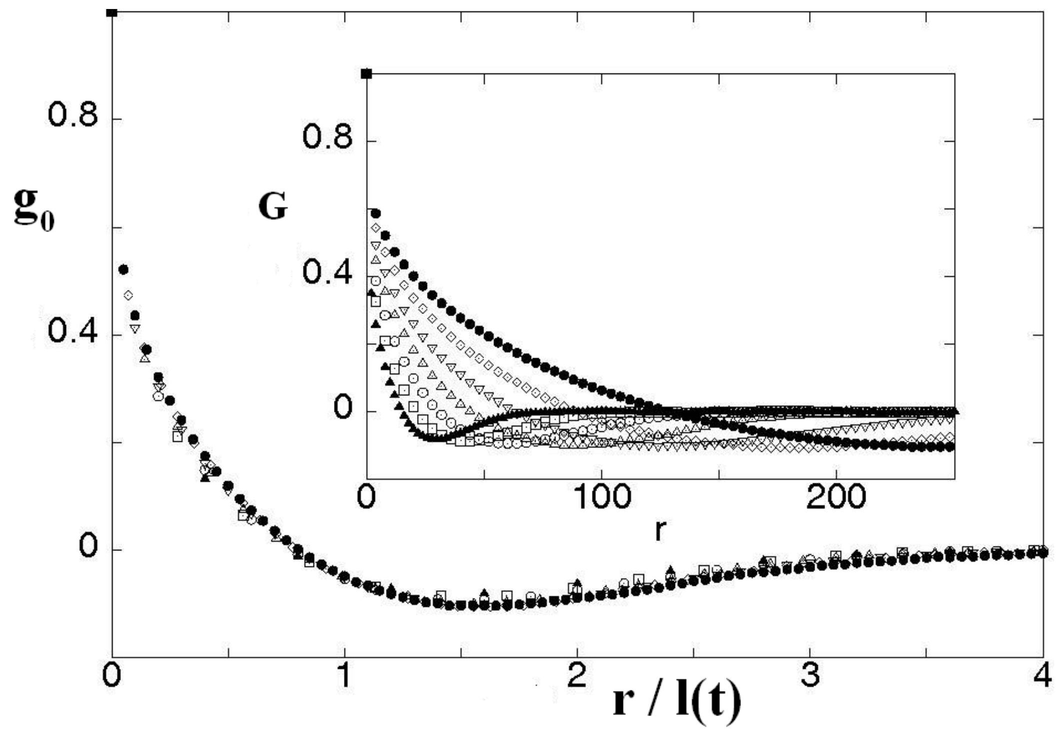
Sliding Particles: Coarsening

Density-density correlation function

$G(r,t) \equiv \langle n(0,t)n(r,t)\rangle - \langle n\rangle\langle n\rangle$ shows scaling

$$= \; g_0\left(\frac{r}{l(t)}\right)$$

with $l(t) \sim t^{1/z}$



Cusp singularity near origin

$g_0(y) \approx m_0{}^2 - cy^{\alpha}$ as $y \to 0$

[$\alpha \cong 0.5$ for EW driving;
$\cong 0.25$ for KPZ driving
$= 0.5$ for Depth model]

Intercept → Long range order

Cusp → Breakdown of Porod Law
→ Ill-defined interfaces

[D. Das et al (2000, 2001)]

Giant Order Parameter Fluctuations

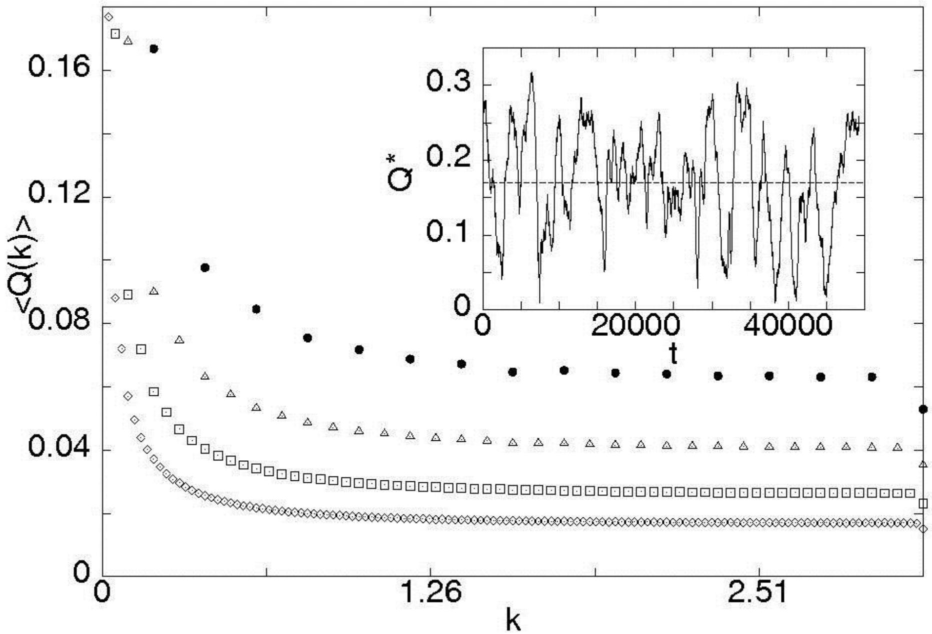
$Q(k) = 1/L \; | \sum_j e^{ikj} n_j |$

with $k = 2\pi m / L$, m=1,2,3...

Observe:

For fixed $k \neq 0$,
$\langle Q(k) \rangle \to 0$ as $L \to \infty$

But: $\langle Q^* \rangle \equiv \langle Q(2\pi/L) \rangle \to 0.18$



Implies: Macroscopic phase separation

$\langle Q^* \rangle \to 0$ for a disordered state
$\to 0.32$ for complete phase separation
When Q^* dips down, m=2,3… modes pick up

Giant fluctuations: Prob(Q^*) remains broad as $L \to \infty$

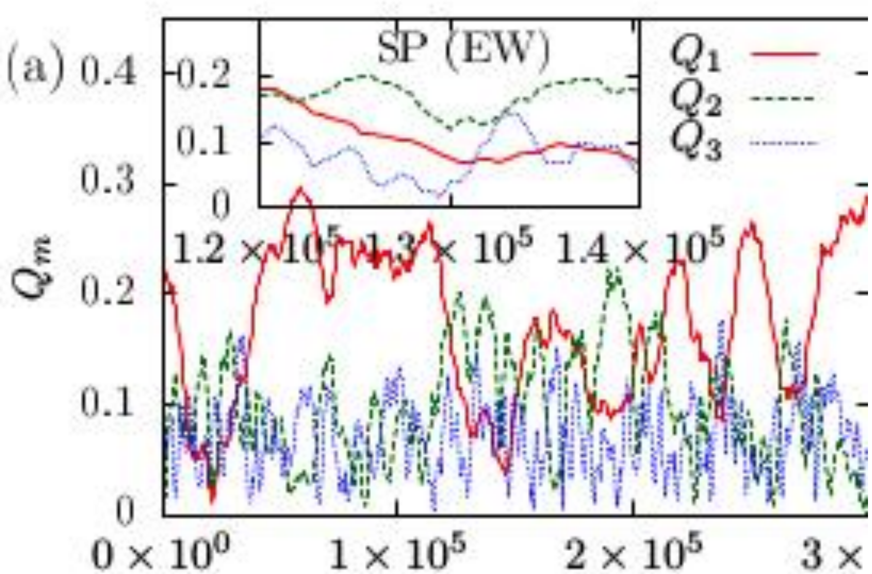
Fluctuation-Dominated Ordering

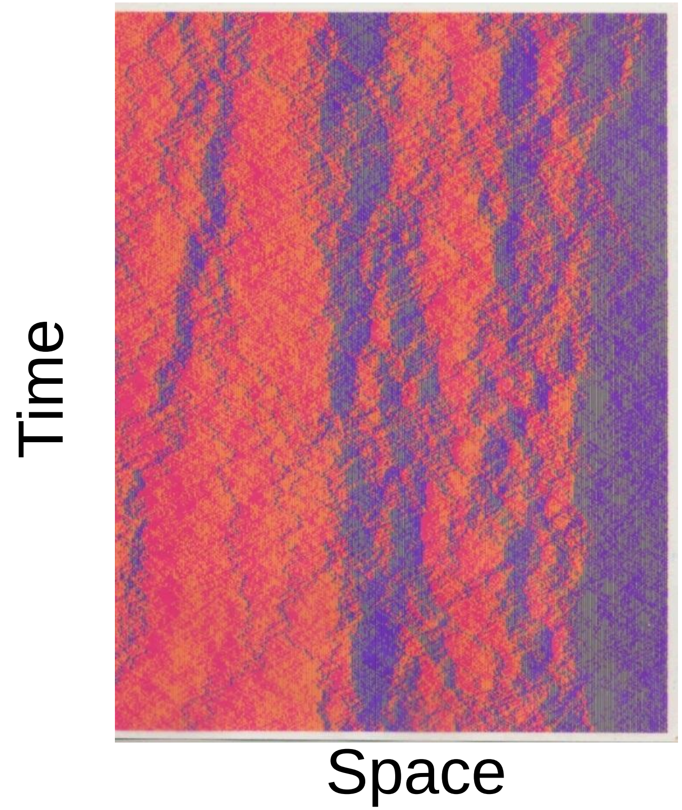
A disordered state implies $Q^* = 0$

But $Q^* = 0$ does not imply a disordered state

When Q^* is small, $Q(2\pi m/L)$ is substantial for m=2,3 …
Several macroscopic patches

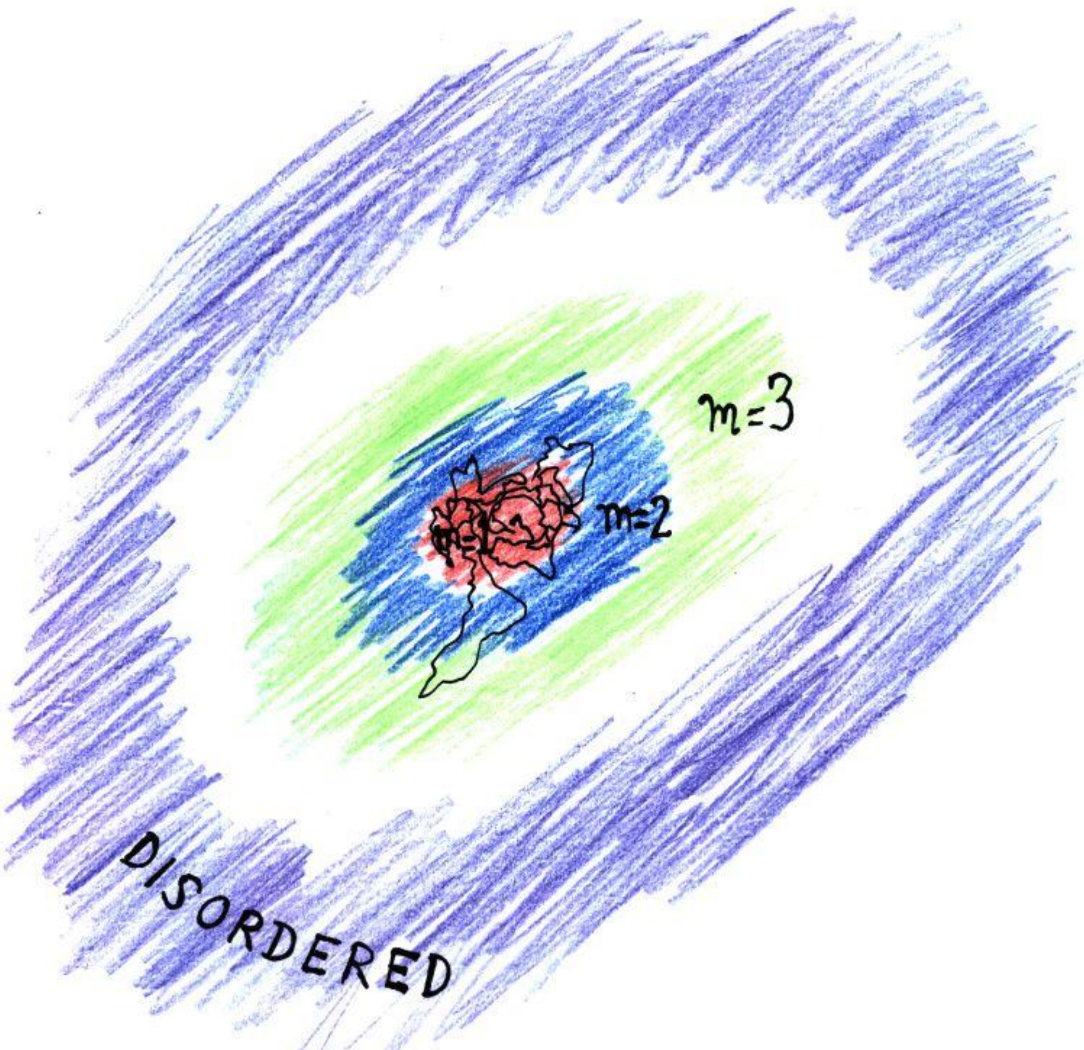
Fourier modes → Order parameter set



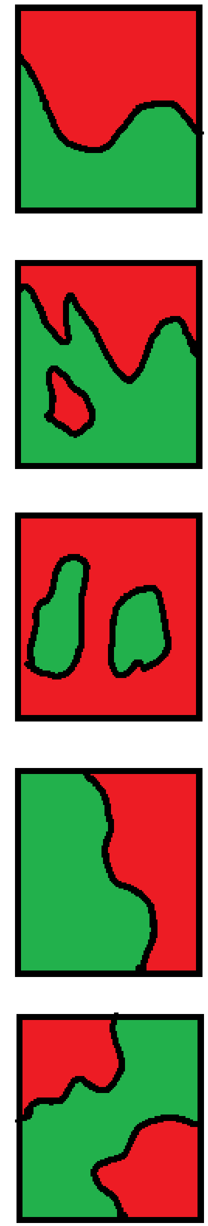


Nature of Ordering

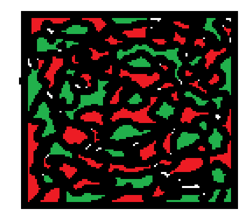
System drawn to an attractor of 'ordered' configurations with macroscopic segments of variable size



Schematic depiction of system evolution

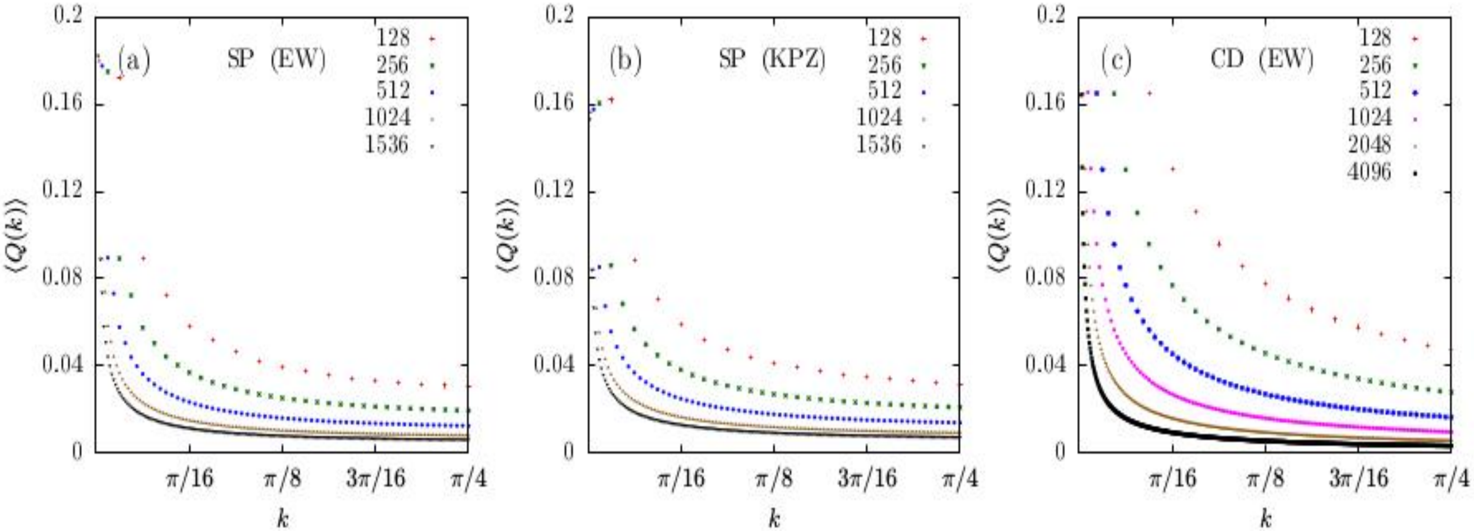


Ordered

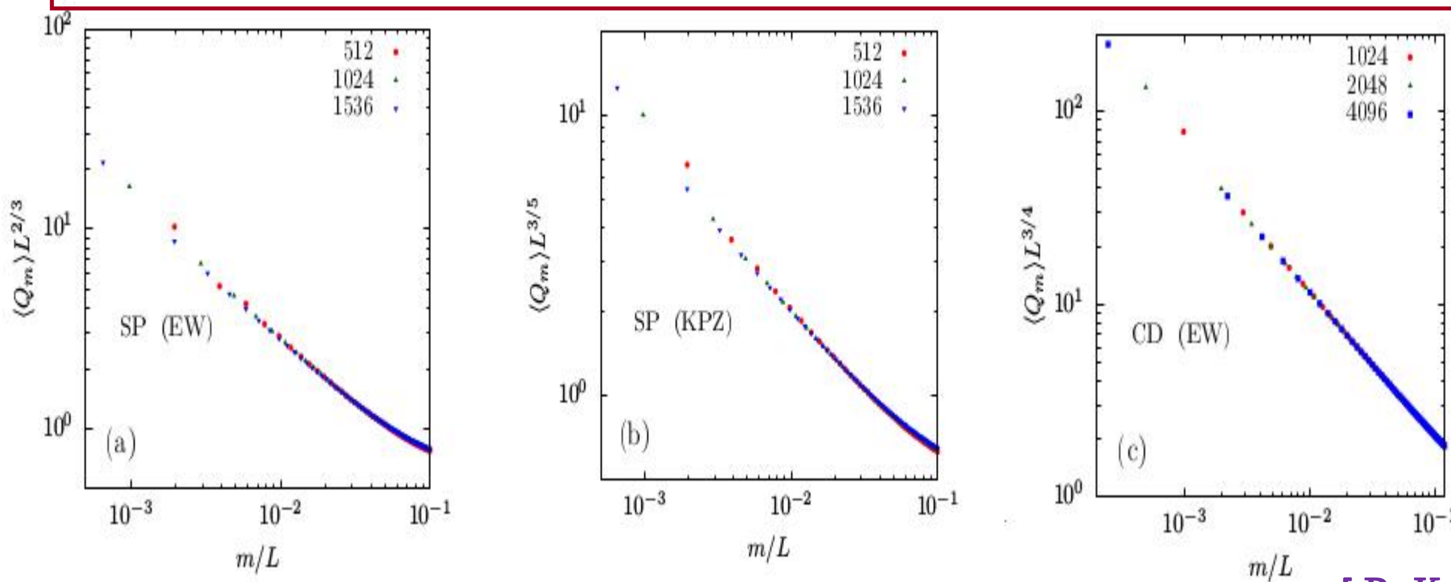


Disordered

Scaling with Mode Number (Mean Values)

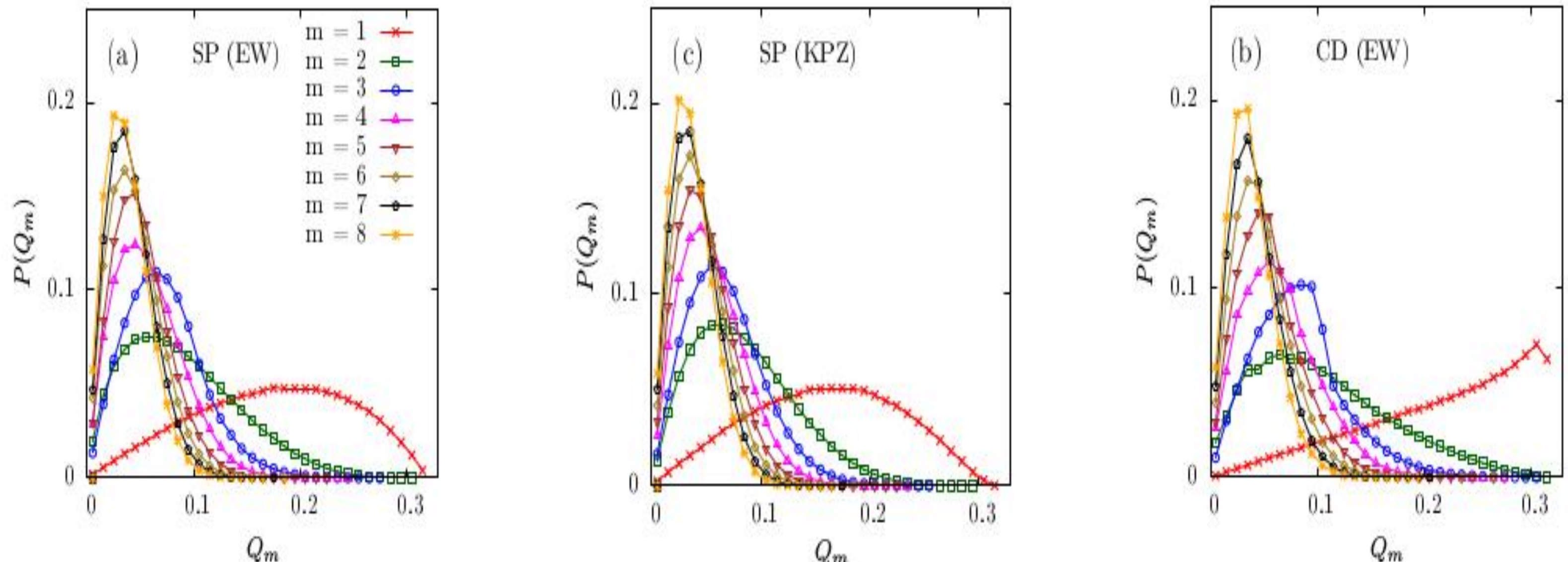


$$< Q_m(L) > \sim L^{-\varphi}\, Y\left(\frac{m}{L}\right) \qquad [\varphi \cong \tfrac{2}{3}\,(EW);\ \tfrac{3}{5}\,(KPZ);\ \tfrac{3}{4}\ (CD)]$$

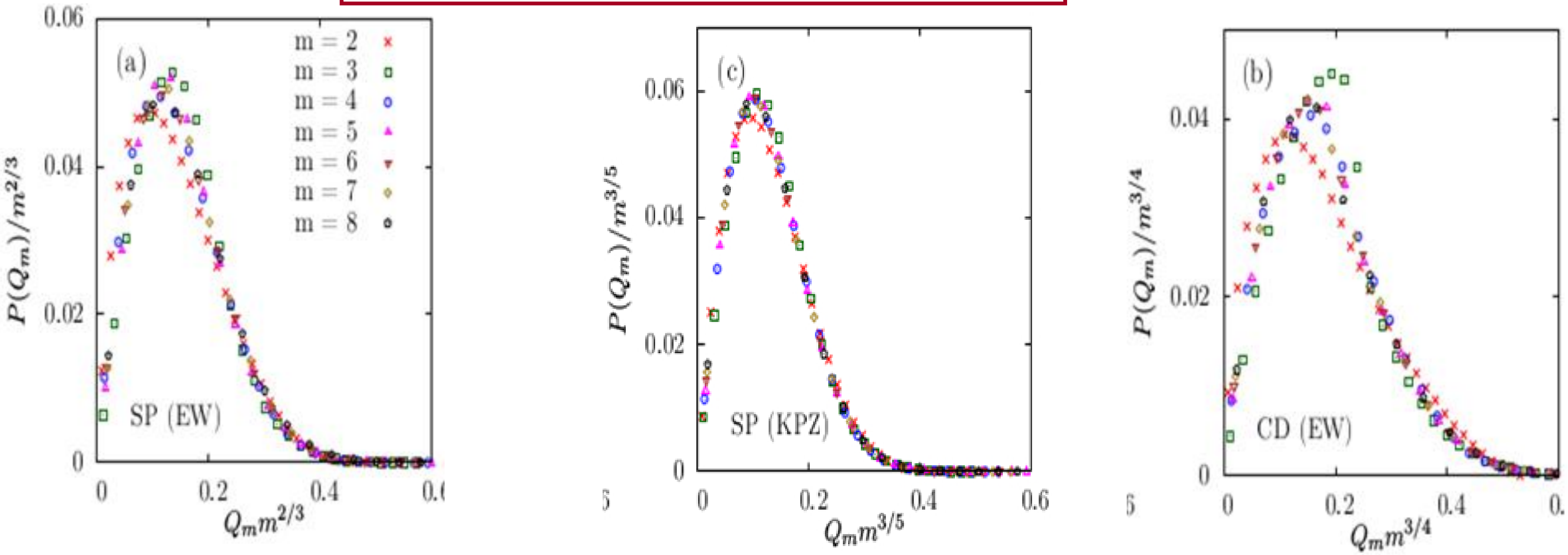


[R. Kapri et al (2015)]

Scaling with Mode Number (Distributions)



$$P(Q_m) \sim m^{\varphi}\, W(Q_m m^{\varphi})$$



Autocorrelation Function

Monitor $A(t, L) \equiv \langle n(r,0)\, n(r,t) \rangle_L - \langle n \rangle^2$

Find $A(t,L) \approx m_0^2 - b\,(t/L^z)^{\beta}$ as $t/L^z \to 0$

Depth model with EW dynamics

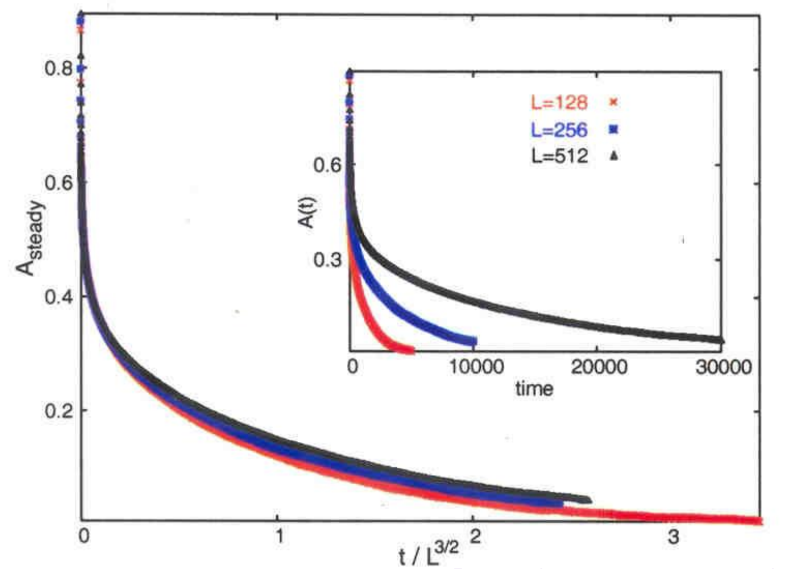
$$A(t, L) = \sin^{-1}[\, e^{-\tau} - \sqrt{\pi\tau}\,(1 - erf\sqrt{\tau}\,)]$$

$$\approx 1 - \frac{4}{\pi^{1/4}}\tau^{1/4} \quad \text{where} \quad \tau = t/L^2,$$

$\beta = 0.25$, $z=2$

Sliding particle model

$\beta \approx 0.2$ for EW; ≈ 0.2 for KPZ

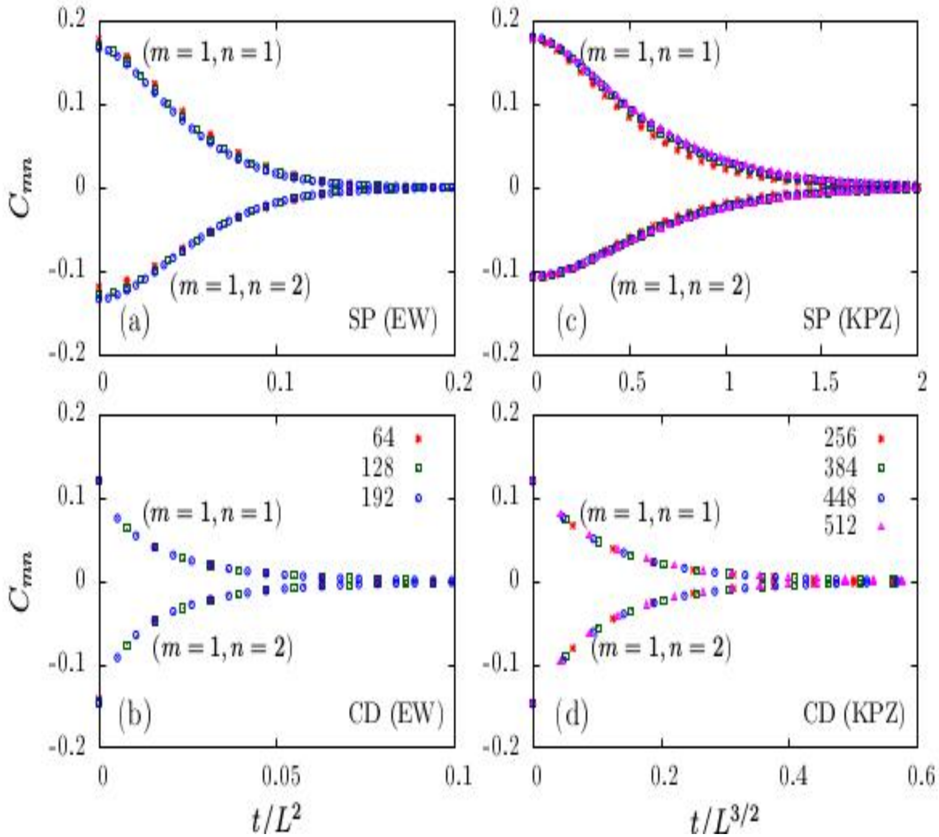


[S. Chatterjee et al (2006)]

Order Parameter Dynamics

Auto-correlation and Cross-correlation Functions for Order Parameter Modes

$$C_{mn} = \frac{< Q_m(0)Q_n(t) > - < Q_m >< Q_n >}{< Q_m >< Q_n >}$$



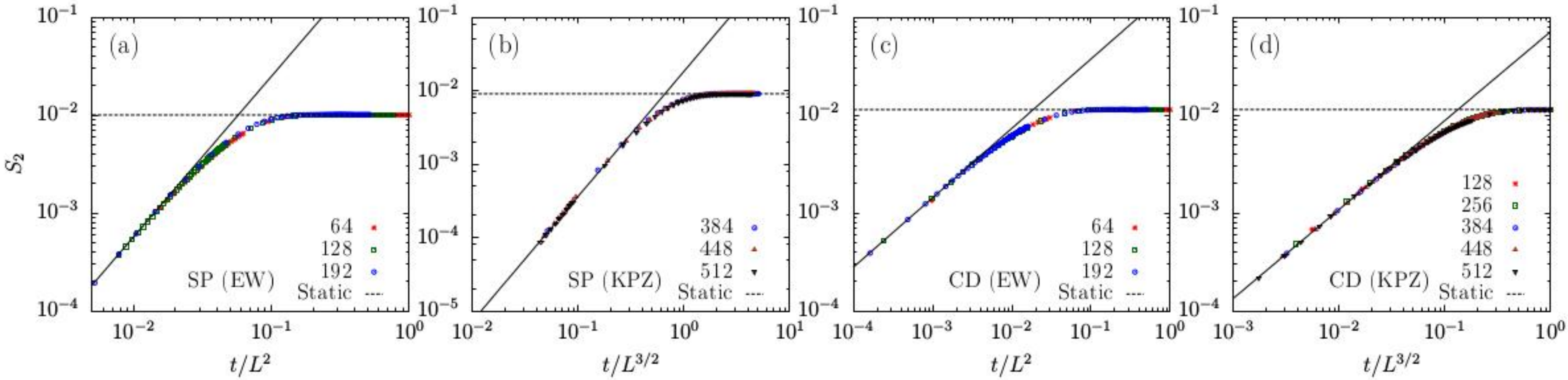
Note

- Scaling
- Cross-correlations are anticorrelated

Structure Functions

$$S_n(t, L) = < [\, Q_1(t) - Q_1(0)]^n >_L$$

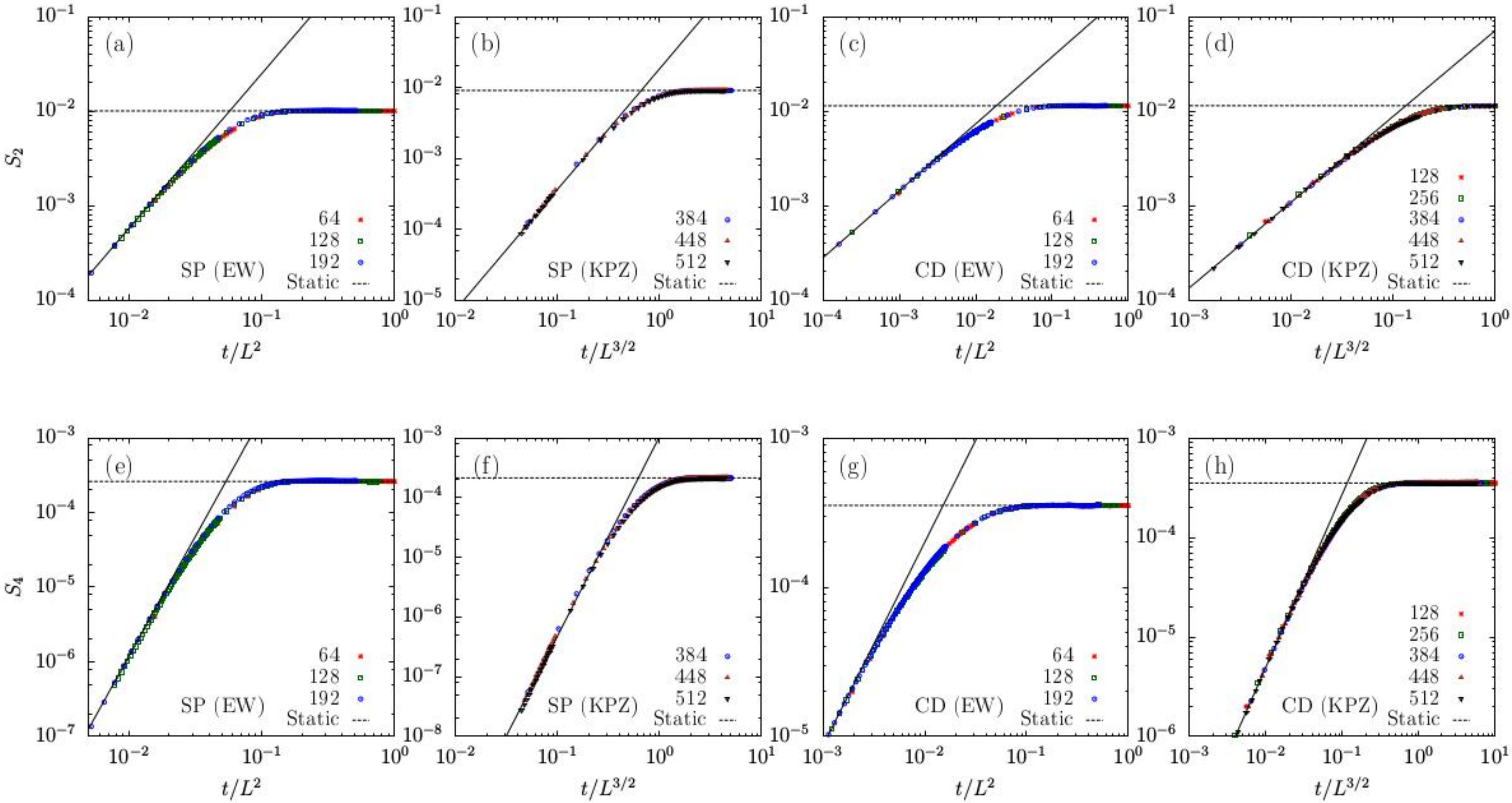
- Scaling
- Initial power law growth
- Asymptotic saturation



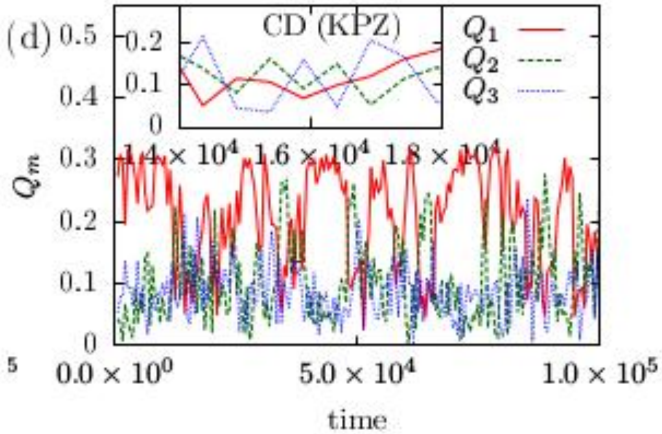
Structure Functions

$$S_n(t, L) = \langle\, [\, Q_1(t) - Q_1(0)]^n \,\rangle_L$$

- Scaling
- Initial power law growth
- Asymptotic saturation

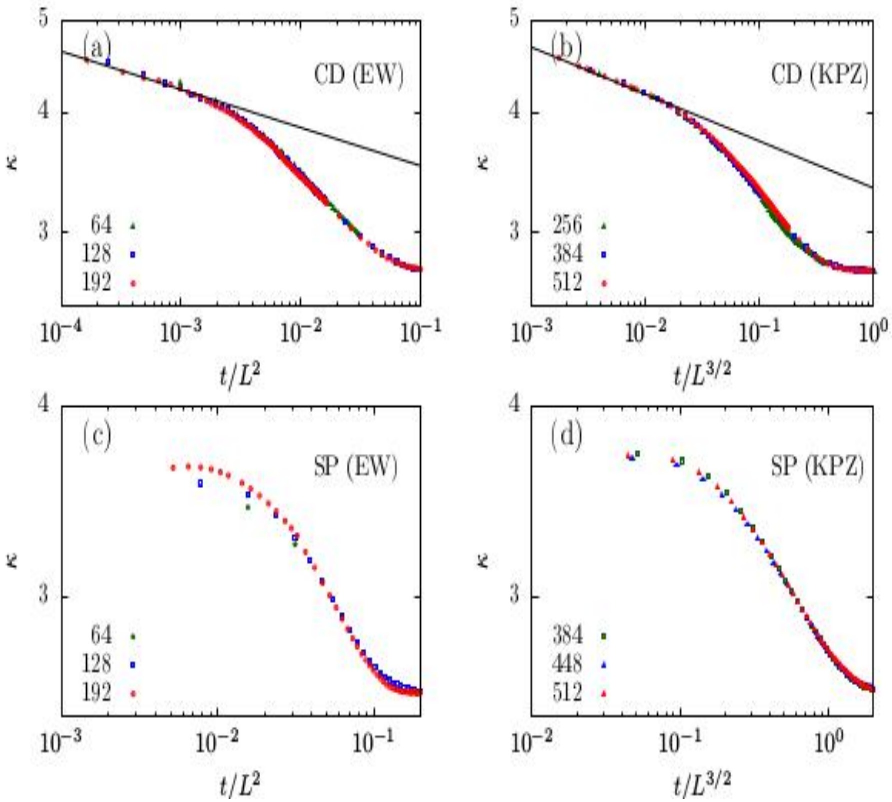


Intermittency Measure: Flatness



Is the time series intermittent?

Measure: Flatness $K\left(\frac{t}{L^z}\right) = \frac{S_4}{{S_2}^2}$



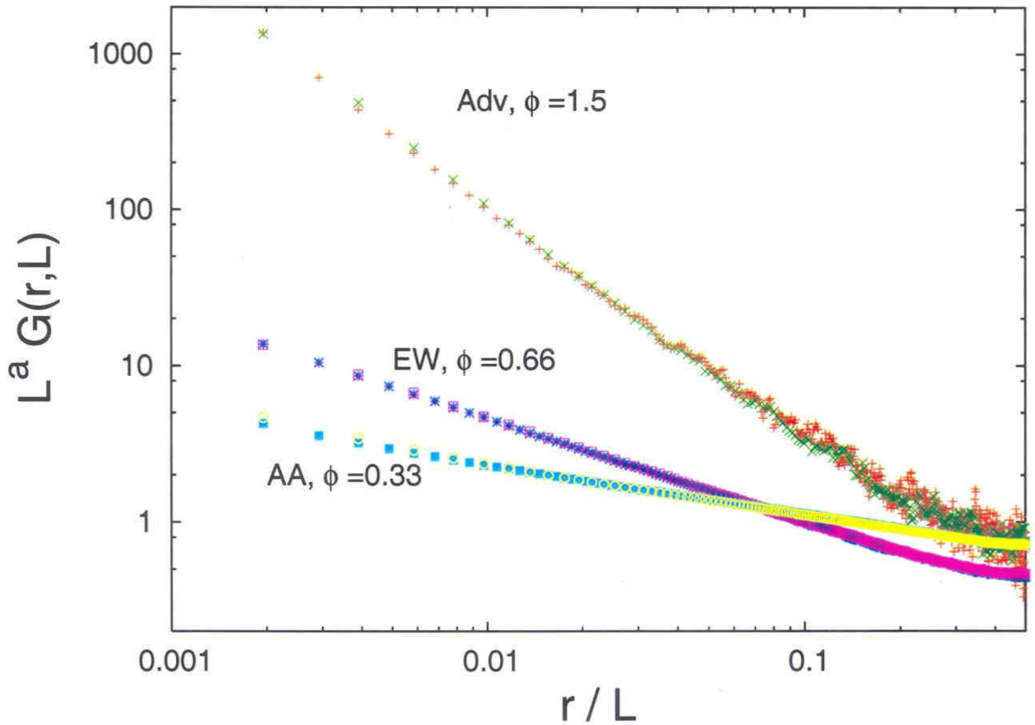
Find:

A weak divergence for the Depth Model

$K(y) \sim y^{-\gamma}$ **with** $\gamma \cong 0.07$

Non-Interacting Particles

Two-point correlation functions
(KPZ Advection, Edwards-Wilkinson, KPZ Anti-advection)



Divergent Scaling Functions

$$L^a\, G \sim (r/L)^{-\varphi} \quad \text{as } r/L \to 0$$

[A. Nagar et al (2006)]

Analytical approach

Adiabatic limit → A problem with quenched disorder (Sinai problem)

$$G(r,L) = (2\pi\beta^2 L)^{-1/2}\left[\frac{r}{L}\left(1-\frac{r}{L}\right)\right]^{-3/2}$$

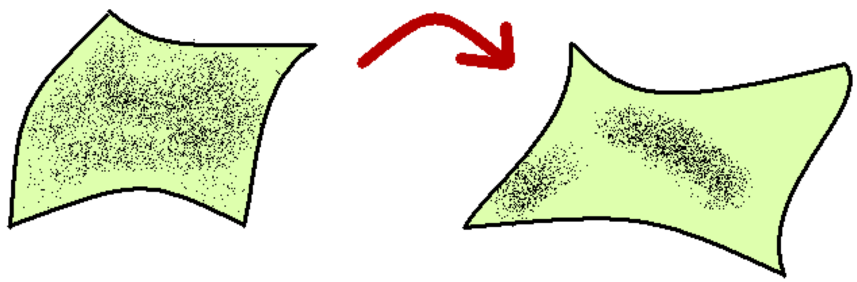
[A. Comtet & Texier (1997)]

Fits KPZ advection data remarkably well

Conclusions

FDPO

Several examples, including passive scalars



- **Giant fluctuations** coexisting with LRO
- **Cusps** in 2-point Correlation Functions (static, dynamic)
- **Order Parameter Set** → Scaling
- **Intermittency** in Depth Model
- **Divergent Scaling Functions** for Noninteracting Particles