Time's Arrow and Entropy: Classical and Quantum

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“What is time? If nobody asks me, I know; but if I were desirous to explain it to one that should ask me, plainly I know not.”

Saint Augustine (354-430)

Some Hindu Concepts of Time

"Time is the Lord of all things
Time was the father of Prajapates."

Athana Veda XXI, 53,1-8

"... time exists in a latent stage during the dissolution of the world,
and is awakened by the god at the moment of recreation."

From the Bhagavata Purana

"... Philosophers tend to be divided into two camps. On one side there are those who regard the passage of time as an objective feature of reality, and interpret the present moment as the marker or leading edge of this advance. Some members of this camp give the present ontological priority, as well, sharing Augustine's view that the past and the future are unreal. Others take the view that the past is real in a way that the future is not, so that the present consists in something like the coming into being of determinate reality "

"... Philosophers in the opposing camp regard the present as a subjective notion, often claiming that **now** is dependent on one's viewpoint in much the same way that **here** is. Just as 'here' means roughly 'this place', so 'now' means roughly 'this time', and in either case what is picked out depends where the speaker stands. In this view there is no more an objective division of the world into the past, the present, and the future than there is an objective division of a region of space into here and there.

Often this is called the **block universe view**, the point being that it regards reality as a single entity of which time is an ingredient, rather than as a changeable entity set in time."

Huw Price, in Time's Arrow and Archimedes' Point

"The Tralfamadorians can look at all different moments just the way we can look at a stretch of the Rocky Mountains, for instance. They can see how permanent all the moments are, and they can look at any moment that interests them. It is just an illusion we have here on earth that one moment follows another like beads on a string ..."

Kurt Vonnegut, Slaughter-House-Five (1969)

Michaelle has left this strange world just before me. This is of no importance. For us convinced physicists the distinction between past, present and future is an illusion, although a persistent one.

Albert Einstein, Letter to Besso's sister

When someone asked Yogi Berra

“What time is it?”,

he replied

“Do you mean now?”.

The laughter evoked by this anecdote shows how strongly we hold a common notion of present.

Jim Hartle in the Physics of Now

American Journal of Physics, 2004

Time present and time past
Are both perhaps present in time future
And time future contained in time past.
If all time is eternally present
All time is unredeemable.

T.S. Eliot, Four Quartets

The Arrow of Time and Quantum Mechanics
A.J.Leggett

I do strongly agree that if in the year 2075 physicists look back on us poor quantum-mechanics-besotted idiots of the twentieth century with pity and head-shaking, an essential ingredient in their new picture of the universe will be a quite new and to us unforeseeable approach to the concept of time: and that to them our current idea about the asymmetry of nature with respect to time will appear as naive as do to us the notions of nineteenth-century physics about simultaneity.

Do you believe time is a truly basic concept that must appear in the foundations of any theory of the world, or is it an effective concept that can be derived from more primitive notions in the same way that a notion of temperature can be recovered in statistical mechanics?

The results were as follows: 20 said there was no time at a fundamental level, 12 declared themselves to be undecided or wished to abstain, and 10 believed time did exist at the most basic level. However, among the 12 in the undecided/abstain column, 5 were sympathetic to or inclined to the belief that time should not appear at the most basic level of theory.

Julian Barbour in The End of Time:
The Next Revolution in Physics
Oxford University Press, 2000

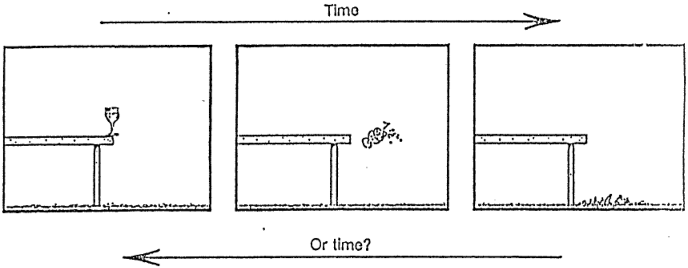
I was among the believers in time as an irreducible element of reality and will, for the purpose of this talk, take as a working hypothesis the intuitive notion of space-time as a primitive undefined concept. This space-time continuum is the arena in which matter, radiation and all kinds of other fields making up this universe of ours, exist and change.

In the world about us the past is distinctly different from the future. Milk spills but doesn't unspill; eggs splatter but do not unsplatter; waves break but do not unbreak; we always grow older, never younger. These processes all move in one direction in time - they are called "time-irreversible" and define the arrow of time.

Gather ye rosebuds while ye may,
Old Time is still a-flying;
And this same flower that smiles today
Tomorrow will be dying.

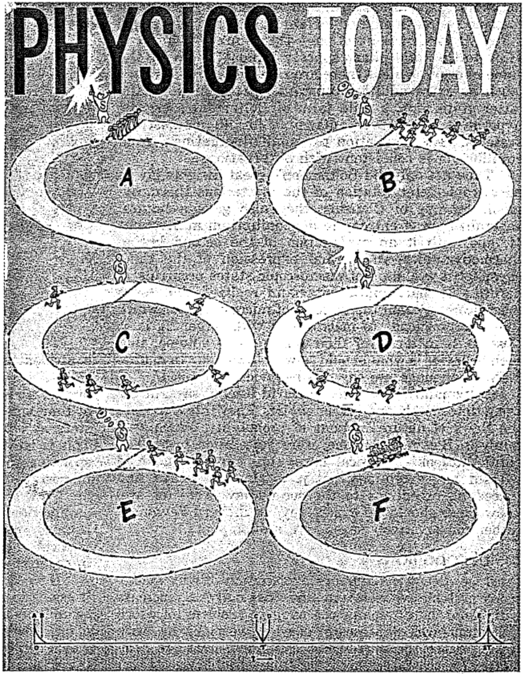
Robert Herrick, 1591-1674

The Emperor's New Mind
Roger Penrose

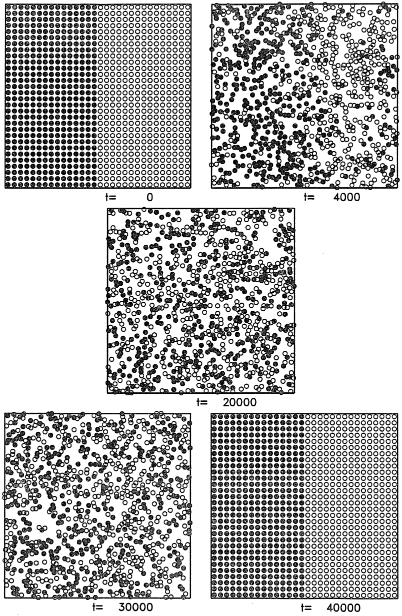


The laws of mechanics are time-reversible; yet the time-ordering of such a scene from the right frame to the left is something that is never experienced, whereas that from the left frame to right would be commonplace.

This would lead us to expect that there is some fundamental law governing the microscopic dynamics of the atoms and molecules which has time asymmetry in it. But, as Brian Greene puts it in his book The Fabric of the Cosmos, "no one has ever discovered any fundamental law which might be called the Law of the Spilled Milk or the Law of the Splattered Egg." It is only "secondary laws" which are time asymmetrical.



Reversing time. PHYSICS TODAY cover from November 1953 shows athletes on a racetrack. At the first gunshot, they start running; at the second, they reverse and run back, ending up again in a line. The drawing, by Kay Kaszas, refers to an article by Erwin L. Hahn on the spin echo effect on page 4 of that issue.


t= 0
t= 4000
t= 20000
t= 30000
t= 40000

Precise formulation of time's arrow problem

Every bit of macroscopic matter is composed of an enormous number of quasi-autonomous units, called atoms. Taking these atoms as classical particles the complete microscopic (or micro) state of an isolated classical system of N particles is specified by the positions and velocities of all the particles. This corresponds to a point X in the phase space Γ,

$$X = (\mathbf{r}_1, \mathbf{v}_1, \ldots, \mathbf{r}_N, \mathbf{v}_N),\ \mathbf{r}_i \in \Lambda \subset \mathbb{R}^d,\ \mathbf{v}_i \in \mathbb{R}^d.$$

Microscopic Reversibility Classical Mechanics

$$\text{Microstate } X = (\mathbf{r}_1, \mathbf{v}_1, \ldots, \mathbf{r}_N, \mathbf{v}_N) \in \Gamma.$$
$$RX = (\mathbf{r}_1, -\mathbf{v}_1, \ldots, \mathbf{r}_N, -\mathbf{v}_N) \in \Gamma.$$
$$\text{Evolution: } x \to T_t X = X(t),\ t \in (-\infty, \infty)$$
$$T_\tau X(t) = X(t + \tau)$$
$$\underline{T_\tau R T_\tau X(t) = RX(t)}.$$

Suppose now that some property of the system, specified by a function $f(X(t))$, behaves in a particular way as t increases, then there is also a trajectory in which the property behaves in the time reversed way. Thus, for example, if particle densities get more uniform, say in a way described by the diffusion equation, then there must be evolutions in which the density gets more nonuniform. So why is one type of evolution, corresponding to "entropy" increase in accord with the second "law", common and the other never seen?

This problem was clearly stated by W.Thomson (later Lord Kelvin) who wrote in 1874 [1].

"The essence of Joule's discovery is the subjection of physical phenomena to dynamical law. If, then, the motions of every particle of matter in the universe were precisely reversed at any instant, the course of nature would be simply reversed for ever after. The bursting bubble of foam at the foot of a waterfall would reunite and descend into the water; the thermal motions would reconcentrate their energy, and throw the mass up the fall in drops re-forming into a close column of ascending water."

[1] *W. Thomson, in The Kinetic Theory of the Dissipation of Energy, Proc. of the Royal Soc. of Edinburgh, 8 325 (1874)*

Resolution

The explanation of this apparent paradox, due to Thomson, Maxwell and Boltzmann, is based on the great disparity between microscopic and macroscopic scales—with the consequent exponentially large ratios between the number of microstates (phase space volume) corresponding to the different macrostates—and the fact that events are determined not only by differential equations, but also by initial conditions.

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Macrostates

To describe the macroscopic state of a system of N atoms in a box V, say $N \gtrsim 10^{20}$, we make use of a much cruder description than that provided by the microstate X. We shall denote by M such a macroscopic description of a macrostate. As an example we may divide V into K cells, where K is large but still $K \ll N$, and specify the number of particles, the momentum and the amount of energy in each cell, with some tolerance.

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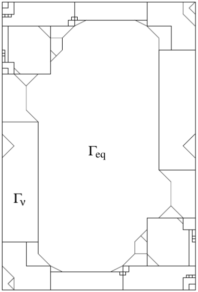
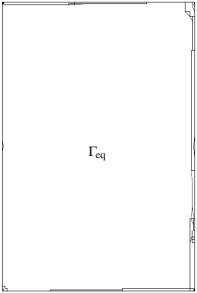
Clearly there are many $X's$ (in fact a continuum) which correspond to the same M. Let Γ_M be the region in Γ consisting of all microstates X corresponding to a given macrostate M and denote by $|\Gamma_M|$ its volume.

The equilibrium macrostate M_{eq} is defined as that state for which $|\Gamma_{Meq}| \sim |\Gamma_E|$, the area of the whole energy surface. When $M(X)$ specifies a nonequilibrium state, $|\Gamma_{M(X)}|$ is much smaller. Thus if the system contains N atoms in a volume V then the ratio of $|\Gamma_{M_{eq}}|$ for the macrostate M_{eq} in which there are $(\frac{1}{2} \pm 10^{-10})N$ particles in the left half of the box to $|\Gamma_M|$ for a macrostate M in which all the particles are in the left half is of order 2^N. For any macroscopic value of N, this is far larger than the ratio of the volume of the known universe to the volume of one proton.

Pictorially





The second picture is slightly more faithful. Neither shows the topology or differences in relative sizes of the different Γ_M's.

Boltzmann then argued that this disparity in the sizes of Γ_M, explains why isolated systems started in a nonequilibrium macro-state M_0 typically evolve into macrostates M_t such that $|\Gamma_{M_t}|$ is increasing and end up in $\Gamma_{M_{\mathrm{eq}}}$, where they typically stay for very, very long times.

To make connection with the Second Law of Thermodynamics, Boltzmann defined the entropy of a macroscopic system with microstate X as

$$S_B(X) = k \log |\Gamma_{M(X)}|$$

and showed that (for a dilute gas) in an equilibrium macrostate M_{eq}, S_B is proportional to the thermodynamic entropy of Clausius. Following O. Penrose, I shall call $k \log |\Gamma_M(X)|$ the Boltzmann entropy of a system in the macrostate $M(X)$.

The Fabric of the Cosmos, by Brian Greene.

Entropy is a concept that makes this idea precise by counting the number of ways, consistent with the laws of physics, in which any given physical situation can be realized. *High entropy means that there are many ways; low entropy means there are few ways.* If the pages of *War and Peace* are stacked in proper numerical order, that is a low-entropy configuration, because there is one and only one ordering that meets the criterion. If the pages are out of numerical order, that is a high-entropy situation, because a little calculation shows that there are
1245521984537783433660029353704988291633611012463890451368
8769126468689559185298450437739406929474395079418933875187
6527656714059286627151367074739129571382353800016108126465
3018234205620571473206172029382902912502131702278211913473
5826558815410713601431193221575341597338554284672986913981
5159925119085867260993481056143034134383056377136715110570
4786941333912934192440961051428879847790853609508954014012
5932850632906034109513149466389839052676761042780416673015
4945522818861025024633866260360150888664701014297085458481
5141598392546876231295293347829518681237077459652243214888
7351679284483403000787170636684623843536242451673622861091
9853939181503076046890466491297894062503326518685837322713
6370247390401891094064988139838026545111487686489581649140
3426444110871911844164280902757137738090672587084302157950
1589916232045813012950834386537908191823777738521437536312
2531641598589268105976528144801387748697026525462643937189
3927305921796747169166978155198569769269249467383642278227
3345776718073316240433636952771183674104284493472234779223
4027225630721193853912472880929072034271692377936207650190

On the basis of the kinetic theory of gases Boltzmann had discovered that, aside from a constant factor, entropy is equivalent to the logarithm of the "probability" of the state under consideration. Through this insight he recognized the nature of course of events which, in the sense of thermodynamics, are "irreversible". Seen from the molecular-mechanical point of view, however all courses of events are reversible.

On the basis of the kinetic theory of gases Boltzmann had discovered that, aside from a constant factor, entropy is equivalent to the logarithm of the "probability" of the state under consideration. Through this insight he recognized the nature of course of events which, in the sense of thermodynamics, are "irreversible". Seen from the molecular-mechanical point of view, however all courses of events are reversible. If one calls a molecular-theoretically defined state a microscopically described one, or, more briefly, micro-state, then an immensely large number (Z) of states belong to a macroscopic condition. Z is then a measure of the probability of a chosen macro-state. This idea appears to be of outstanding importance also because of the fact that its usefulness is not limited to microscopic description on the basis of mechanics.

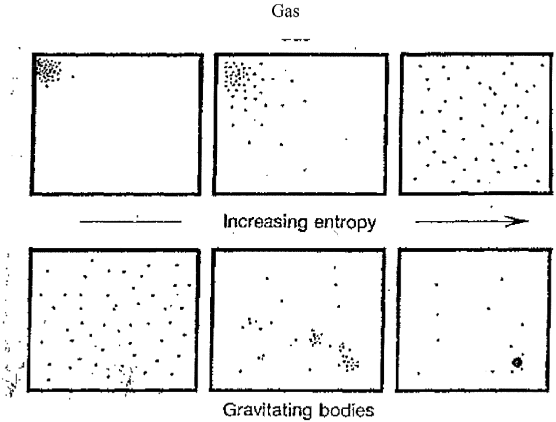
A. Einstein, Autobiographical notes

Boltzmann's grave in the Zentalfriedhof, Vienna, with bust and entropy formula.

I would like to note, however, that large entropy of a macrostate, i.e. the large number of microstates corresponding to the macrostate, is not necessarily the same as what one would normally call large disorder. Thus at small energy (or temperature) the maximal entropy state of most systems is that of a crystal, which one would normally think of as being ordered. A striking example of this occurs when gravity plays an important role in the macroscopic structure.

Clustering in Gravitational Systems at all Energies

R. Penrose: Inflationary Cosmology



With a gas in a box, the maximum entropy state (thermal equilibrium) has the gas distributed uniformly; however, with a system of gravitating bodies, entropy can be increased from the uniform state by gravitational clumping to a black hole finally. Roughly speaking this is due to the fact that the potential energy due to gravity goes roughly like N^2 while kinetic energy goes like N .

“First, my good friend, you state that the two directions of your time variables, from $-t$ to $+t$ and from $+t$ to $-t$ are a priori equivalent. Then by fine arguments appealing to common sense you show that disorder (or ‘ entropy’) must with overwhelming probability increase with time. Now, if you please, what do you mean by ‘ with time’? Do you mean in the direction $-t$ to $+t$? But if your interferences are sound, they are equally valid for the direction $+t$ to $-t$. If these two directions are equivalent a priori, then they remain so a posteriori. The conclusions can never invalidate the premise. Then your inference is valid for both directions of time, and that is a contradiction. ”

E. Schrödinger

Initial Conditions

From the fact that the differential equations of mechanics are left unchanged by reversing the sign of time without changing anything else, Herr Ostwald concludes that the mechanical view of the world cannot explain why natural processes always run preferentially in a definite direction. *But such a view appears to me to overlook that mechanical events are determined not only by differential equations, but also by initial conditions.*

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L. Boltzmann

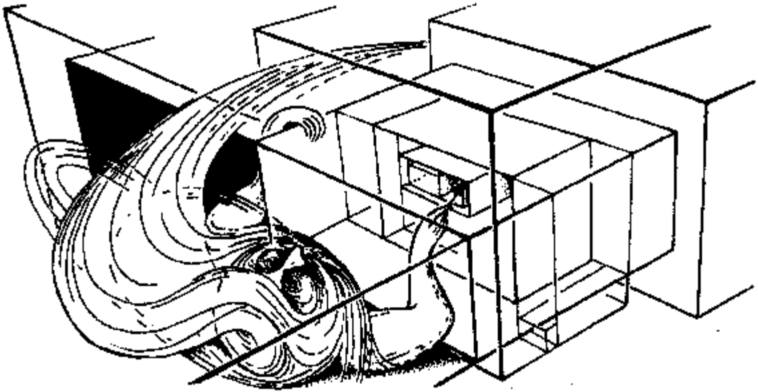
But why were the initial conditions of our world so special?

" One may speculate that the universe as a whole is in thermal equilibrium and therefore dead, but there will be local deviations from equilibrium which may last for the relatively short time of a few eons. For the universe as a whole, there is no distinction between the "backwards" and "forwards" directions of time, but for the worlds on which living beings exist, and which are therefore in relatively improbable states, the direction of time will be determined by the direction of increasing entropy, proceeding from less to more probable states."

L. Boltzmann

Since we always make the prediction that in a place where we have not looked we shall see stars in a similar condition, or find the same statement about Napolean, or that we shall see bones like the bones that we have seen before, the success of all those sciences indicates that the wold did not come from a fluctuation... Therefore I think it is necessary to add to the physical laws the hypothesis that in the past the universe was more ordered... than it is today. I think this is the additional statement that is needed to make sense, and to make an understanding of the irreversibility.

Feynman, The Character of Physical Law



Creation of the universe: a fanciful description! The Creator's pin has to find a tiny box, just 1 part in $10^{10^{123}}$ of the entire phase-space volume, in order to create a universe with as special a Big Bang as that we actually find.

R.Penrose, The Emperor's New Mind

Boltzmann's explanation of the time-reversal asymmetry of natural process is that "this one-sidedness lies uniquely and solely in the initial conditions", by which he means "not ... that for each experiment one must specially assume that certain initial conditions exist" but rather that " it is sufficient to have a uniform basic assumption about the initial properties of the mechanical picture of the world". He proposes to "conceive of the wold as an enormously large mechanical system ... which starts from a ... ordered initial state, and even at present is still in a substantially ordered state. "

From Oliver Penrose

It is this fact that we are still in a state of low entropy that permits the existence of relatively stable neural connections, of marks of ink on paper, which retain over relatively long periods of time shapes related to their formation. Such nonequilibrium states are required for memories - in fact for the existence of living beings and of the earth itself.
We have no such records of the future and the best we can do is use statistical reasoning which leaves much room for uncertainty.
Equilibrium systems, in which the entropy has its maximal value, do not distinguish between past and future.

The general struggle for existence of living beings is therefore not a fight for the elements- the elements of all organisms are available in abundance in air, water, and soil-, nor for energy, which is plentiful in the form of heat, unfortunately untransformably, in every body. Rather, it is a struggle for entropy [more accurately: negative entropy] that becomes available through the flow of energy from the hot Sun to the cold Earth. To make the fullest use of this energy, the plans spread out the immeasurable areas of their leaves and harness the Sun's energy by a process as yet unexplored, before it sinks down to the temperature level of our Earth, to drive chemical synthesis of which one has no inkling as yet in our laboratories. The products of this chemical kitchen are the object of the struggles in the animal world.

L. Boltzmann, The Second Law of the Mechanical

Quantum systems: microstates

Let me turn now to the quantum world, which is, to the best of our knowledge, the world we live in. The first question to answer is: what takes the place of the point $X \in \Gamma$ as the microstsate of a macroscopic system?

Sad to say there is not, at the present time, almost a century after the Schrödinger's Equation

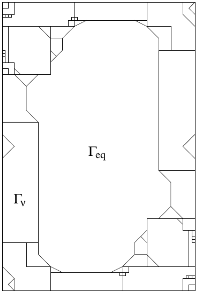
$$i\hbar \frac{\partial \Psi(q_1, \ldots, q_N; t)}{\partial t} = -\sum_{k=1}^{N} \frac{\hbar^2}{m_k} \nabla_k^2 \Psi + V(\mathbf{q})\Psi,$$

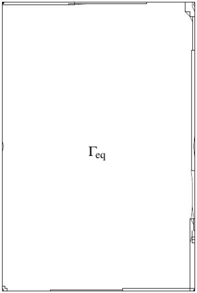
a satisfying generally accepted answer to this question. What "everyone" does agree on is that the wave function Ψ, $\Psi \in \mathscr{H}$, the Hilbert space of the system, is an important ingredient of the microstate. Let us assume for the moment that Ψ is a sufficiently good description, i.e. it will give, via the usual rules, the properties of our isolated system.

The Schrödinger equation is time reversible in the same way as the Hamiltonian equations of classical mechanics with $\Psi \to \bar{\Psi}$ corresponding to velocity reversal $X \to RX$.
Thus the problem of time's arrow is the same as before. The answer is also the same involving macrostates.

Macrostates: quantum

The next question then is: what takes the place of the $\Gamma_M \subset \Gamma$,





The second picture is slightly more faithful. Neither shows the topology or differences in relative sizes of the different Γ_M's.

The answer, as given by von Neumann, is to first “round” the operators corresponding to the macro-variables so they all commute, then let $\mathscr{H}_\nu$ be the linear subspace of $\mathscr{H}$ specified by their simultaneous eigenvalues, ν. Thus $\Psi \in \mathscr{H}_\nu$ corresponds to the system being in the macrostate M_ν. We then have

$$\mathscr{H}_E = \bigoplus \mathscr{H}_\nu$$

with the $\mathscr{H}_\nu$ forming an orthogonal decomposition of the very large but still finite dimensional Hilbert space $\mathscr{H}_E$, with a basis given by the energy eigenstates $|\phi_\alpha\rangle$ with $E_\alpha \in [E, E + \delta E]$.

Accepting this rounding, the orthogonal decomposition of $\mathscr{H}$ corresponds to the classical decomposition of the energy shell Γ_E into regions Γ_M with $\Gamma_E = \cup \Gamma_M$. The analog of the Liouville volume $|\Gamma_M|$ is the dimension of $\mathscr{H}_\nu$, denoted by $|\mathscr{H}_\nu|$. The Hamiltonian time evolution preserving Liouville volume now corresponds to the unitary evolution, obtained from the solution of the Schrödinger equation, which preserves the dimension of subspaces of $\mathscr{H}$.

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The linear subspace, $\mathscr{H}_{eq}$, corresponding to the equilibrium macrostate is again (like in the classical case) characterized by the fact that its dimension is, for a macroscopic system, almost the same as the dimension of the whole (microcanonical) Hilbert space,

$$|\mathscr{H}_{eq}| \approx |\mathscr{H}_E|.$$

All other $|\mathscr{H}_\nu|$ are much, much smaller.

The analog of the Boltzmann entropy for the system with a wave function $\Psi \in \mathscr{H}_\nu$ would then be, according (more or less) to von Neumann[2], given by

$$S_B(\nu) = \log(|\mathscr{H}_\nu|)$$

Using this prescription the reasoning of Boltzmann about the approach to equilibrium and the microscopic origin of the second law for classical systems goes through, essentially unchanged, also for macroscopic quantum systems.

[2] One always talks of von Neumann's Gibbs entropy $S_G = -\operatorname{tr}(\rho \log(\rho))$ which gives zero entropy to a pure state and never of his Boltzmann entropy discussed in chapter 5 of his book Foundations of Quantum Mechanics.

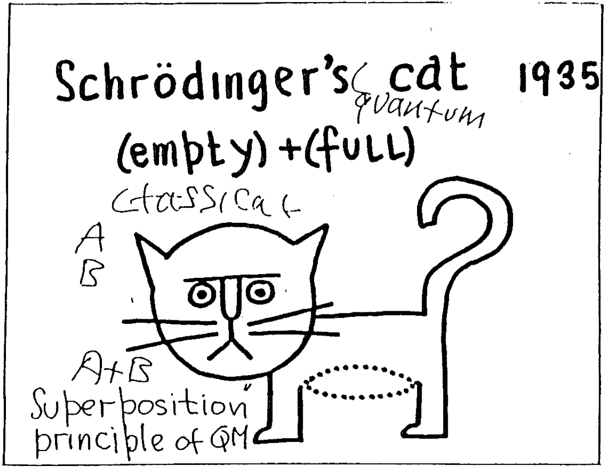
The only "fly in the ointment" in this analysis is that while for classical systems the microstate X uniquely specifies the macrostate $M(X)$ (with possible exceptions of measure zero which are generally not important) this is no longer true in quantum mechanics:

A "typical" wave function Ψ will (in general) not be just in one linear subspace but will have projections $P_\nu|\Psi\rangle \neq 0$, for essentially all ν.

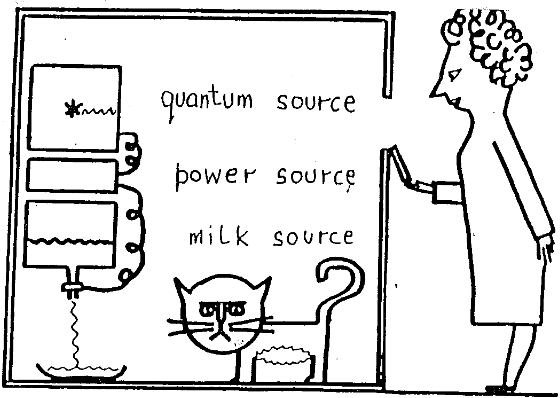
Thus, giving Ψ does not uniquely specify the macrostate of the system. This is the problem (or paradigm) of Schrödinger's cat, where the Ψ of the cat clearly corresponds to a superposition of incompatible macrostates: one in which the cat drank the milk and one in which it did not.

My only answer to this, at the moment, is that there is always a "fact of the matter" about macrostates, i.e. of whether the cat has drunk its milk or has not. How this "fact of the matter" for macrostates is to be understood or reconciled with the autonomous Schrödinger evolution of the wave function is still very much (as far as I am concerned) an unsettled matter. I quote from J. S. Bell's talk in 1989 on the occasion of J. Schwinger's 70th birthday celebration.

“After Einstein and Bohr the most famous person [in the foundations of quantum mechanics] is Schrödinger’s cat



Now Schrödinger's cat is a quantum cat. It is like the traditional or classical cat in that it has nine lives; but unlike the traditional classical cat it can have all of these lives at the same time. This is a consequence of the superposition principle of quantum mechanics. If a state A is possible and a state B, then a state $A + B$ is also possible, and so on. This particular cat is at the same time thin and fat - it has had supper <u>and</u> it has not had supper. Now this is now just an academic possibility - it happens all the time, and Schrödinger pointed out that it could be realized under controlled laboratory conditions."


quantum source
power source
milk source

Fundamental ambiguity:
Nobody knows what
quantum mechanics says
exactly about any situation.

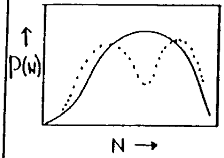
For nobody knows where
the boundary really is,
between wavy quantum
system and the world of
particular events.

THIS IS THE PROBLEM
OF QUANTUM MECHANICS

It is no problem
in practice — because
practice is not accurate
enough — and maybe never
will be.

ψ not real ?   what is ?

information ?   what about ?



} soldiers
horses
kicks

ψ ?   } ? "allowed states"
? jumps

picture good only for weakly interacting system

ψ is not all | what else?

allowed states? jumps?

$\{\psi(t, r\ldots), x(t)\ldots\}$

de Broglie Bohm 1926, 1952

x's are particle pos

'pilot-wave picture'

$m\dot{x}(t) = \frac{\partial}{\partial x} \mathrm{im} \log \psi(t, x(t)\cdot)$

no jumps $\rho(0,x) = |\psi(0,x)|^2$

rational, clear, exact

agrees with experiment

Lorentz-invariance?

Bohmian QM: $(\mathbf{Q}, \psi)$

$$
\begin{aligned}
i\hbar \frac{\partial \psi(q_1, \ldots, q_N; t)}{\partial t} &= -\sum_{k=1}^{N} \frac{\hbar^2}{m_k} \nabla_k^2 \psi + V(\mathbf{q})\psi \\
\frac{dQ_k(t)}{dt} &= \frac{\hbar}{m_k} I_m \frac{\nabla_k \psi(Q_1, \ldots, Q_N; t)}{\psi}.
\end{aligned}
$$

"No one can understand this theory until (s)he is willing to think of ψ as a real objective field rather than just a ' probability amplitude.' Even though it propagates not in 3-space but in 3^N-space ". ... "for instantaneous macroscopic configurations the pilot-wave theory gives the same distribution [of configurations] as the orthodox theory insofar as the latter is unambiguous."

J.S. Bell, QM for Cosmologists

Large Quantum Superpositions and Interference of Massive Nanometer-Sized Object

O. Romero-Isart, A. C. Pflanzer, F. Blaser, R. Kaltenbaek, N. Kiesel, M. Aspelmeyer, and J. I. Cirac

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A fundamental feature of quantum physics is superposition of states, such as the double slit experiment in which a particle passes through both slits at the same time to interfere downstream. This kind of spatially separated quantum superposition has been observed for particles from electrons to complex molecules, but what about larger macroscopic systems?

The biggest obstacle is decoherencepreparing and measuring a superposition of so many atoms requires minimizing environmental interactions that would otherwise rapidly destroy this fragile state. Writing in Physical Review Letters, Oriol Romero-Isart from the Max Planck Institute for Quantum Optics in Garching, Germany, and colleagues propose a method for creating and observing quantum superpositions of objects with millions of atoms.

Romero-Isart et al. consider the case of a nanometer-size dielectric sphere, trapped and cooled in an optical cavity, which prepares an initially pure quantum state of the center of mass. The sphere can then be released from the first trap to enter a second cavity, in which a carefully designed laser pulse creates a superposition of the two distinct spatial positions (similar to a double slit). As the superposition evolves in time, measurement of the particles center of mass reveals the interference pattern of the two wave functions centered at different locations. The authors note that the achievement of a large superposition of such massive objects should enable more stringent tests of quantum mechanics, especially of theories predicting a spontaneous collapse of the wave function.

-David Voss

Anyway let me leave this open and assert that the second law of non-decrease in (Boltzmann) entropy will apply to any time sequence of macrostates with a typical Ψ. That is, if a system in a macrostate ν_1 at time t_1 evolves into a macrostate ν_2 at time t_2, $t_2 > t_1$, then the entropy cannot decrease, i.e. if

$$\langle \Psi_{t_1} | P_{\nu_1} | \Psi_{t_1} \rangle \sim 1$$

and

$$\langle \Psi_{t_2} | P_{\nu_2} | \Psi_{t_2} \rangle > \varepsilon,$$

then

$$S_B(\nu_2) \gtrsim S_B(\nu_1),$$

where ε is a very small number, $\varepsilon \to 0$ as $N \to \infty$.

By typical here I mean with respect to some appropriate measure on wave functions, as I shall soon discuss.

I will now describe some recent work in which the difference between the classical $X \in \Gamma$ and $\Psi \in \mathscr{H}$ plays a central role.

Canonical Typicality

Consider an isolated system consisting of two parts. Call them system 1 and 2 or system and reservoir. Then,

Theorem Let H be the Hamiltonian of the whole system and let the number of particles in system 1 and 2 be $N_1 \ll N_2$. Let $\mathscr{H}_E \subset \mathscr{H}_1 \otimes \mathscr{H}_2$ be an energy shell. Then for most $\Psi \in \mathscr{H}_E$ with $\|\Psi\| = 1$,

$$\mathrm{tr}_2 |\Psi\rangle\langle\Psi| \approx \mathrm{tr}_2\, \rho_{mc},$$

where ρ_{mc} is the microcanonical density matrix of the whole system at energy E, i.e. equal weight to all energy eigenstates in $\mathscr{H}_E$.

There is no analog to this for a classical system where any subsystem of a composite system in state $X^{(1,2)}$ is also in a unique state $X^{(2)}$.

When the interaction between systems 1 and 2 is weak, $H \approx H_1 \otimes I_2 + I_1 \otimes H_2$, then , as is well known,

$$\mathrm{tr}_2\ \rho_{mc} \approx \frac{1}{Z} e^{-\beta H_1},$$

for $\beta = \beta(E) = \mathrm{d}S_{eq}(E)/\mathrm{d}E$.

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Typicality

When I say "for most Ψ" of the composite system, I mean with respect to a uniform measure on the unit sphere in $\mathscr{H}_E$. This measure on "wave functions" for a system by a microcanonical density matrix was considered already by Schrödinger and particularly Felix Bloch. It is analogous to the microcanonical measure on Γ_E.

Summary of Boltzmann's Ideas (also Maxwell, Kelvin, Feynman)

Time-asymmetric behaviour as embodied in the second law of thermodynamics is observed in *individual macroscopic* systems. It can be understood as arising naturally from time-symmetric microscopic laws when account is taken of a) the great disparity between microscopic and macroscopic sizes, b) initial conditions, and c) that what we observe is "typical" behaviour of real systems- not all imaginable ones. Common alternate explanations, such as those based on equating irreversible macroscopic behaviour with ergodic or mixing properties of ensembles (probability distributions) already present for chaotic dynamical systems having only a few degrees of freedom or on the impossiblity of having a truly isolated system, are either unnecessary, misguided or misleading.

We have not succeeded in answering all our problems. The answers we have found only serve to raise a whole set of new questions. In some ways we feel we are as confused as ever, but we believe we are confused on a higher level and about more important things

Posted outisde the mathematics reading room Tromsø University

Time is getting short so let me end my talk with a quote from Schrödinger :
"I am born into an environment - I know not whence I came nor whither I go nor who I am. This is my situation as yours, every single one of you. The fact that everyone always was in this same situation, and always will be, tells me nothing. Our burning question as to the whence and whither - all we can ourselves observe about it is the present environment. That is why we are eager to find out about it as much as we can. That is science, learning, knowledge, this is the true source of every spiritual endeavour of man."