

Total cost of operating an information engine

Jaegon Um (KIAS)

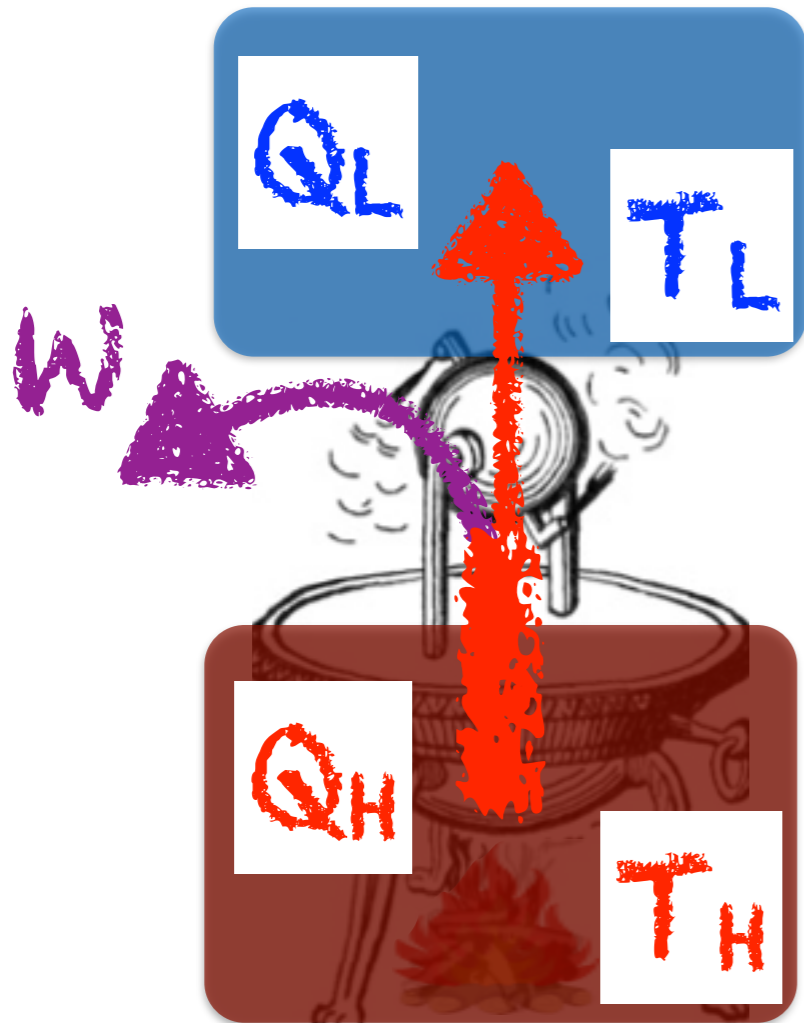
Haye Hirichsen (Würzburg Univ.)

Chulan Kwon (Myongji Univ.)

Hyunggyu Park (KIAS)

30/10/2015, NESP 2015, ICTS

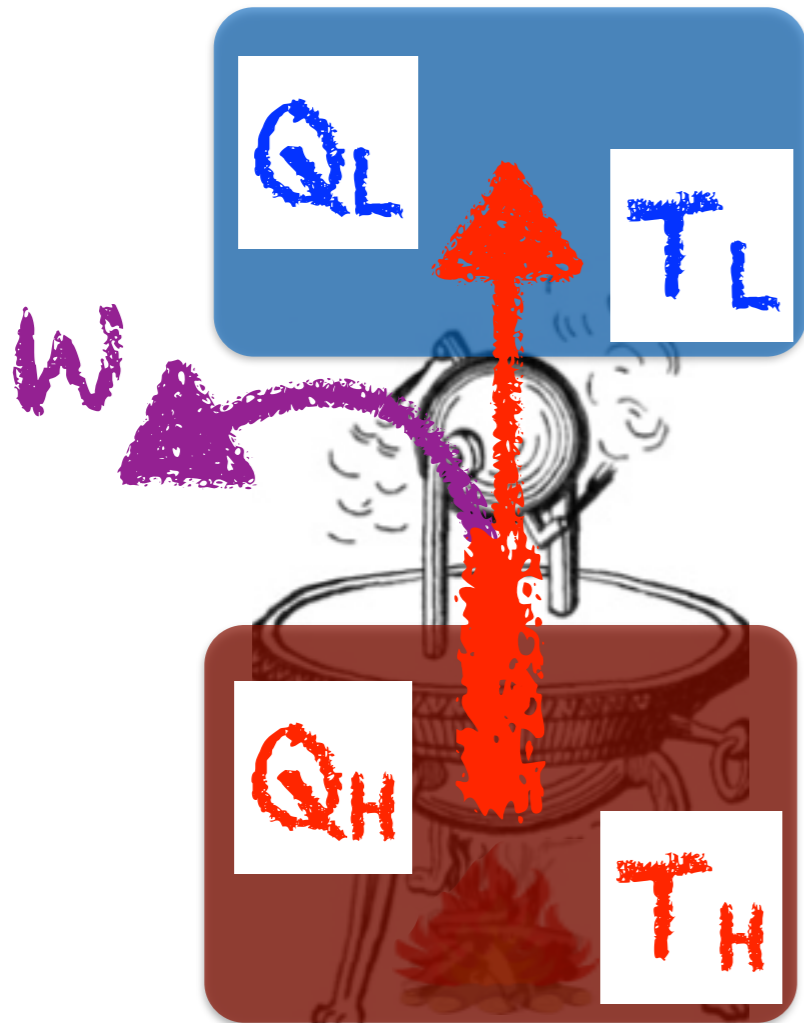
Heat engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

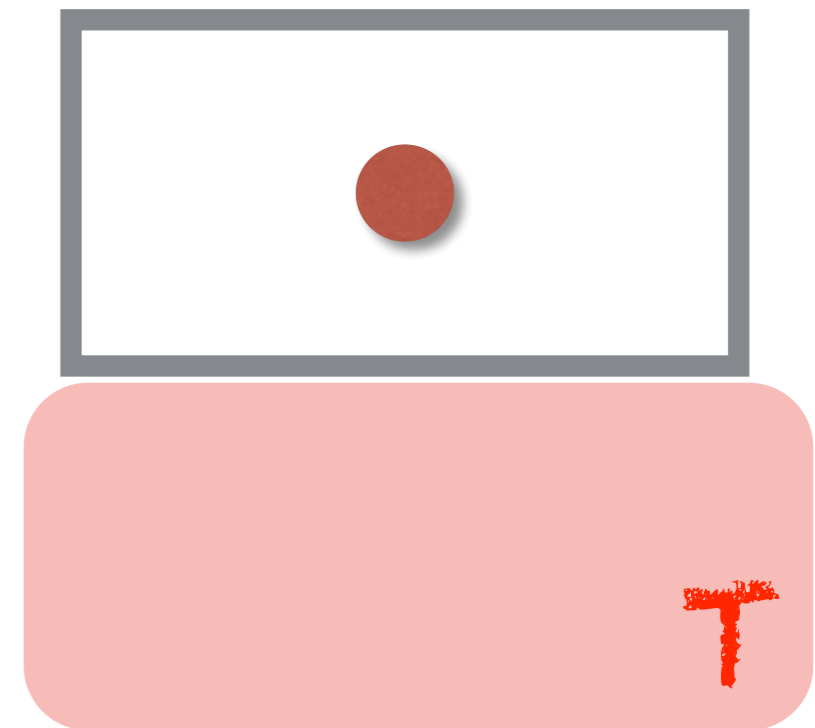
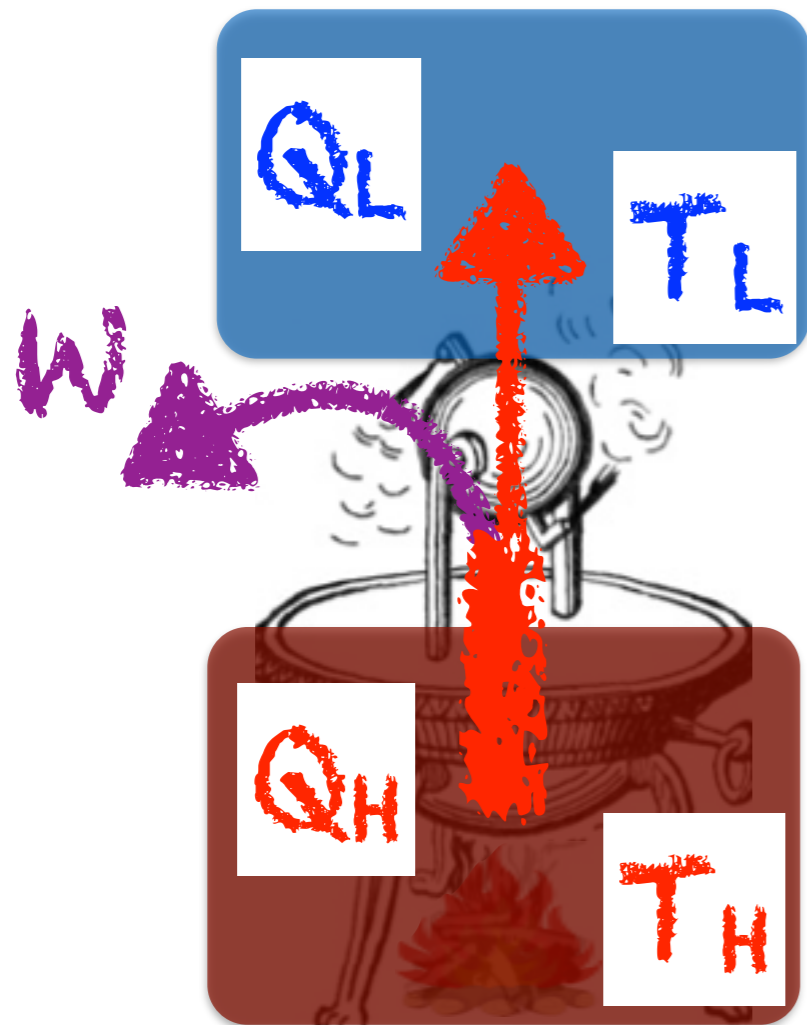
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

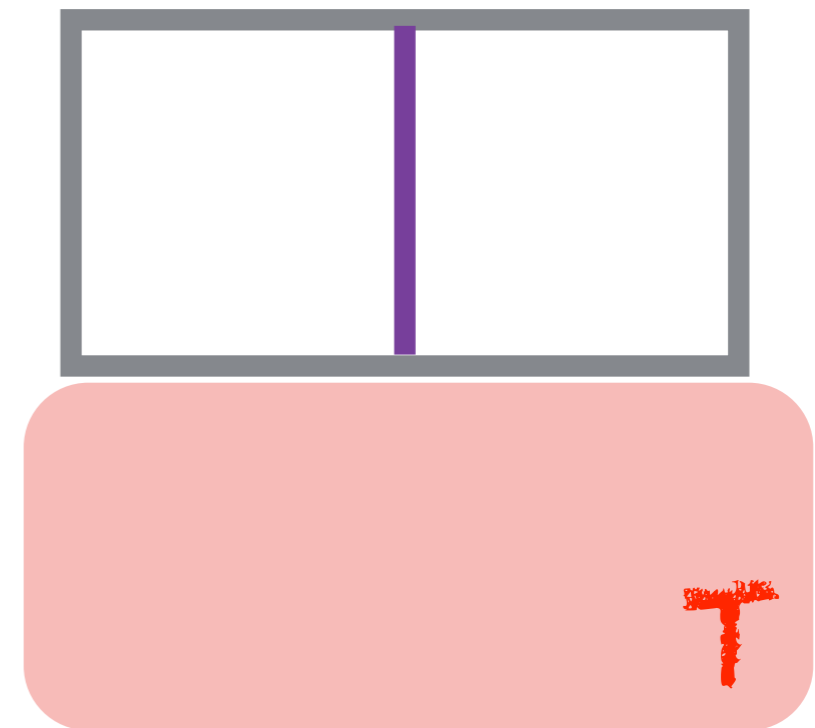
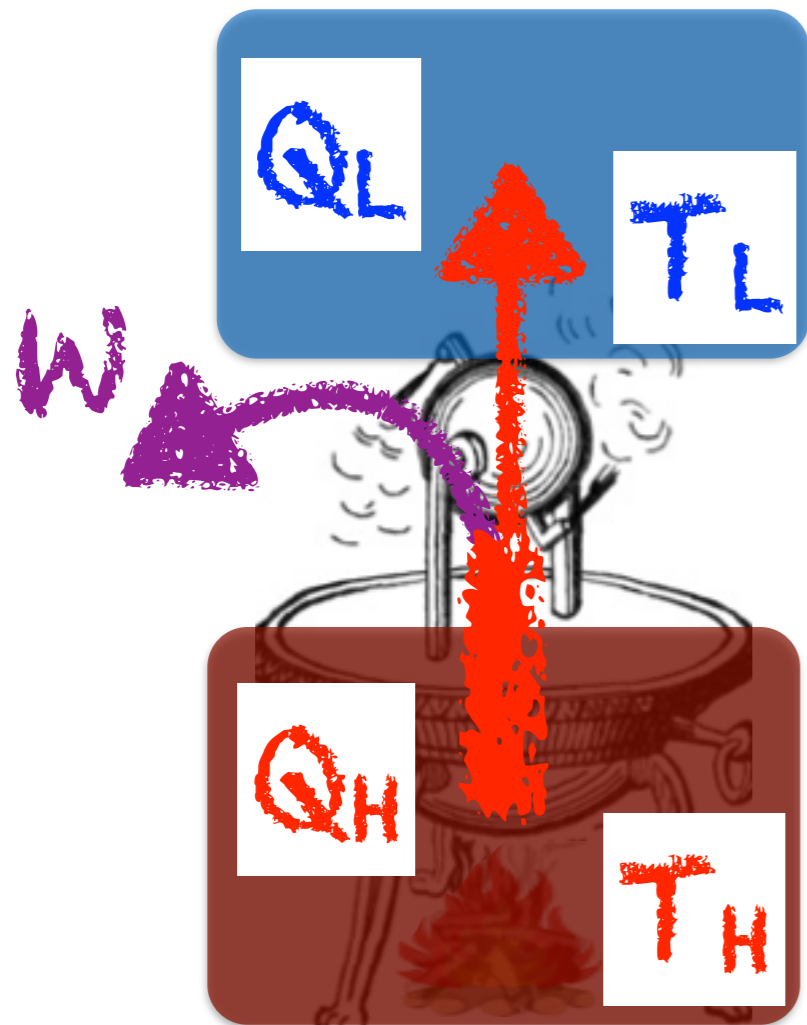
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

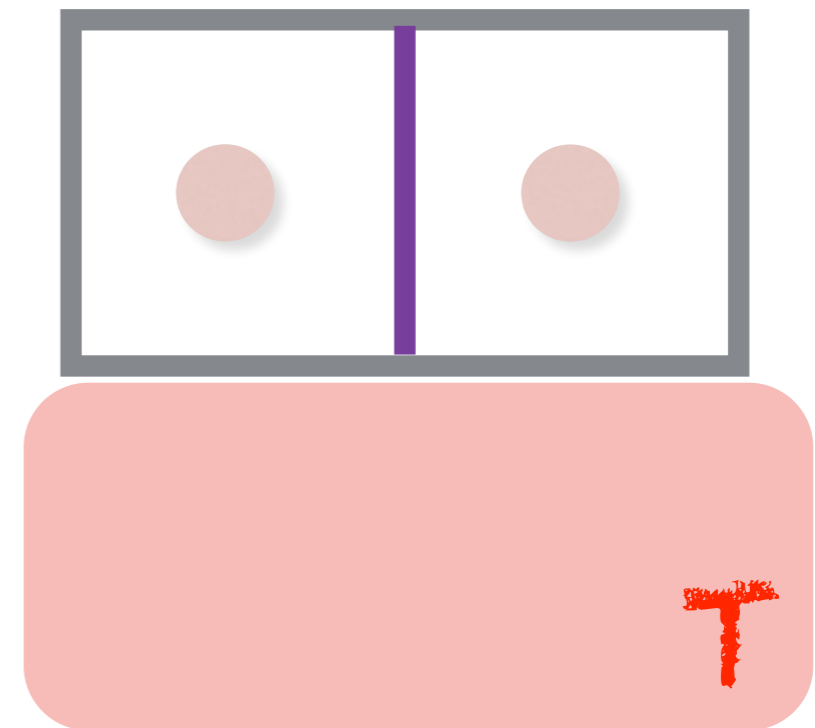
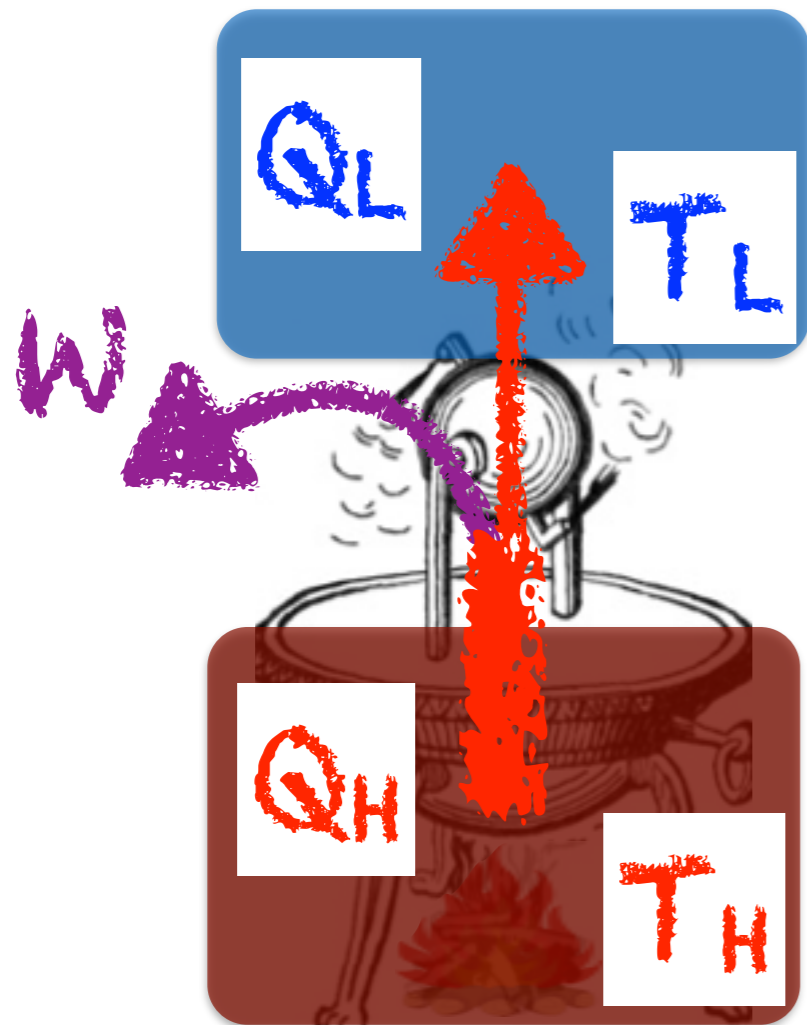
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

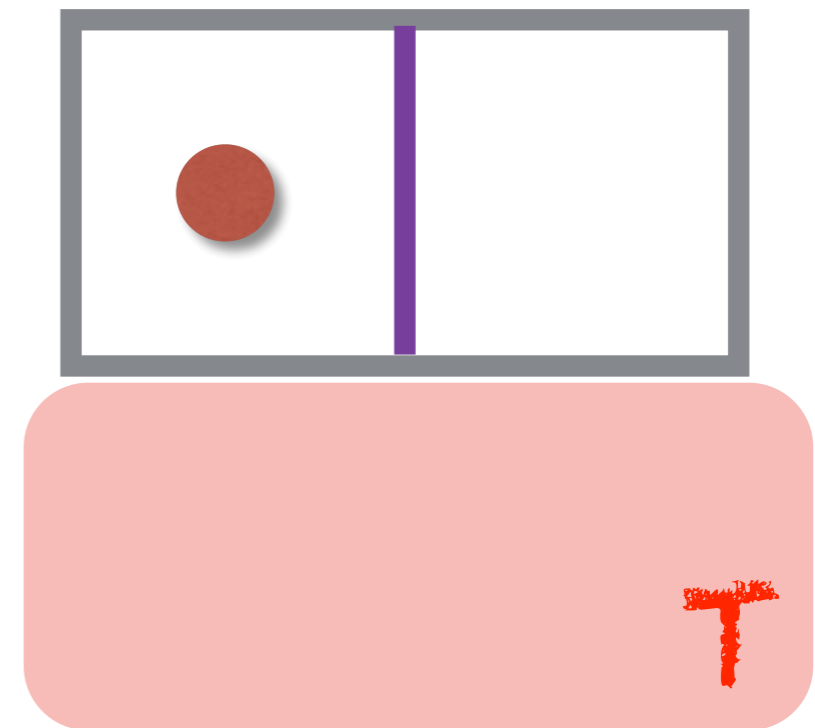
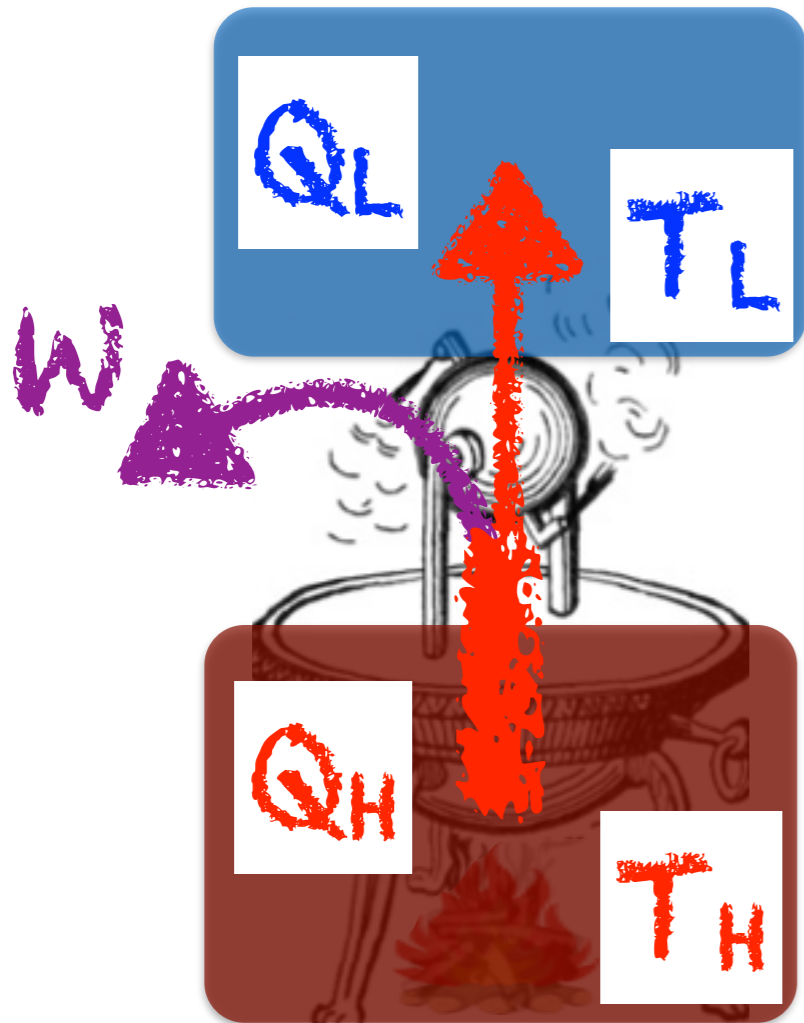
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

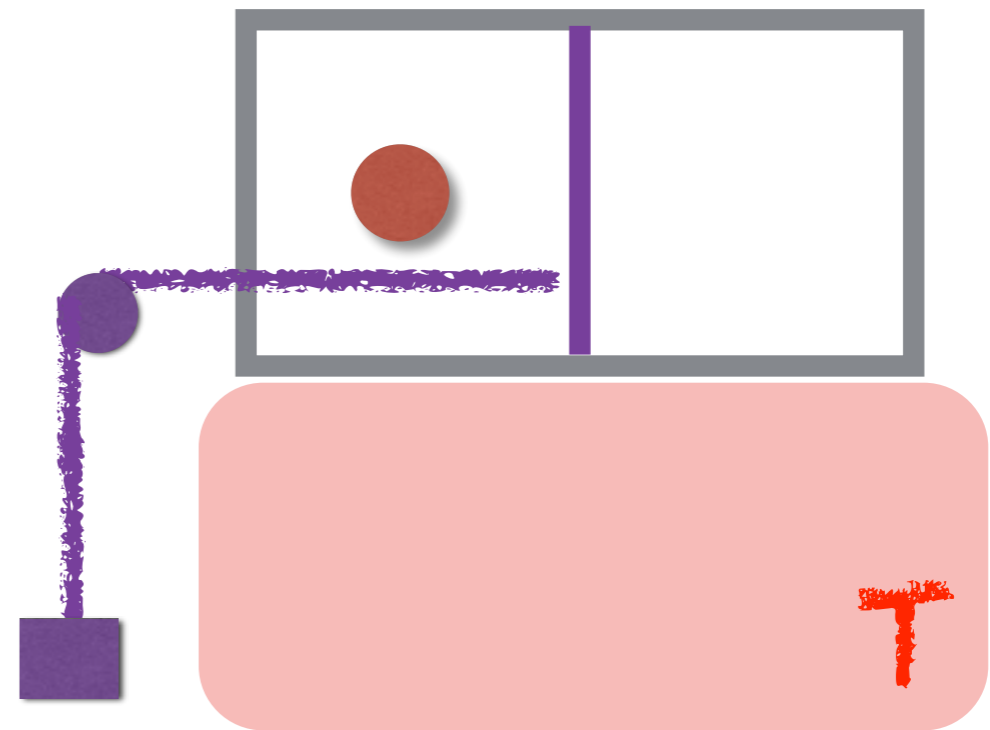
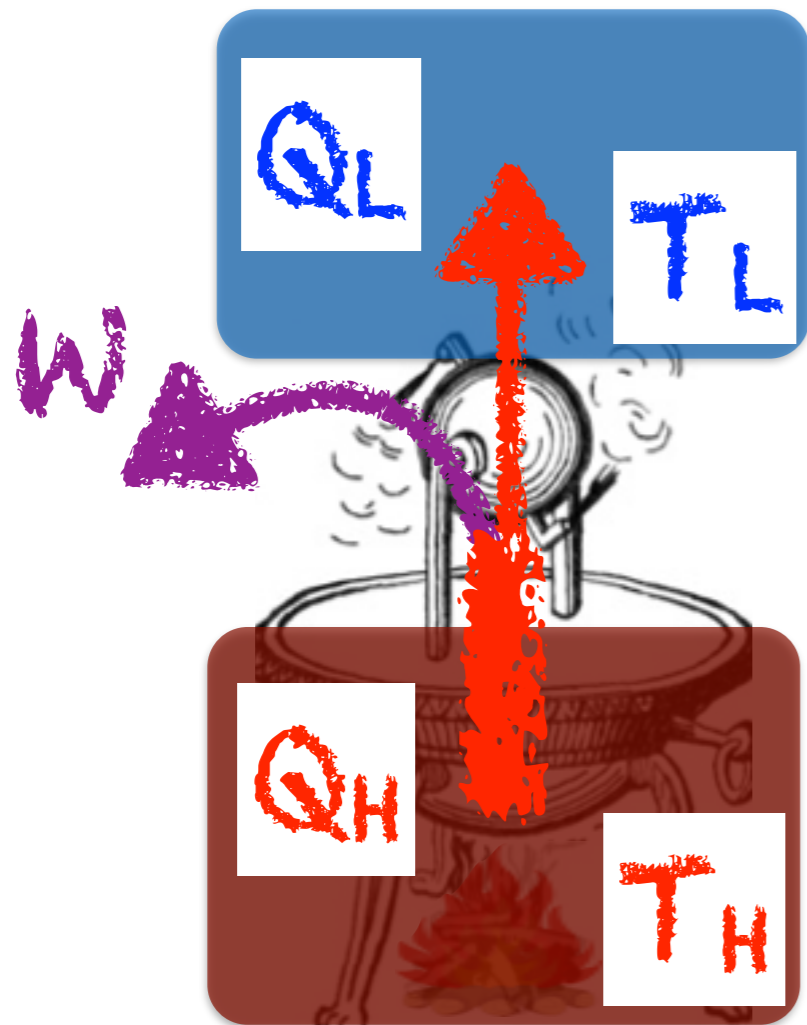
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

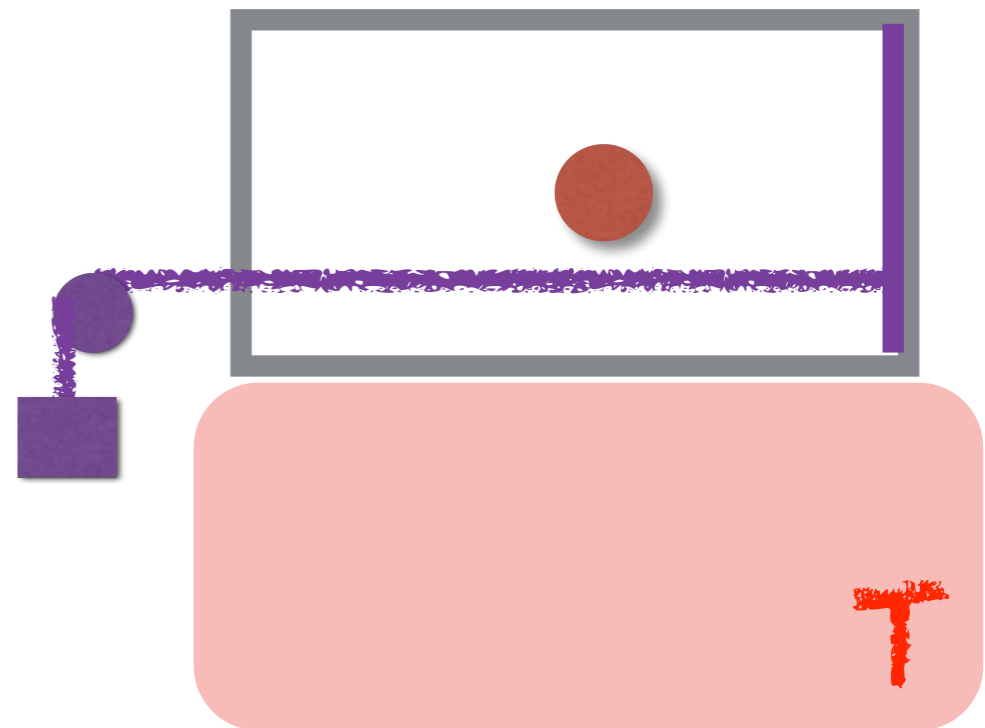
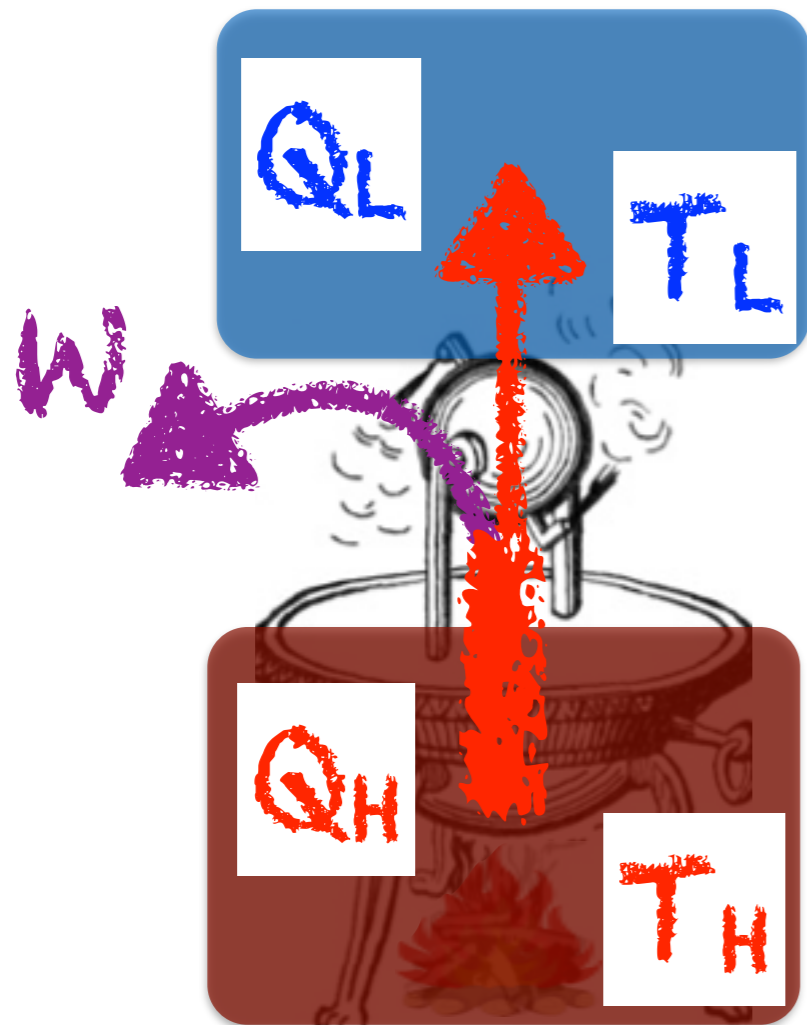
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

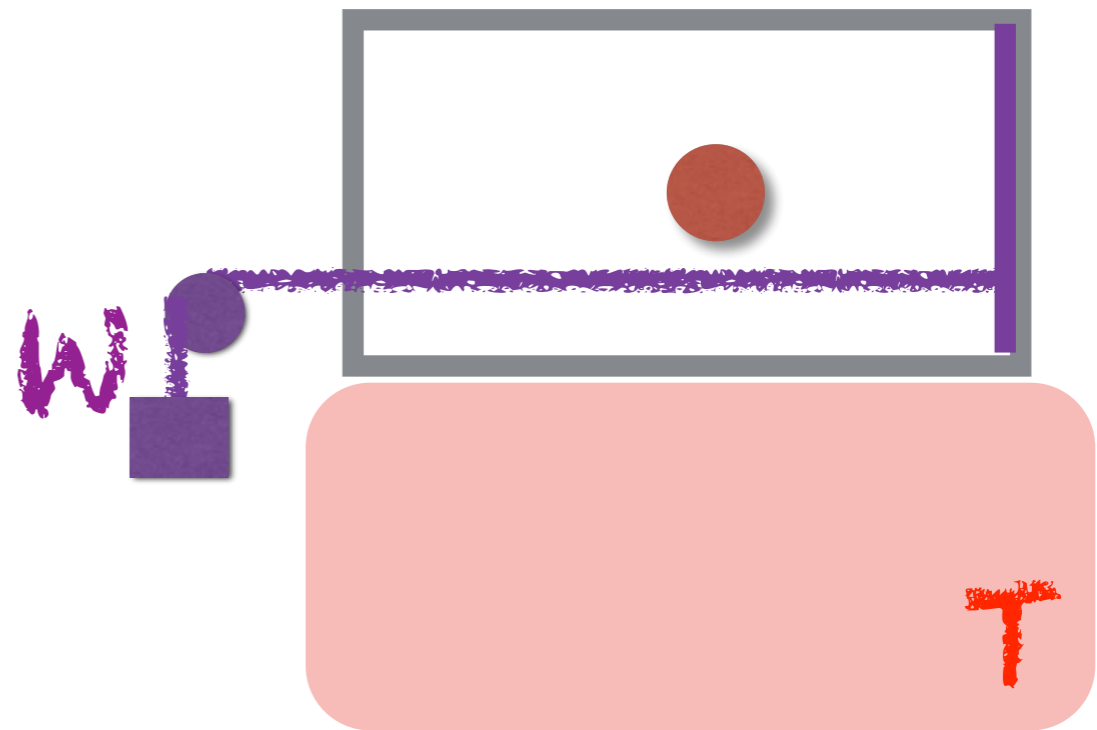
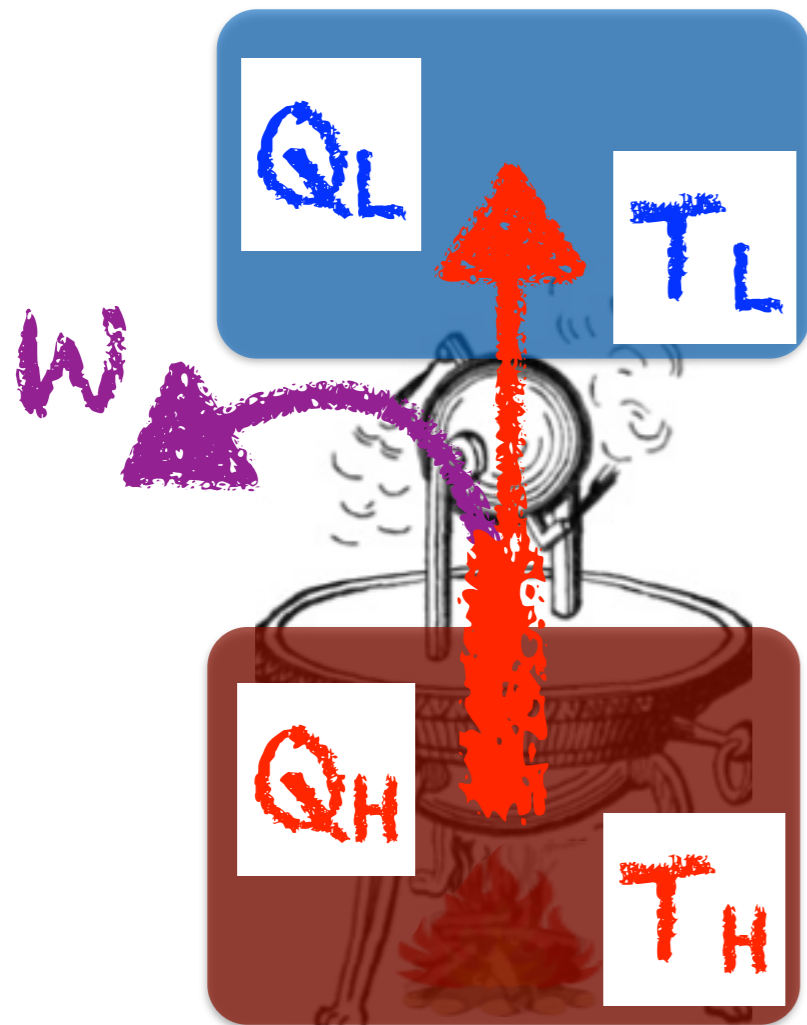
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

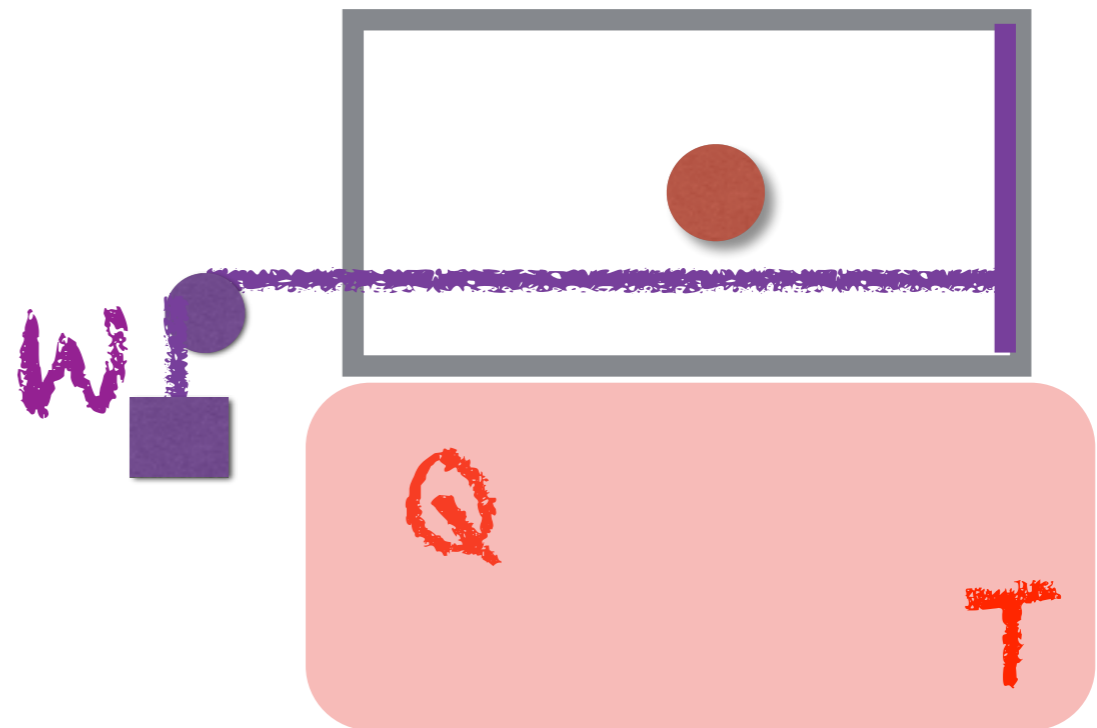
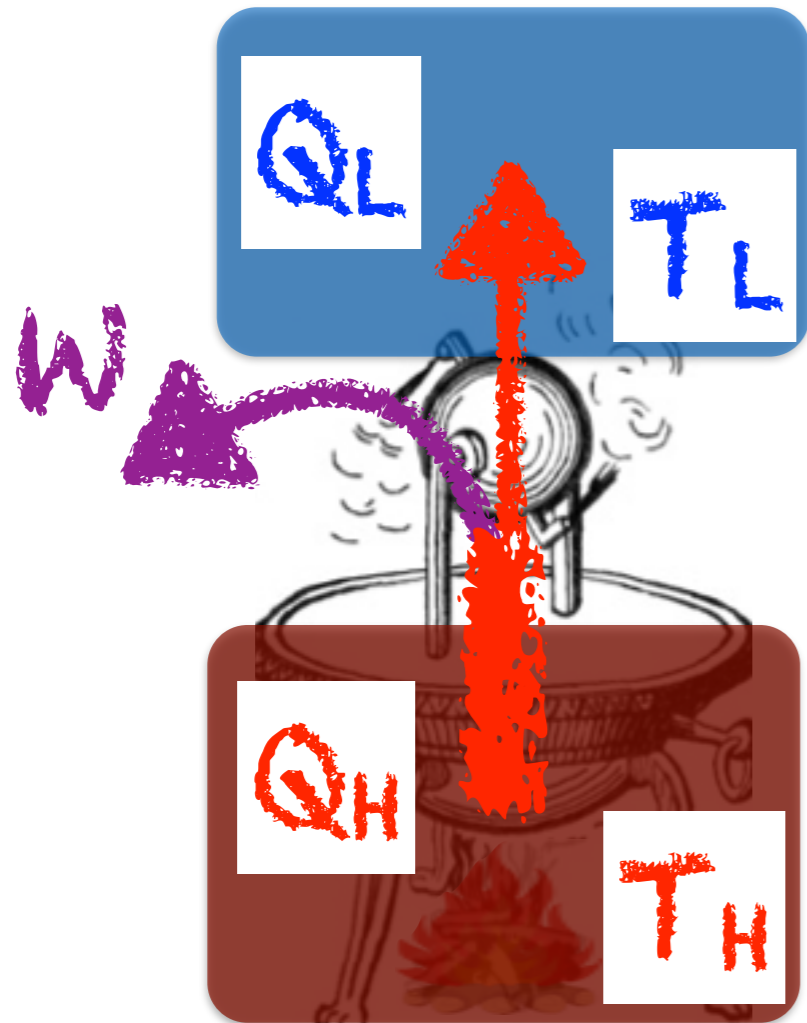
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

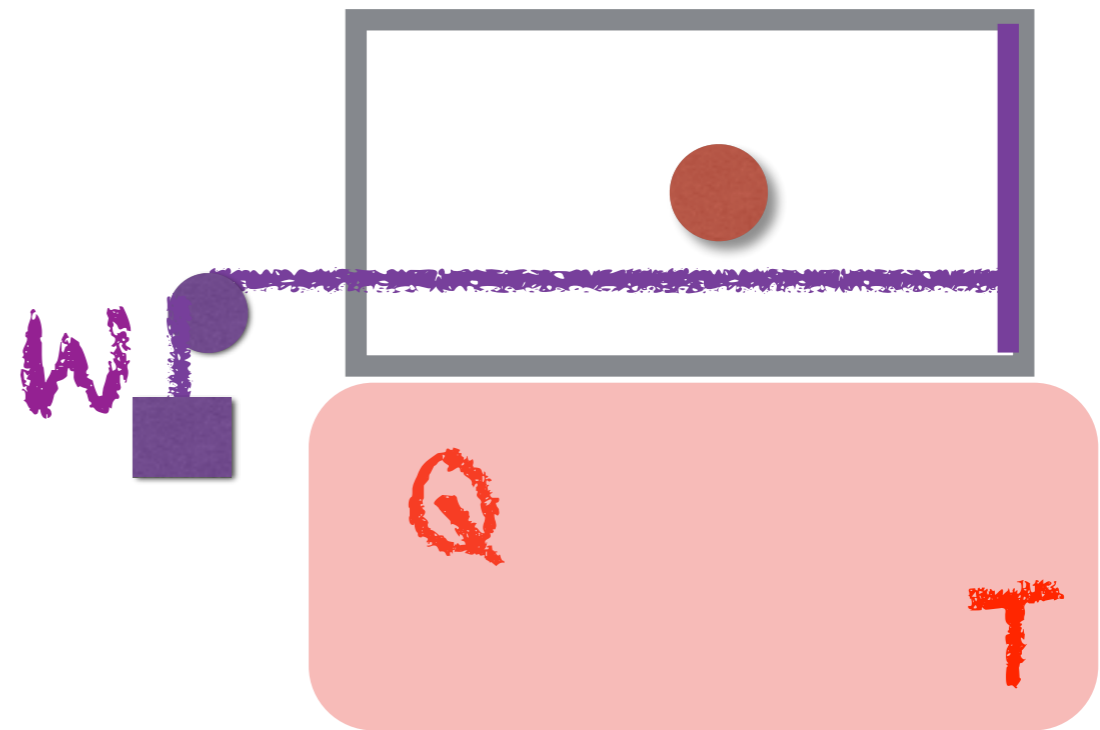
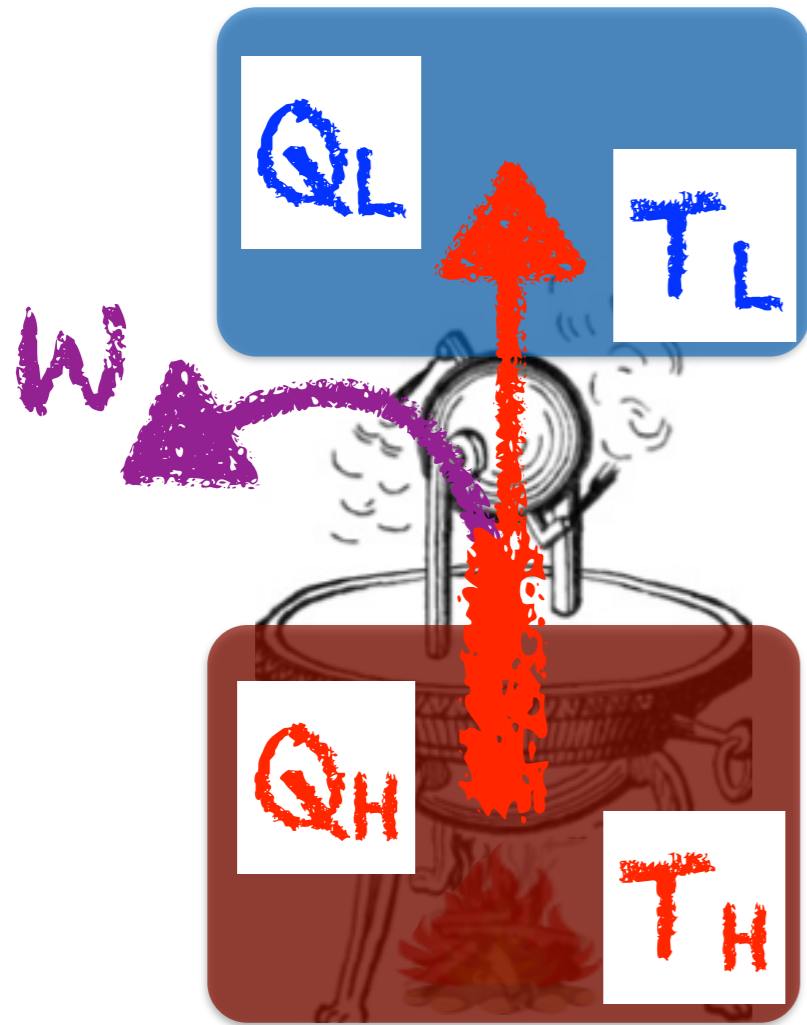
Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

Heat engine vs. Info engine

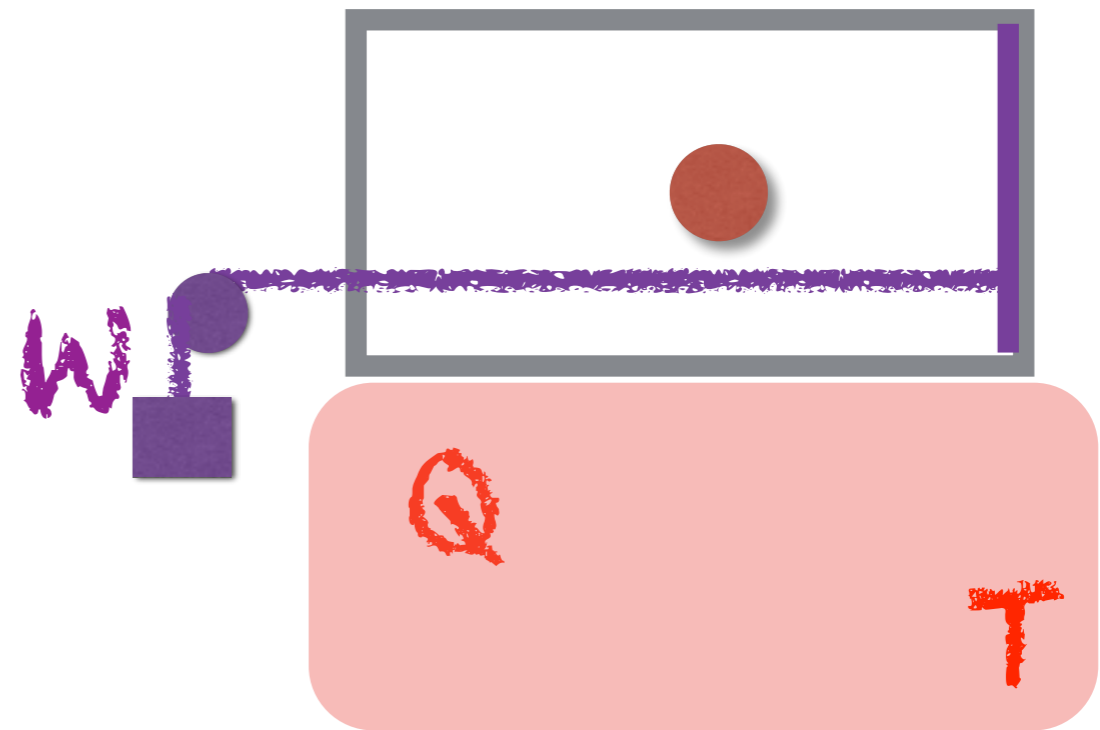
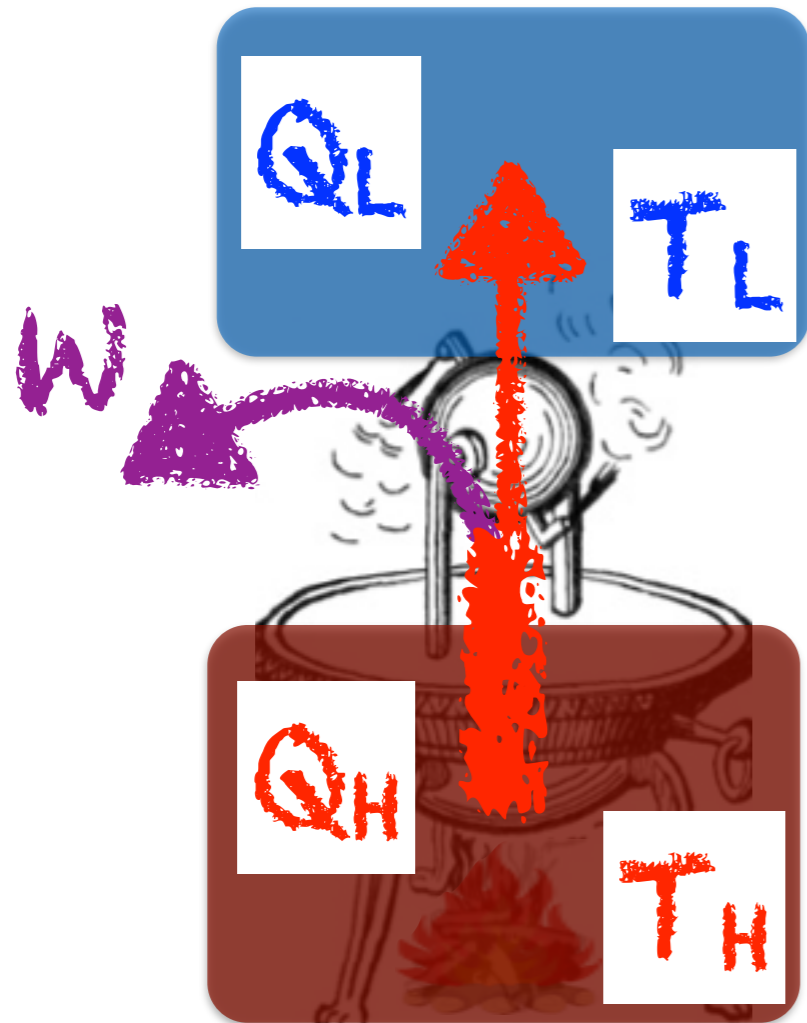


$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta \langle W \rangle = -\beta \langle Q \rangle = \ln 2$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

Heat engine vs. Info engine

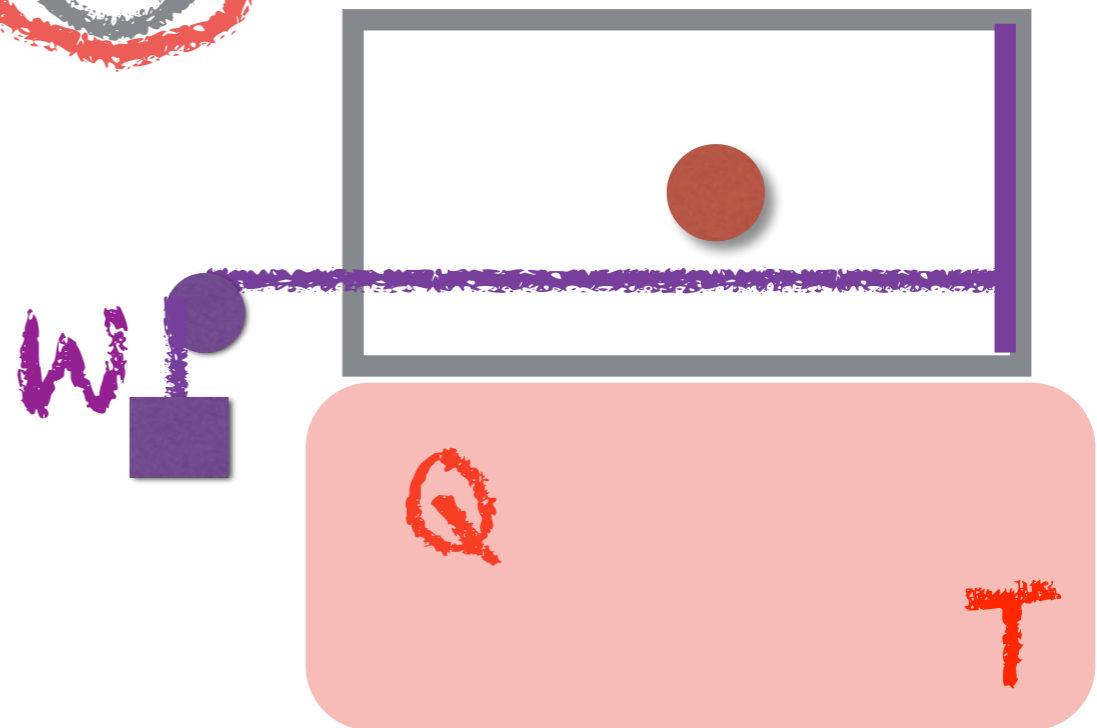
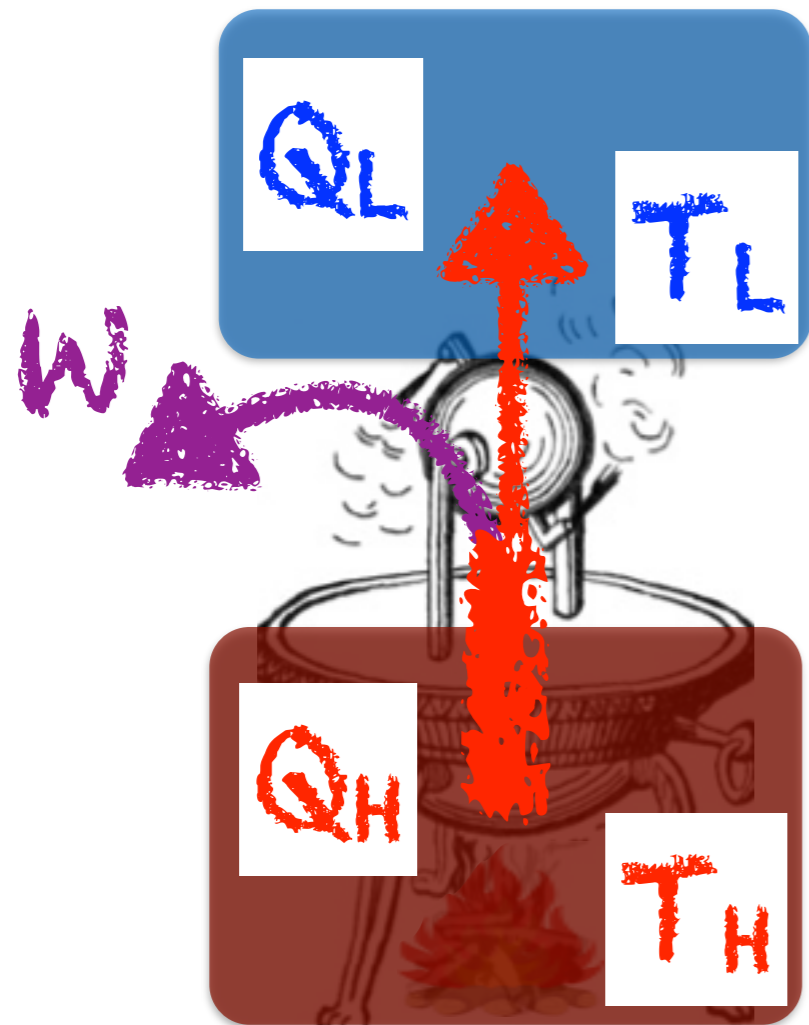


$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

$$\beta \langle W \rangle = \underline{-\beta \langle Q \rangle} = \ln 2 \quad ?$$

Heat engine vs. Info engine

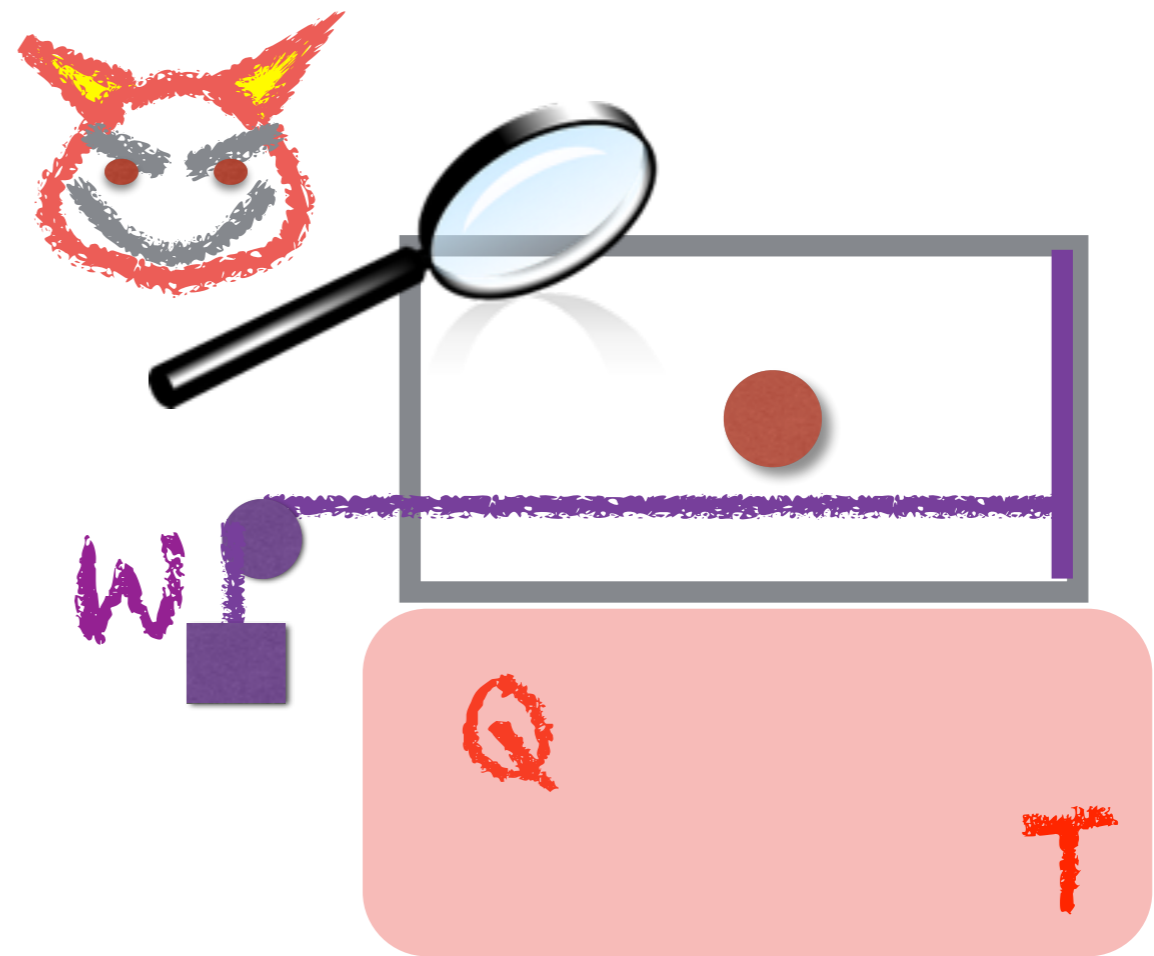
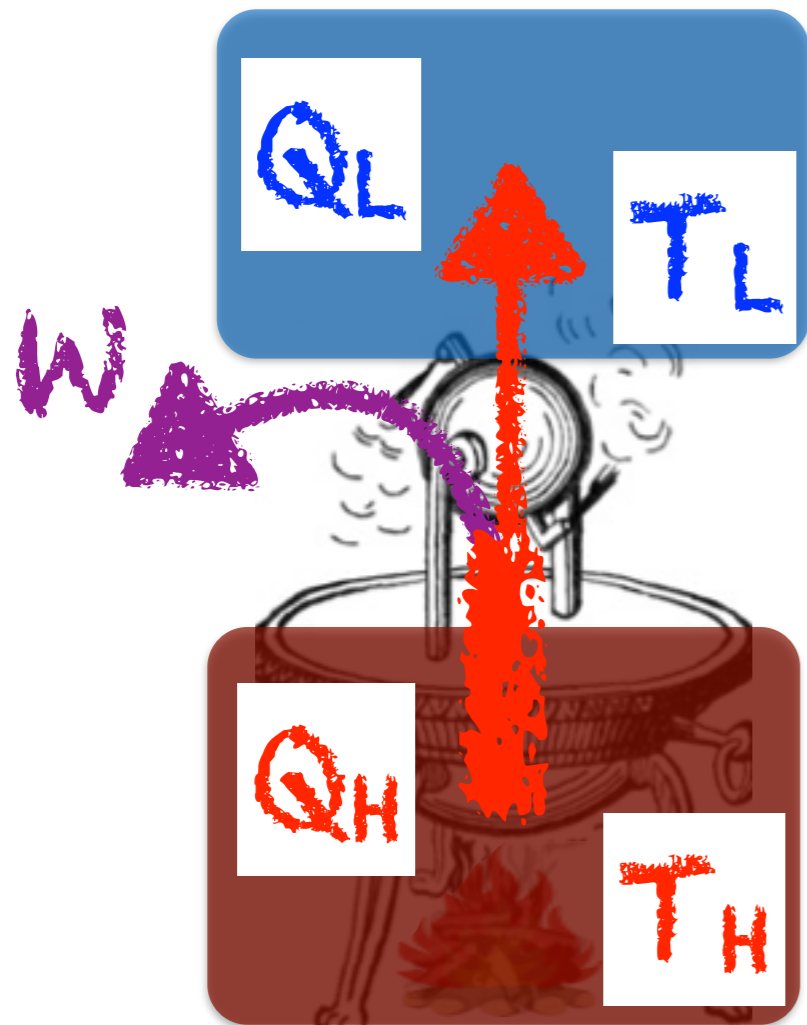


$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

$$\beta \langle W \rangle = \underline{-\beta \langle Q \rangle} = \ln 2 \quad ?$$

Heat engine vs. Info engine

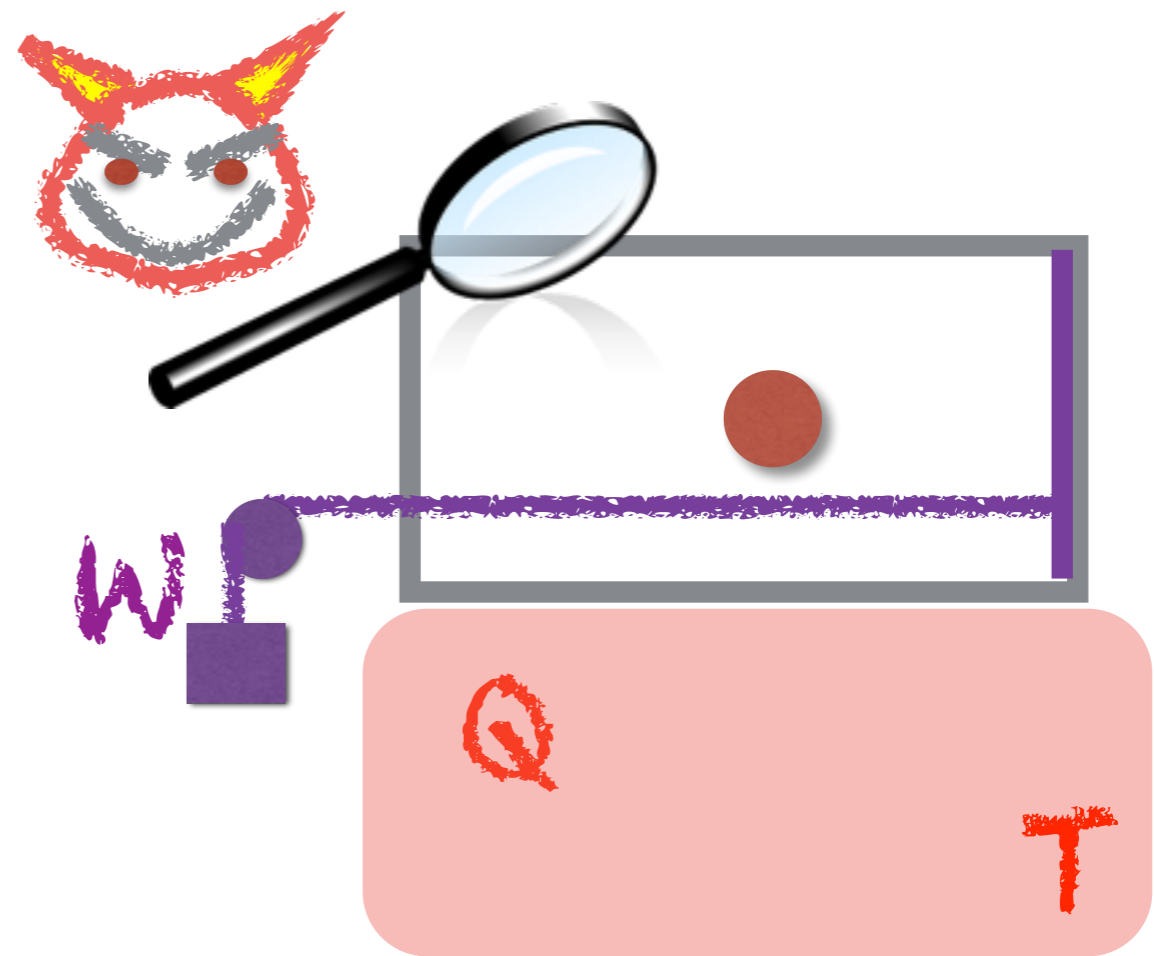
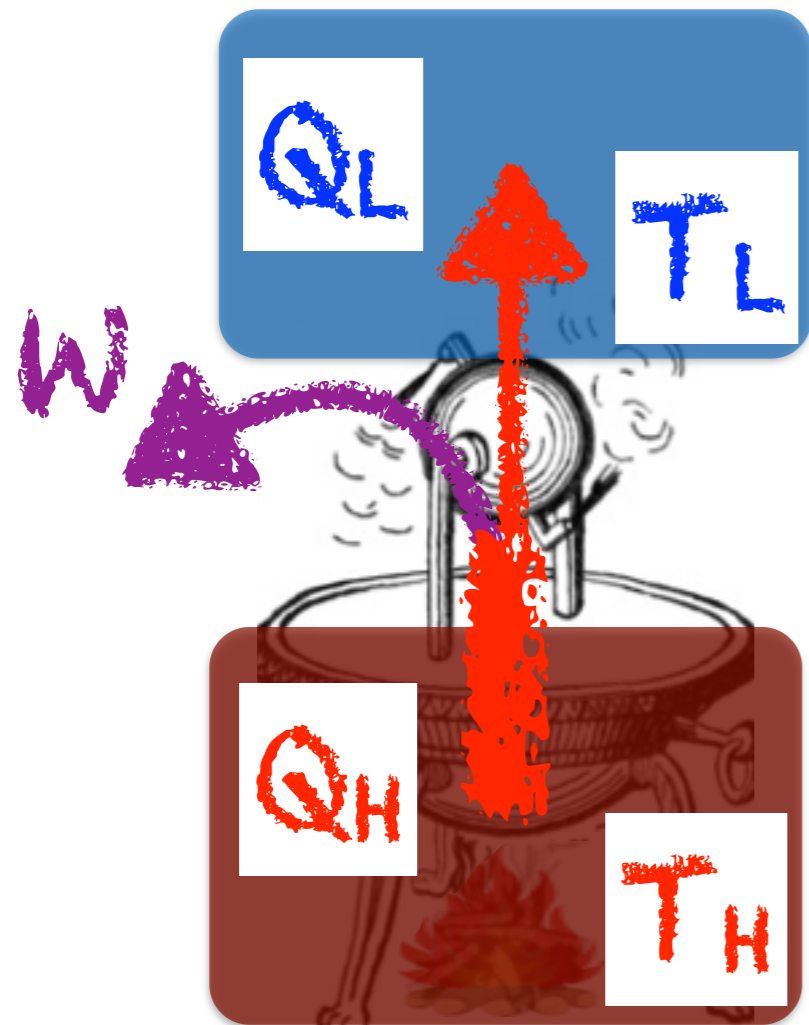


$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

$$\beta \langle W \rangle = \underline{-\beta \langle Q \rangle} = \ln 2 \quad ?$$

Heat engine vs. Info engine



$$\langle W \rangle = -\langle Q_H \rangle - \langle Q_L \rangle$$

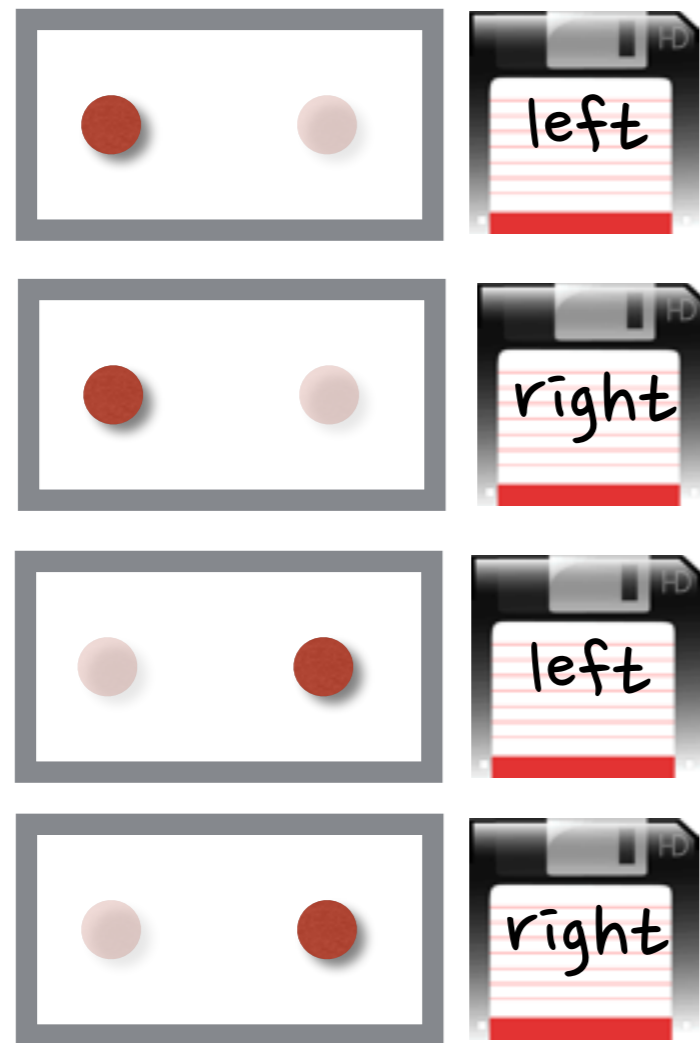
$$\beta_H \langle Q_H \rangle + \beta_L \langle Q_L \rangle \geq 0$$

$$\beta \langle W \rangle = \underline{-\beta \langle Q \rangle} = \ln 2 \quad ?$$

composite system!!

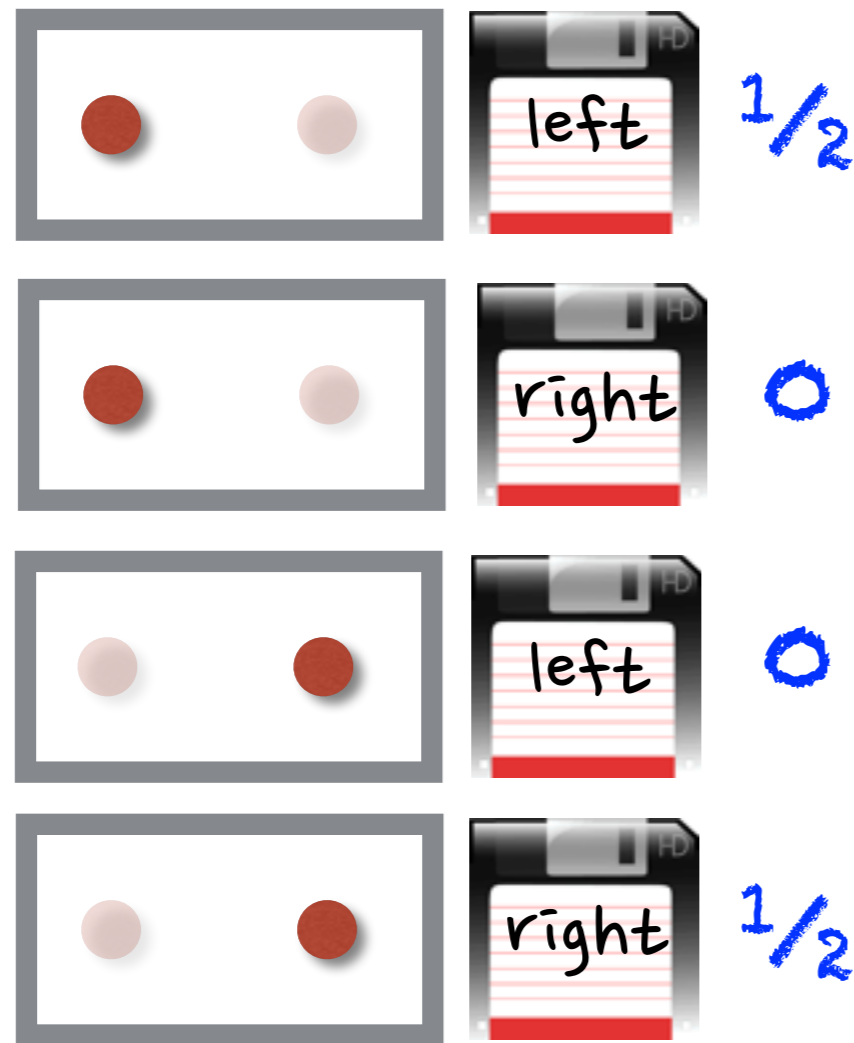
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



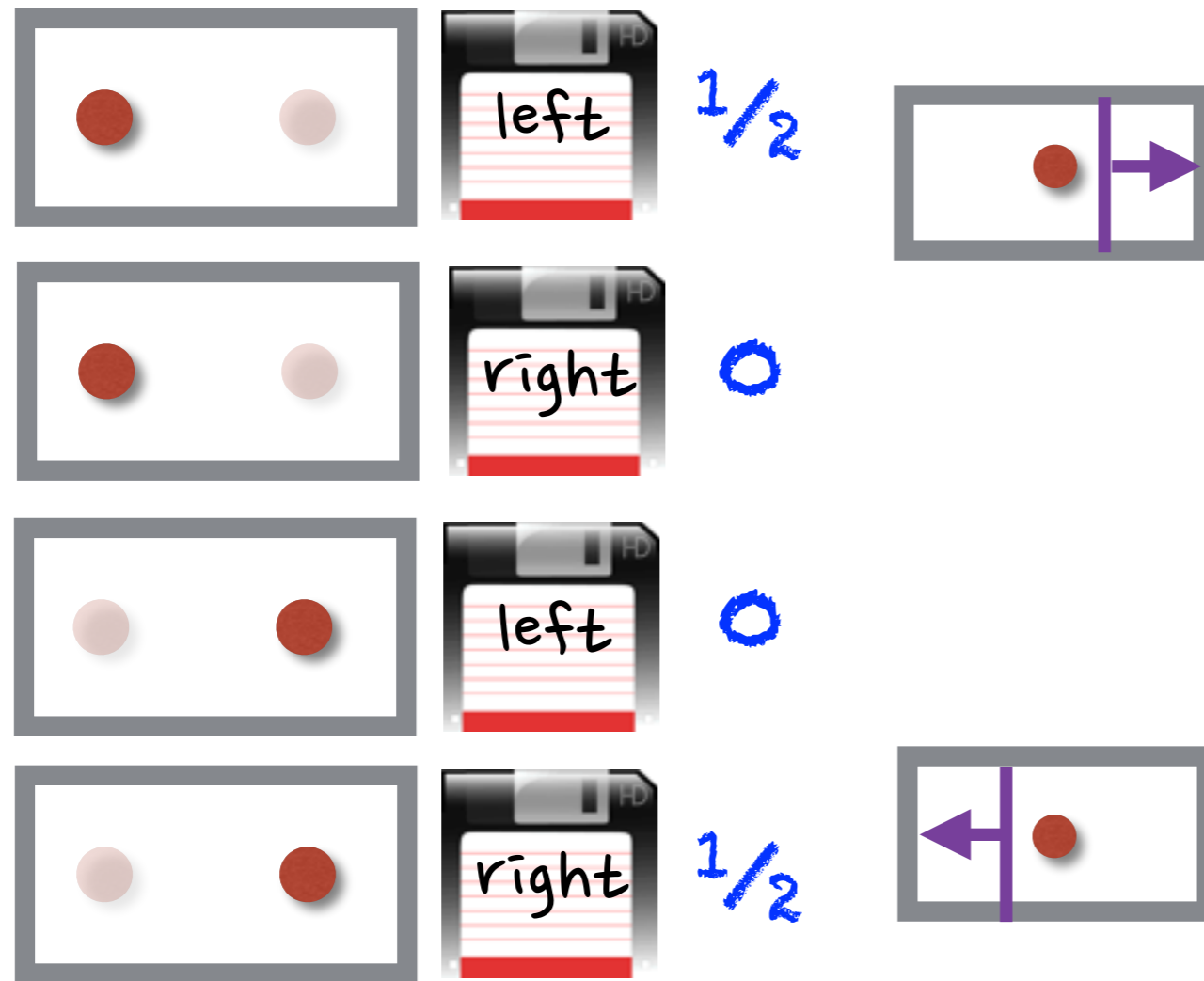
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



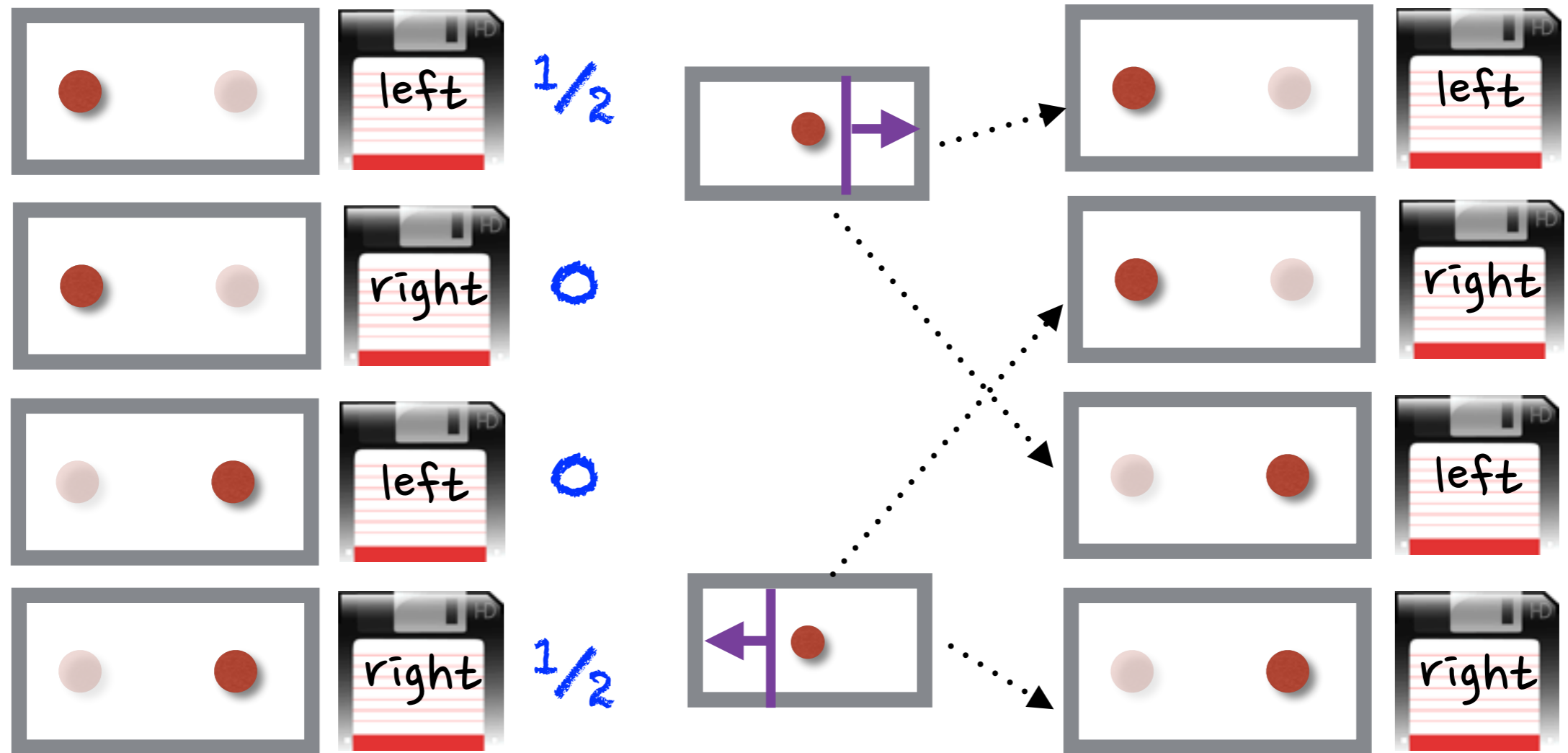
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



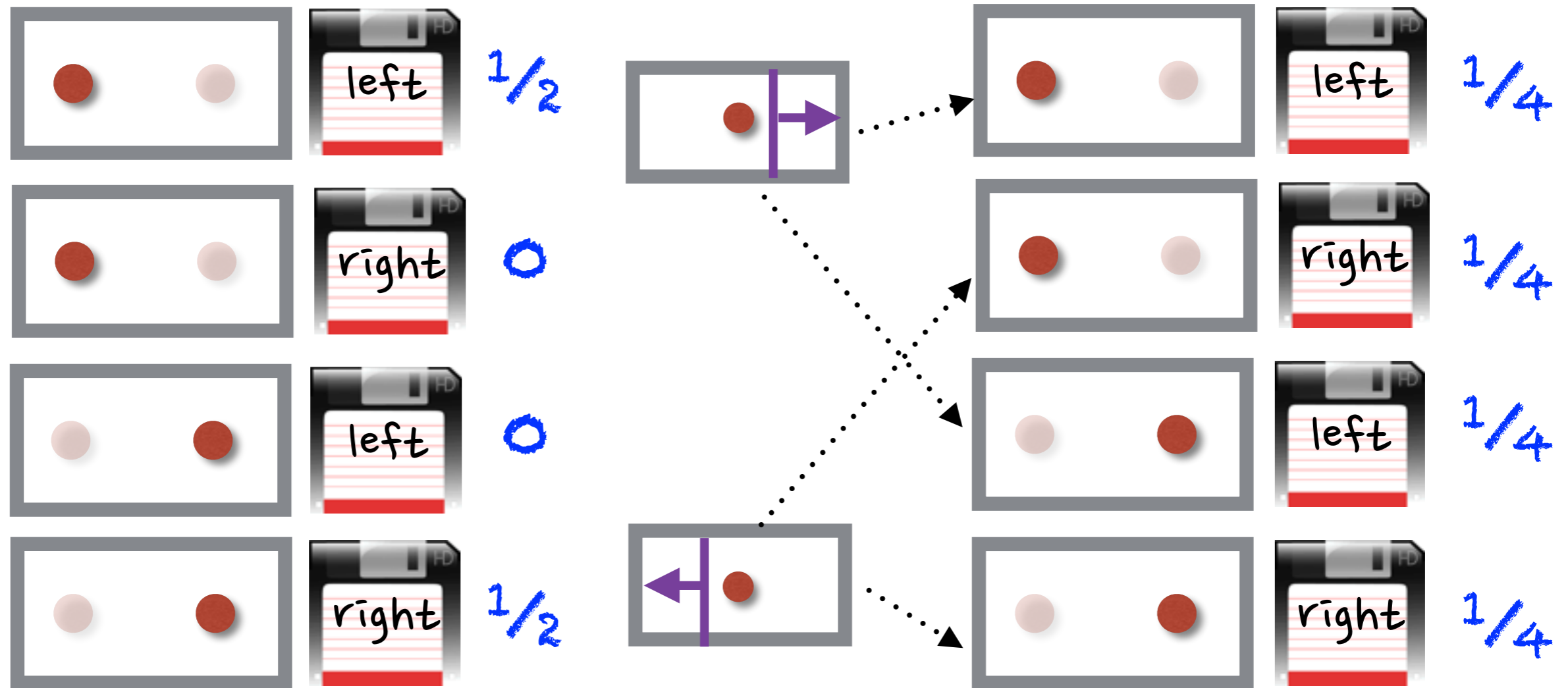
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



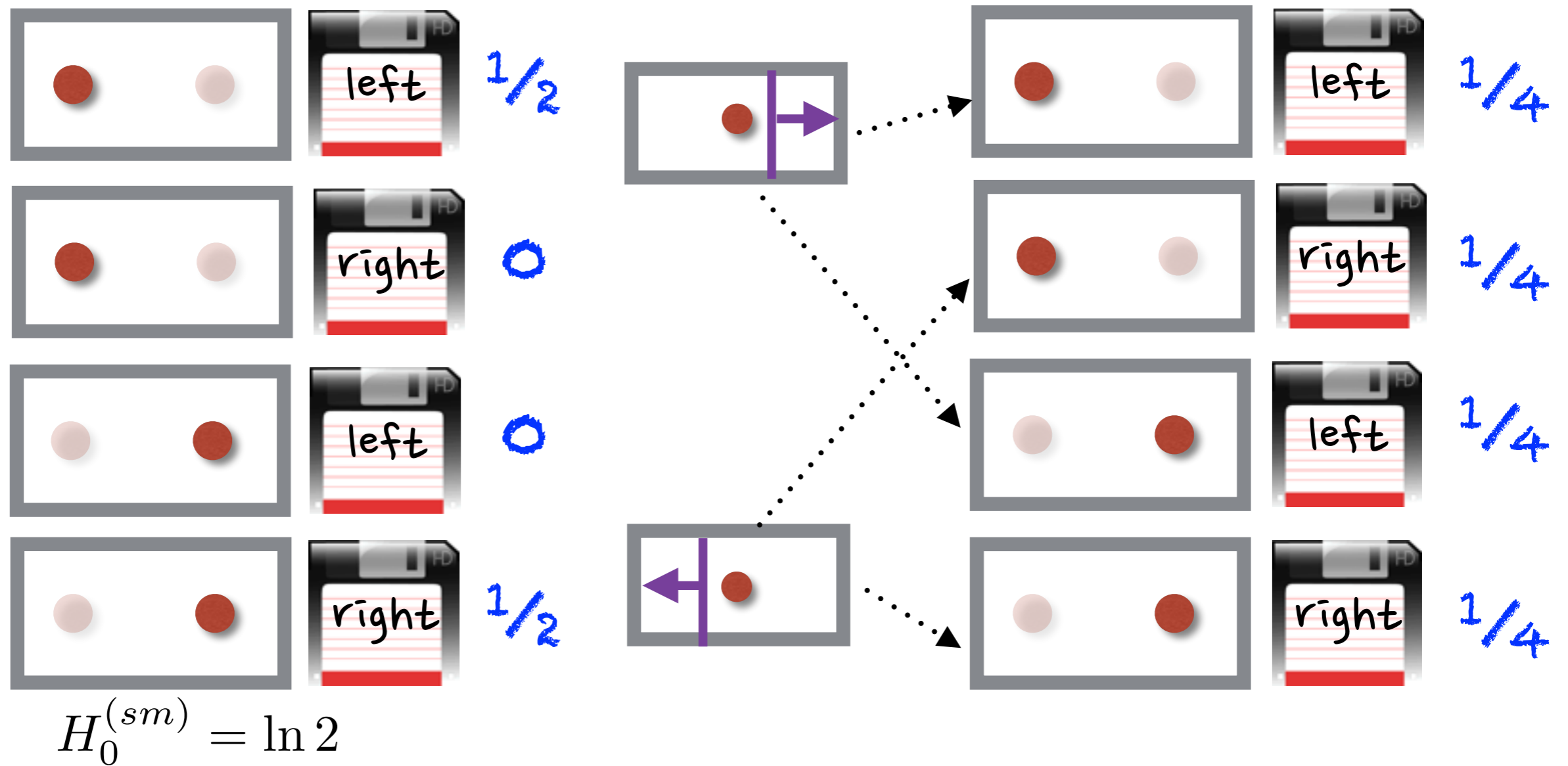
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



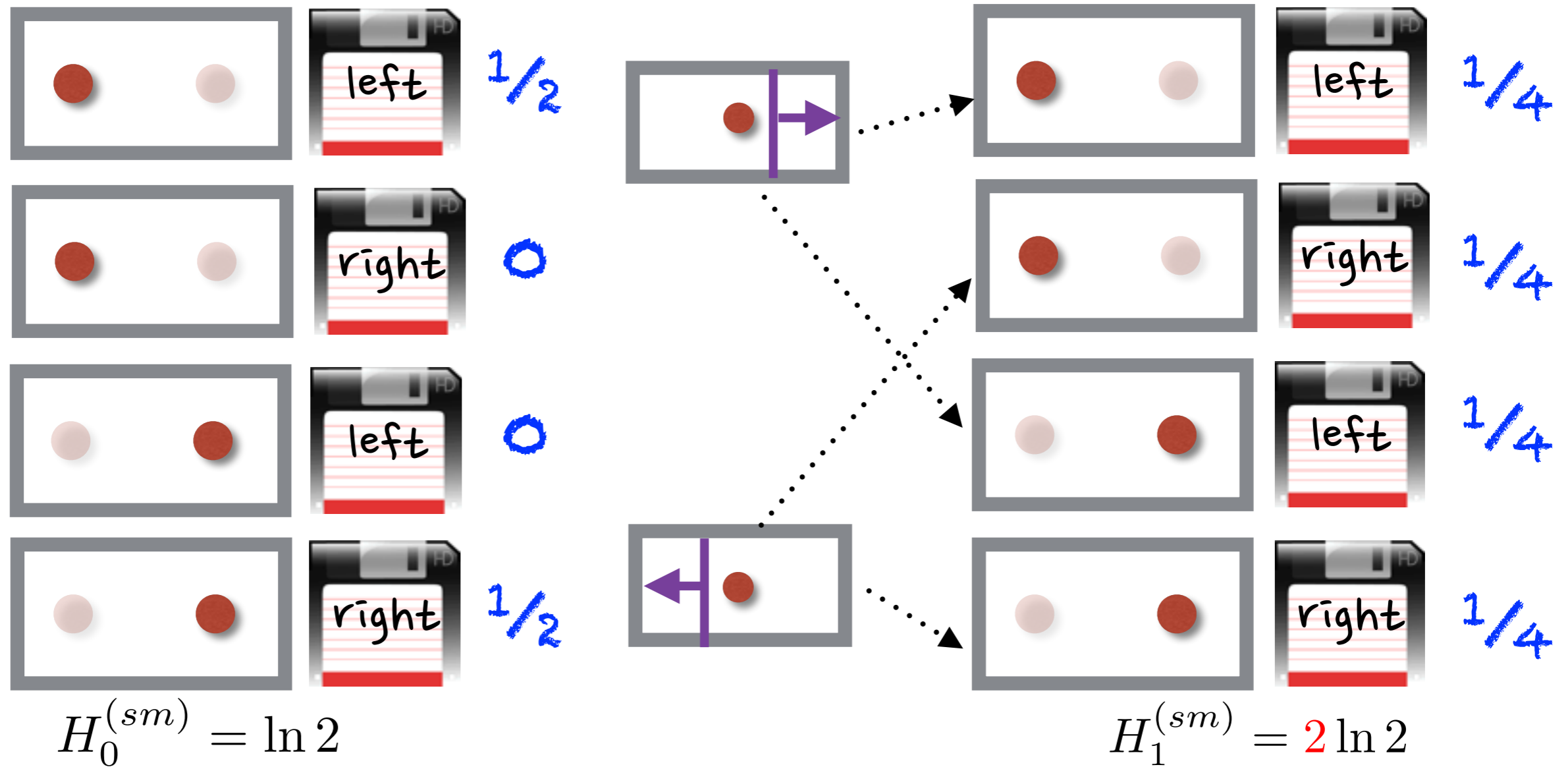
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



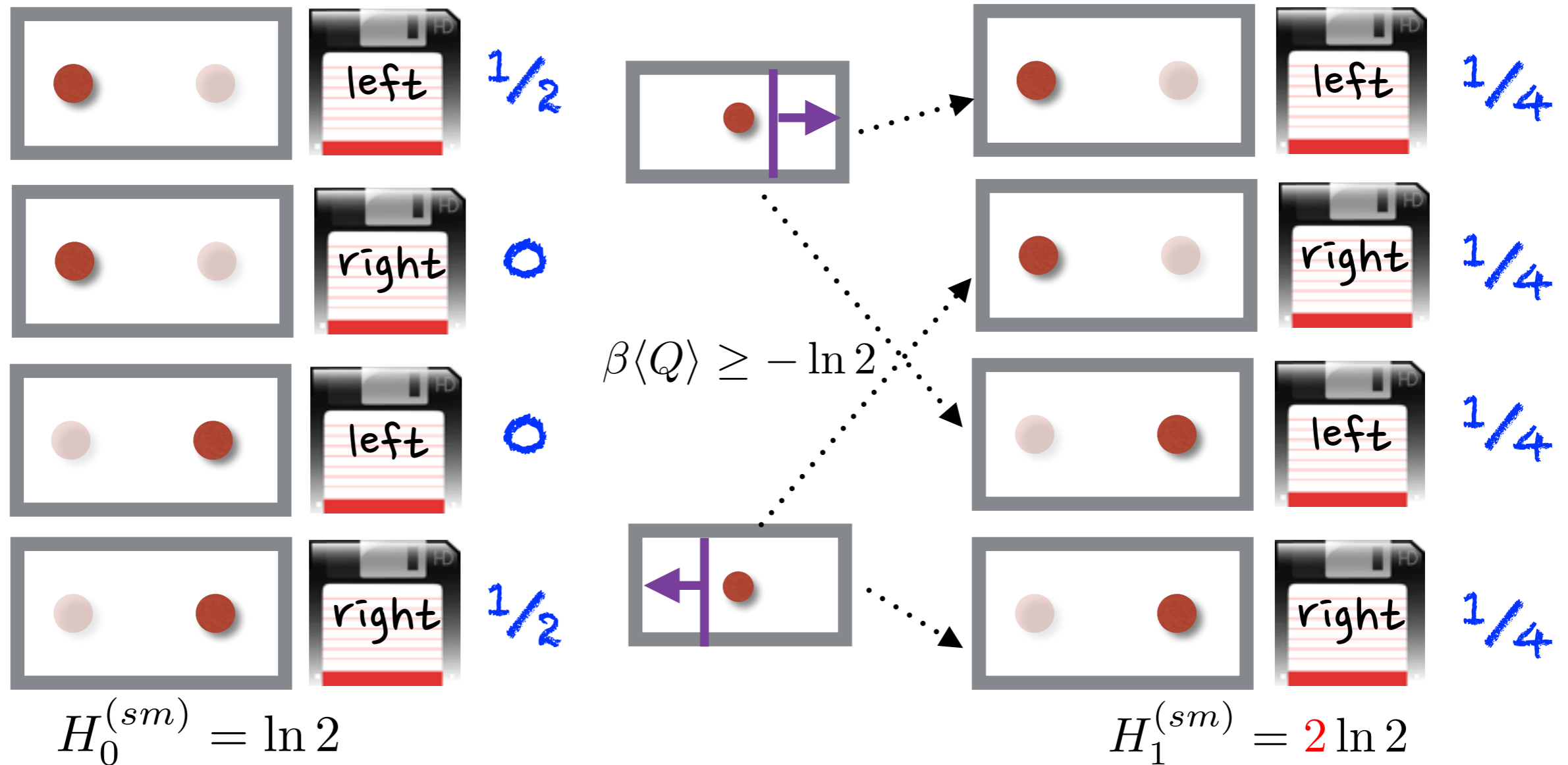
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



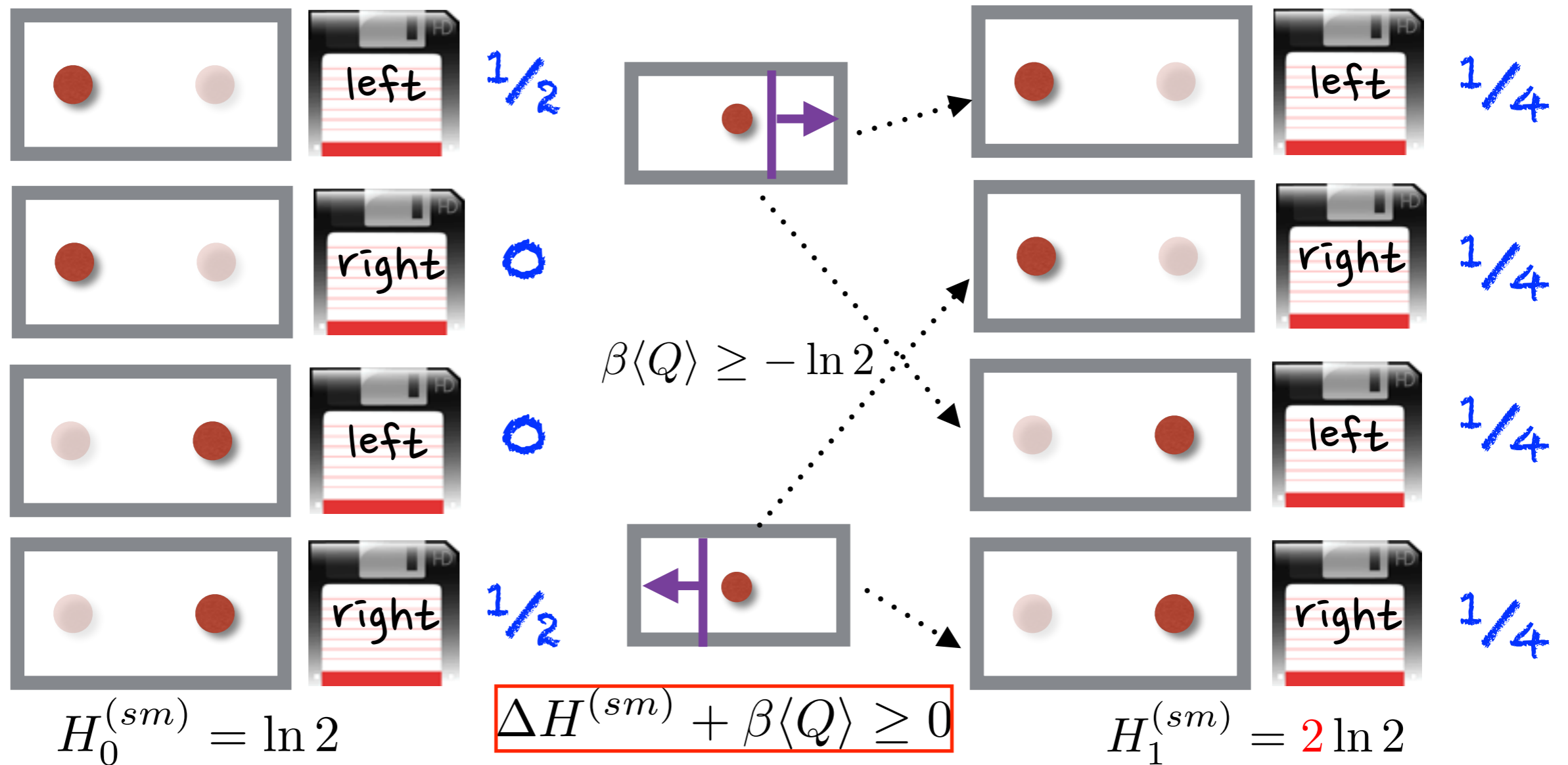
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



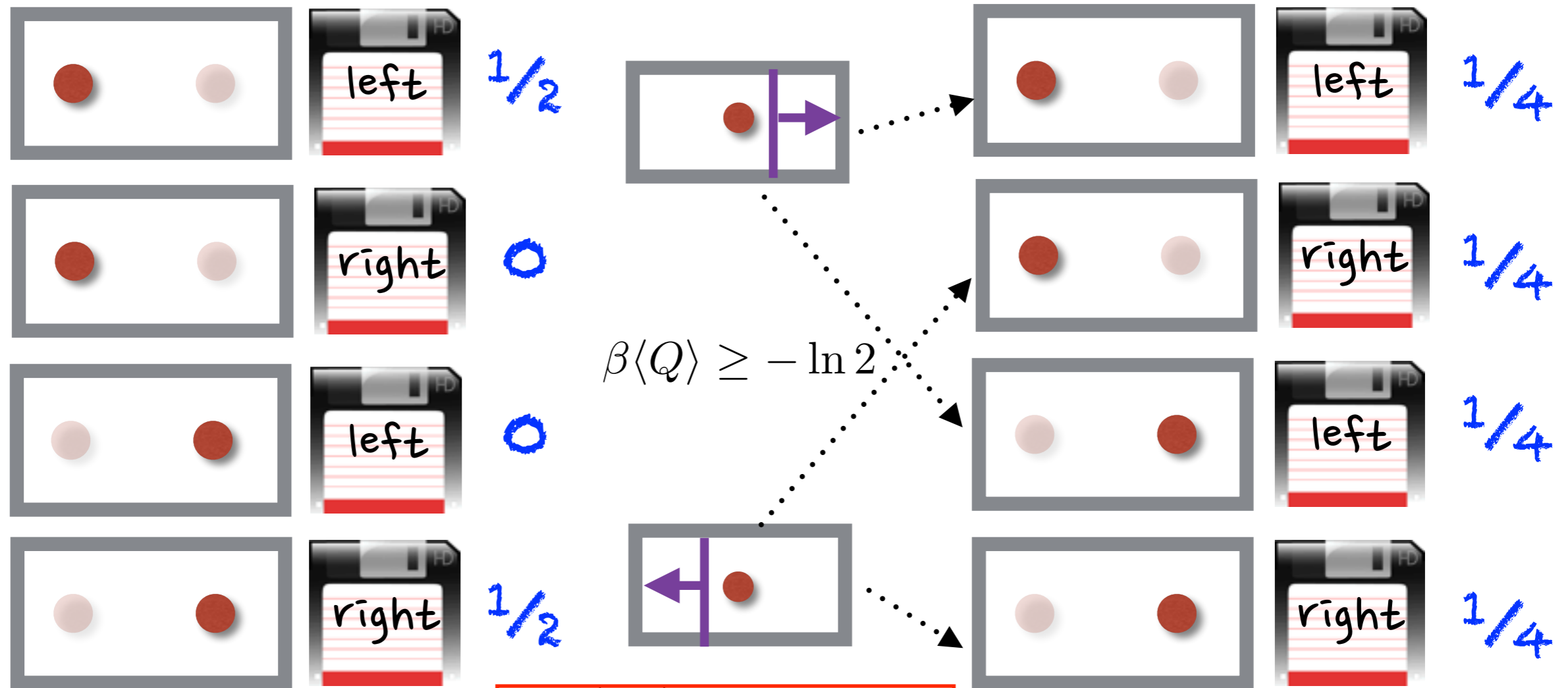
Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



Composite system & Shannon entropy

$$H = - \sum_c P^c \ln P^c$$



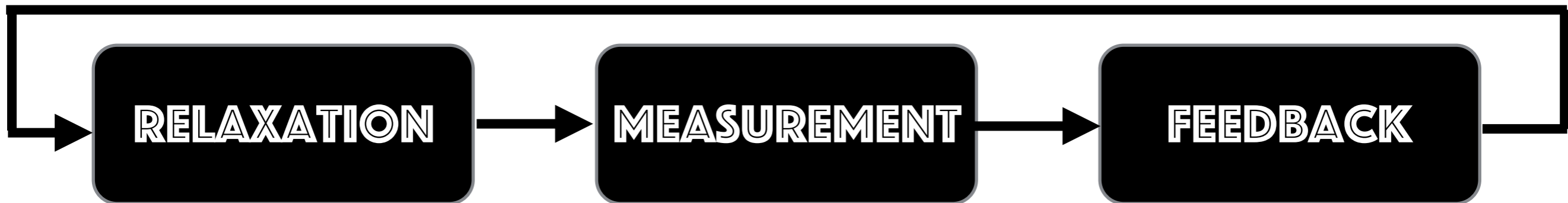
$$H_0^{(sm)} = \ln 2$$

$$\Delta H^{(sm)} + \beta \langle Q \rangle \geq 0$$

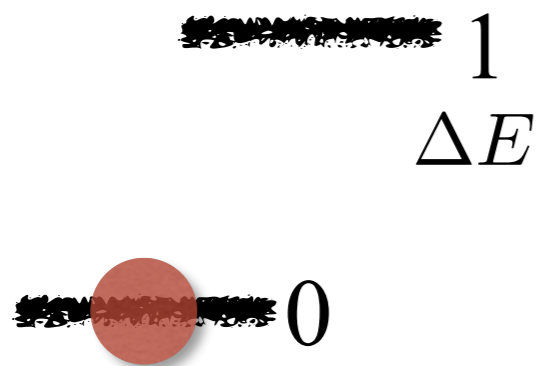
Sagawa & Ueda

$$H_1^{(sm)} = 2 \ln 2$$

Two-state model



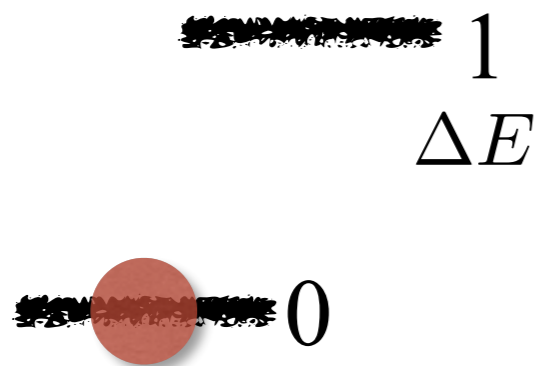
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



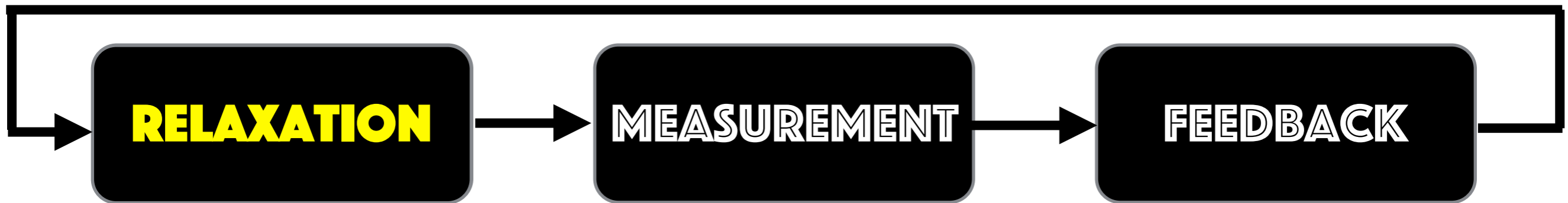
Two-state model



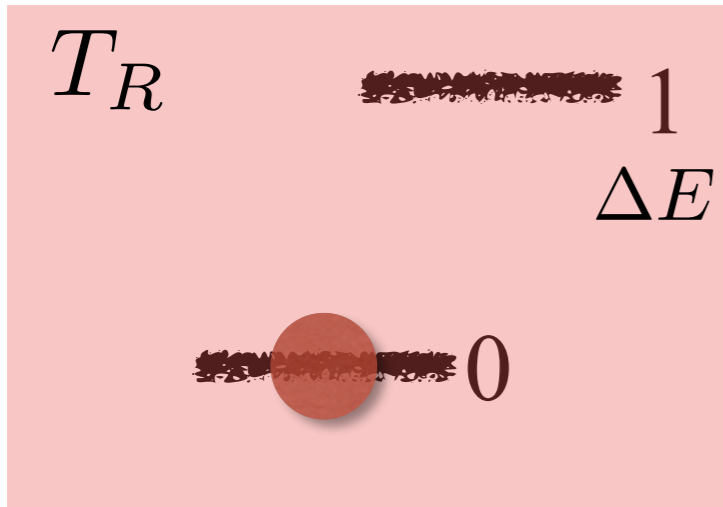
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



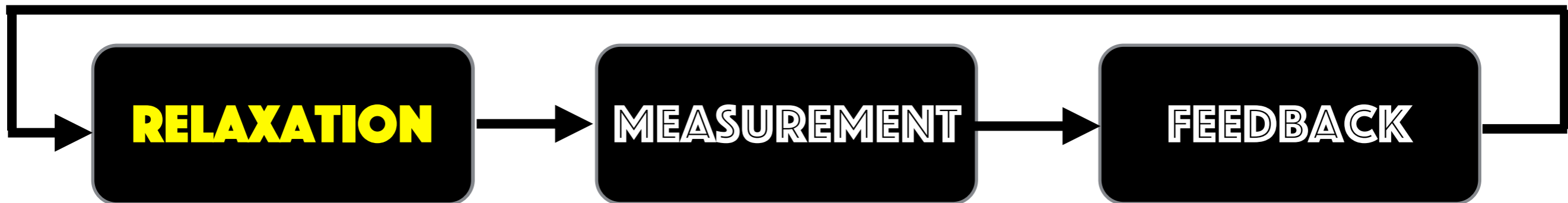
Two-state model



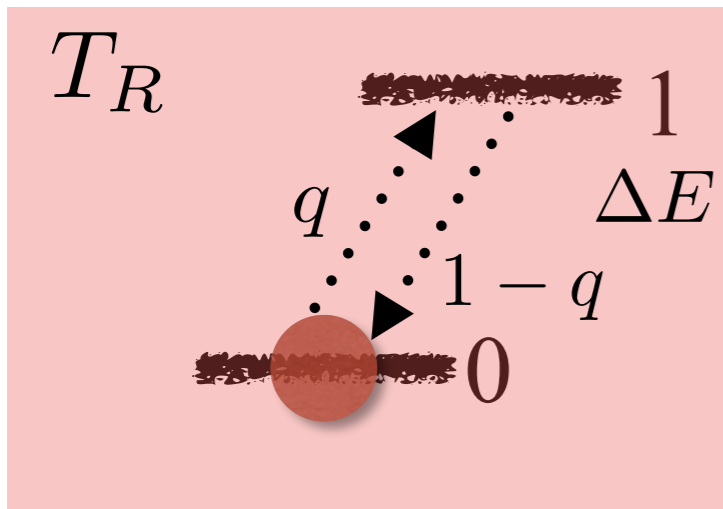
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



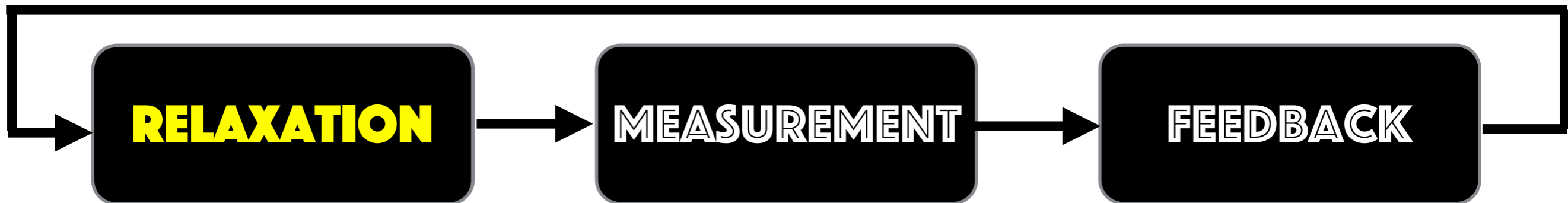
Two-state model



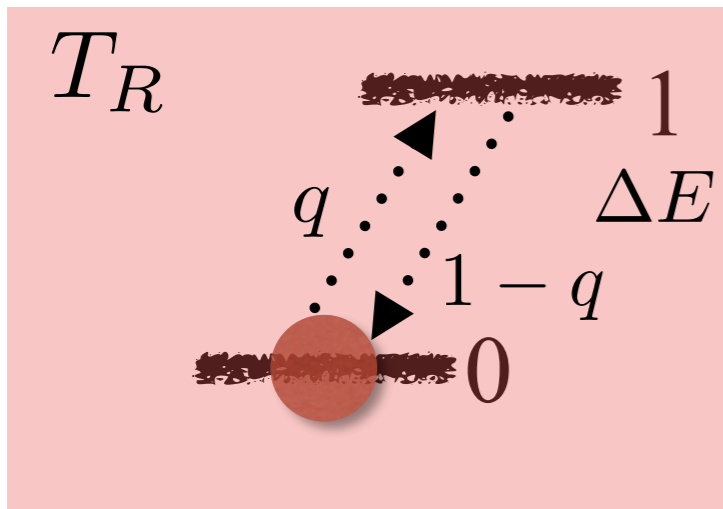
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



Two-state model

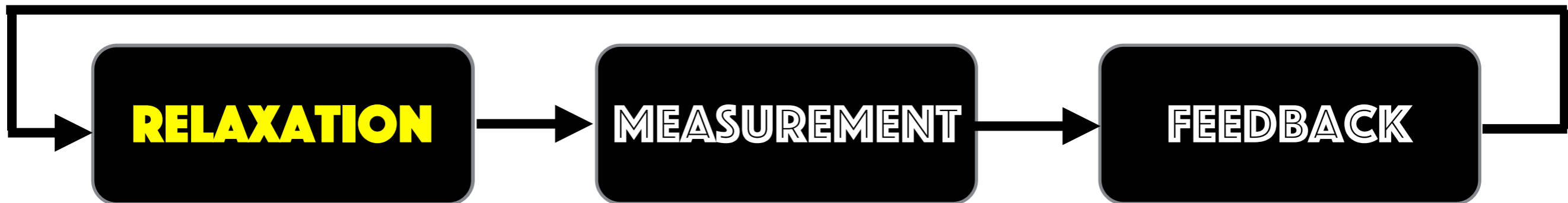


$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



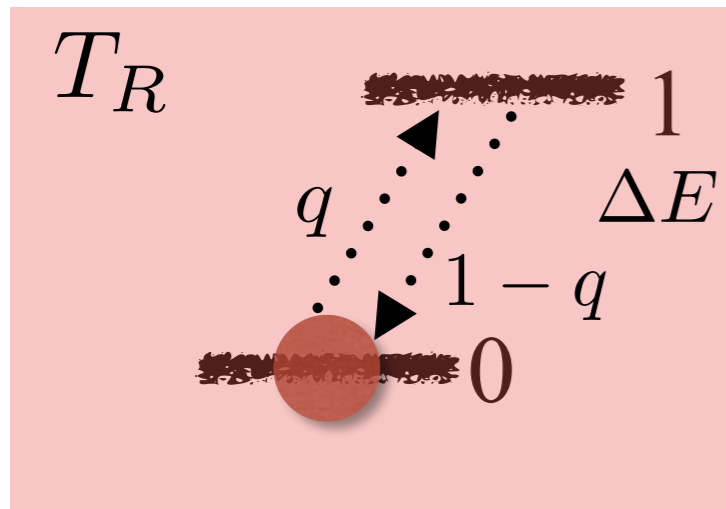
$$q/(1 - q) = e^{-\Delta E/T_R}$$

Two-state model



$$|P_0\rangle \quad \mathcal{T}_R(t_R)|P_0\rangle = |P_1\rangle$$

$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



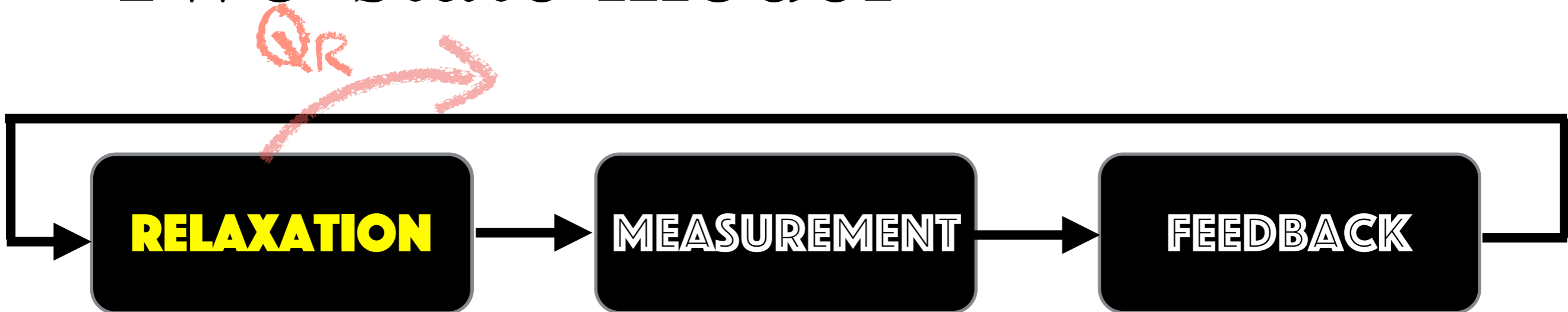
$$q/(1-q) = e^{-\Delta E/T_R}$$



$$\mathcal{R} \equiv e^{-t_R} \quad \bar{\mathcal{R}} \equiv 1 - \mathcal{R} \quad \bar{q} \equiv 1 - q$$

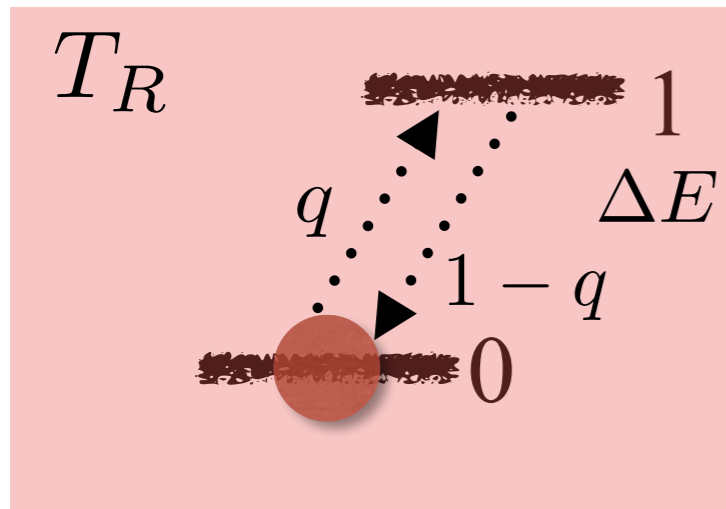
$$\mathcal{T}_R(t_R) = \begin{pmatrix} \mathcal{R} + \bar{\mathcal{R}}\bar{q} & 0 & \bar{\mathcal{R}}\bar{q} & 0 \\ 0 & \mathcal{R} + \bar{\mathcal{R}}q & 0 & \bar{\mathcal{R}}q \\ \bar{\mathcal{R}}q & 0 & \mathcal{R} + \bar{\mathcal{R}}q & 0 \\ 0 & \bar{\mathcal{R}}q & 0 & \mathcal{R} + \bar{\mathcal{R}}q \end{pmatrix}$$

Two-state model



$$|P_0\rangle \quad \mathcal{T}_R(t_R)|P_0\rangle = |P_1\rangle$$

$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



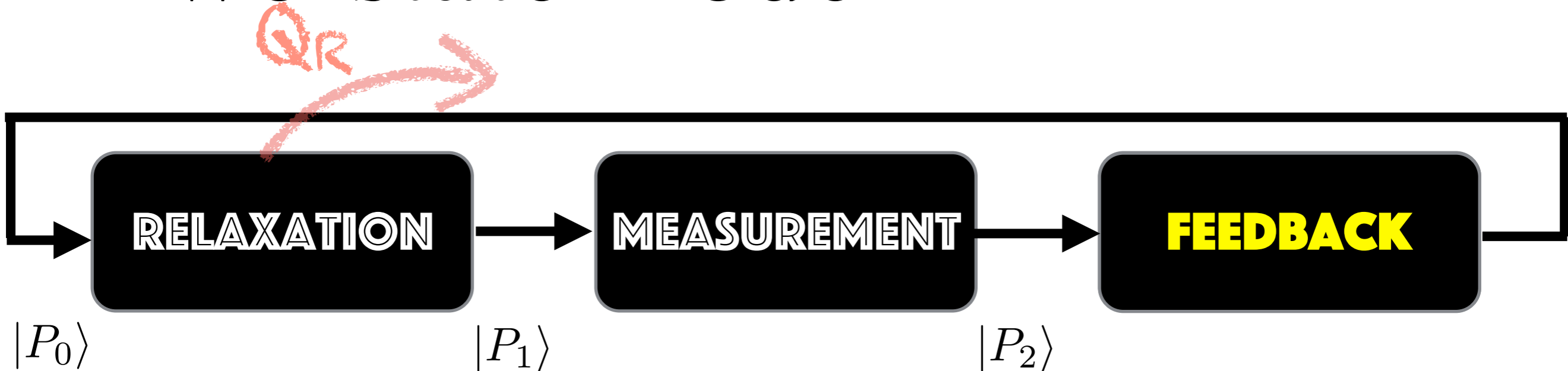
$$q/(1-q) = e^{-\Delta E/T_R}$$



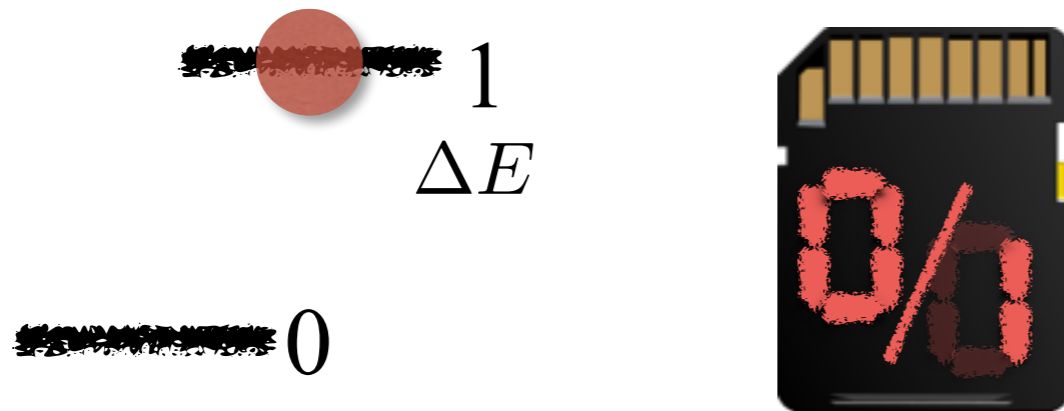
$$\mathcal{R} \equiv e^{-t_R} \quad \bar{\mathcal{R}} \equiv 1 - \mathcal{R} \quad \bar{q} \equiv 1 - q$$

$$\mathcal{T}_R(t_R) = \begin{pmatrix} \mathcal{R} + \bar{\mathcal{R}}\bar{q} & 0 & \bar{\mathcal{R}}\bar{q} & 0 \\ 0 & \mathcal{R} + \bar{\mathcal{R}}q & 0 & \bar{\mathcal{R}}q \\ \bar{\mathcal{R}}q & 0 & \mathcal{R} + \bar{\mathcal{R}}q & 0 \\ 0 & \bar{\mathcal{R}}q & 0 & \mathcal{R} + \bar{\mathcal{R}}q \end{pmatrix}$$

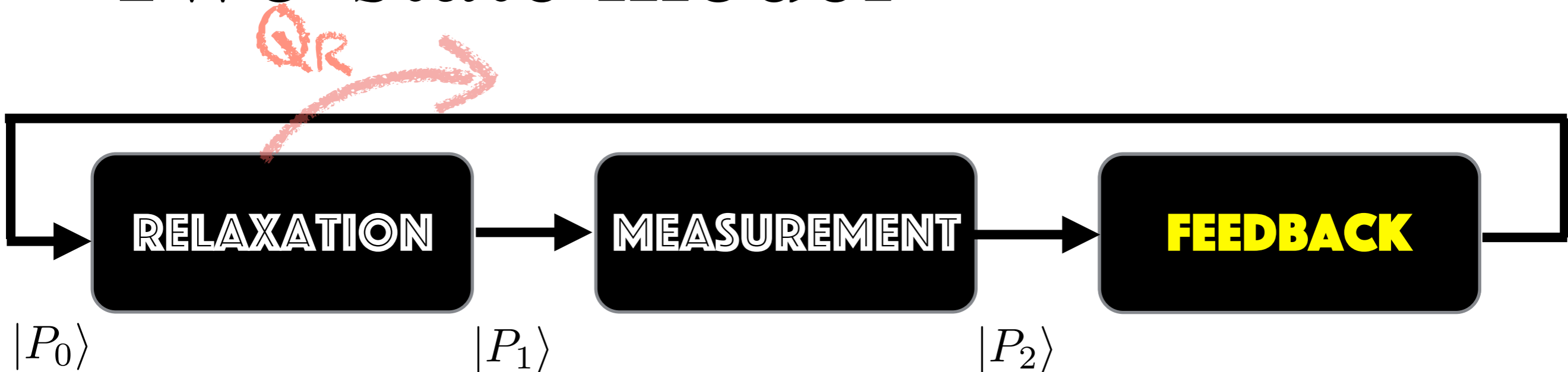
Two-state model



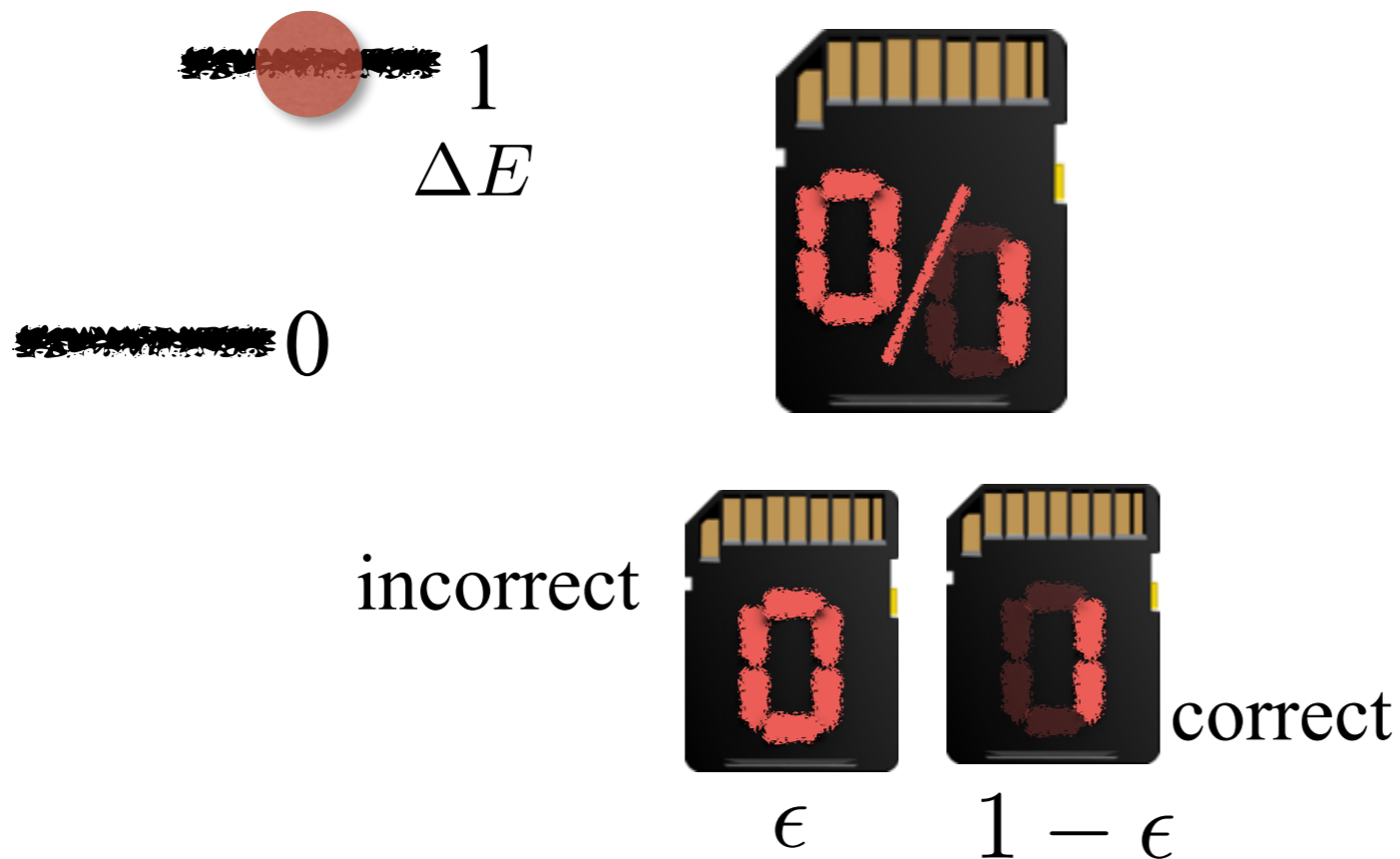
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



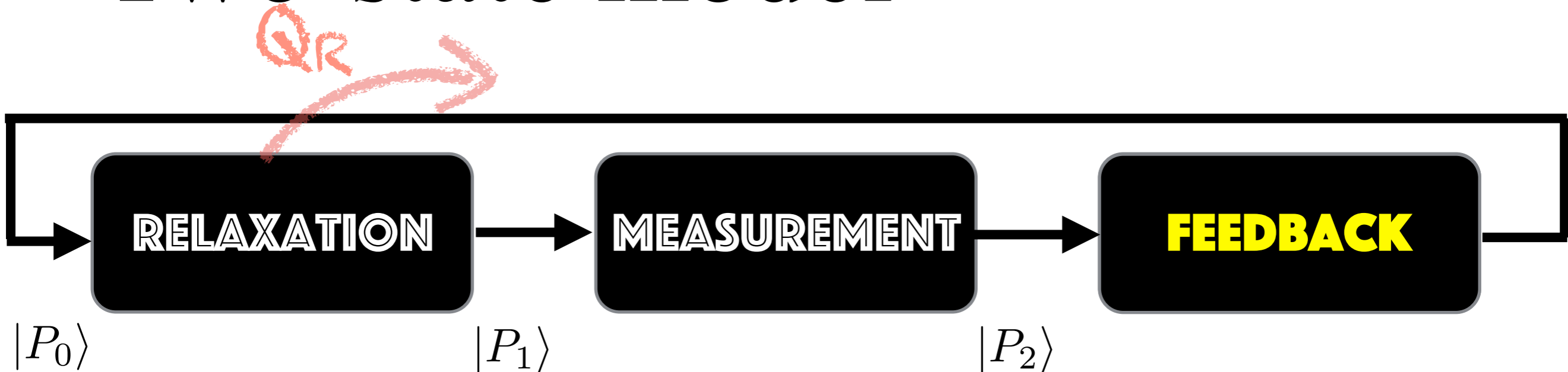
Two-state model



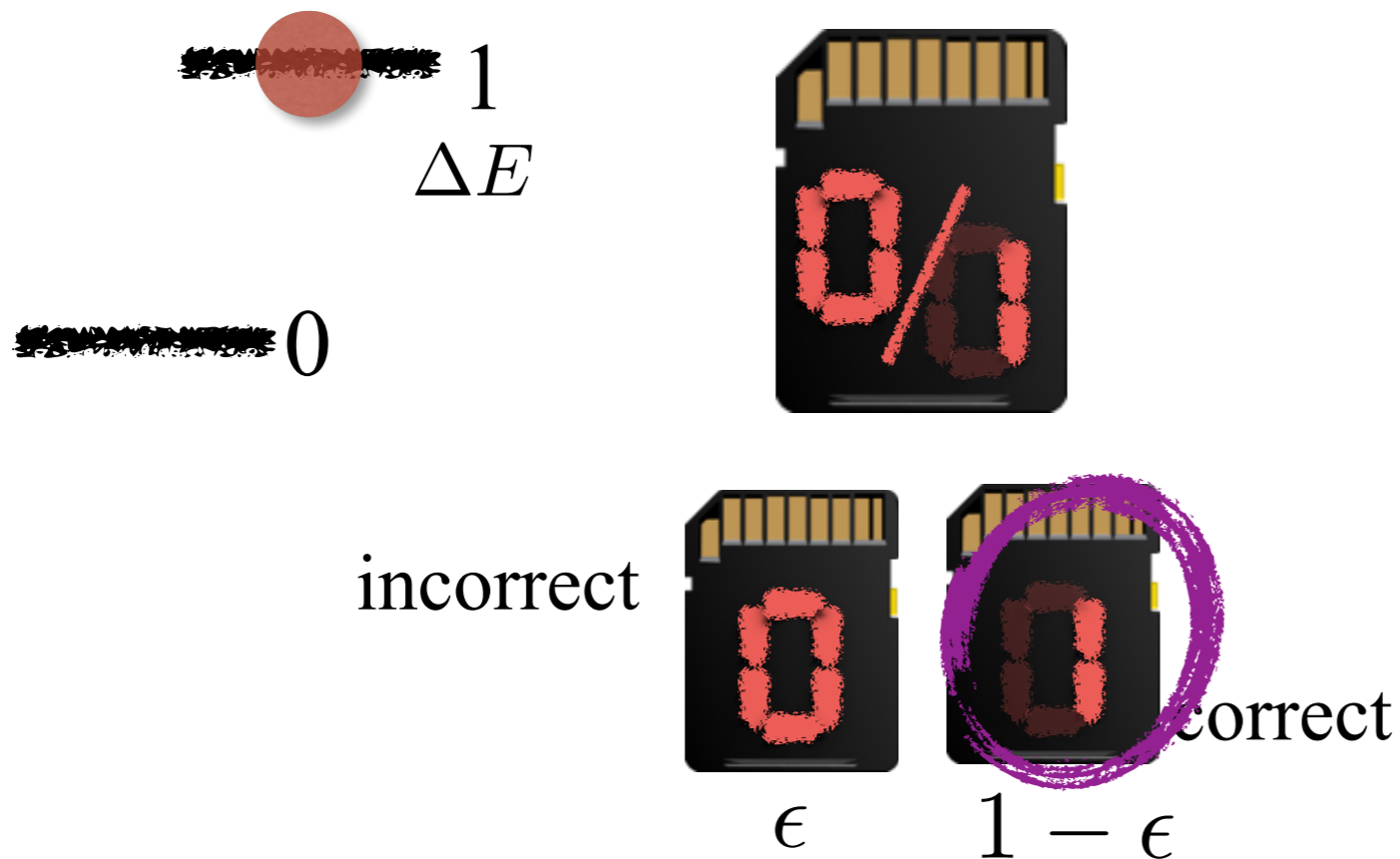
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



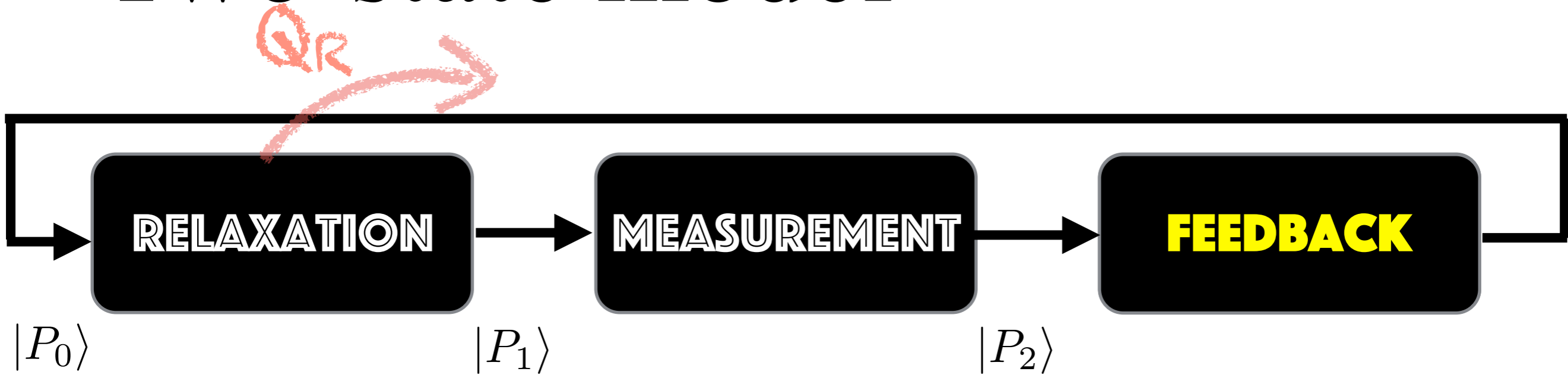
Two-state model



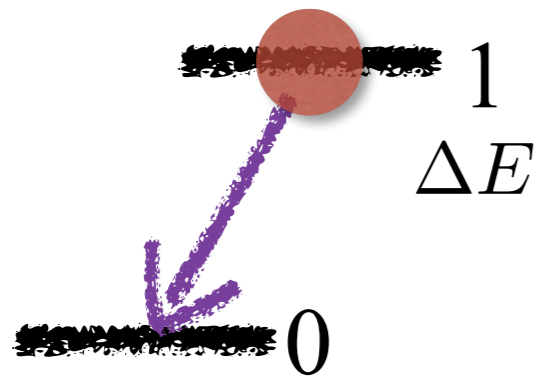
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



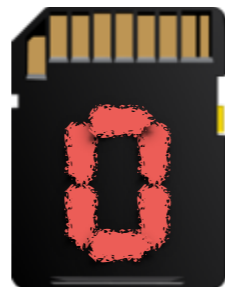
Two-state model



$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



incorrect

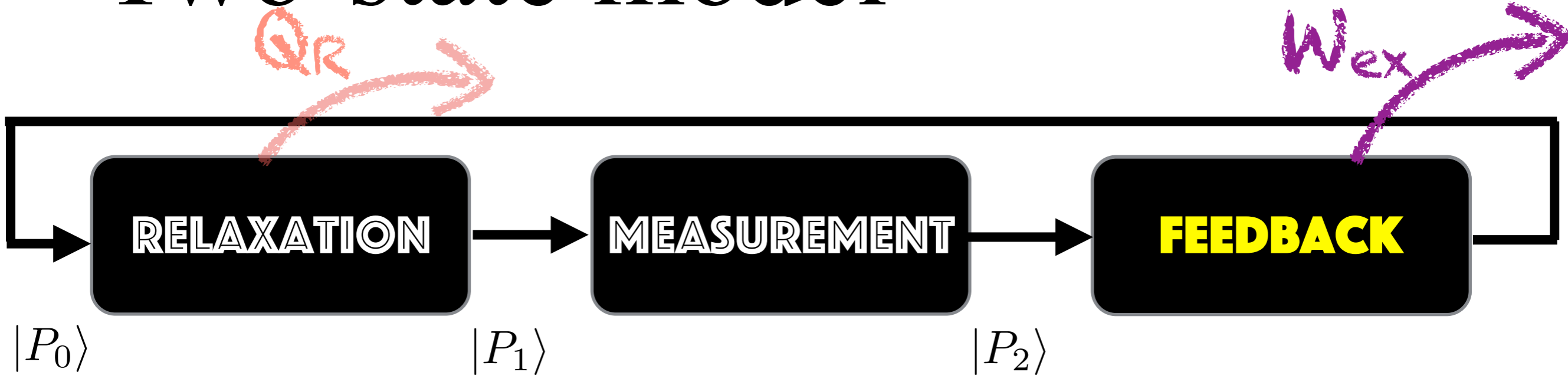


correct

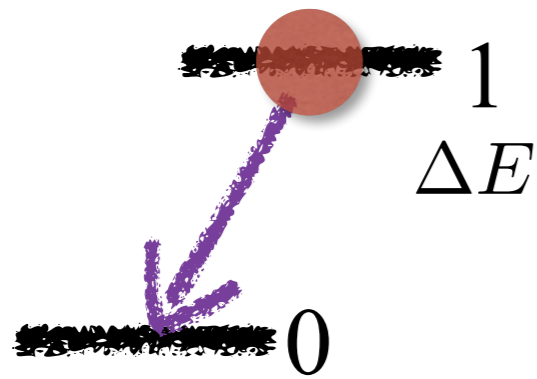
ϵ

$1 - \epsilon$

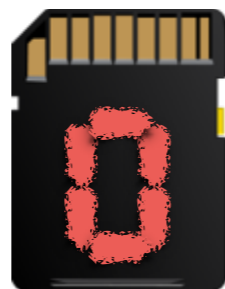
Two-state model



$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



incorrect

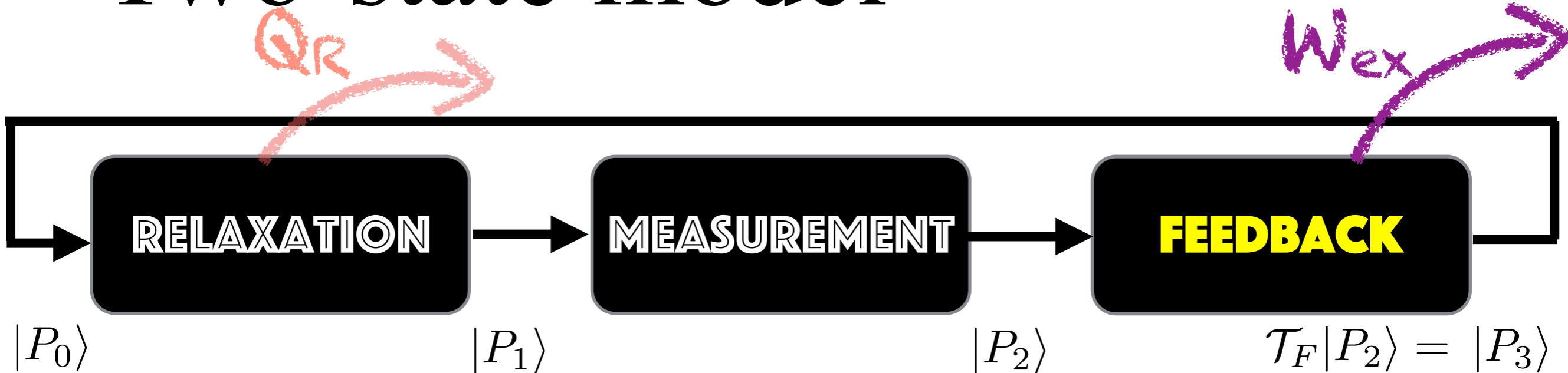


correct

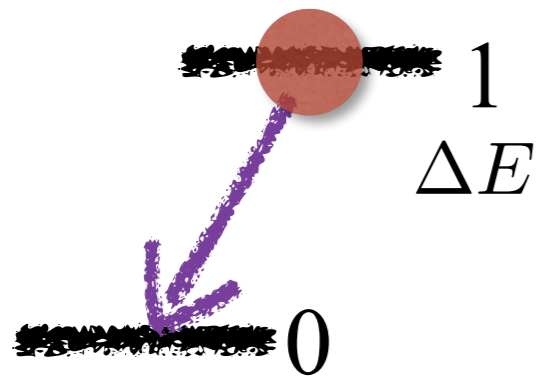
ϵ

$1 - \epsilon$

Two-state model

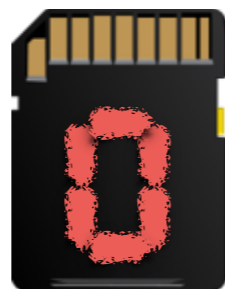


$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



$$\mathcal{T}_F = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

incorrect

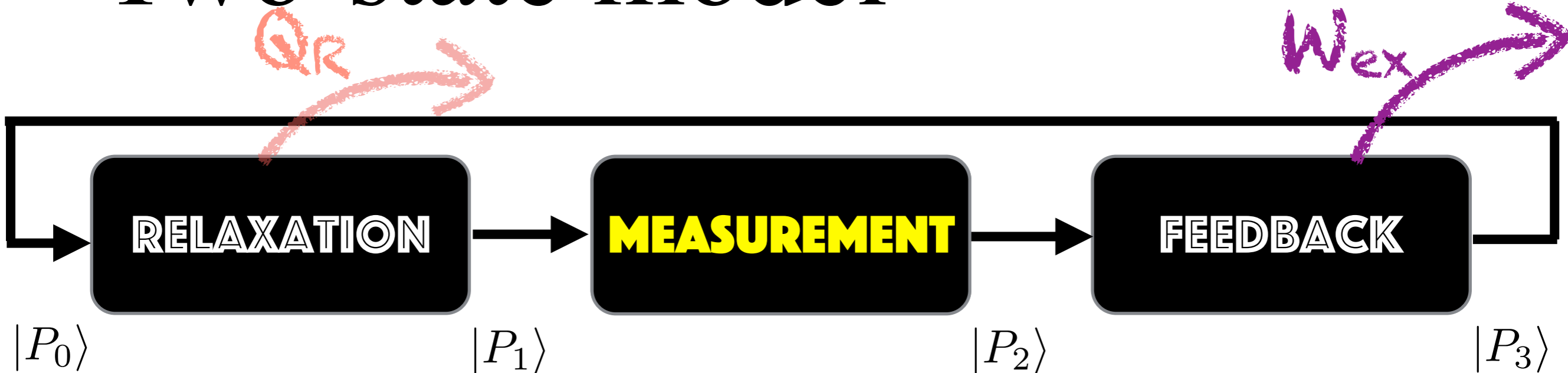


correct

ϵ

$1 - \epsilon$

Two-state model



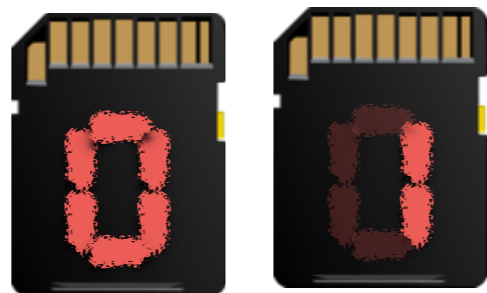
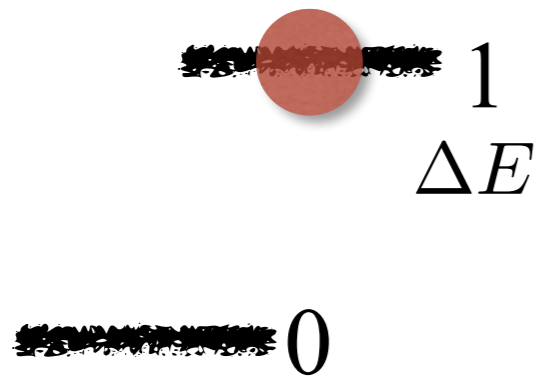
$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_3\rangle$

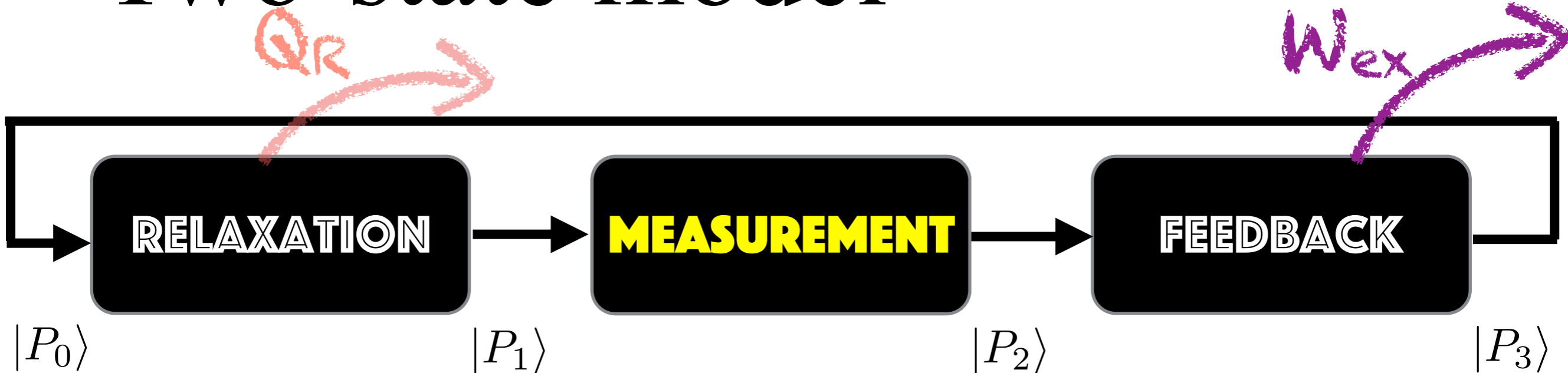
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



ϵ

$1 - \epsilon$

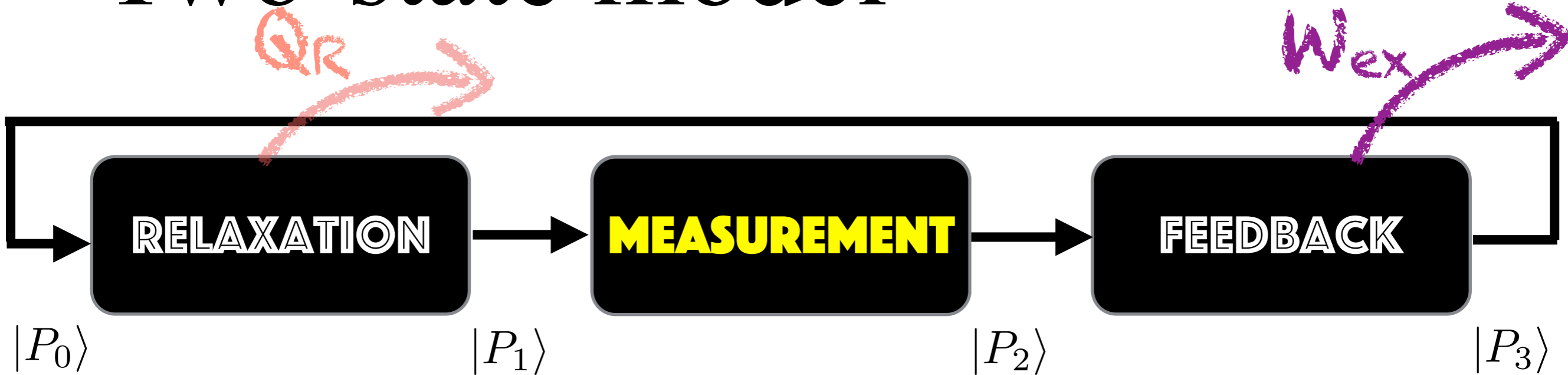
Two-state model



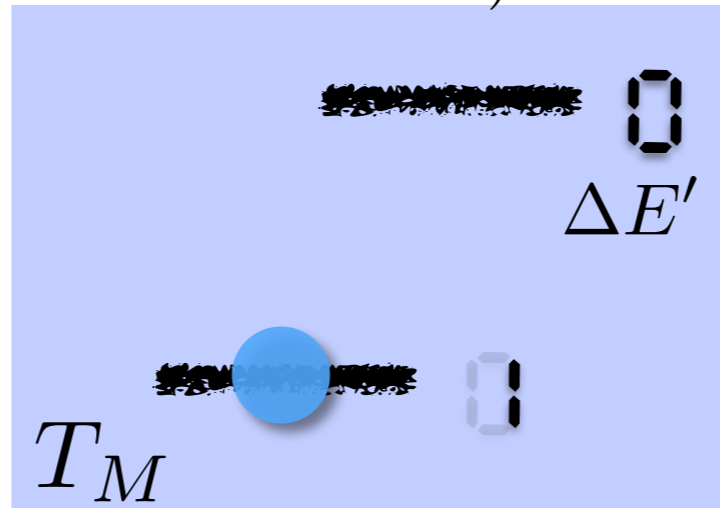
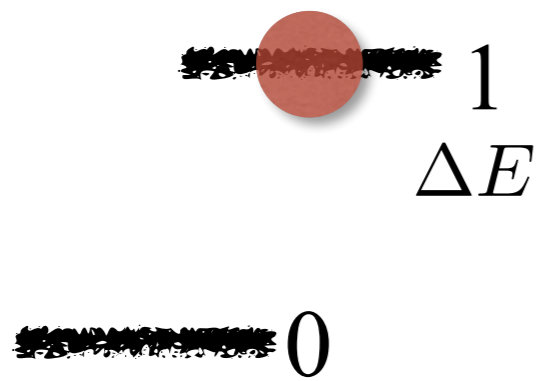
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



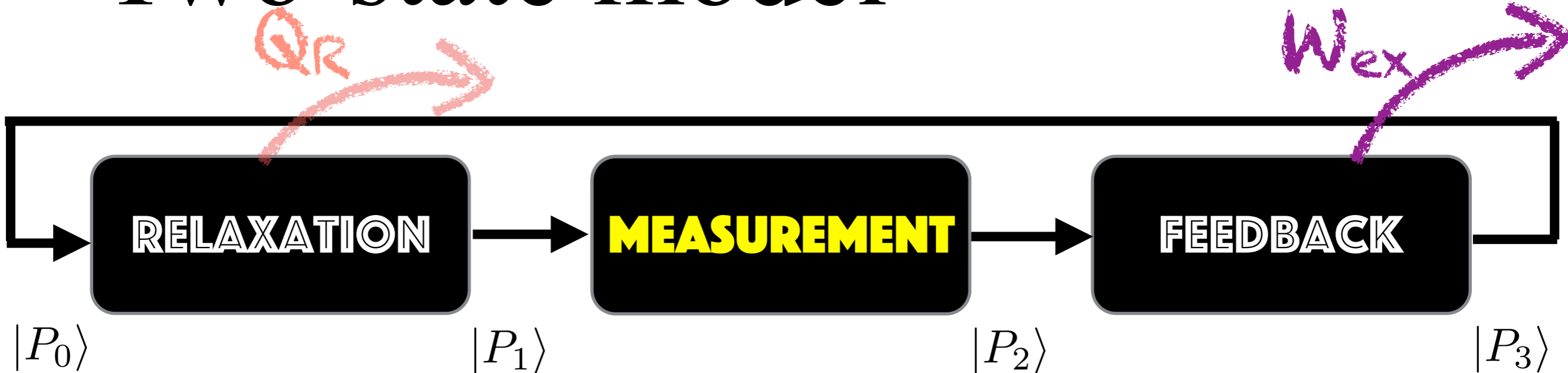
Two-state model



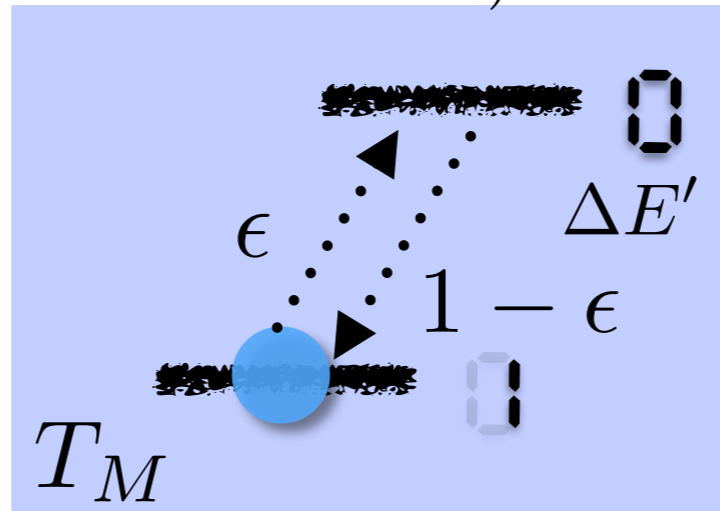
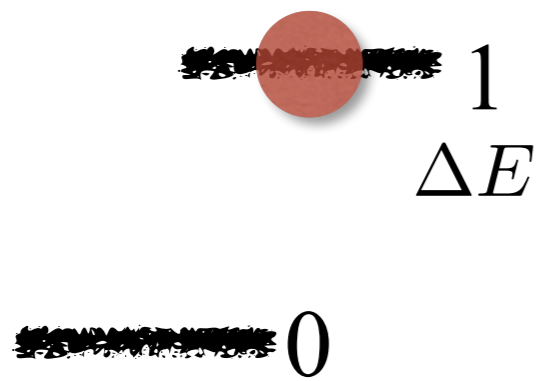
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



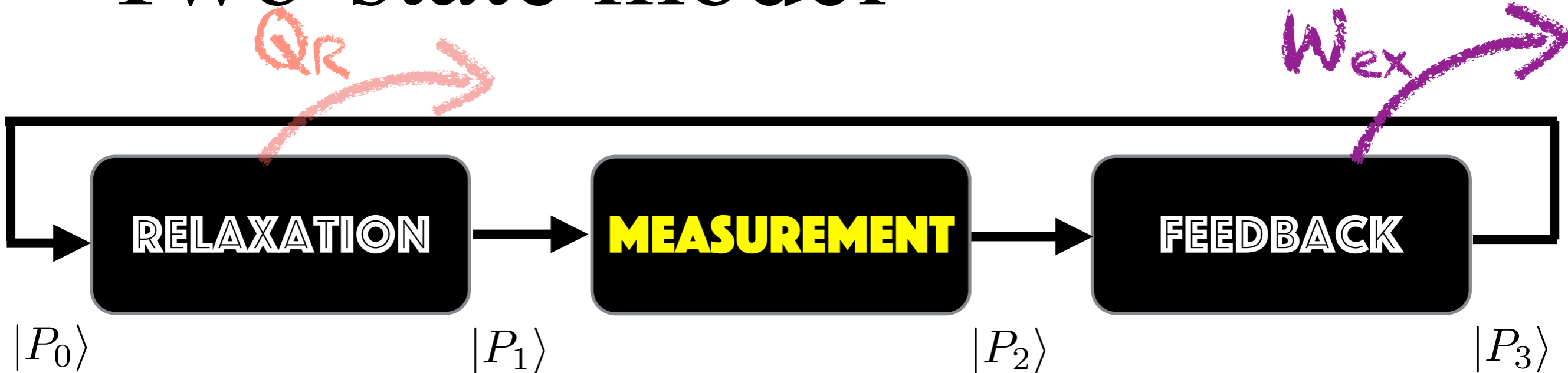
Two-state model



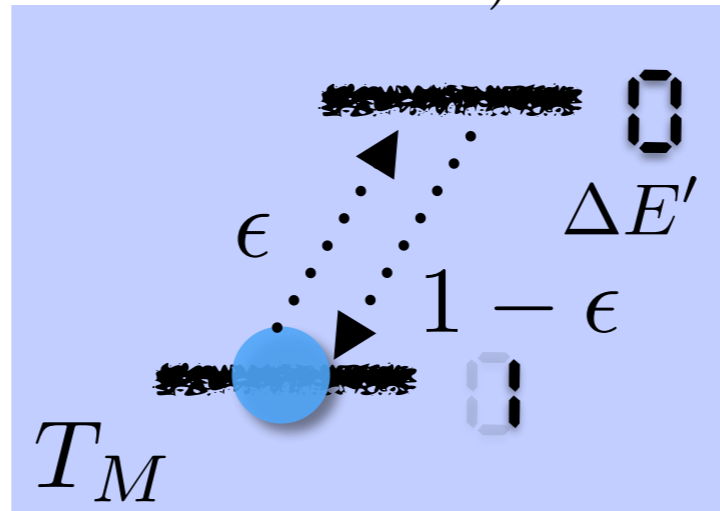
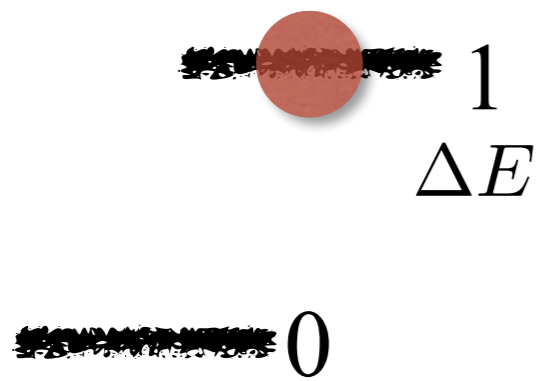
$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



Two-state model

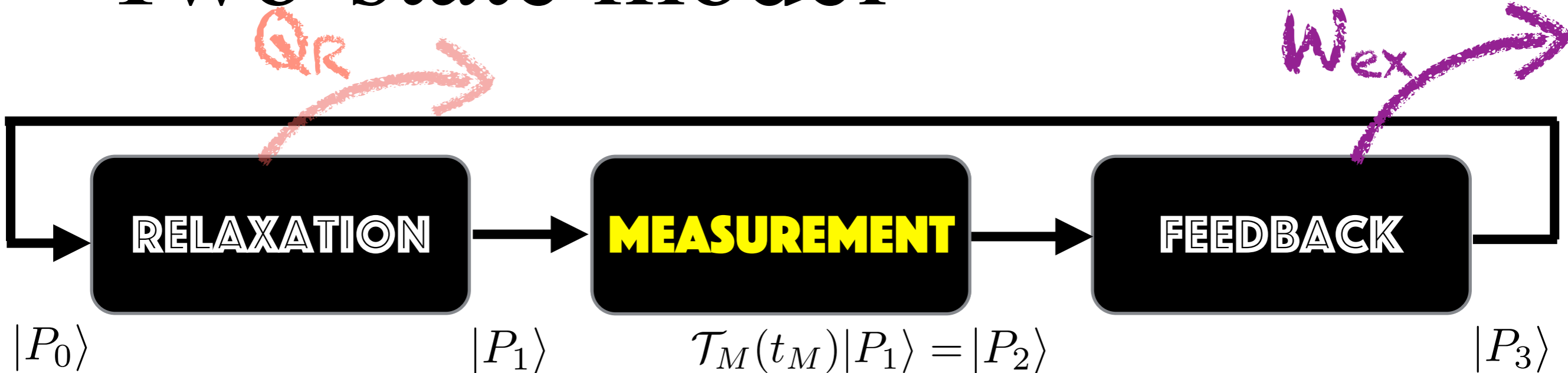


$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$

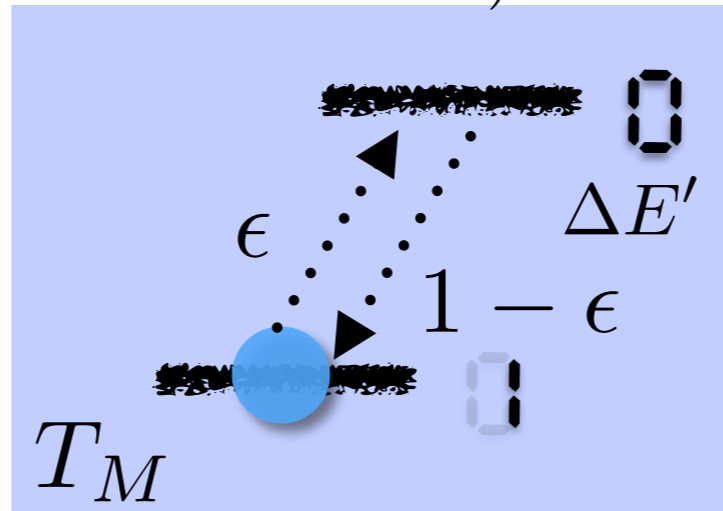
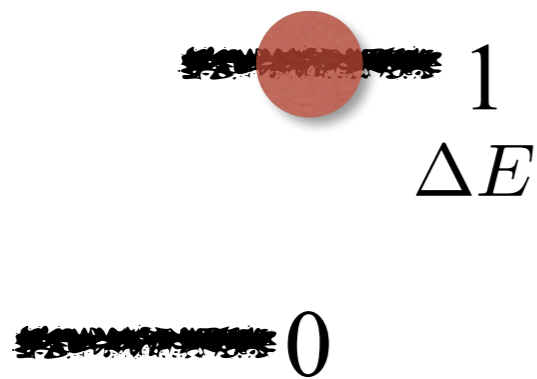


$$\epsilon / (1 - \epsilon) = e^{-\Delta E' / T_M}$$

Two-state model



$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



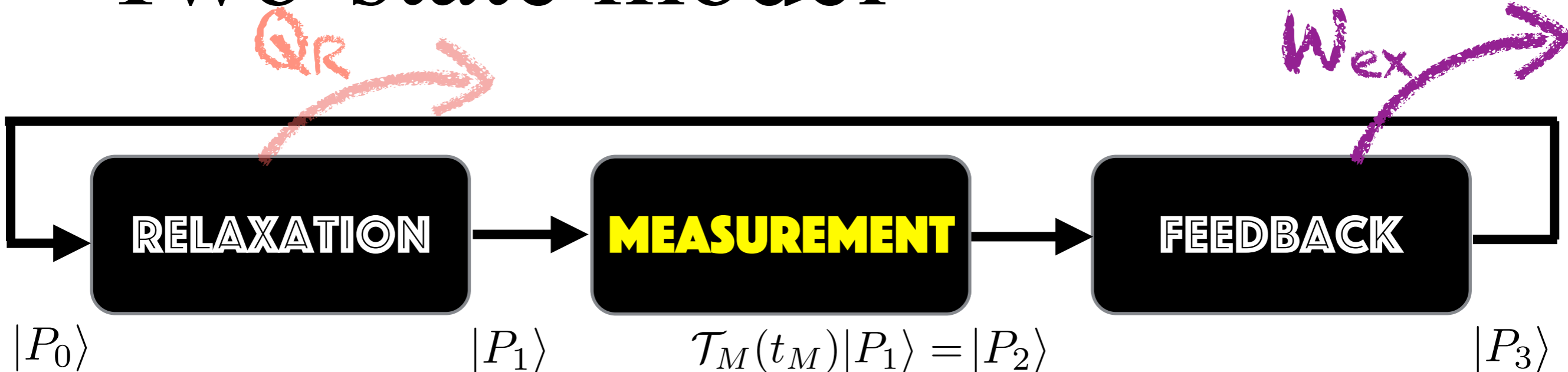
$$\epsilon / (1 - \epsilon) = e^{-\Delta E' / T_M}$$

$$\bar{\epsilon} \equiv 1 - \epsilon$$

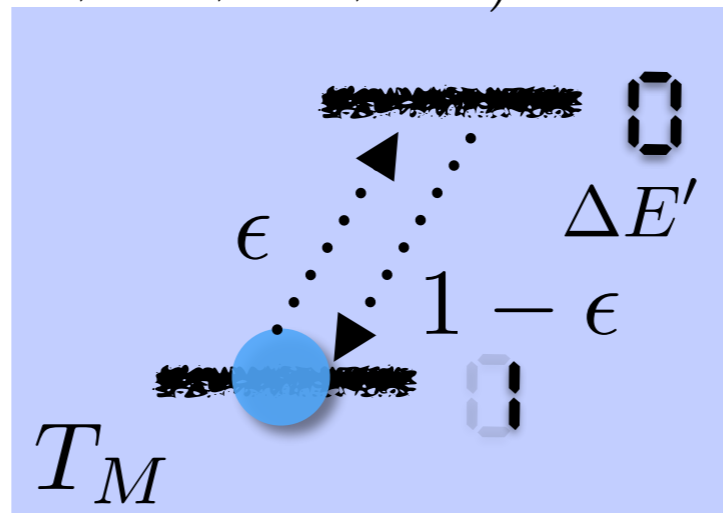
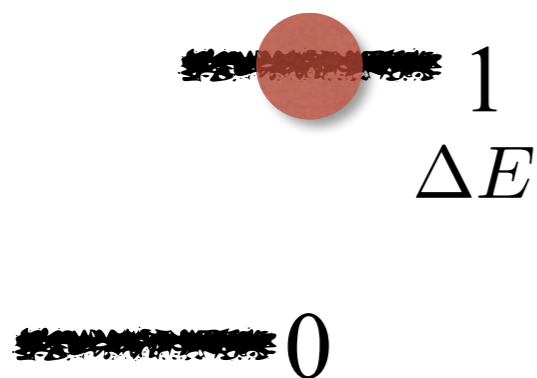
$$\mathcal{M} \equiv e^{-t_M} \quad \bar{\mathcal{M}} \equiv 1 - \mathcal{M}$$

$$\mathcal{T}_M(t_M) = \begin{pmatrix} \mathcal{M} + \bar{\mathcal{M}}\bar{\epsilon} & \bar{\mathcal{M}}\bar{\epsilon} & 0 & 0 \\ \bar{\mathcal{M}}\epsilon & \mathcal{M} + \bar{\mathcal{M}}\epsilon & 0 & 0 \\ 0 & 0 & \mathcal{M} + \bar{\mathcal{M}}\epsilon & \bar{\mathcal{M}}\epsilon \\ 0 & 0 & \bar{\mathcal{M}}\bar{\epsilon} & \mathcal{M} + \bar{\mathcal{M}}\bar{\epsilon} \end{pmatrix}$$

Two-state model



$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



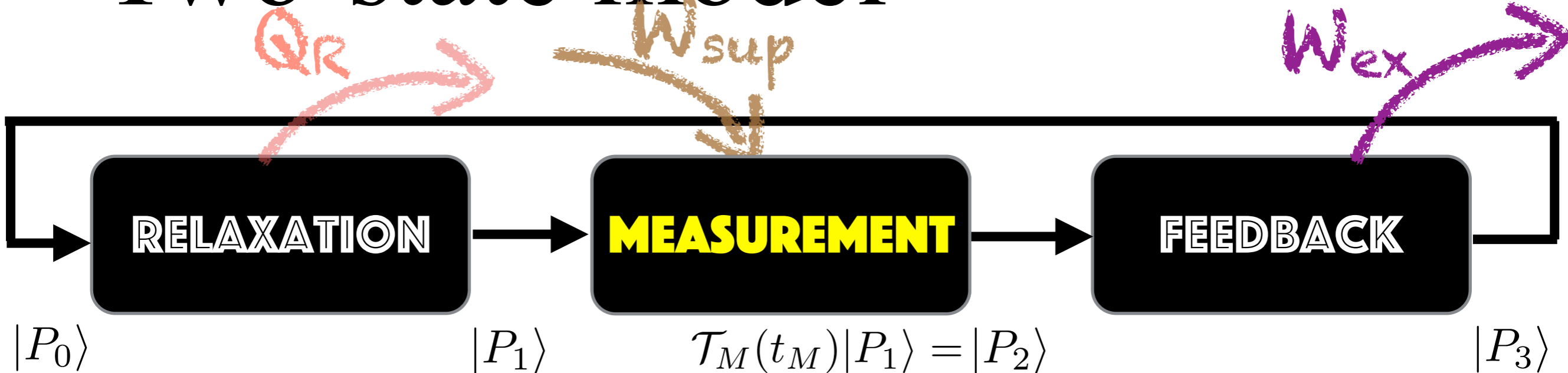
$$\epsilon / (1 - \epsilon) = e^{-\Delta E' / T_M}$$

$$\bar{\epsilon} \equiv 1 - \epsilon$$

$$\mathcal{M} \equiv e^{-t_M} \quad \bar{\mathcal{M}} \equiv 1 - \mathcal{M}$$

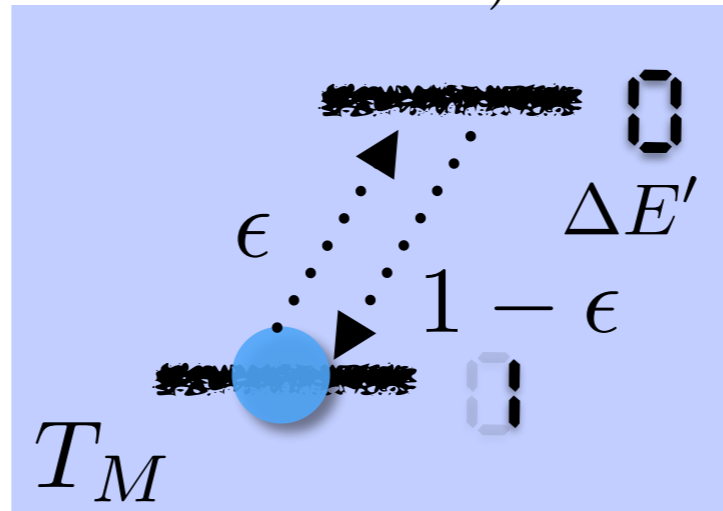
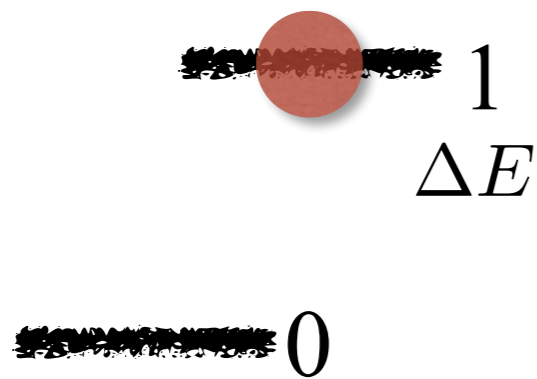
$$\begin{array}{c}
 \mathcal{T}_M(t_M) \\
 t_M \rightarrow \infty
 \end{array}
 \begin{array}{cc}
 \begin{array}{c} \epsilon \\ 1 - \epsilon \end{array} & \begin{array}{c} \epsilon \\ 1 - \epsilon \end{array}
 \end{array}
 \begin{pmatrix}
 \bar{\mathcal{M}}\bar{\epsilon} & 0 & 0 \\
 \mathcal{M} + \bar{\mathcal{M}}\epsilon & 0 & 0 \\
 0 & \mathcal{M} + \bar{\mathcal{M}}\epsilon & \bar{\mathcal{M}}\epsilon \\
 0 & \bar{\mathcal{M}}\bar{\epsilon} & \mathcal{M} + \bar{\mathcal{M}}\bar{\epsilon}
 \end{pmatrix}$$

Two-state model



$$\mathcal{T}_M(t_M)|P_1\rangle = |P_2\rangle$$

$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



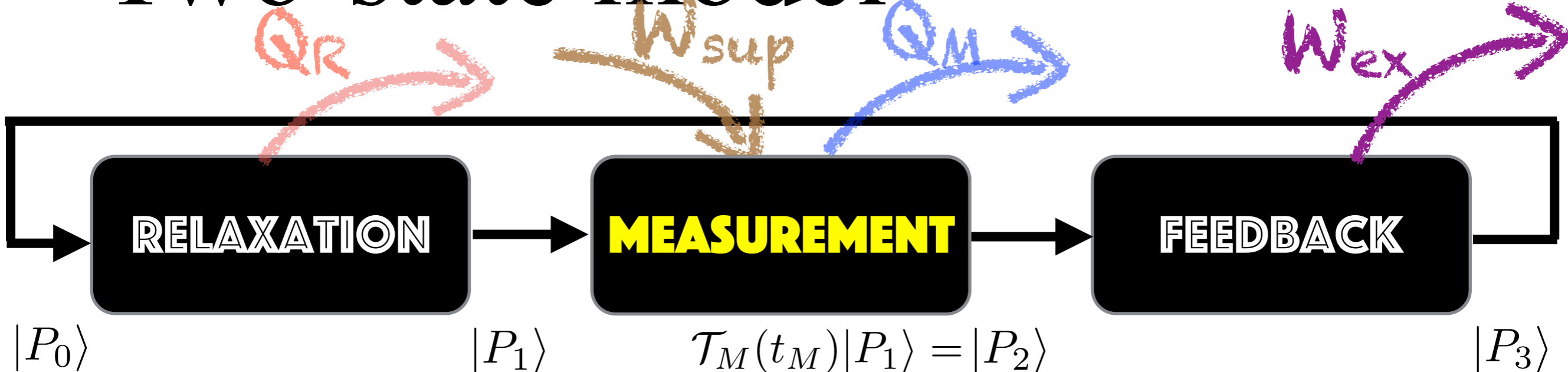
$$\epsilon/(1 - \epsilon) = e^{-\Delta E'/T_M}$$

$$\bar{\epsilon} \equiv 1 - \epsilon$$

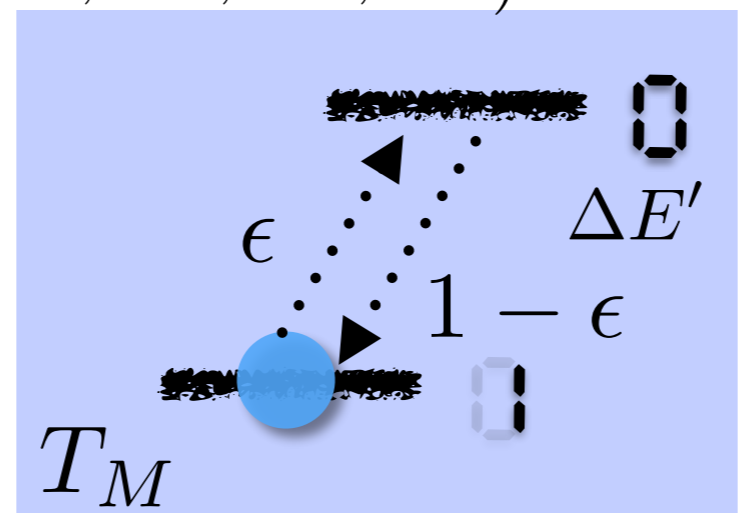
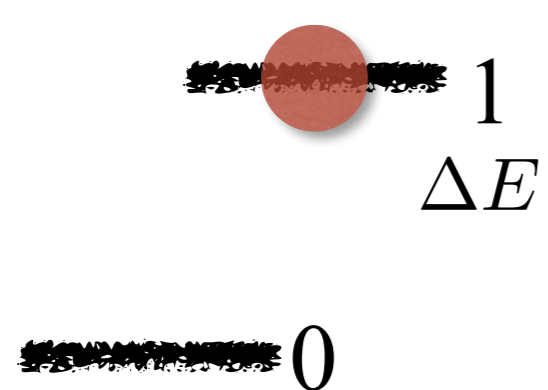
$$\mathcal{M} \equiv e^{-t_M} \quad \bar{\mathcal{M}} \equiv 1 - \mathcal{M}$$

$$\begin{array}{c}
 \mathcal{T}_M(t_M) \\
 t_M \rightarrow \infty
 \end{array}
 \begin{array}{cc}
 \begin{array}{c} \epsilon \\ \text{0} \end{array} & \begin{array}{c} 1 - \epsilon \\ \text{0} \end{array}
 \end{array}
 \begin{pmatrix}
 \bar{\mathcal{M}}\bar{\epsilon} & 0 & 0 \\
 \mathcal{M} + \bar{\mathcal{M}}\epsilon & 0 & 0 \\
 0 & \mathcal{M} + \bar{\mathcal{M}}\epsilon & \bar{\mathcal{M}}\epsilon \\
 0 & \bar{\mathcal{M}}\bar{\epsilon} & \mathcal{M} + \bar{\mathcal{M}}\bar{\epsilon}
 \end{pmatrix}$$

Two-state model



$$|P\rangle = (P^{00}, P^{01}, P^{10}, P^{11})^T$$



$$\epsilon / (1 - \epsilon) = e^{-\Delta E' / T_M}$$

$$\bar{\epsilon} \equiv 1 - \epsilon$$

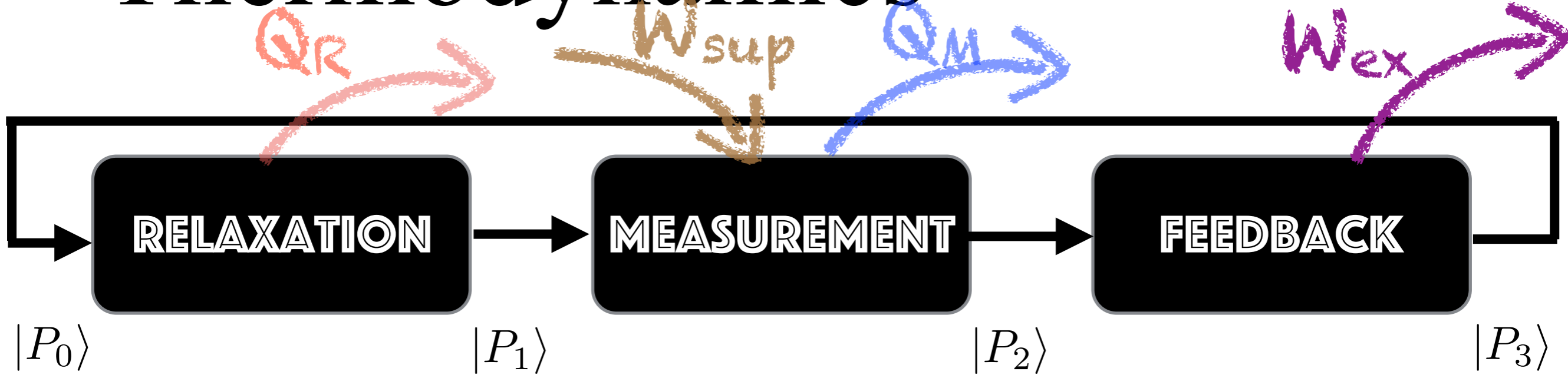
$$\mathcal{M} \equiv e^{-t_M} \quad \bar{\mathcal{M}} \equiv 1 - \mathcal{M}$$

$t_M \rightarrow \infty$

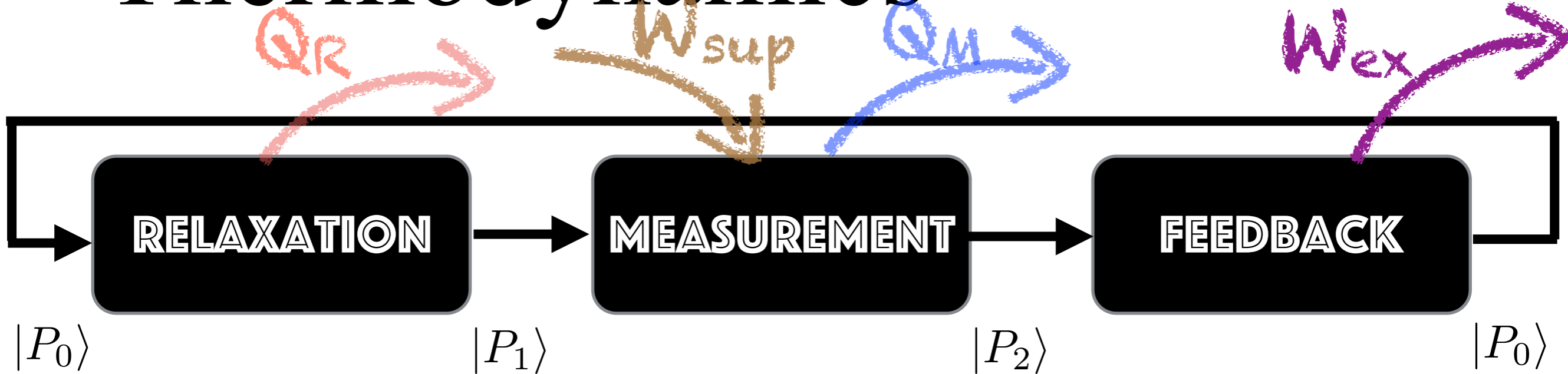
| | | | |
|-------------------------------------------|-------------------------------------------|-------------------------------------------------|---|
| \mathcal{M} | $\bar{\mathcal{M}}\bar{\epsilon}$ | 0 | 0 |
| $\mathcal{M} + \bar{\mathcal{M}}\epsilon$ | 0 | 0 | 0 |
| 0 | $\mathcal{M} + \bar{\mathcal{M}}\epsilon$ | $\bar{\mathcal{M}}\epsilon$ | |
| 0 | $\bar{\mathcal{M}}\bar{\epsilon}$ | $\mathcal{M} + \bar{\mathcal{M}}\bar{\epsilon}$ | |

ϵ $1 - \epsilon$

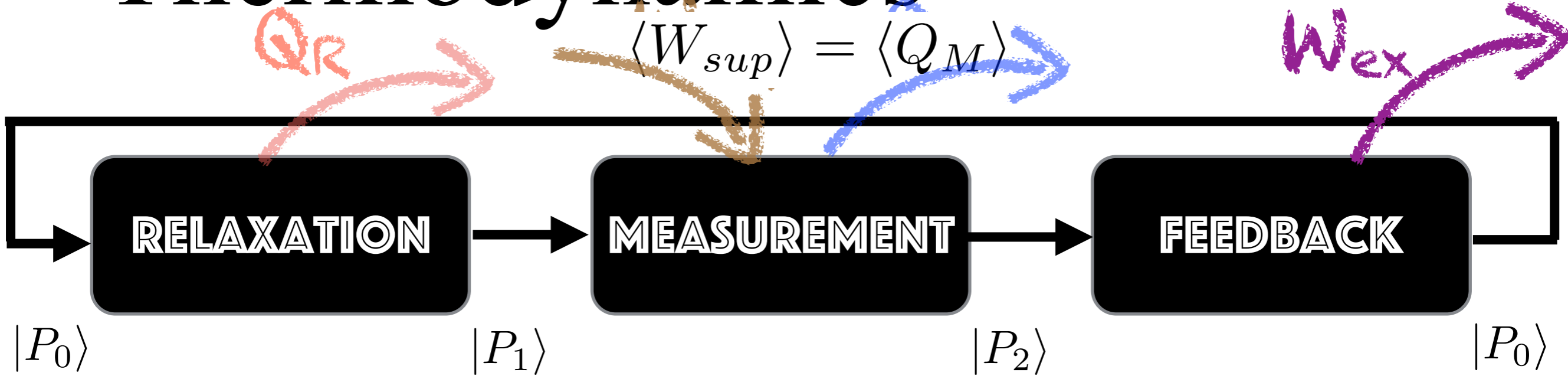
Thermodynamics



Thermodynamics



Thermodynamics



Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →

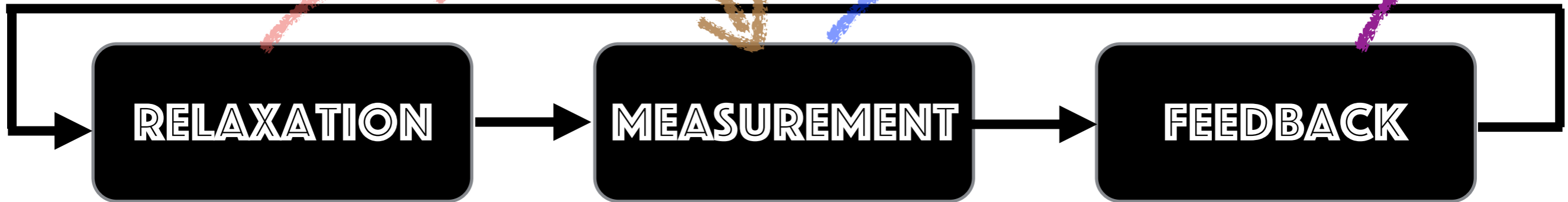


Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

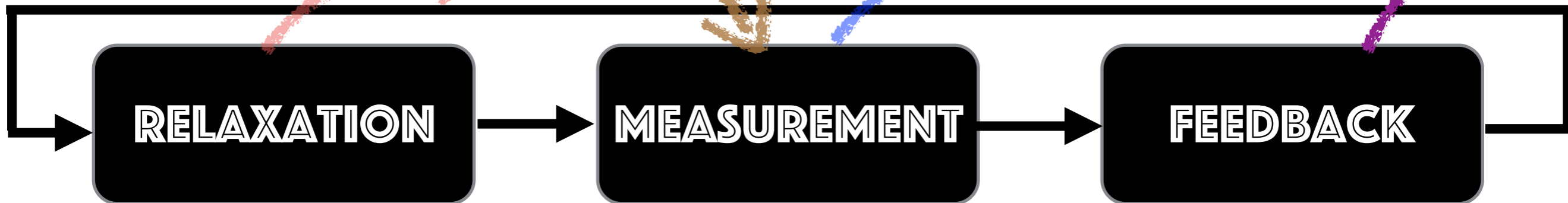
$H_0^{(sm)}$

Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

$H_0^{(sm)}$

$$\Delta H_R^{(sm)} + \beta_R \langle Q_R \rangle \geq 0$$

Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

$H_0^{(sm)}$

$$\Delta H_R^{(sm)} + \beta_R \langle Q_R \rangle \geq 0 \quad \Delta H_M^{(sm)} + \beta_M \langle Q_M \rangle \geq 0$$

Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

$H_0^{(sm)}$

$$\Delta H_R^{(sm)} + \beta_R \langle Q_R \rangle \geq 0$$

$$\Delta H_M^{(sm)} + \beta_M \langle Q_M \rangle \geq 0$$

$$\Delta H_F^{(sm)} = 0$$

Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

$H_0^{(sm)}$

$$= -\Delta H_M^{(sm)}$$

$$\Delta H_R^{(sm)} + \beta_R \langle Q_R \rangle \geq 0$$

$$\Delta H_M^{(sm)} + \beta_M \langle Q_M \rangle \geq 0$$

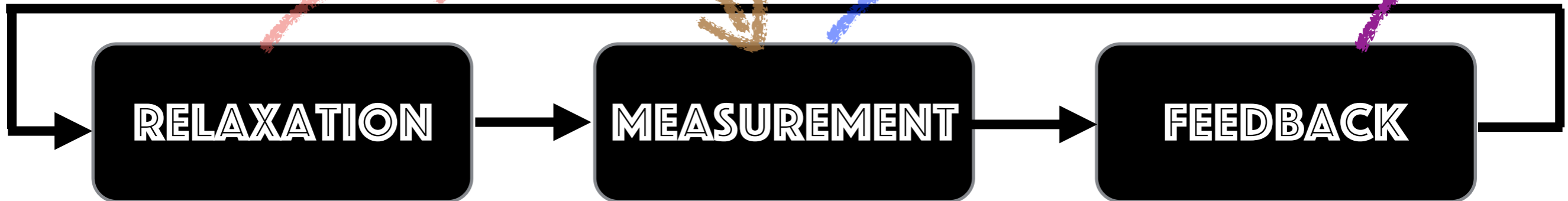
$$\Delta H_F^{(sm)} = 0$$

Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

$H_0^{(sm)}$

$$= -\Delta H_M^{(sm)}$$

$$\Delta H_R^{(sm)} + \beta_R \langle Q_R \rangle \geq 0$$

$$\Delta H_M^{(sm)} + \beta_M \langle Q_M \rangle \geq 0$$

$$\Delta H_F^{(sm)} = 0$$

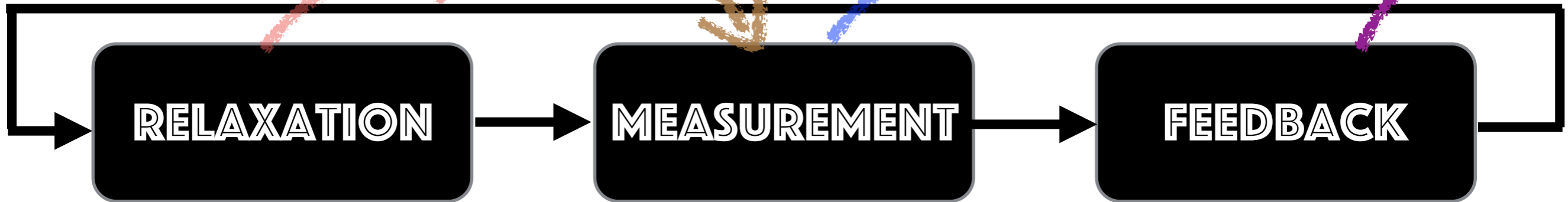
$$(1) \quad \beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

$H_0^{(sm)}$

$$\Delta H_R^{(sm)} + \beta_R \langle Q_R \rangle \geq 0$$

$$\Delta H_M^{(sm)} + \beta_M \langle Q_M \rangle \geq 0$$

$$\Delta H_F^{(sm)} = 0$$

$$(1) \quad \beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

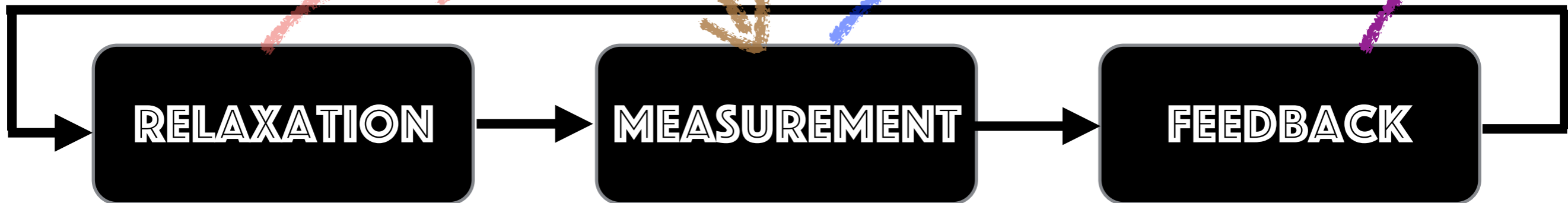
$$(2) \quad -\beta_R \langle Q_R \rangle \leq -\Delta H_M^{(sm)} \leq \beta_M \langle Q_M \rangle$$

Thermodynamics

$$-\langle W_{ex} \rangle = \langle Q_R \rangle$$

$$\langle W_{sup} \rangle = \langle Q_M \rangle$$

W_{ex} →



$|P_0\rangle$

$|P_1\rangle$

$|P_2\rangle$

$|P_0\rangle$

$H_0^{(sm)}$

$H_1^{(sm)}$

$H_2^{(sm)}$

$H_0^{(sm)}$

$$= -\Delta H_M^{(sm)}$$

$$\Delta H_R^{(sm)} + \beta_R \langle Q_R \rangle \geq 0$$

$$\Delta H_M^{(sm)} + \beta_M \langle Q_M \rangle \geq 0$$

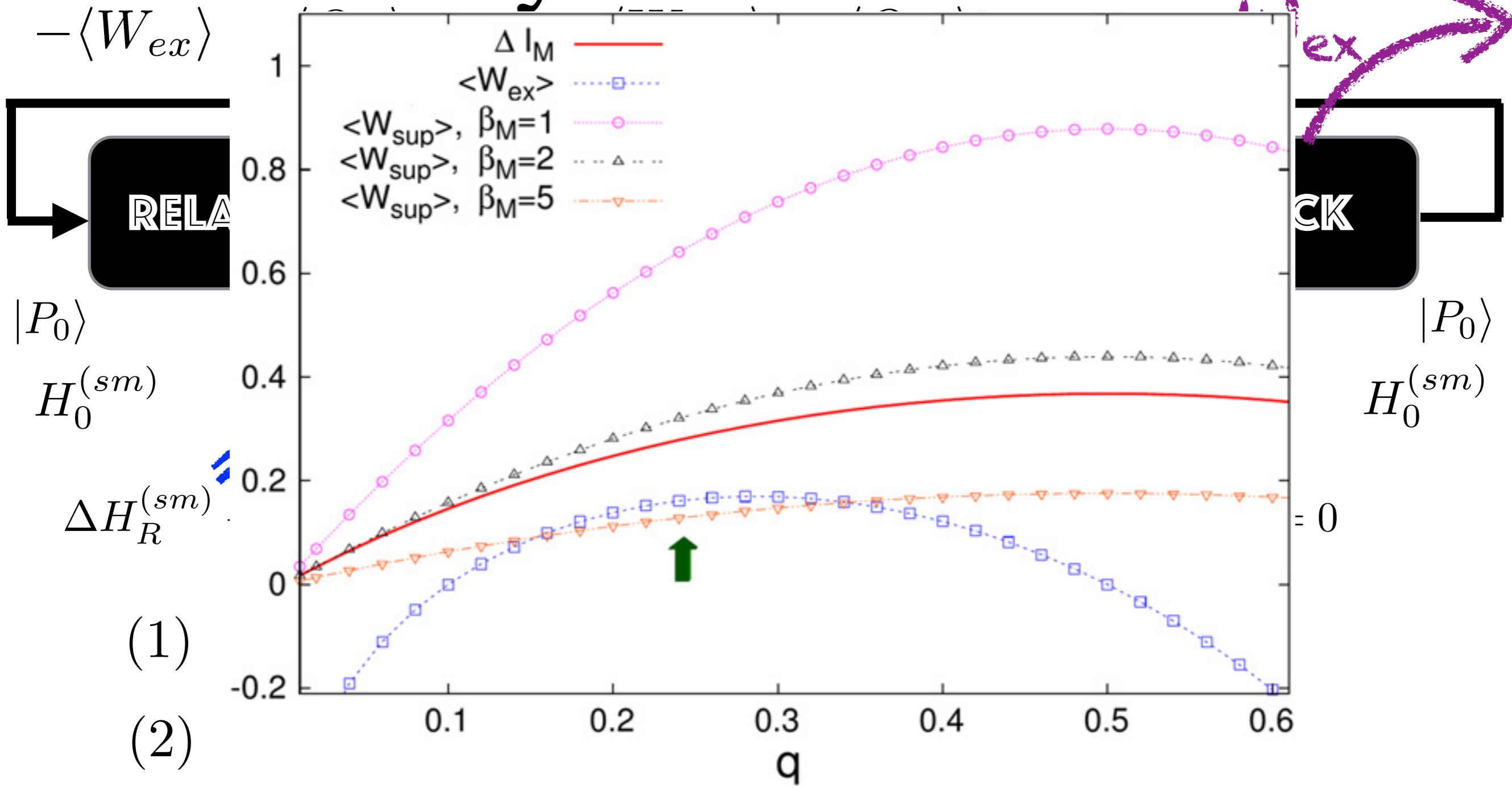
$$\Delta H_F^{(sm)} = 0$$

$$(1) \quad \beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$(2) \quad -\beta_R \langle Q_R \rangle \leq -\Delta H_M^{(sm)} \leq \beta_M \langle Q_M \rangle$$

$$\beta_R \langle W_{ex} \rangle \leq \Delta I_M \leq \beta_M \langle W_{sup} \rangle$$

Thermodynamics

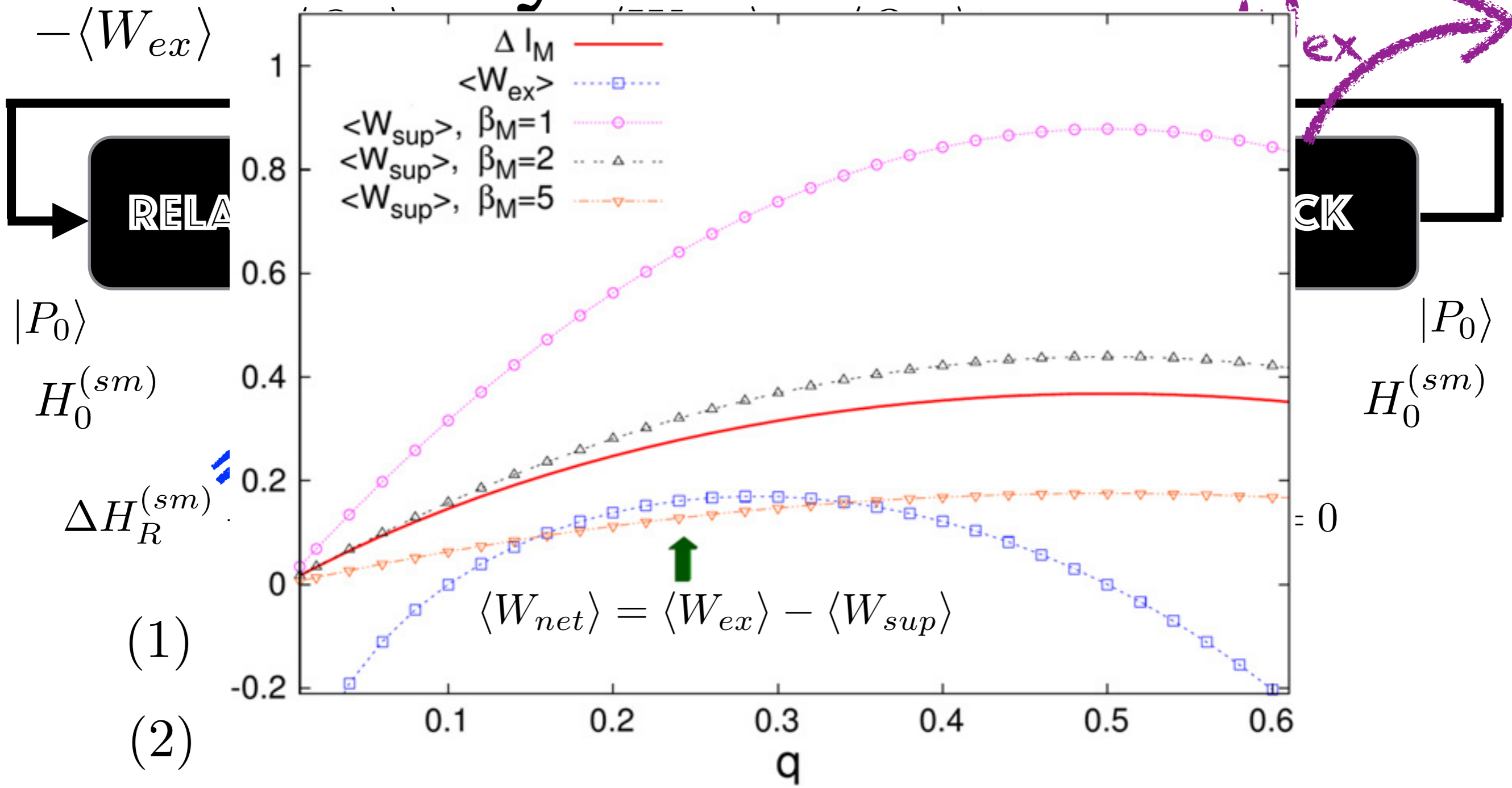


(1)

(2)

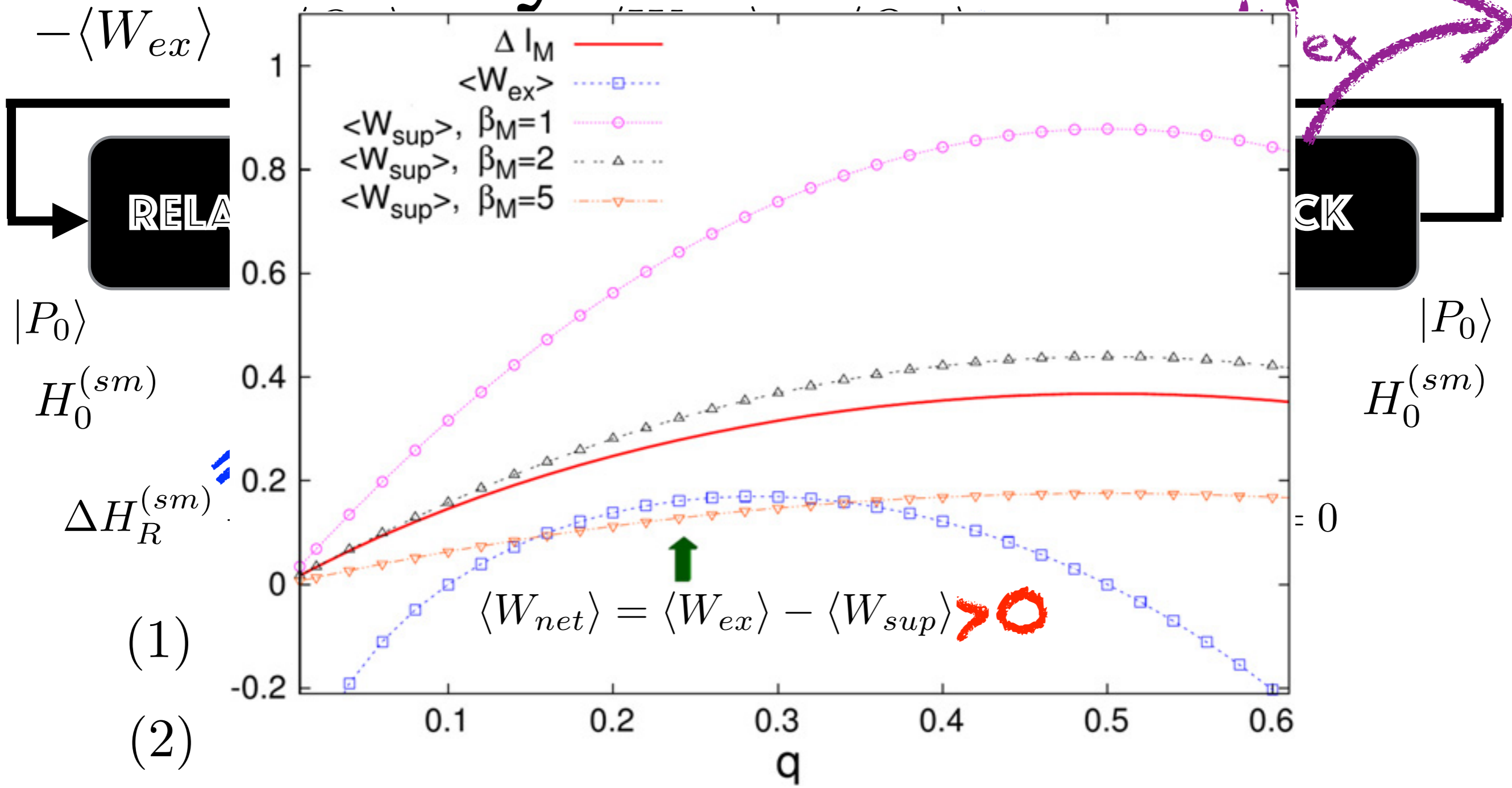
$$\beta_R \langle W_{ex} \rangle \leq \Delta I_M \leq \beta_M \langle W_{sup} \rangle$$

Thermodynamics



$$\beta_R \langle W_{ex} \rangle \leq \Delta I_M \leq \beta_M \langle W_{sup} \rangle$$

Thermodynamics



$$\beta_R \langle W_{ex} \rangle \leq \Delta I_M \leq \beta_M \langle W_{sup} \rangle$$

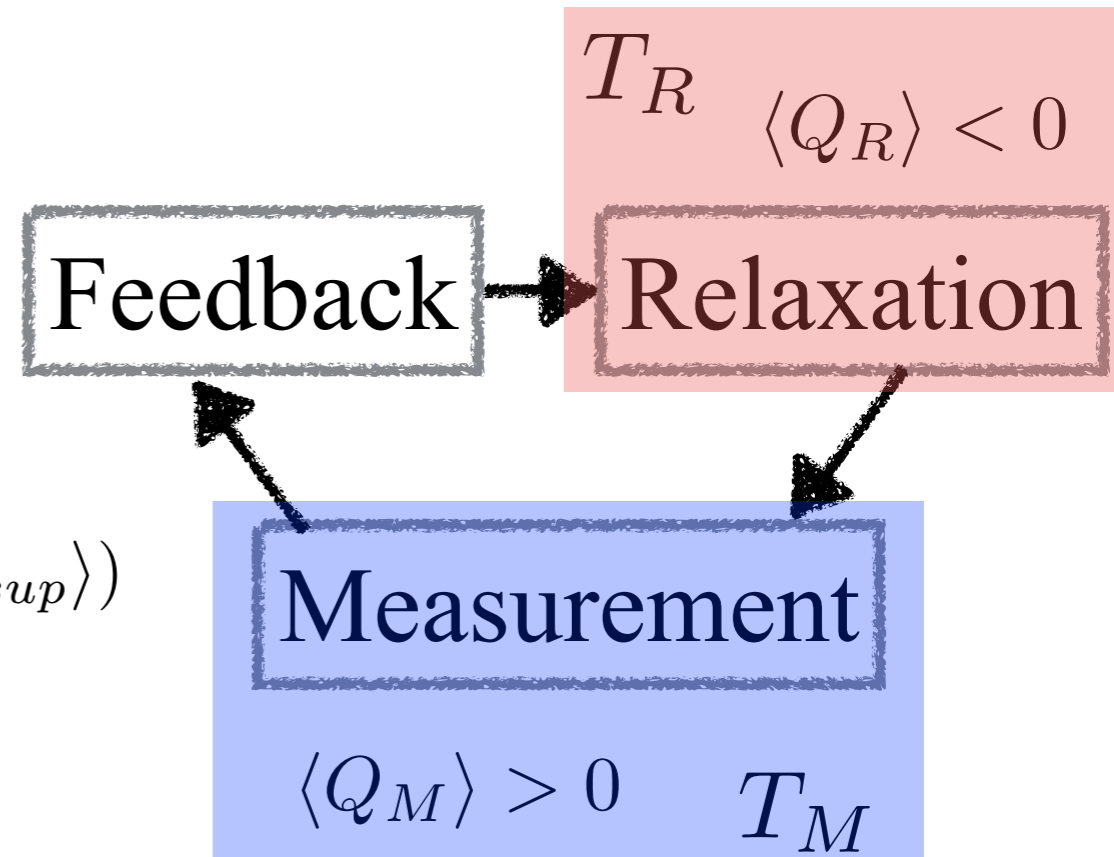
Efficiency

- information assisted **heat engine**
- efficiency

$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$



Efficiency

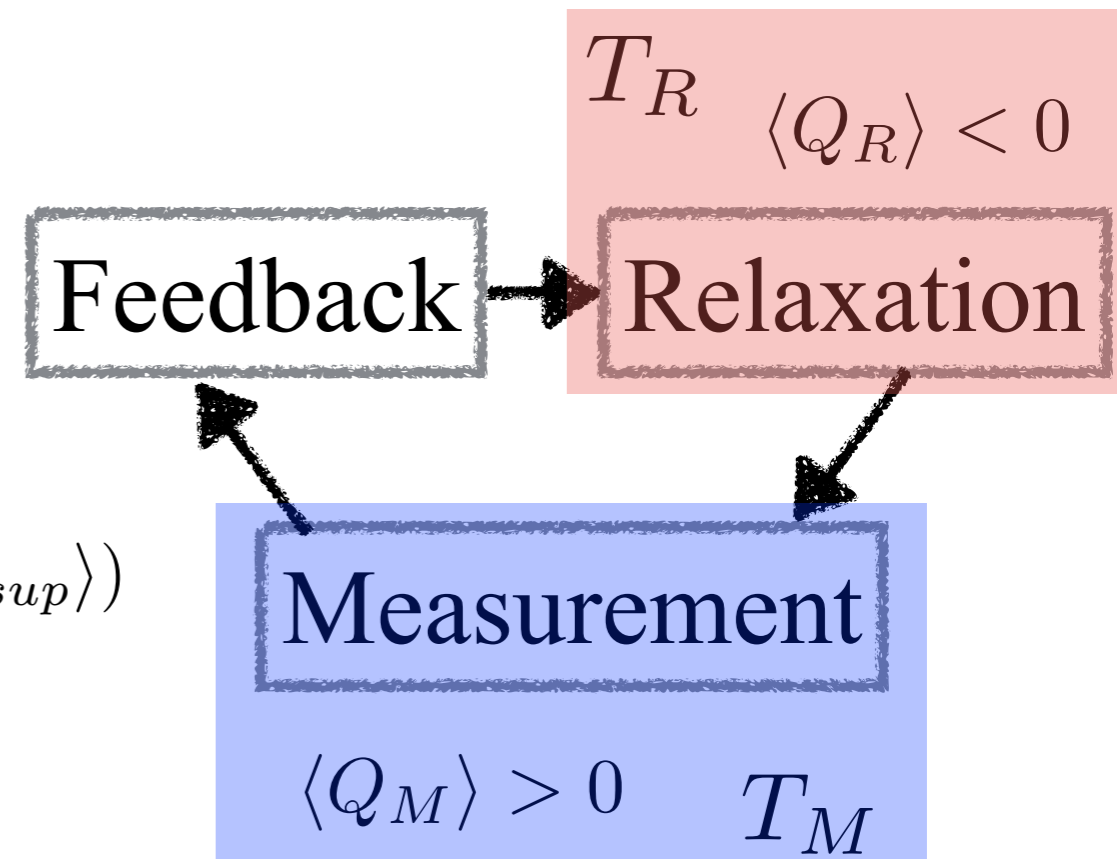
- information assisted **heat engine**
- efficiency

$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \lambda(t_R, t_M)$$



Efficiency

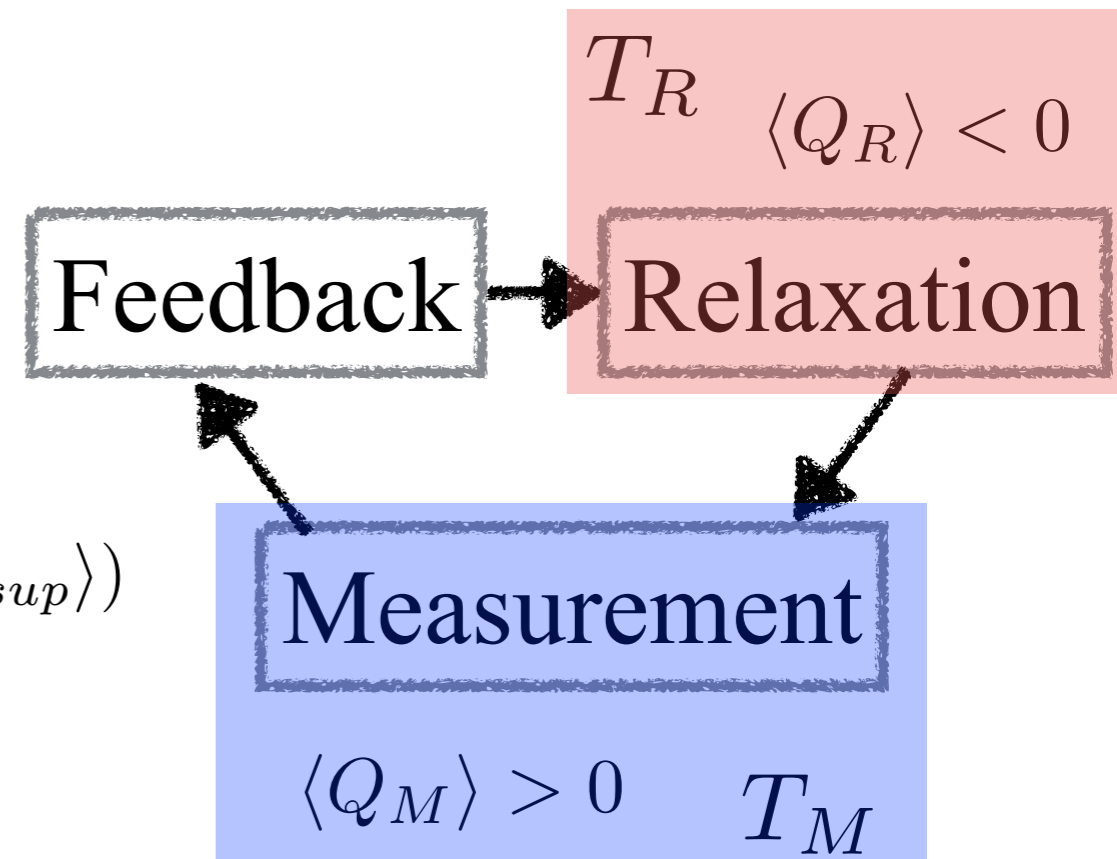
- information assisted **heat engine**
- efficiency

$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

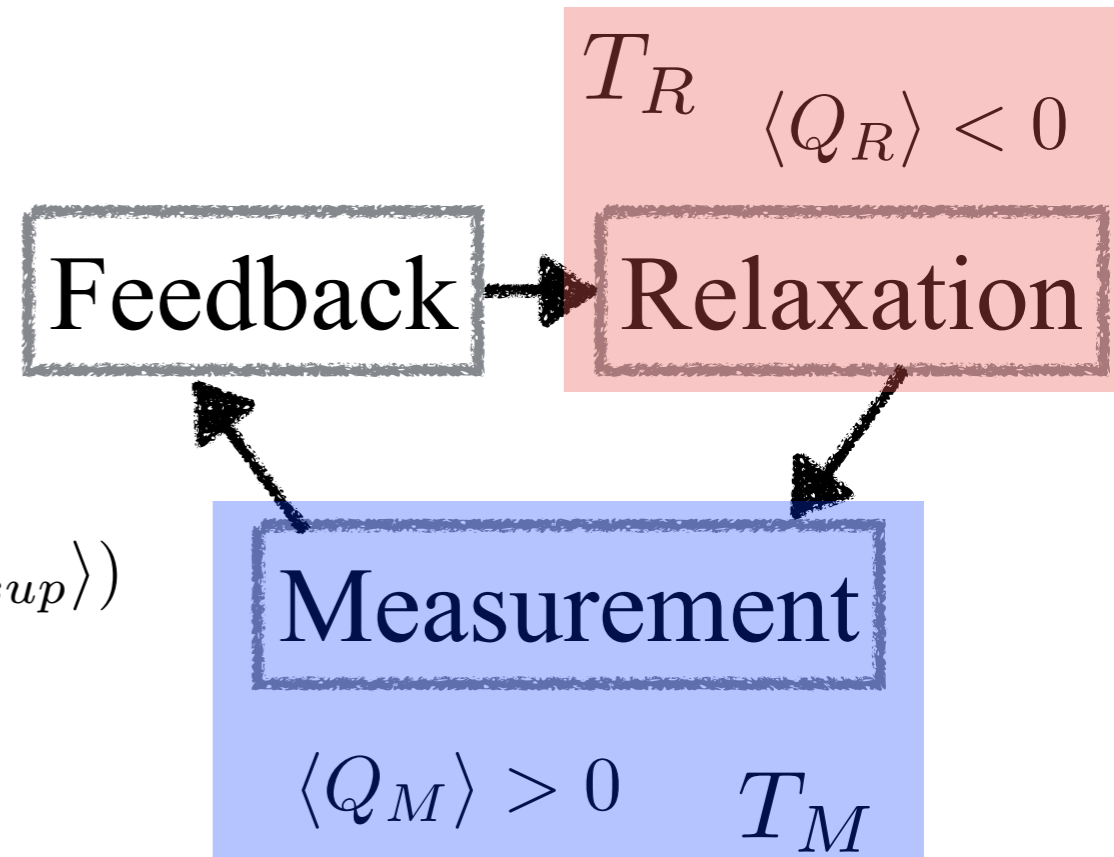
$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \lambda(t_R, t_M)$$



Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

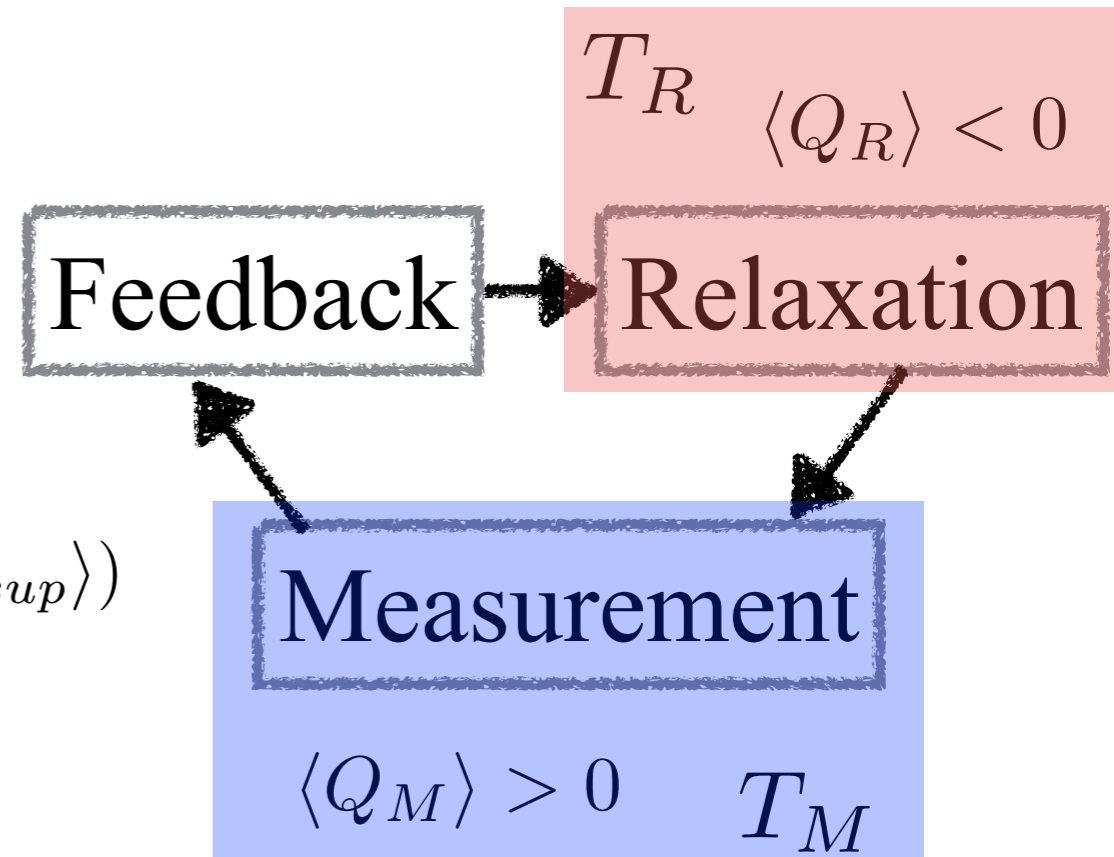
$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) (\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)}$$

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

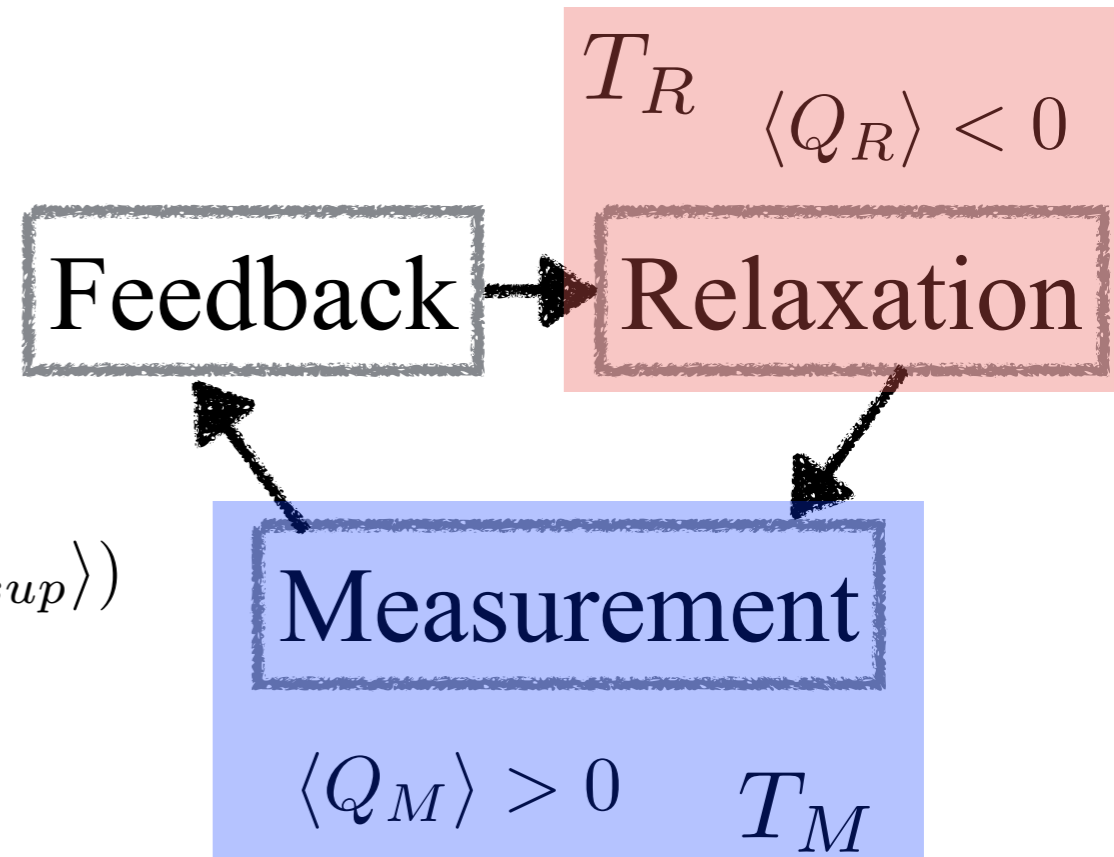
$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) + (\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)}$$

$$\frac{\bar{R}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{R}q + \mathcal{R}\bar{M}\epsilon)}{1 - \alpha\mathcal{R}\mathcal{M}}$$

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

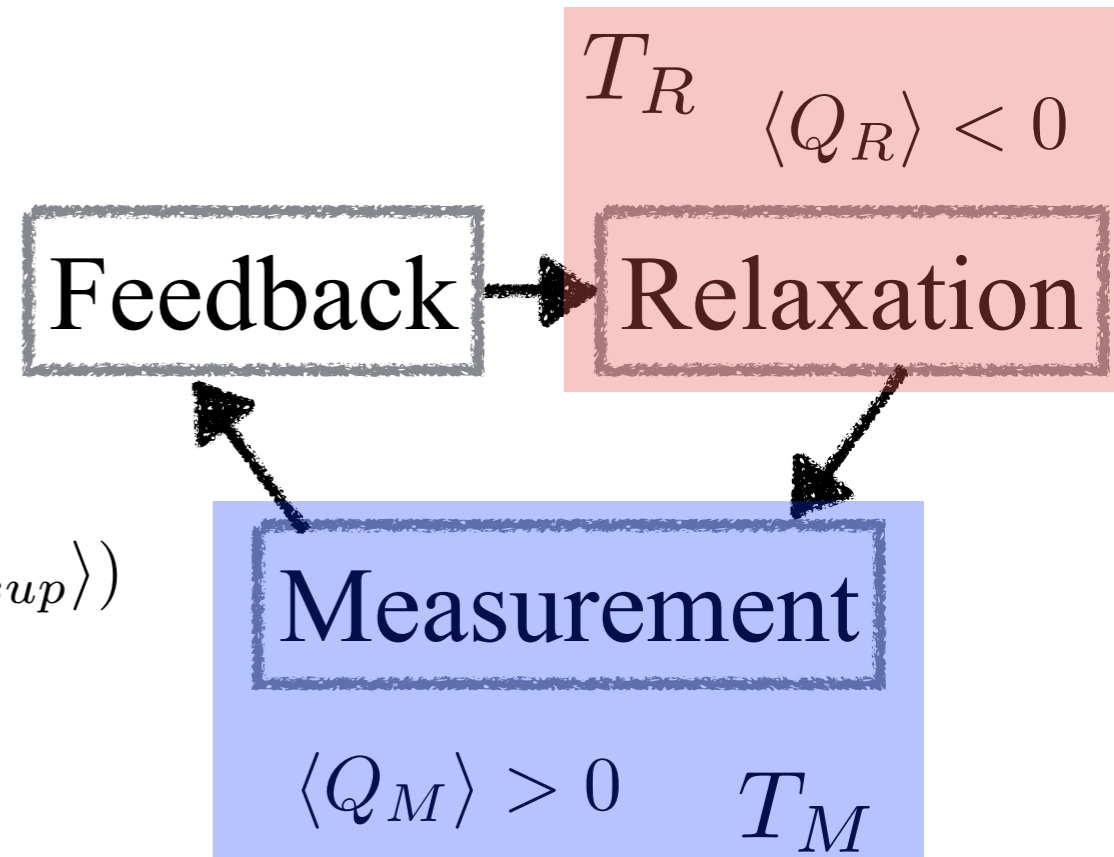
$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) \frac{(\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)} + \frac{\bar{R}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{R}q + \mathcal{R}\bar{M}\epsilon)}{1 - \alpha\mathcal{R}\mathcal{M}}}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)}$$

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

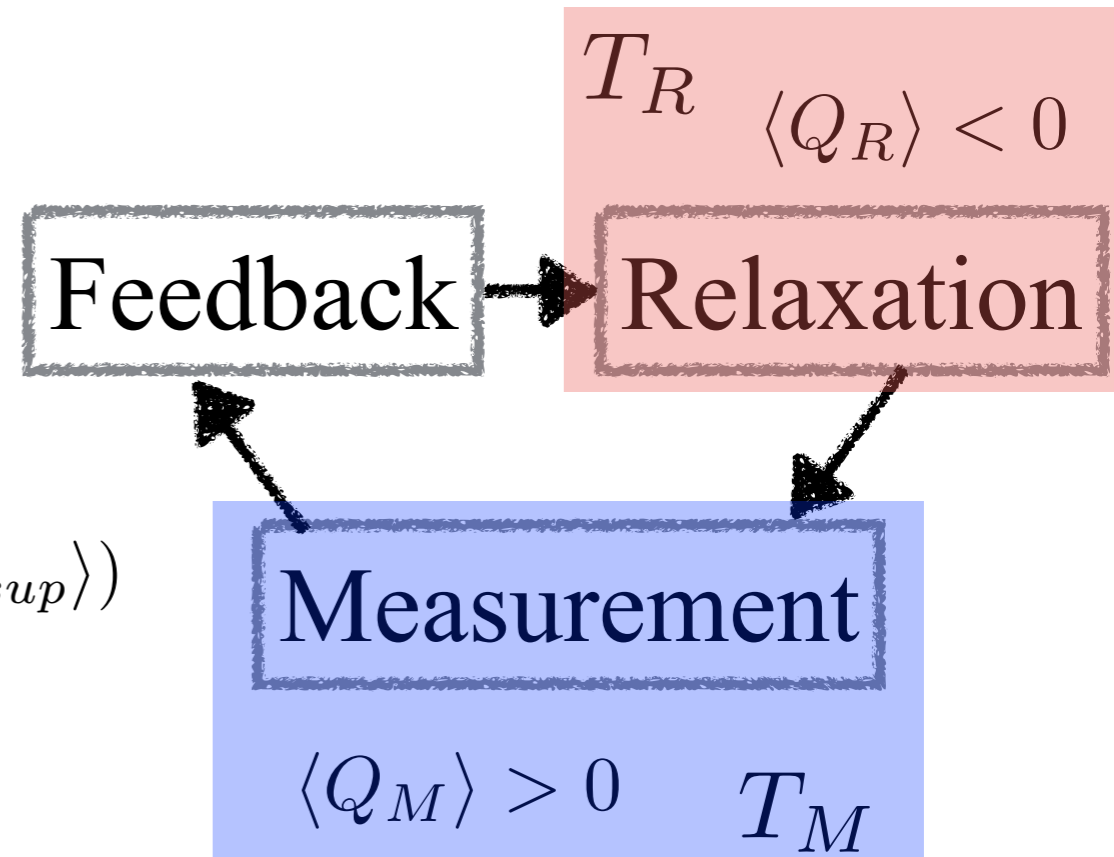
$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) \frac{(\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)} + \frac{\bar{\mathcal{R}}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{\mathcal{R}}q + \mathcal{R}\bar{M}\epsilon)}{1 - \alpha\mathcal{R}\mathcal{M}}}{\mathcal{R} + \bar{\mathcal{R}}(\bar{q} - q)(\bar{\epsilon} - \epsilon)}$$

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

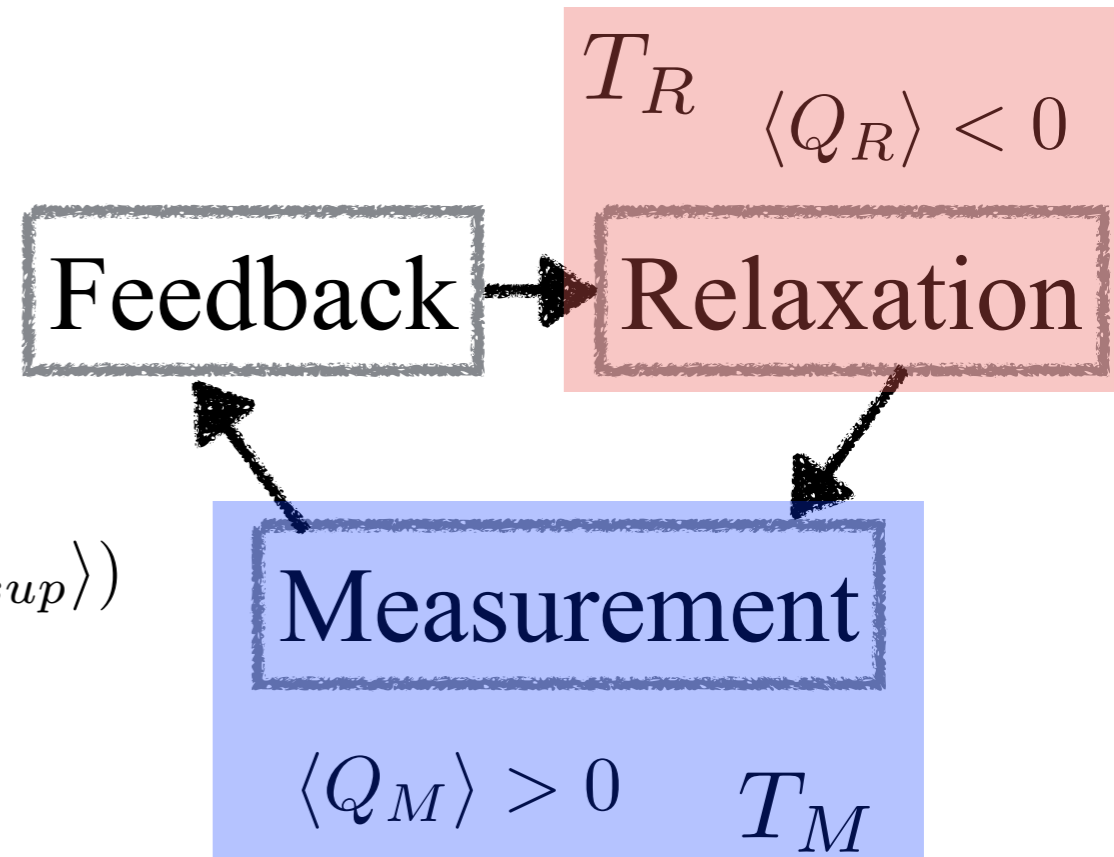
$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) \frac{(\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)} + \frac{\bar{R}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{R}q + \mathcal{R}\bar{M}\epsilon)}{1 - \alpha\mathcal{R}\mathcal{M}}}{\mathcal{R} + \bar{R}(\bar{q} - q)(\bar{\epsilon} - \epsilon)}$$

$\lambda(t_R, t_M) \geq 2$

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

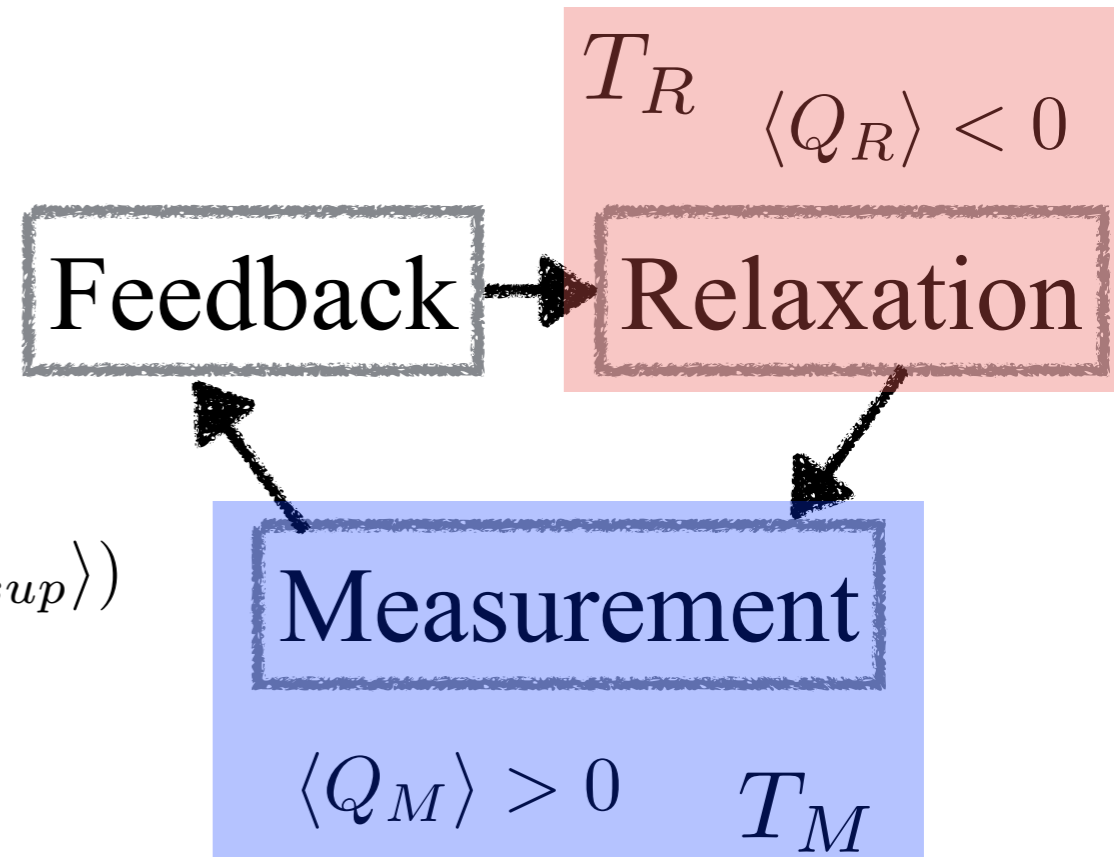
$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) (\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)} \frac{\bar{\mathcal{R}}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{\mathcal{R}}q + \mathcal{R}\bar{M}\epsilon)}{1 - \alpha\mathcal{R}\mathcal{M}} \frac{1}{\mathcal{R} + \bar{\mathcal{R}}(\bar{q} - q)(\bar{\epsilon} - \epsilon)}$$

$$\lambda(t_R, t_M) \geq 2$$

- maximum efficiency

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

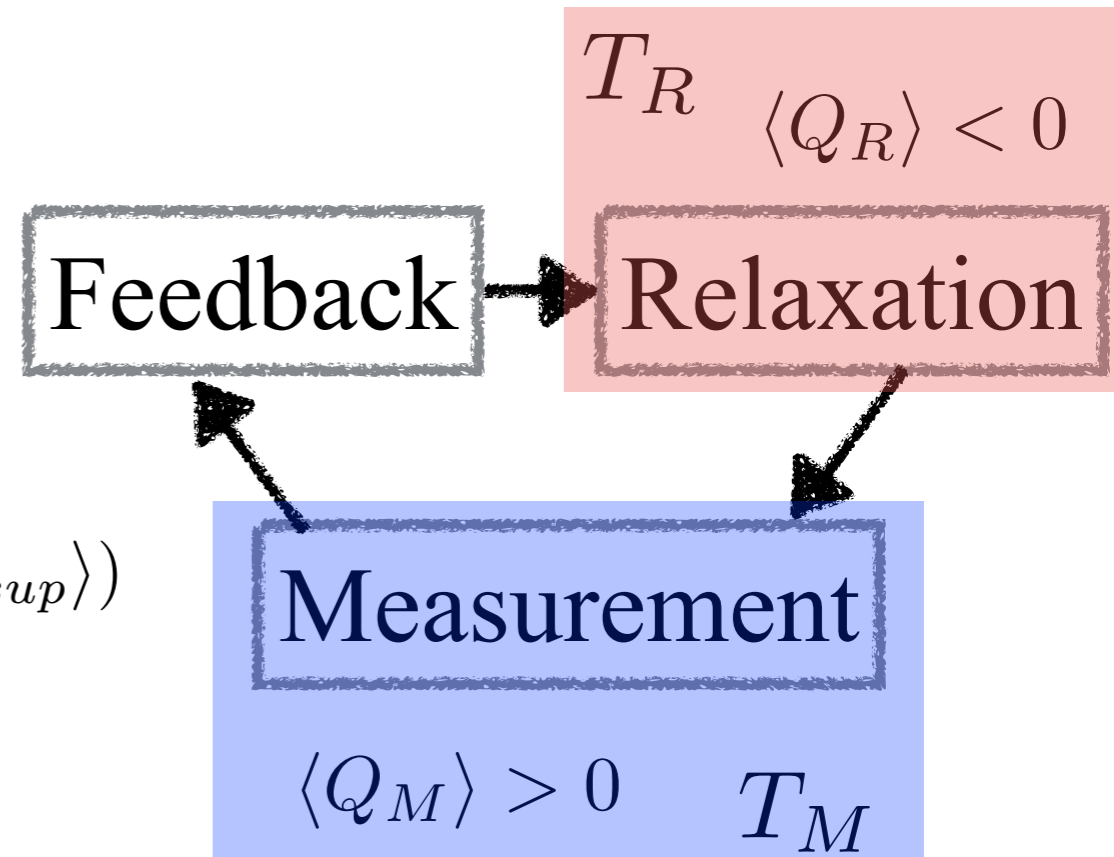
$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) \frac{(\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)} + \frac{\bar{R}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{R}q + \mathcal{R}\bar{M}\epsilon)}{1 - \alpha\mathcal{R}\mathcal{M}}}{\mathcal{R} + \bar{R}(\bar{q} - q)(\bar{\epsilon} - \epsilon)}$$

$$\lambda(t_R, t_M) \geq 2$$

- maximum efficiency $\eta_{max} = \lim_{\substack{t_R \rightarrow \infty \\ t_M \rightarrow \infty}} \eta(t_R, t_M)$

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

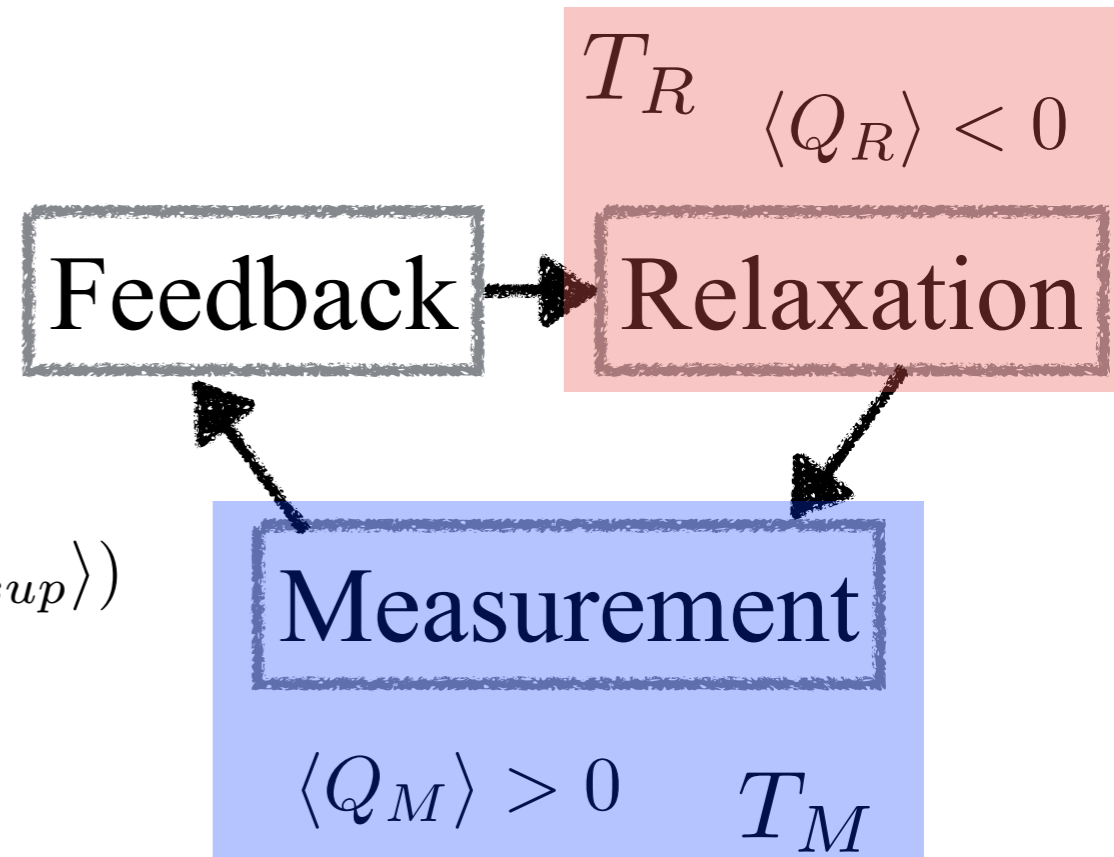
$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) \frac{(\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon)}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)} \frac{\bar{R}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{R}q + \mathcal{R}\bar{M}\epsilon)}{1 - \alpha\mathcal{R}\mathcal{M}}}{\mathcal{R} + \bar{R}(\bar{q} - q)(\bar{\epsilon} - \epsilon)}$$

$$\lambda(t_R, t_M) \geq 2$$

- maximum efficiency $\eta_{max} = \lim_{\substack{t_R \rightarrow \infty \\ t_M \rightarrow \infty}} \eta(t_R, t_M) < 1 - \frac{T_M}{T_R}$

Efficiency

- information assisted **heat engine**
- efficiency



$$\langle W_{net} \rangle = -\langle Q_R \rangle - \langle Q_M \rangle \quad (= \langle W_{ex} \rangle - \langle W_{sup} \rangle)$$

$$\eta = \frac{\langle W_{net} \rangle}{-\langle Q_R \rangle} \leq 1 - \frac{T_M}{T_R}$$

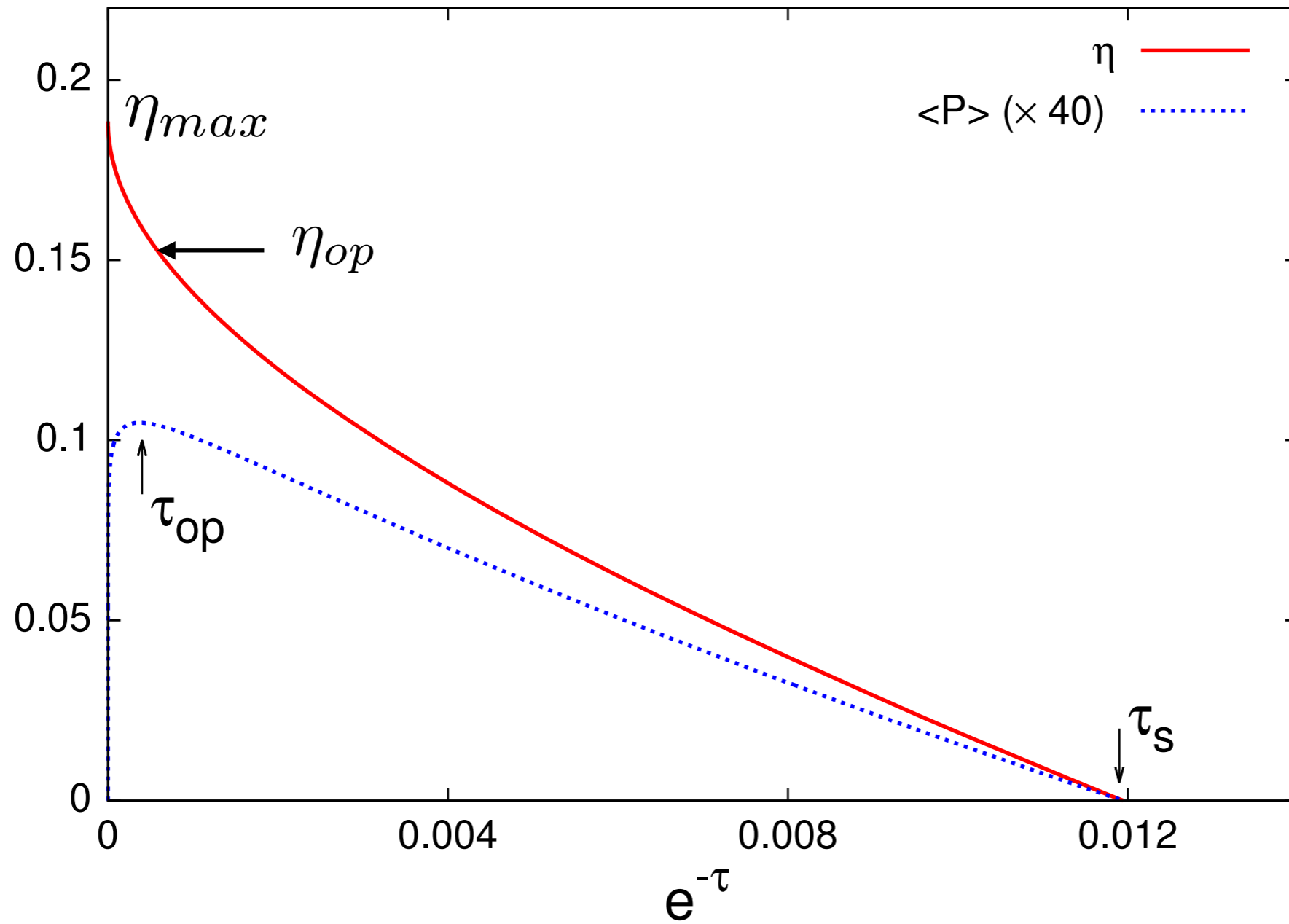
$$\beta_R \langle Q_R \rangle + \beta_M \langle Q_M \rangle \geq 0$$

$$\eta(t_R, t_M) = 1 - \frac{T_M}{T_R} \frac{\lambda(t_R, t_M) \frac{(\mathcal{E} - \epsilon) \bar{M} \ln(\bar{\epsilon}/\epsilon) + \frac{\bar{R}(\bar{\epsilon}q + \epsilon\bar{q}) + \alpha(\bar{R}q + R\bar{M}\epsilon)}{1 - \alpha R M}}{[q - \epsilon - \mathcal{M}(\mathcal{E} - \epsilon)] \bar{R} \ln(\bar{q}/q)}}{\mathcal{R} + \bar{\mathcal{R}}(\bar{q} - q)(\bar{\epsilon} - \epsilon)}$$

$$\lambda(t_R, t_M) \geq 2$$

- maximum efficiency $\eta_{max} = \lim_{\substack{t_R \rightarrow \infty \\ t_M \rightarrow \infty}} \eta(t_R, t_M) < 1 - \frac{T_M}{T_R}$
not quasi-static process

Efficiency at max power



$$\beta_M / \beta_R = 5$$

$$q = 0.2$$

$$\epsilon = 0.1$$

$$t_R = t_M = \tau/2$$

$$\eta_{op} \approx \eta_{max} \frac{\eta_{max}}{|\ln \eta_{max}|}$$

Summary

- We have investigated an information assisted heat engine in terms of composite system.
- It is found that the mutual information developed by measurement gives the upper and lower bound for work extraction and supply, respectively.
- Efficiency is found to be distinguished from the efficiency of usual heat engine.