2D Hard-disk fluid

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Non-equilibrium statistical physics ICTS-India2015

Outline

1. Decay of velocity autocorrelation function
2. Decay of energy current autocorrelation function
3. Mechanisms
4. Bridge the kinetics and hydrodynamics
5. Influences of hydrodynamics tails on the observed values of transport coefficients
6. Summary

Background & problems

Complex Systems Group, Xiamen University
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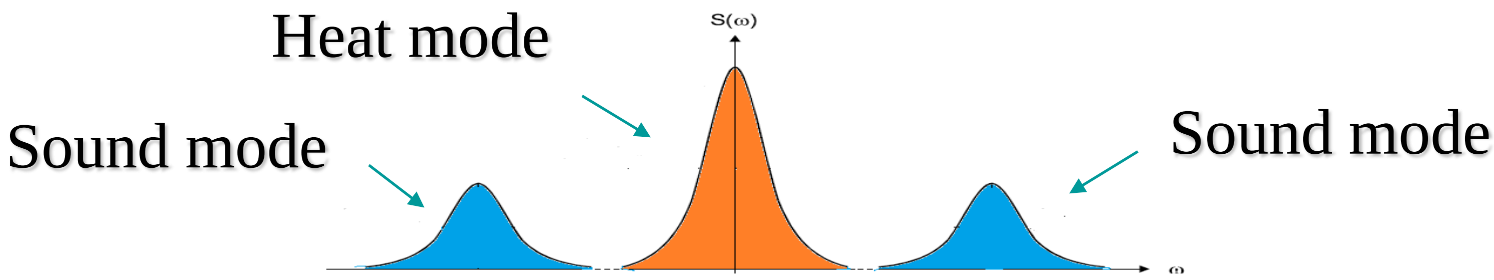
Problems:

(a) How do autocorrelation functions relax?

$$c_v(t) = \langle v(t)\, v(0) \rangle \qquad D_{G-K} = \lim_{\tau\to\infty} \int_0^{\tau} c_v(t)\,dt$$

$$c_J(t) = \langle J(t) J(0) \rangle \qquad \kappa_{G-K} = \lim_{\tau\to\infty} \lim_{L\to\infty} \frac{1}{2k_B T^2 V} \int_0^{\tau} C(t)\,dt$$

(b) How do density fluctuations relax?



Prähofer - Spohn scaling function :

$$S(x,t) \sim t^{-\lambda} f_{KPZ}(t^{-\lambda}(\bar{x}))$$

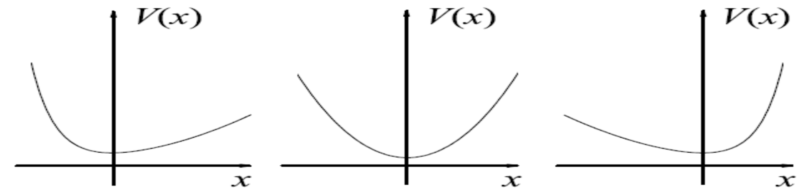
heat mode： $\bar{x} = x$, sound mode： $\bar{x} = x - vt$,

Focused topic

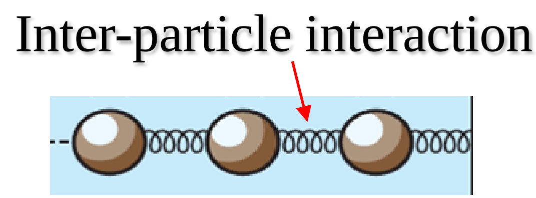
Asymmetric inter-particle interactions play a key role in transport behavior

theoretical predictions for energy current in 1D momentum conserved lattices:

$$c_J(t) = \langle J(t)J(0) \rangle \sim t^{-\gamma}$$



$$\gamma = \begin{cases} 2/3 & \text{for asymmetric potential} \\ 1/2 & \text{for symmetric potential} \end{cases}$$

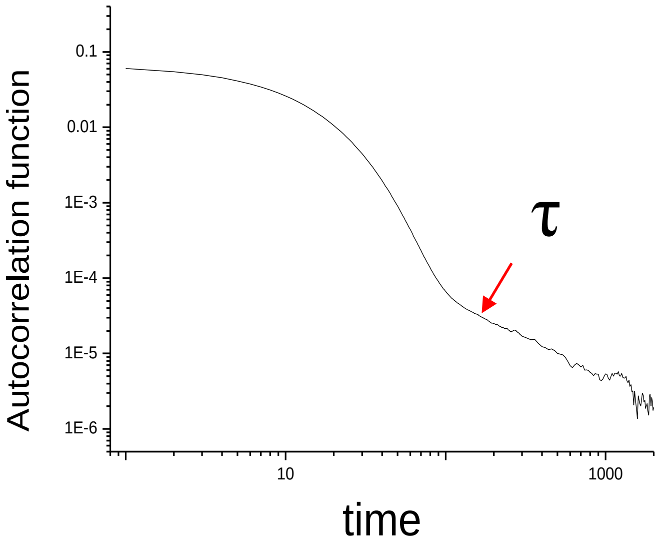


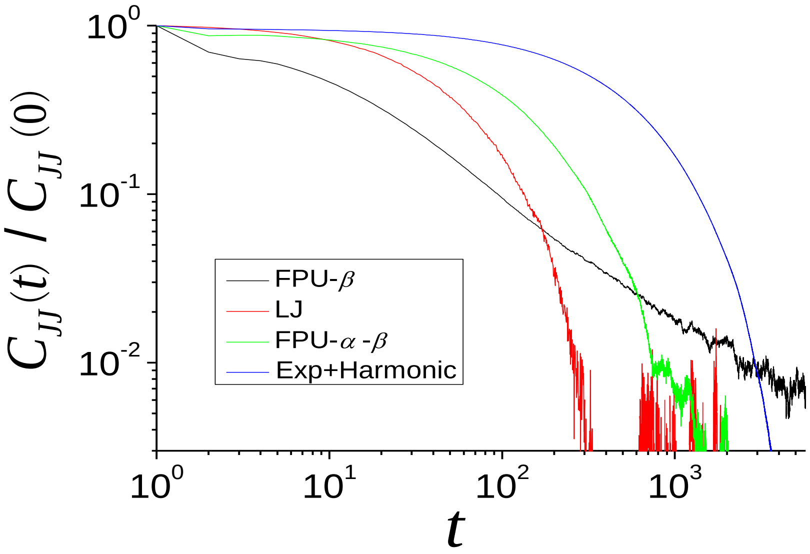
We do not change these predictions of long time behavior – We want to emphasize the important difference in the 'kinetic' stage or the transient period

L. Delfini, S. Lepri, R. Livi, A. Politi. J Stat Mech, P02007 (2007);
H. van Beijeren. Phys Rev Lett, 108, 180601 (2012); H. Spohn, PRL (2014)

Why is the transient stage important?

$$D_{G-K} = \int_0^{\tau} c_v(t)dt + \int_{\tau}^{tr} c_v(t)dt = D_{G-K}^{\tau} + \int_{\tau}^{tr} c_v(t)dt$$





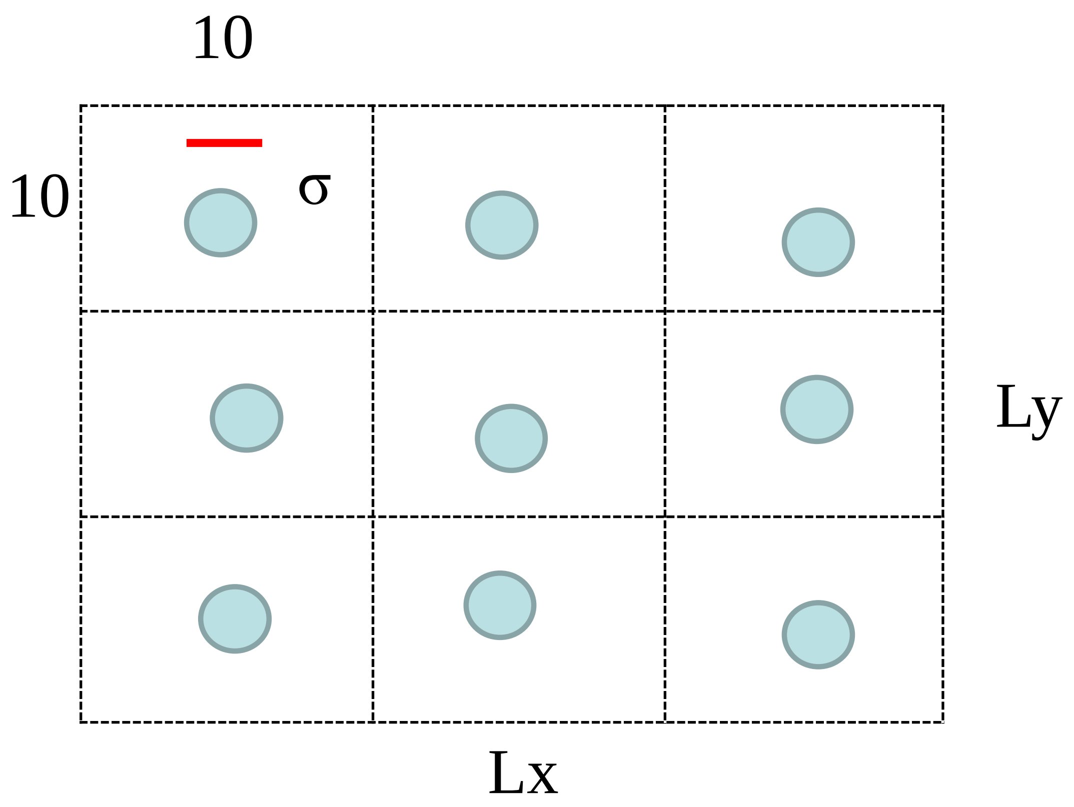
If only the rapid decay extends a sufficient long time, a size-independent transport coefficient may be observed practically, and the contribution of the tail is negligible

S. Chen Y. Zhang, J. Wang and H. Zhao, arXiv:1204.5933 (2012); arXiv:1511.00292(2015)

2D Hard-disk fluid

H. Q. Zhao and H. Zhao, arXiv:1511.00292

Disk- fluid model



particle density: n=0.01

Periodic boundaries,
Temperature=1

packing friction: $\nu = \frac{\sigma^2}{4} n\pi, \quad \nu = 0.03(\sigma = 2), 0.13(\sigma = 3), 0.28(\sigma = 6)$

Previous theoretical predictions:

$$c(t) = \begin{cases} b_k e^{-\xi t}, t \sim 10 t_0 \\ b_h t^{-1}, t \sim \mathrm{P}\, t_0 \\ \dfrac{b_s}{t\sqrt{\ln t}}, t >> t_0 \end{cases}$$

Kinetic theory

Hydrodynamics

Self-consistent model-coupling

t_0 : the mean free time, P: large positive number

Kinetics and Hydrodynamics theories: B. S. Alder and T. E. Wainwright, Phys. Rev. Lett. 18, 988 (1969),S. R. Dorfman and E. G. D. Cohen, PRL 25,1257(1970); M, H. Ernst, E. H. Hauge, and J. M. J. van Leeuwen, Phys. Rev. Lett. 25, 1254 (1970); M. H. Ernst, E. H. Hauge, and J. M. J. van Leeuwen, Phys. Rev. A 4, 2055 (1971). J. R. Dorfman and E. G. D. Cohen, Phys. Rev. A 6, 776 (1972).

Mode-coupling theory: D. Forster, D. R. Nelson, and M. J. Stephen, Phys. Rev. A **16**, 732(1977);J. J. Erpenbeck and W. W. Wood, Phys. Rev. A **26**, 1648(1982); **32**, 412 (1985).

Open problems: (a) a unified c(t)

$$\mathrm{c}(t)=\begin{cases} b_k e^{-\xi t}, t \sim 10t_0 \\ b_h t^{-1}, t \sim \mathrm{P}\, t_0 \\ \dfrac{b_s}{t\sqrt{\ln t}}, \mathrm{t} >> \mathrm{t}_0 \end{cases} \Rightarrow \mathrm{c}(t) = c(t), t > 0$$

A most recent work (2015-10-30 PRE) attacked this goal,
But with a set of fitting parameters

S. Bellissima, M. Neumann, E. Guarini, U. Bafile and F. Barocchi1, Phys. Rev. E 92, 042166(2015)

Open problems: (b) The validation of theoretical predictions , qualitatively and quantitatively

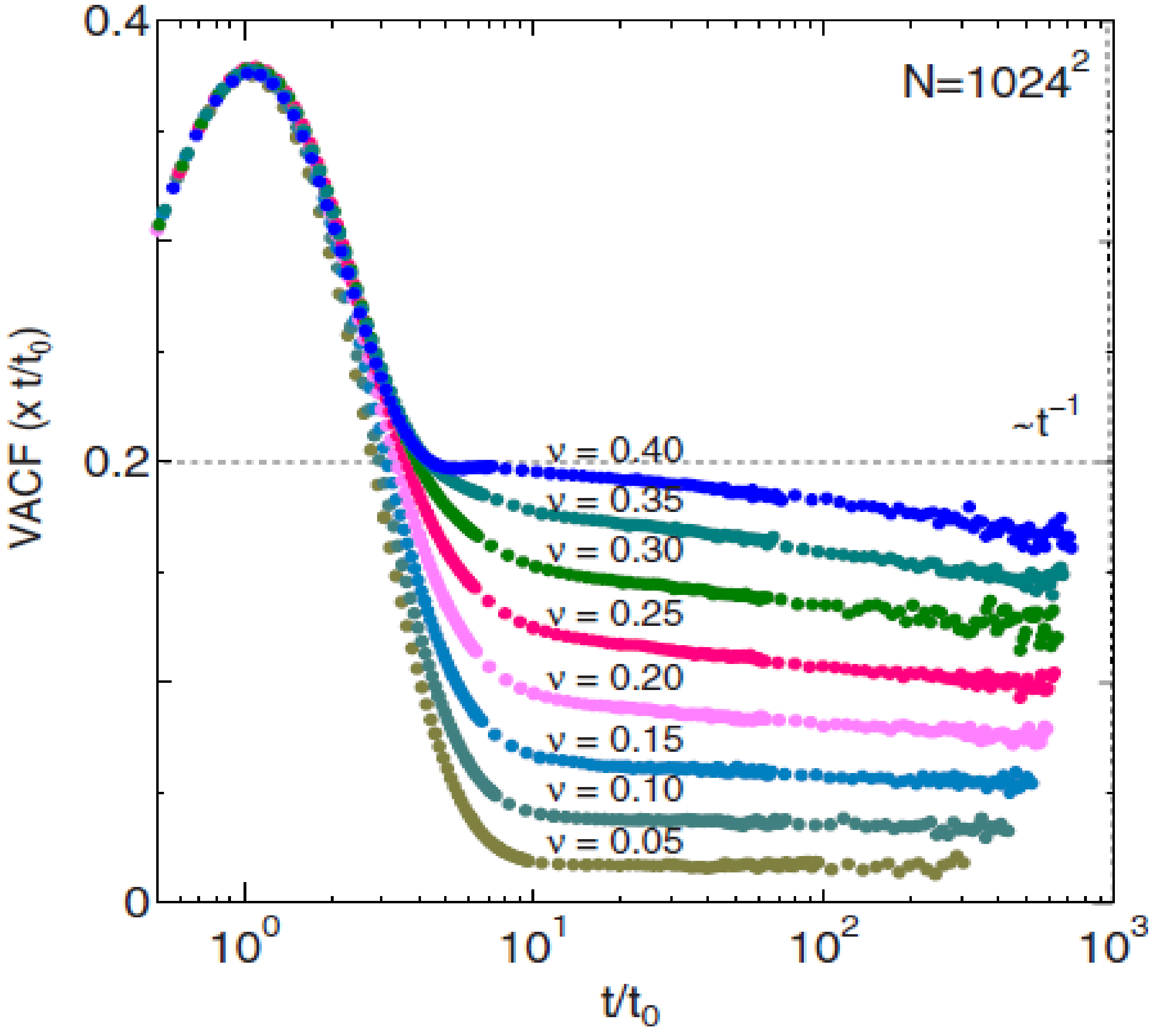
$$c(t) = \begin{cases} b_k e^{-\xi t}, t \sim 10 t_0 \\ b_h t^{-1}, t \sim \mathrm{P}\, t_0 \\ \dfrac{b_s}{t\sqrt{\ln t}}, \mathrm{t} >> \mathrm{t}_0 \end{cases}$$

The most recent verification (M. Isobe, PRE 77, 2008) for the velocity autocorrelation :

$$\text{low densities:} \quad c(t) \sim t^{-1};$$

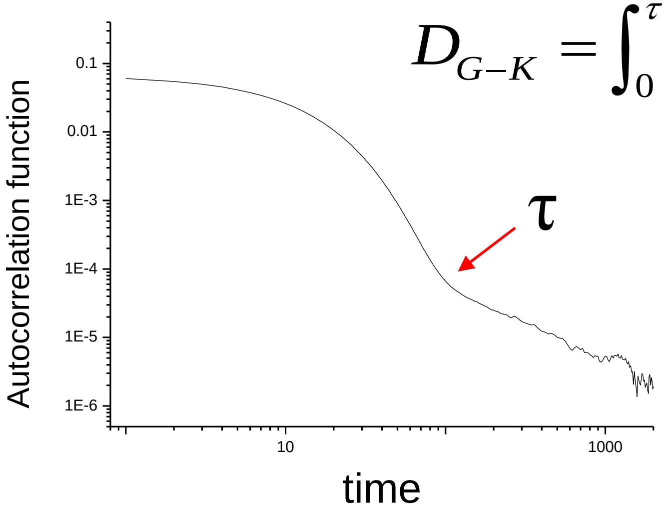
$$\text{moderated densities:} \, c(t) \sim (t\sqrt{\ln t})^{-1}$$

b_k, b_h and b_s are not checked



M. Isobe, PRE 77, 2008

Open problems: (c) influence of the long-time tail on the observed values of transport coefficients.

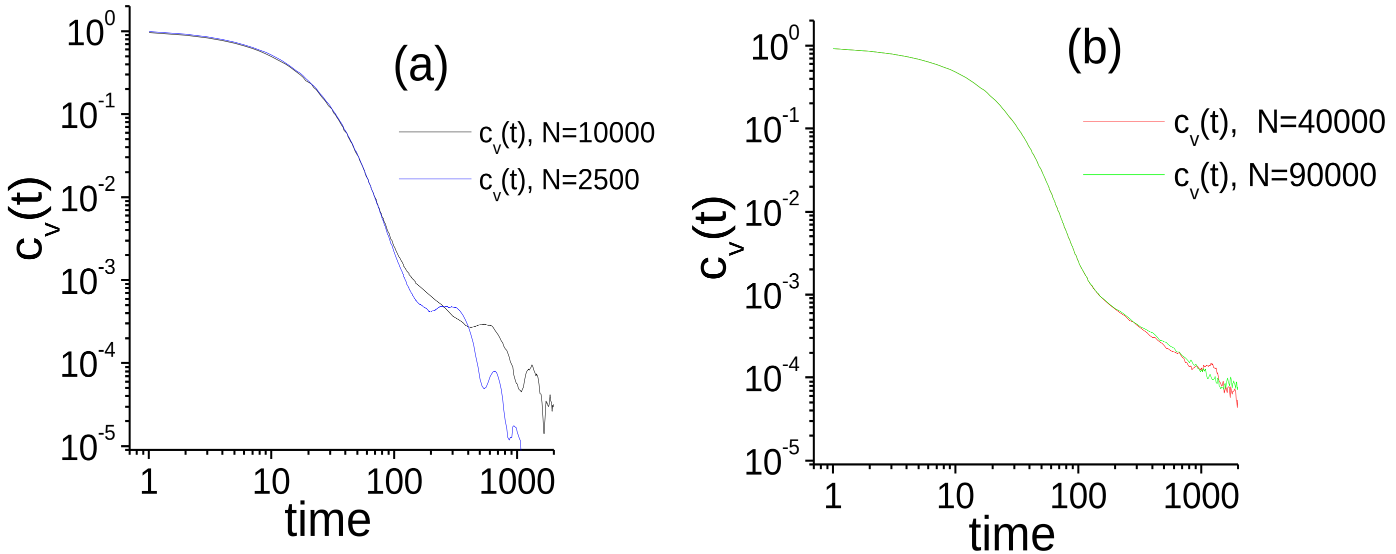


$$D_{G-K} = \int_0^{\tau} c_v(t)dt + \int_{\tau}^{tr} c_v(t)dt = D_{G-K}^{\tau} + \int_{\tau}^{tr} c_v(t)dt$$

Few discussions have focused on this topic. The influence is supposed to be negligible for practical applications

S. Viscardy and P. Gaspard, Phys. Rev. E 68,041204 (2003); R. Garciu-Rojo, S. Luding, J. Javier Brey, Phys. Rev. E 74, 061305 (2006)

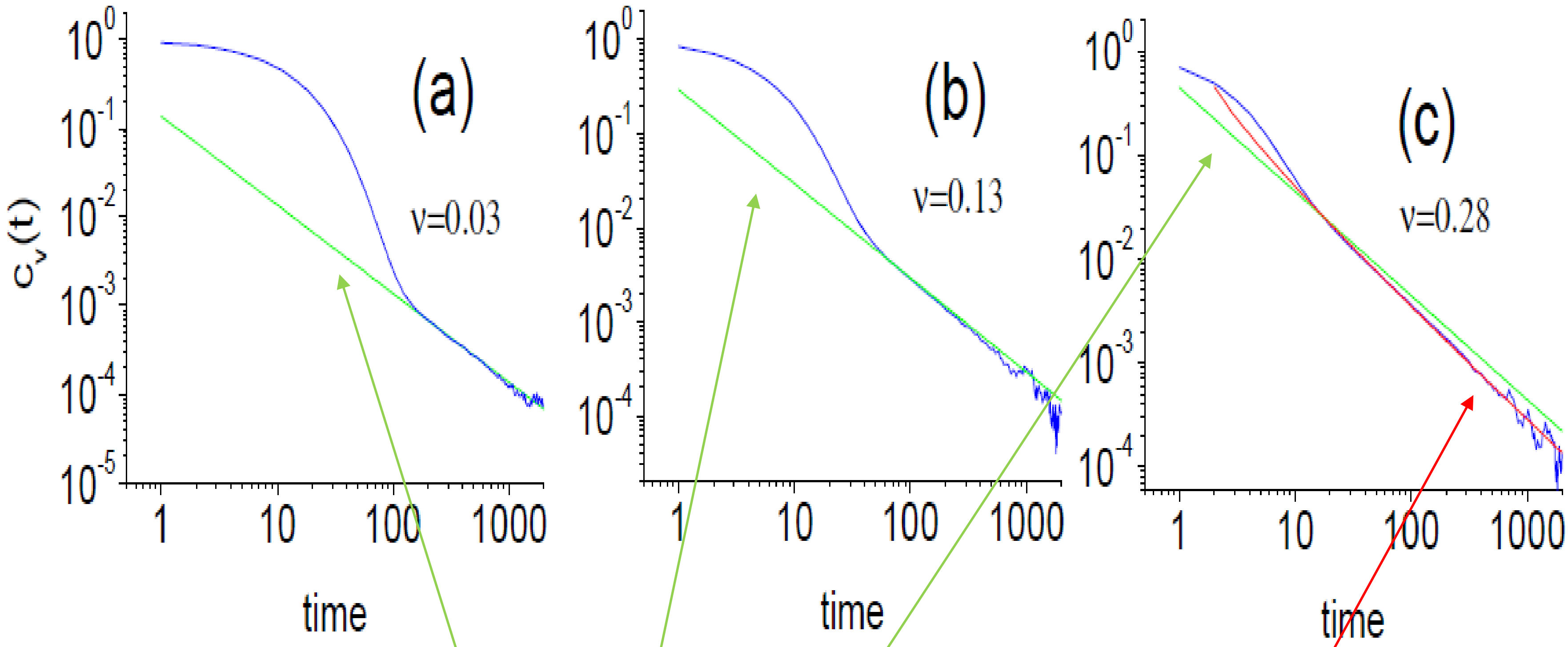
Result: finite-size effect and the minimum system size for simulation study



$$\text{Minimum size: } L \geq 20000, N \geq 40000$$

S. Chen, Y. Zhang, J. Wang, and H. Zhao, Phys. Rev. E 87, 032153 (2013)

Result: velocity autocorrelation

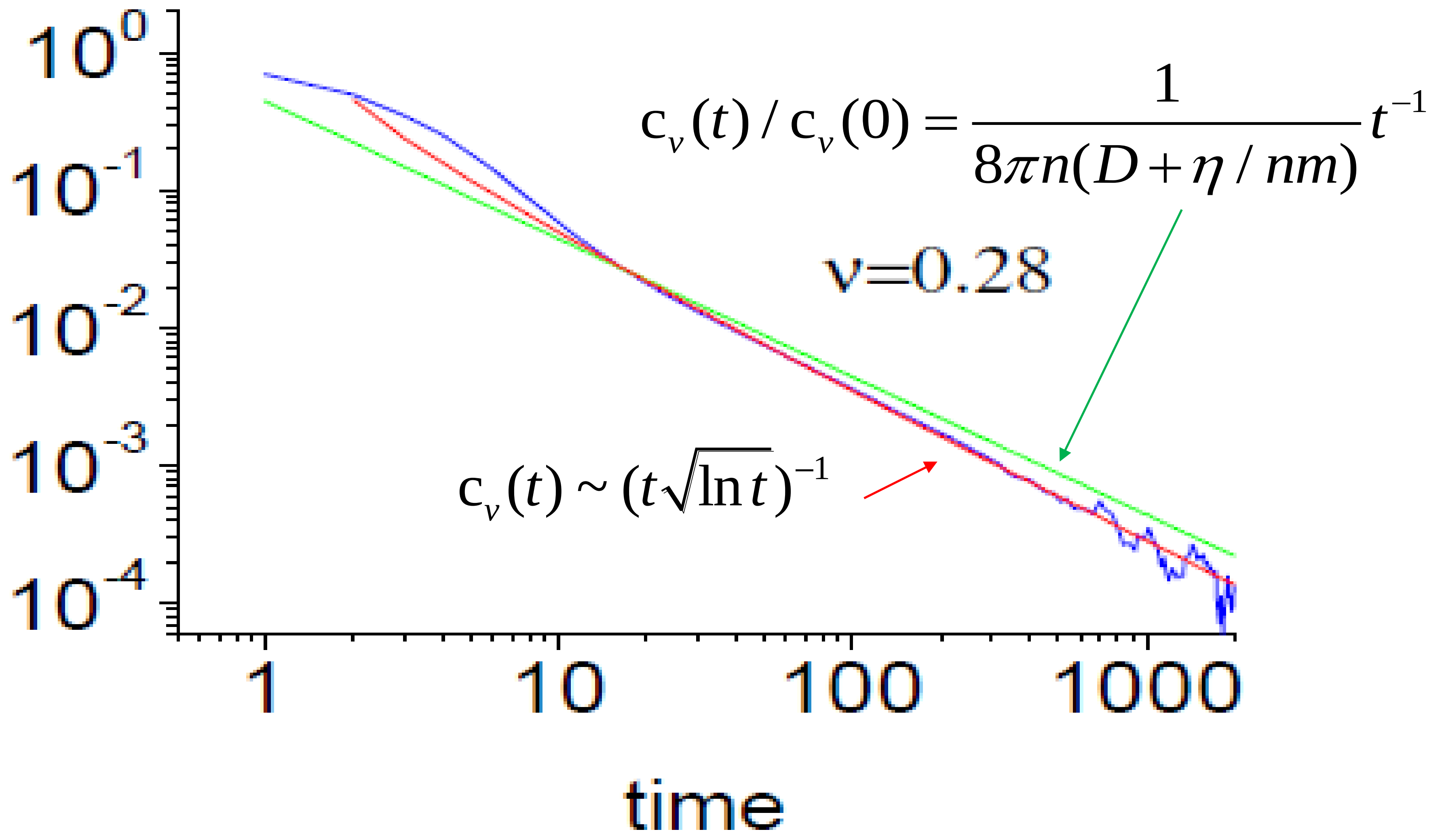


$$c_v(t)/c_v(0) = \frac{1}{8\pi n(D+\eta/nm)} t^{-1}$$

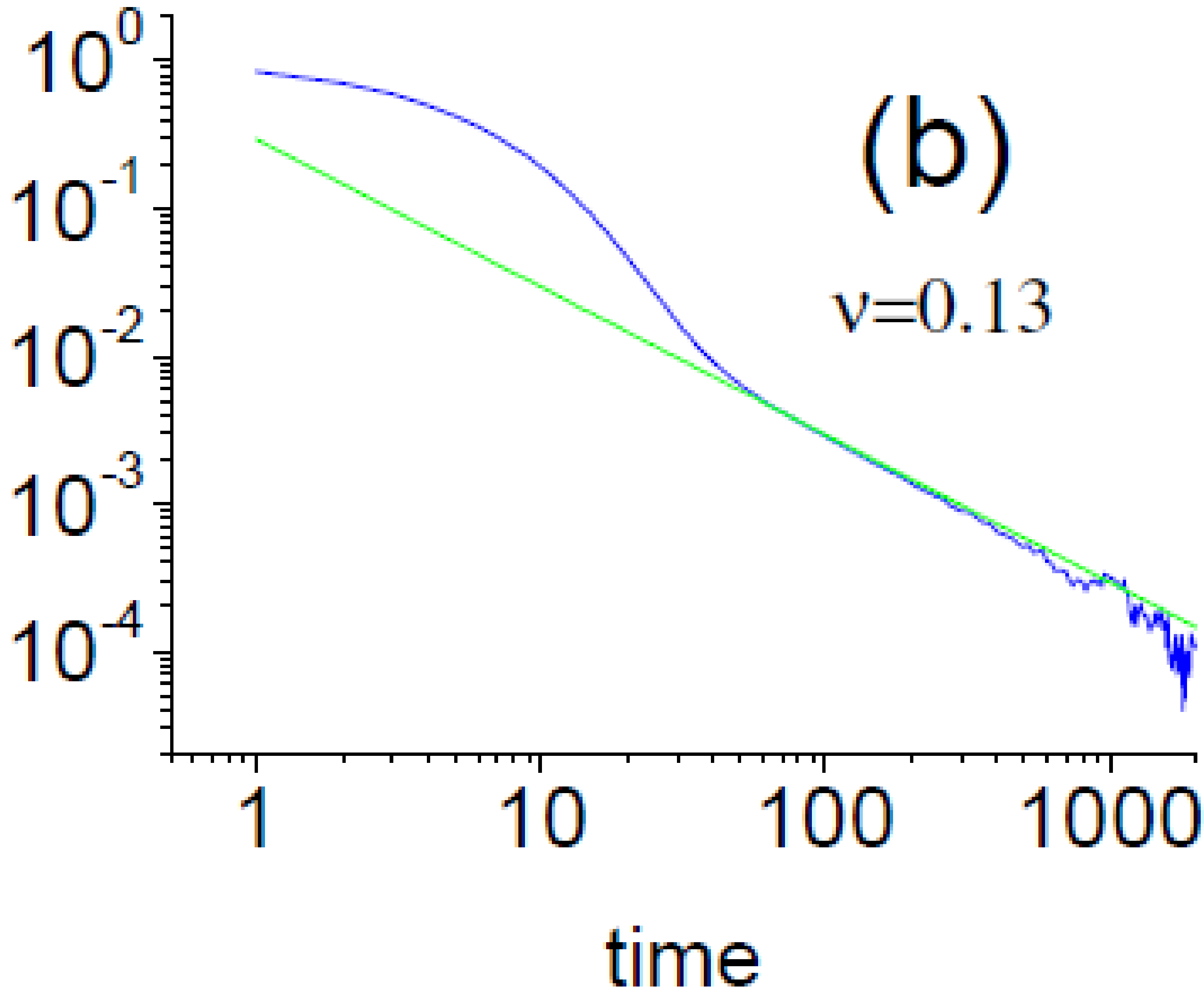
$$c_v(t) \sim (t\sqrt{\ln t})^{-1}$$

(1) The amplitude of 1/t is verified in low densities, $(t\sqrt{\ln t})^{-1}$ law is evidenced at moderate density. However…

(2) It seems that the 1/t low is not exact

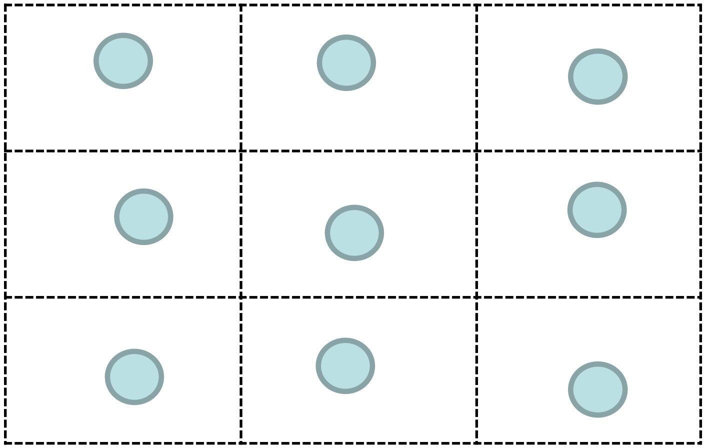


10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
1
10
100
1000
time
$c_v(t)/c_v(0) = \frac{1}{8\pi n(D+\eta/nm)} t^{-1}$
ν=0.28
$c_v(t) \sim (t\sqrt{\ln t})^{-1}$



(b)
ν=0.13
10^0
10^-1
10^-2
10^-3
10^-4
1
10
100
1000
time

Discussion: why the calculation of the current autocorrelation is so difficult?



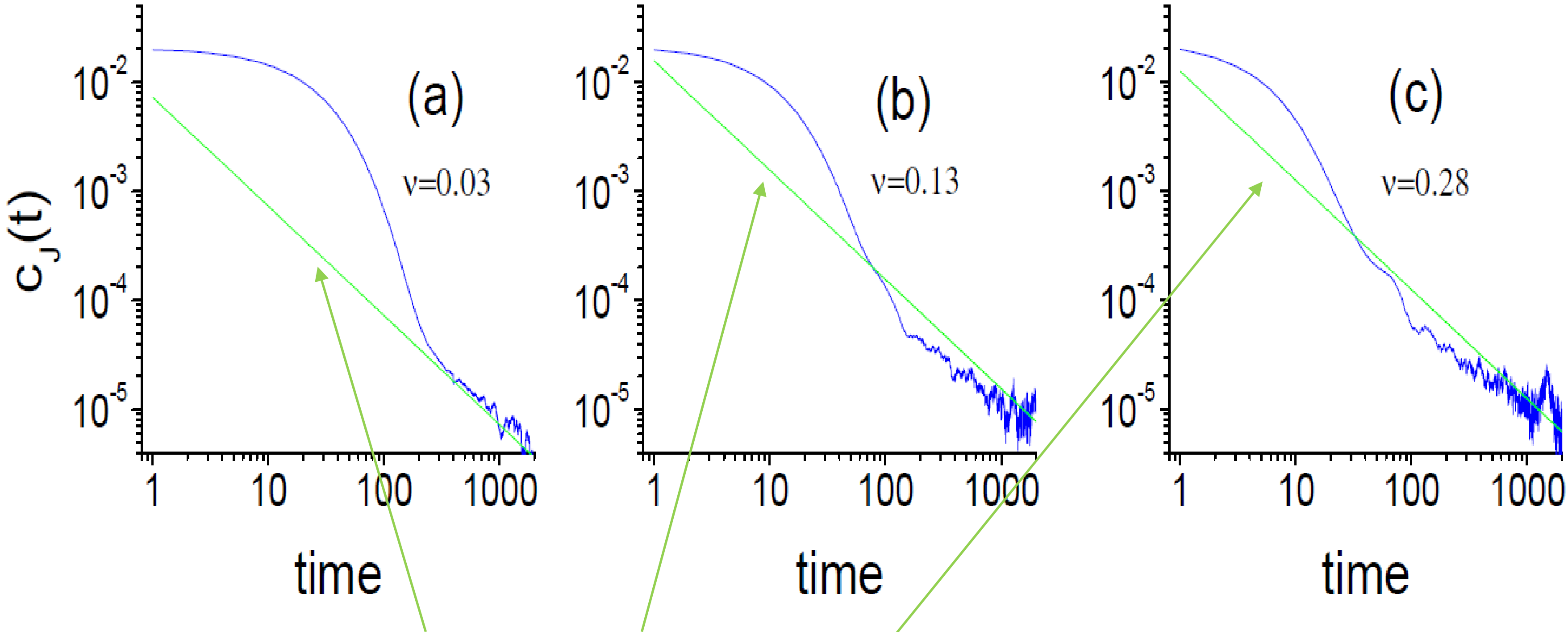
Evolve a system a period, one obtains N v(t)v(0) for $c_v(t)$

While one can obtains only one J(t)J(0) for $c_J(t)$

$$J^x(\mathrm{t}) = \sum_{k=1}^{N} j_k^x(\mathrm{t})$$

So one needs 40000 times of the computer time for calculating the current autocorrelation function

Result: energy current autocorrelation

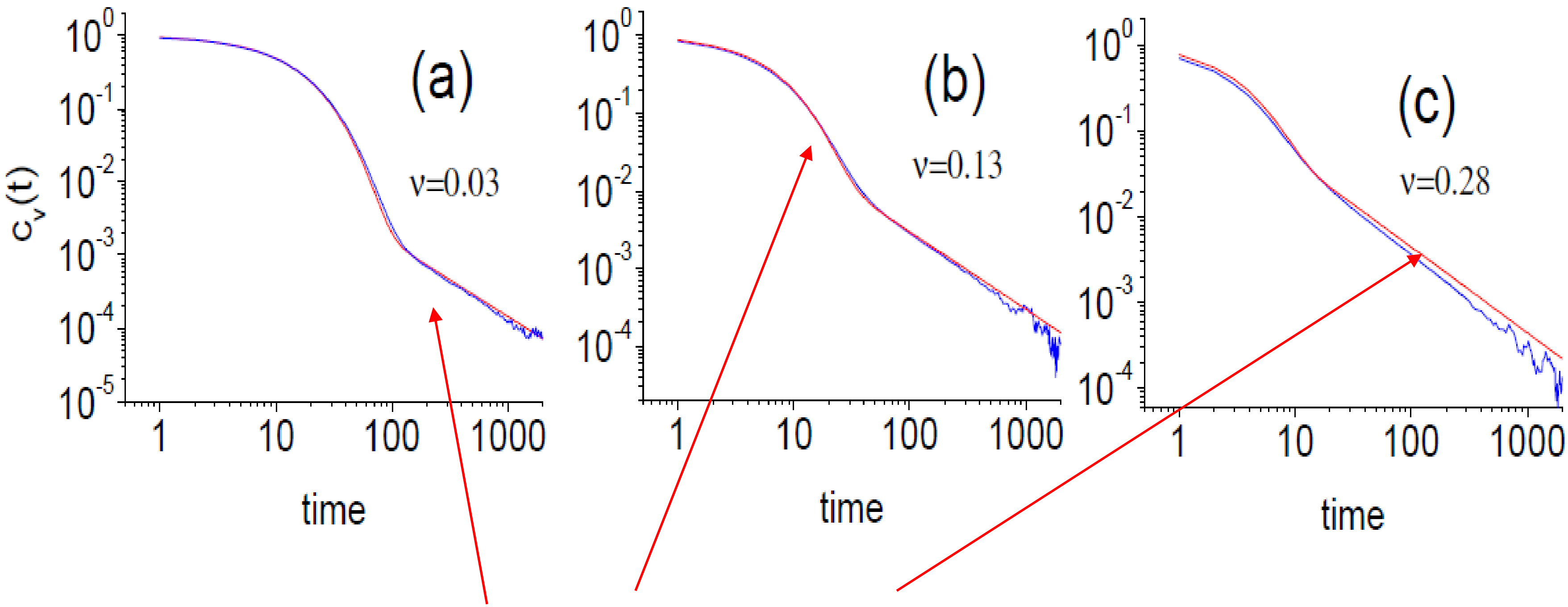


$$c_J(t)/c_J(0) = \frac{1}{2\pi(\kappa/2k_B + \eta/m)} t^{-1}$$

(1) The 1/t tail as well as the amplitude are verified in the dilute limit

(2) Significant divergence evidenced with the increase of the density

Result: unified formula for $c_v(t)$

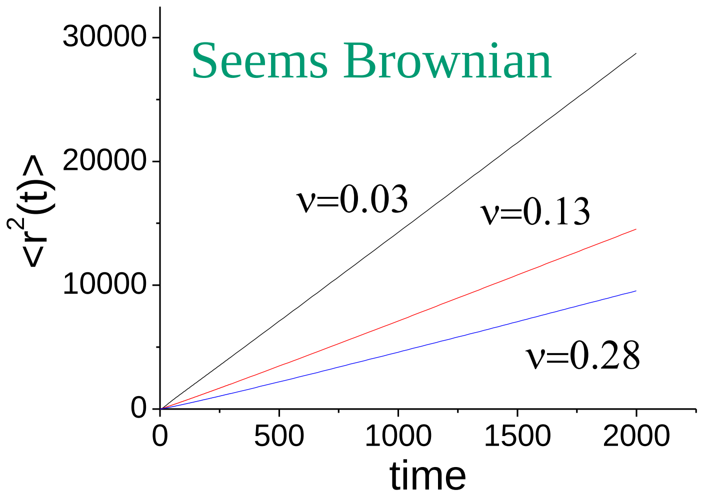


$$c_v(t)/c_v(0) = e^{-\frac{k_B T}{mD}t} + \frac{1}{8\pi n(D+D_\eta)}(1-e^{-\frac{k_B T}{mD_\eta}t})t^{-1}, \quad c_v(0) = \frac{k_B T}{m}, D_\eta = \eta / nm$$

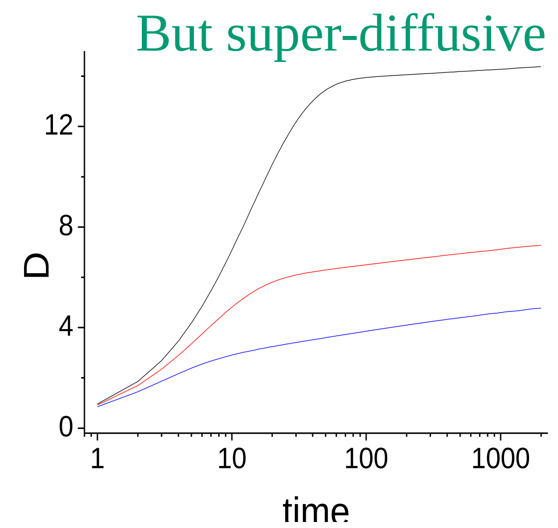
D, η Enskog values

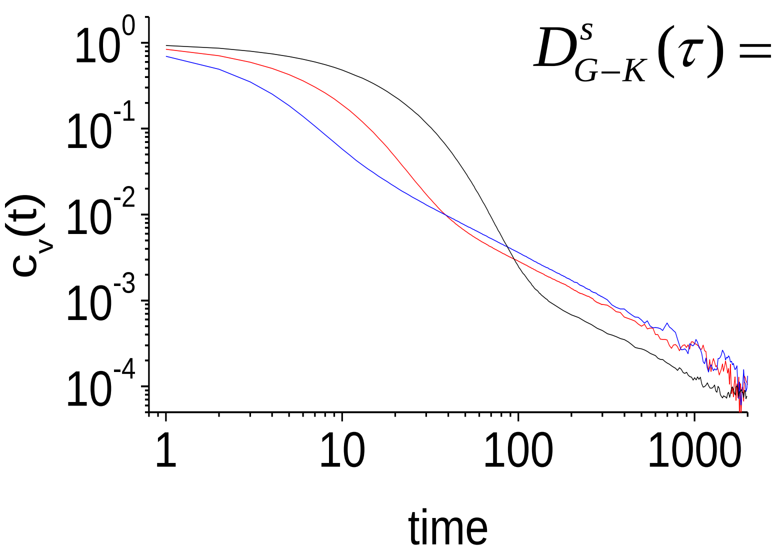
It works well in the low density regime

Consistency between observed diffusion constant and that obtained using Green-Kubo formula

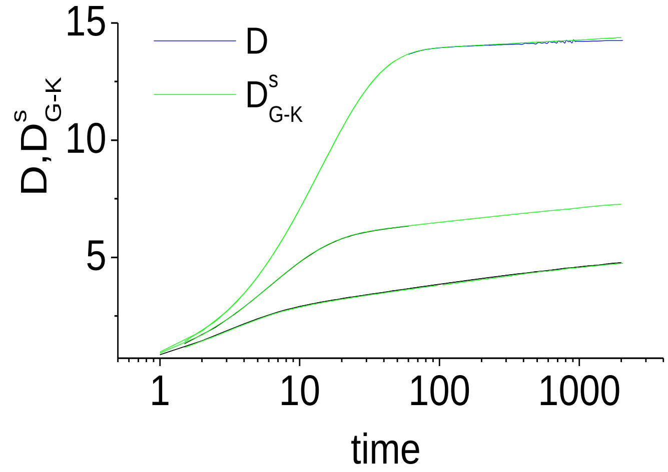


$$D = \frac{1}{4}\frac{d}{dt}\left\langle r^2(t)\right\rangle$$



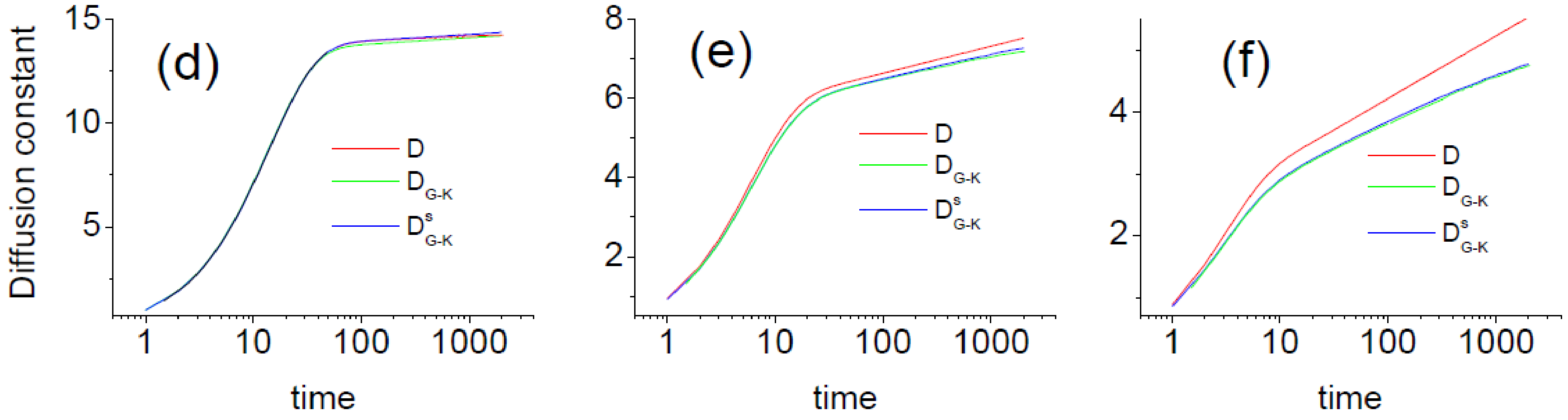


$$D^{s}_{G-K}(\tau) = \int_0^{\tau} c_v(t)\,dt$$



Green-Kubo formula is precisely correct

Result: influence on transport coefficients



D: by tracing the tagged particle

D^{s}_{G-K}: G-K formula using simulation $c_v(t)$

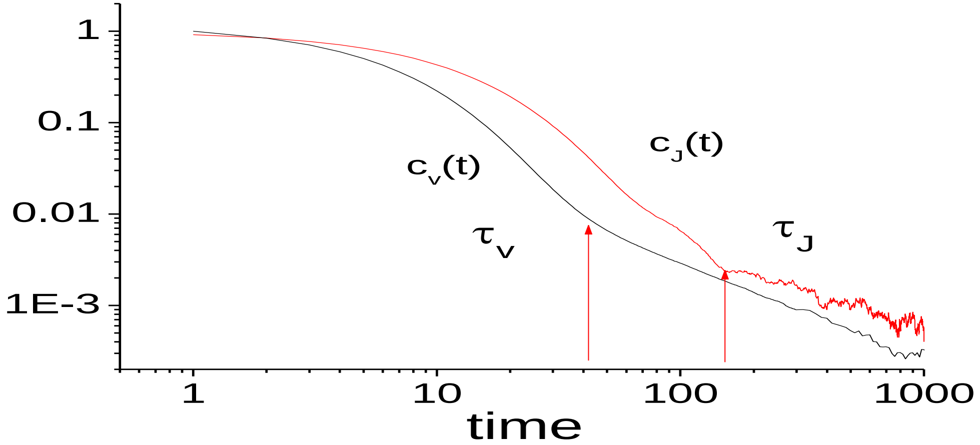
D_{G-K}: G-K formula by asuming the t^{-1}

$$D_{G-K} = \int_0^{\tau} c_v(t)dt + \int_{\tau}^{tr} c_v(t)dt = D^{\tau}_{G-K} + \int_{\tau}^{tr} c_v(t)dt$$

The contribution of the hydrodynamics tail can not be ignored

Result: influence on transport coefficients

ν	D_0	D_E	$D_{G-K}^{\tau_v}$	τ_v	t_{10}	t_{100}
0.03	14.10	13.42	13.96	110	10^5	10^{35}
0.13	7.05	5.71	6.40	40	200	10^9
0.28	4.70	2.76	2.85	10	16	1000



κ_0	κ_E	$\kappa_{G-K}^{\tau_J}$	τ_J	t_{10}	t_{100}
0.564	0.592	0.580	280	10^5	10^{39}
0.282	0.354	0.258	160	6000	10^{16}
0.188	0.360	0.139	110	1000	10^{10}

Influence to the thermal conductivity is relatively small

Discussion: why $c_J(t)$ and $c_v(t)$ are so different?

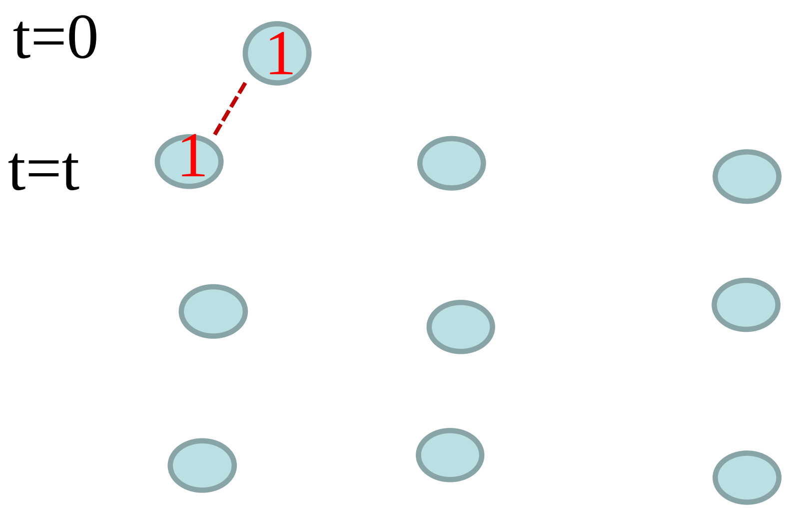
$$c_J(t) =< J^x(0) J^x(\mathrm{t}) >=< \sum_{k=1}^{N} j_k^x(0) \sum_{k=1}^{N} j_k^x(\mathrm{t}) >$$

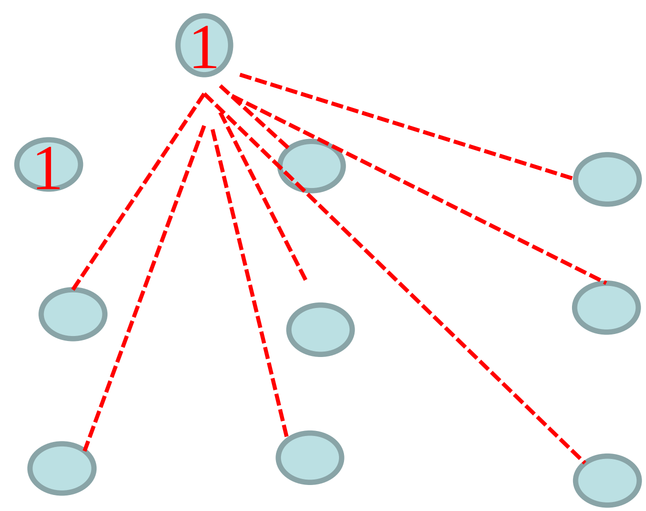
$$= N(< j_1^x(0) j_1^x(\mathrm{t}) > + < j_1^x(0) \sum_{k \neq 1}^{N} j_k^x(\mathrm{t}) >) \equiv N(c_j(t) + \tilde{c}_j(t))$$

$$\boxed{c_J(t) / \mathrm{N} = c_j(t) + \bar{c}_j(t)}$$

$$c_j(t) =< j_1^x(t) j_1^x(0) >$$

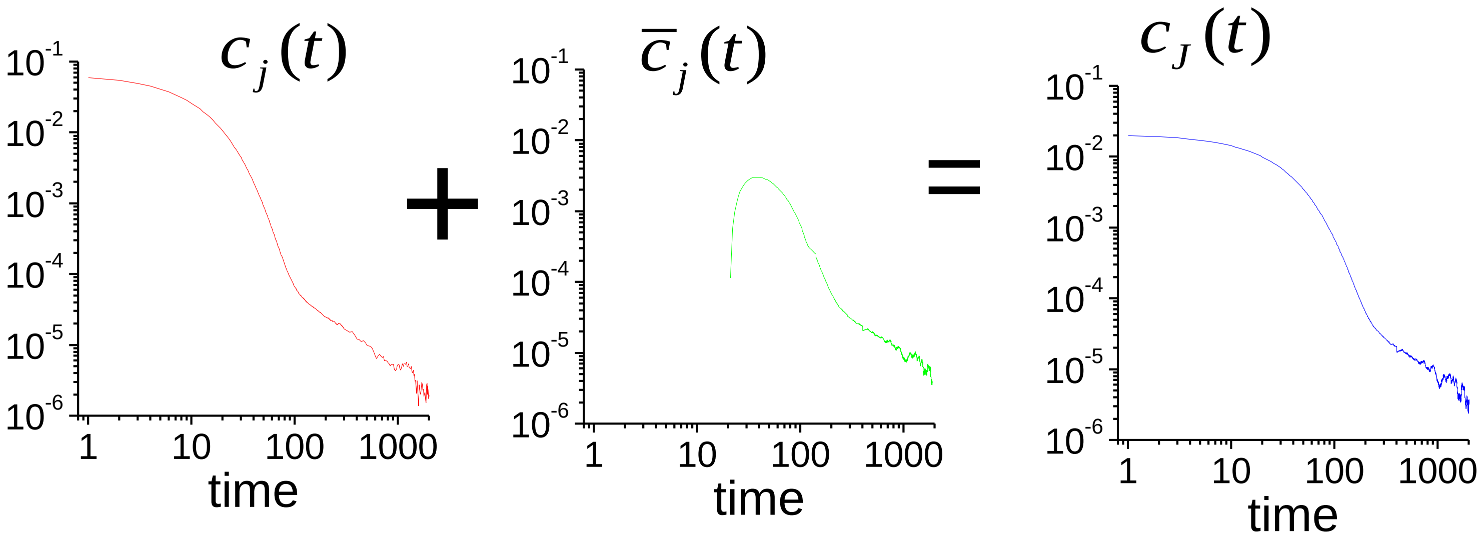
$$\bar{c}_j(t) =< j_1^x(0) \sum_{k \neq 1}^{N} j_k^x(\mathrm{t}) >$$



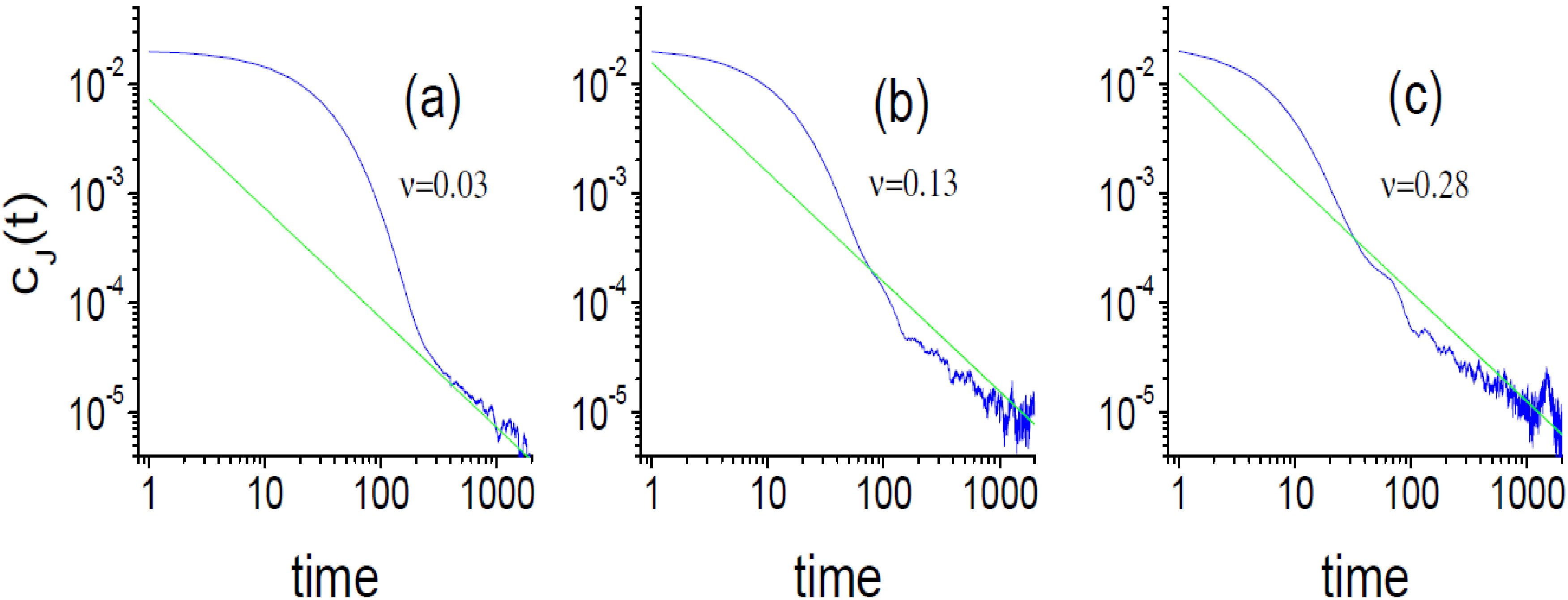


Discussion: why $c_J(t)$ and $c_v(t)$ are so difference

$$c_j(t) + \overline{c}_j(t) = c_J(t) / \mathrm{N}$$

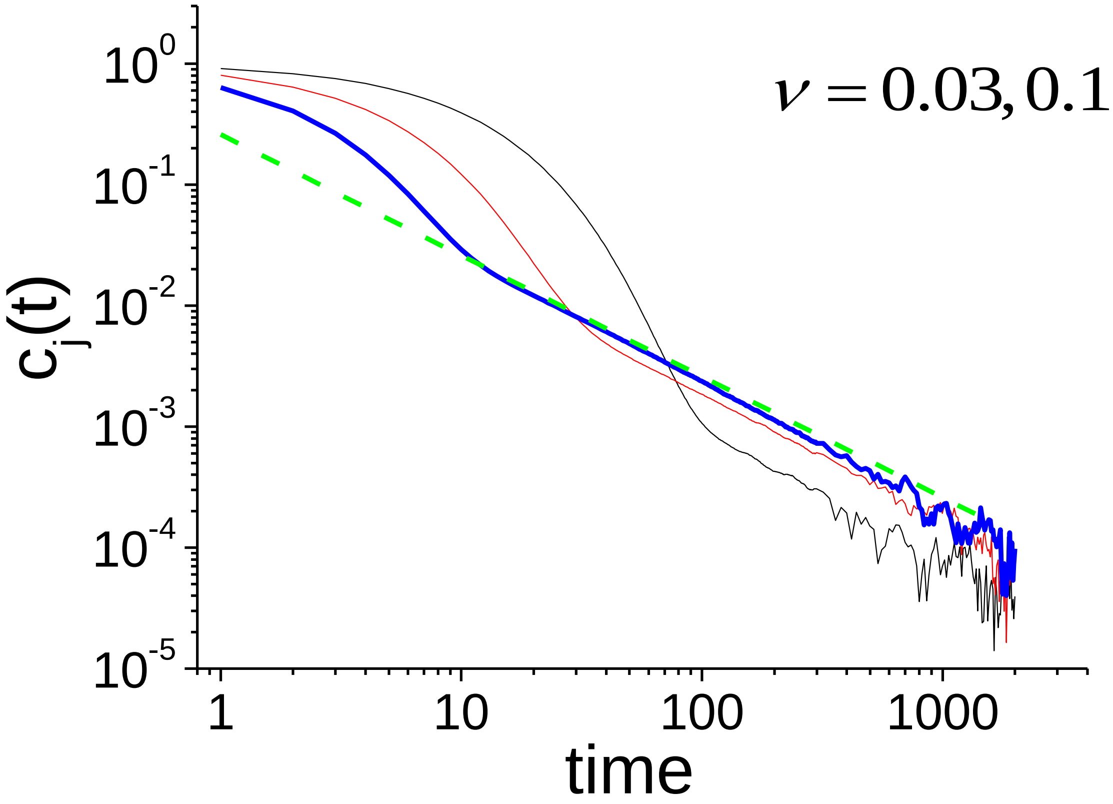


$c_J(t)$ Could be a mutli-stage function



$c_J(t)$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
(a)
ν=0.03
(b)
ν=0.13
(c)
ν=0.28
1
10
100
1000
time
time
time

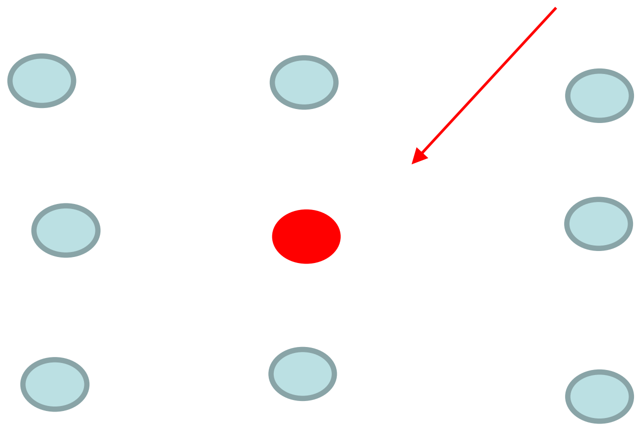
Discussion: why $c_J(t)$ and $c_\nu(t)$ are so different?



Therefore $\bar{c}_j(t)$ play the key role in determining $c_J(t)$

Discussion: mechanisms

tagged particle [mv(0), j(0)]



How does the tagged particle, the momentum mv(0), and the local current j(0) diffusion as a function of time?

$$c_A(\mathbf{r},t)=\frac{\langle \mathbf{A}(0)\bullet\mathbf{A}(\mathbf{r},t)\rangle}{\langle \mathbf{A}^2(0)\rangle}+c$$

H. Zhao, Phys. Rev. Lett. 96, 140602 (2006); Shunda Chen, Yong Zhang, Jiao Wang, and Hong Zhao,Phys. Rev. E 87, 032153 (2013)

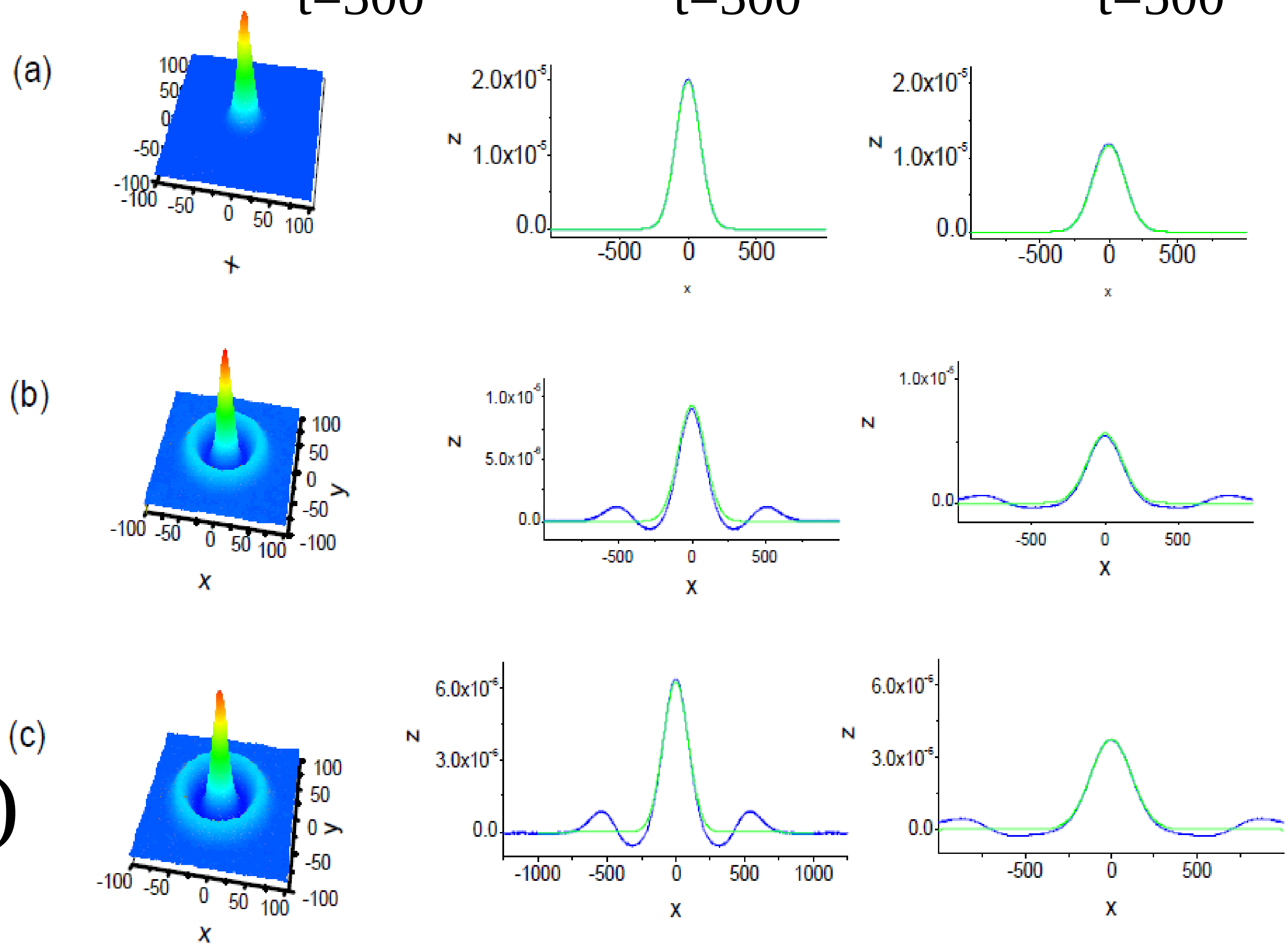
Discussion: mechanisms

t=300 t=300 t=500

$\rho(\mathbf{r},t)$

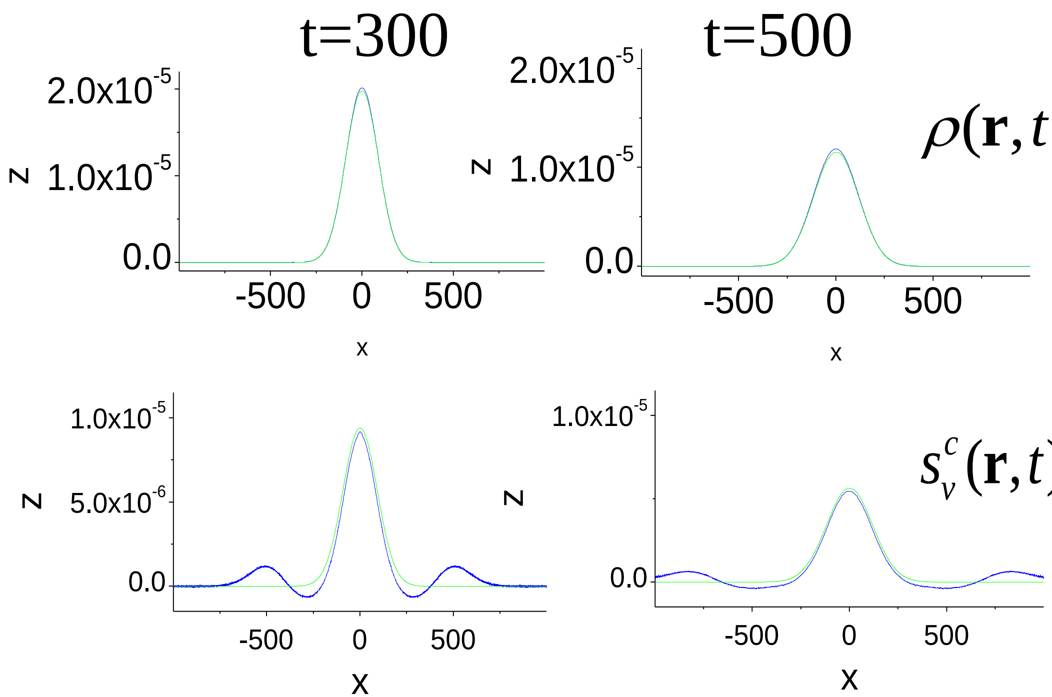
$s_v(\mathbf{r},t)$

$s_j(\mathbf{r},t)$



$\nu = 0.03$

Discussion: mechanisms

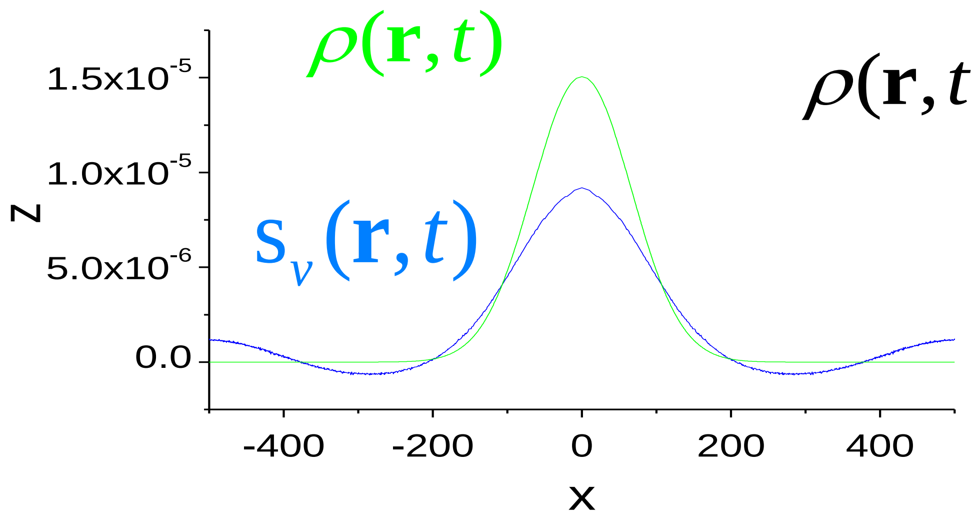


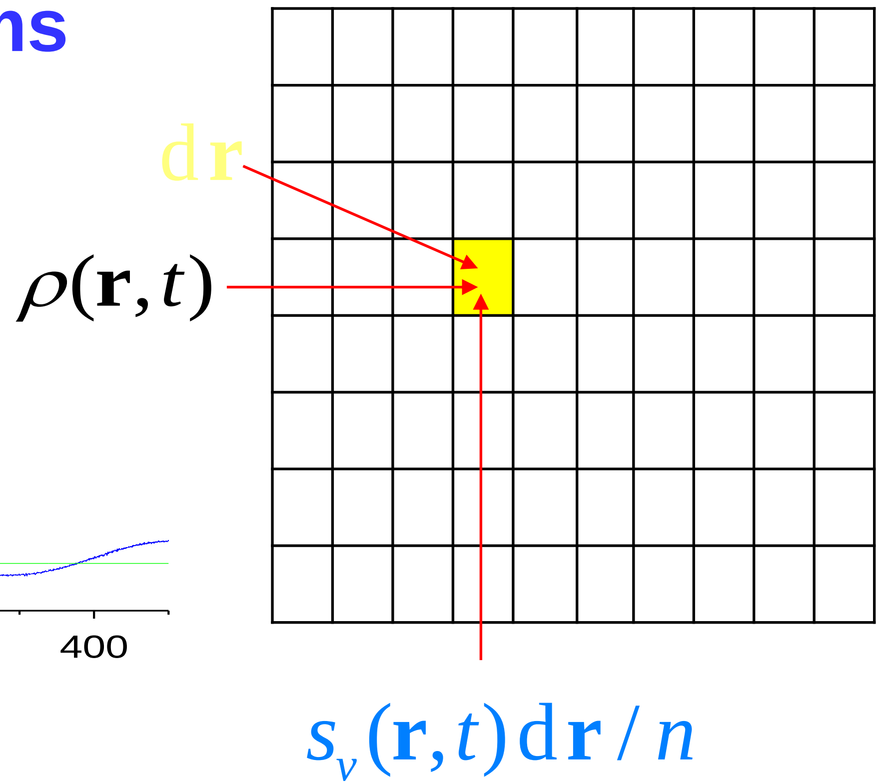
$$\rho(\mathbf{r},t) = \frac{1}{4\pi D t} e^{-\frac{r^2}{4Dt}}, \quad D = D_E = 13.4$$

$$s_v^c(\mathbf{r},t) = \frac{a_v}{4\pi D_v t} e^{-\frac{r^2}{4D_v t}}, \quad D_v = \eta_E / nm = 14.1$$

D and η_E are Enskog values

Discussion: mechanisms





The tagged particle have the probability $\rho(\mathbf{r},t)$ at $(\mathbf{r},t)$
where there is $s_v(\mathbf{r},t)\,d\mathbf{r}$ momentum.
It thus get a feedback momentum $s_v(\mathbf{r},t)\,d\mathbf{r}/n$
since there are $d\mathbf{r}/n$ disks in this area

Discussion: mechanisms

$$\mathrm{c}_v(t)/\mathrm{c}_v(0) = \frac{1}{n}\int \rho(\mathbf{r},t)\mathrm{s}_v(\mathbf{r},t)\,\mathrm{d}\,\mathbf{r}$$

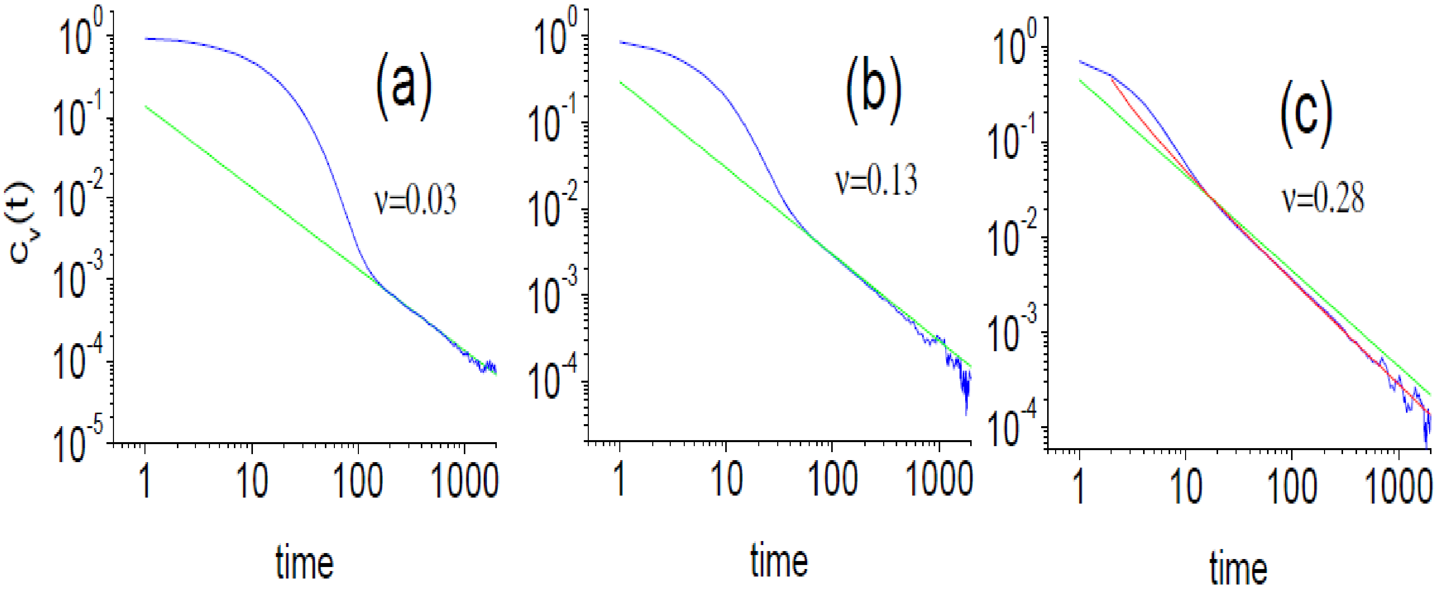
$$\sim \frac{1}{n}\int \rho(\mathbf{r},t)\mathrm{s}_v^c(\mathbf{r},t)\,\mathrm{d}\,\mathbf{r} = \frac{a_v}{4\pi n(D+D_v)}\mathrm{t}^{-1}$$

$$\frac{\int s_v^c(\mathbf{r},t)d\mathbf{r}}{\int s_v^{rest}(\mathbf{r},t)d\mathbf{r}} = \text{ Landau--Placzek ratio} = \frac{C_P}{C_V} - 1$$

$$\int s_v^c(\mathbf{r},t)d\mathbf{r} + \int s_v^{rest}(\mathbf{r},t)d\mathbf{r} = 1$$

$$\Rightarrow a_v = \frac{C_V}{C_P} = 0.5(d=2)$$

$$c_v(t)/c_v(0)=\frac{a_v}{4\pi n(D+D_v)}t^{-1},\quad a_v=\frac{C_V}{C_P}$$



B. S. Alder and T. E. Wainwright, Phys. Rev. Lett. 18, 988 (1969),S. R. Dorfman and E. G. D. Cohen, PRL 25,1257(1970); M, H. Ernst, E. H. Hauge, and J. M. J. van Leeuwen, Phys. Rev. Lett. 25, 1254 (1970); M. H. Ernst, E. H. Hauge, and J. M. J. van Leeuwen, Phys. Rev. A 4, 2055 (1971). J. R. Dorfman and E. G. D. Cohen, Phys. Rev. A 6, 776 (1972).

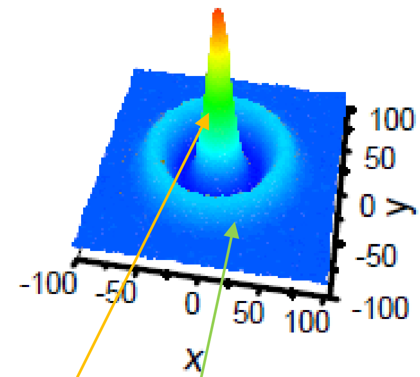
The momentum carried by the tagged particle at time t.

t=0

t=t

losted

$$c_v(0)(1-e^{-\frac{k_B T}{m D_\eta}t})$$



$$c_v(0)$$

remained

$$c_v(t)=c_v(0)e^{-\frac{k_B T}{m D}t}$$

+

$$c_v(t)/c_v(0)=e^{-\frac{k_B T}{m D}t}+\frac{1}{8\pi n(D+D_\eta)}(1-e^{-\frac{k_B T}{m D}t})t^{-1}$$

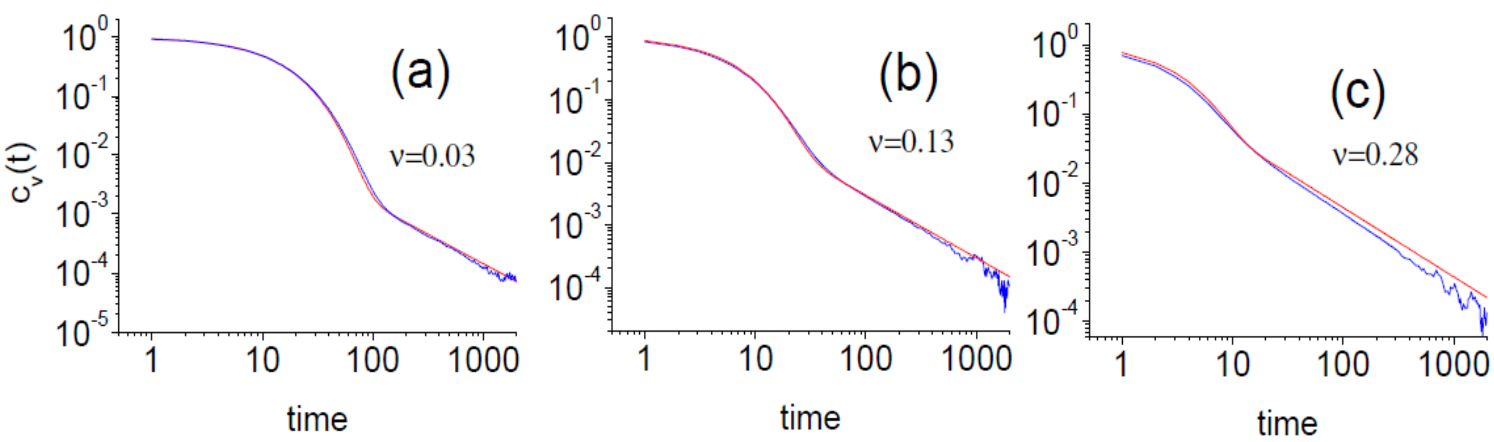
The momentum carried by the tagged particle at time t.

The global picture is thus as following:

(1) The tagged particle loss its memory as $c_v(t)/c_v(0) = e^{-\frac{k_B T}{mD}t}$

(2) The lost part $1 - e^{-\frac{k_B T}{mD_\eta}t}$ is split into two parts, the center peak and the ring peak, the momentum remain in the center peak may feedback to the tagged particle, thus at time t the momentum remain on the tagged particle is

$$c_v(t)/c_v(0) = e^{-\frac{k_B T}{mD}t} + \frac{1}{8\pi n(D + D_\eta)}(1 - e^{-\frac{k_B T}{mD}t})t^{-1}$$



Summary

(1) Velocity autocorrelation function:

(a) A unified formula is obtained, it works well at low density

(b) The 1/t tail as well as its amplitude are verified in the low-density regime, the logarithmic decay is evidenced at moderated density.

(c) The 1/t law is not exact even at low densities

(d) The hydrodynamics tail may dramatically influence the diffusion constant.

(2) Energy-current autocorrelation function:

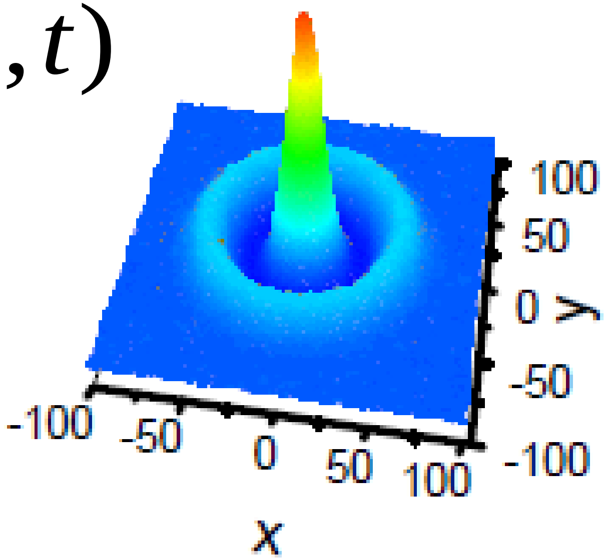
(a) The 1/t tail as well as its amplitude are verified in the dilute density. Significant divergence observed even in low-density regime

(b) The influence of the tail to the thermal conductivity is not significant as in the case of diffusion constant.

Thanks for your attention!

Discussion: the tail of $c_J(t)$ and $c_v(t)$ have different mechanism

$s_j(\mathbf{r},t)$



$$c_J(t) = \int s_j(\mathbf{r},t)d\mathbf{r}$$

The mechanism of tails may be different

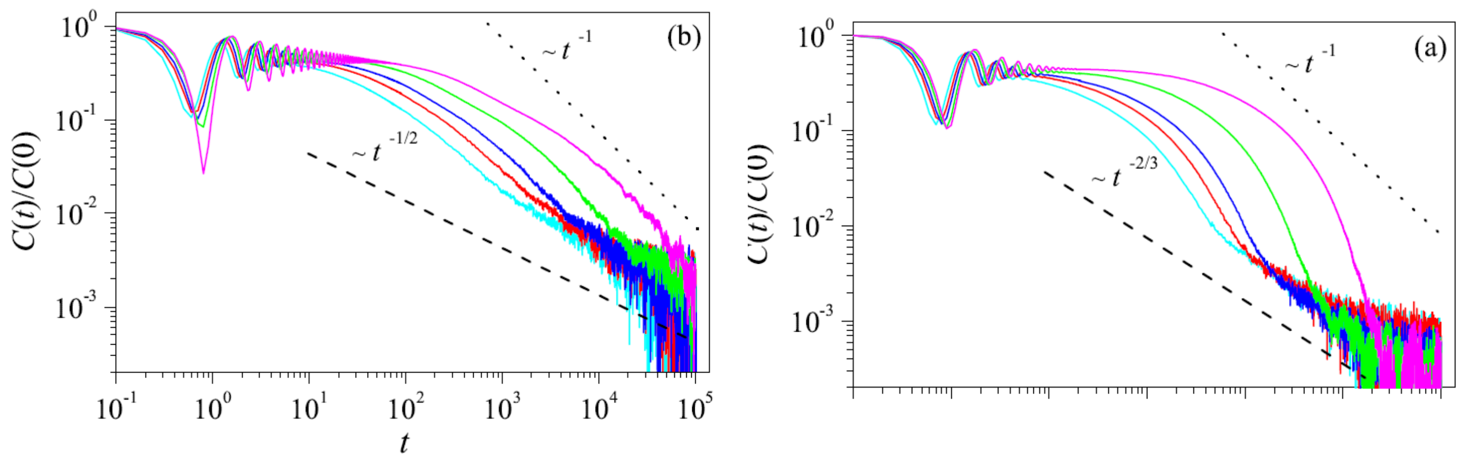
Simulation results for lattice: the energy current decay

The $\mathrm{FPU}-\beta$ model (symmetric)

$$H=\sum_{i=1}^{N}\frac{p_i^2}{2m}+\frac{1}{2}(x_i-x_{i-1})^2+\frac{1}{4}(x_i-x_{i-1})^4$$

The $\mathrm{FPU}-\alpha-\beta$ model (asymmetric)

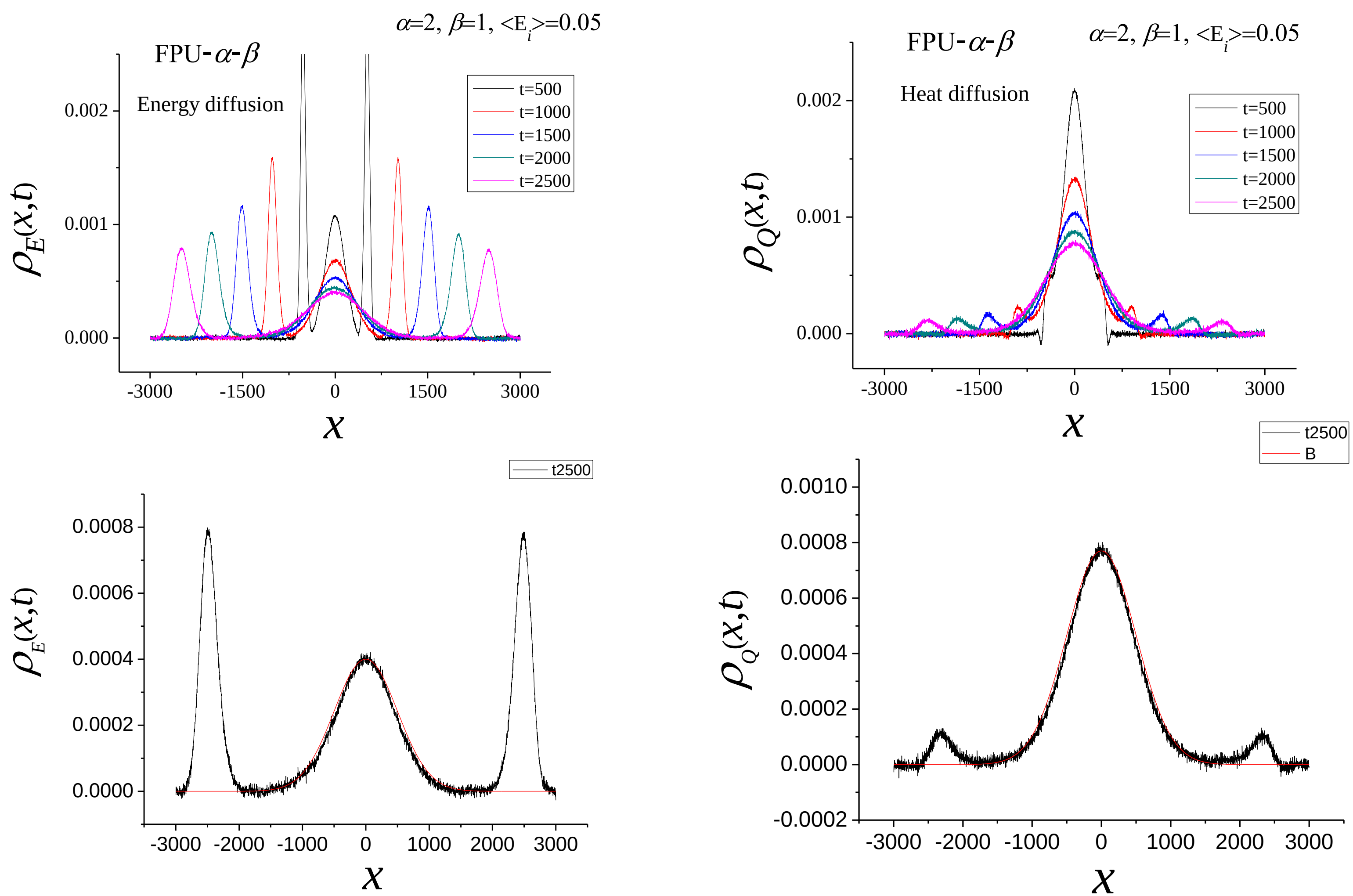
$$H=\sum_{i=1}^{N}\frac{p_i^2}{2m}+\frac{1}{2}(x_i-x_{i-1})^2+\frac{1}{3}(x_i-x_{i-1})^3+\frac{1}{4}(x_i-x_{i-1})^4$$



Temperature 0.56,0.32, 0.21,0.1 and 0.05

S. Chen Y. Zhang, J. Wang and H. Zhao, arXiv:1204.5933 (2012); arXiv:1511.00292(2015)

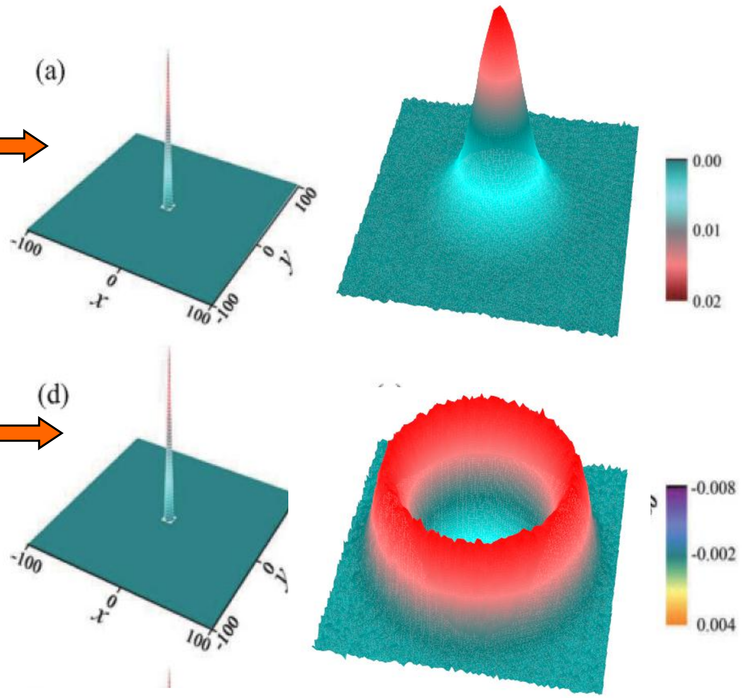
Focus topics
(b) Distinct the concepts of energy and heat



2D Gas (hard discs) Model

heat

energy



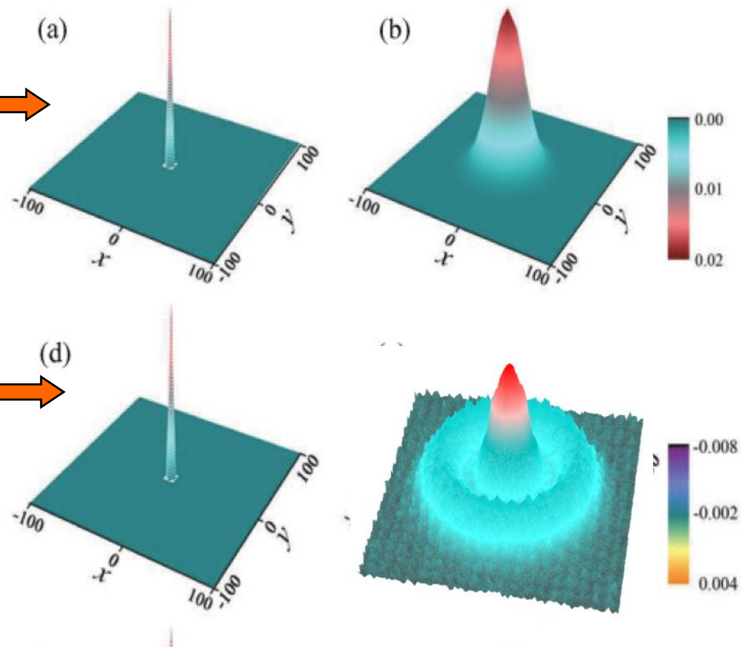
Normal or supperdiffusion

Ballistic diffusion

Results for 2D Gas

Particle

Normal diffusion



Mass-density

Heat + sound

Heat and energy

$$\frac{\partial}{\partial t}[e(x,t) - \frac{(e+P)\rho(x,t)}{\rho}] + \frac{\partial}{\partial x} j^q(x,t) = 0,$$

$$q(\mathbf{r},t) = e(\mathbf{r},t) - \left(\frac{e+P}{\rho}\right)\rho(\mathbf{r},t)$$

Basically they are conceptually different quantities

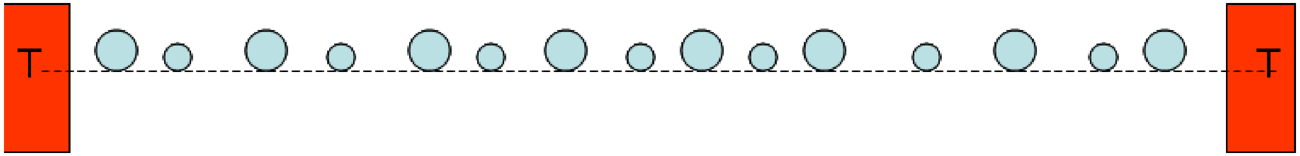
The local heat current and local energy current:

$$j_i^q(t) = j_i^e(t) - \frac{(e+P)}{\rho}\rho_i(r,t)v_i$$

$$J^q(t) = J^e(t)$$

Models consistent to theoretical prediction

（a） 1D gas with alternative masses (asymmetric, m1/m2=1/3)



(b) The FPU $-\beta$ model (symmetric)

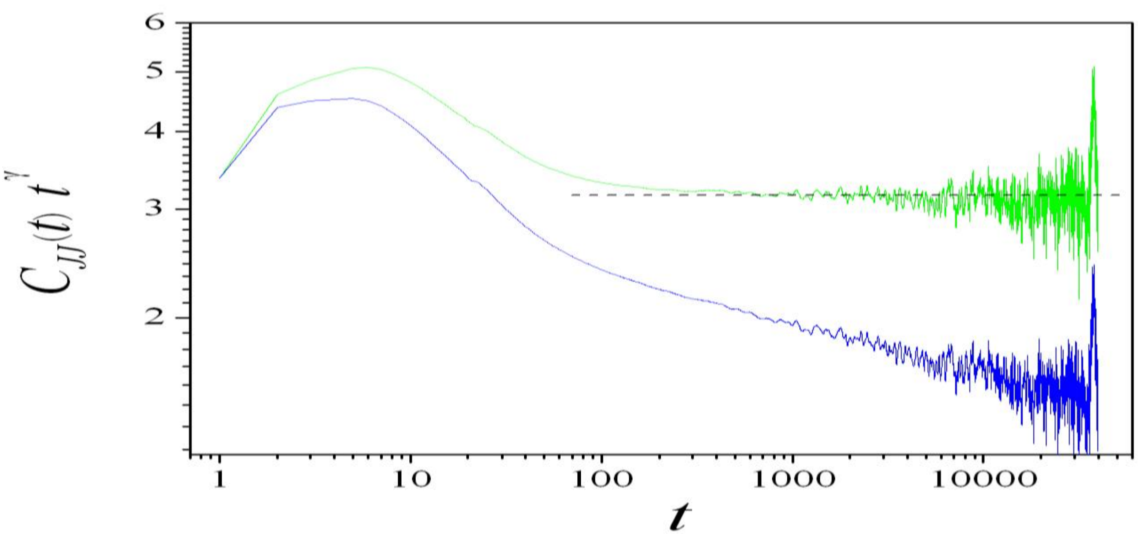
$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2}(x_i - x_{i-1})^2 + \frac{\beta}{4}(x_i - x_{i-1})^4$$

Models consistent to theoretical prediction

（1） 1D gas with alternating masses

$$C(t) = < J(t)J(0) > \sim t^{-\gamma}$$

simulation size	L	4096	8192	16384	32768	65536
gas model($t_0 = 100$)	γ	0.725	0.699	0.683	0.679	0.672
gas model($t_0 = 1000$)	γ	0.762	0.704	0.691	0.681	0.675



$\gamma = 2/3$

$\gamma = 2/5$

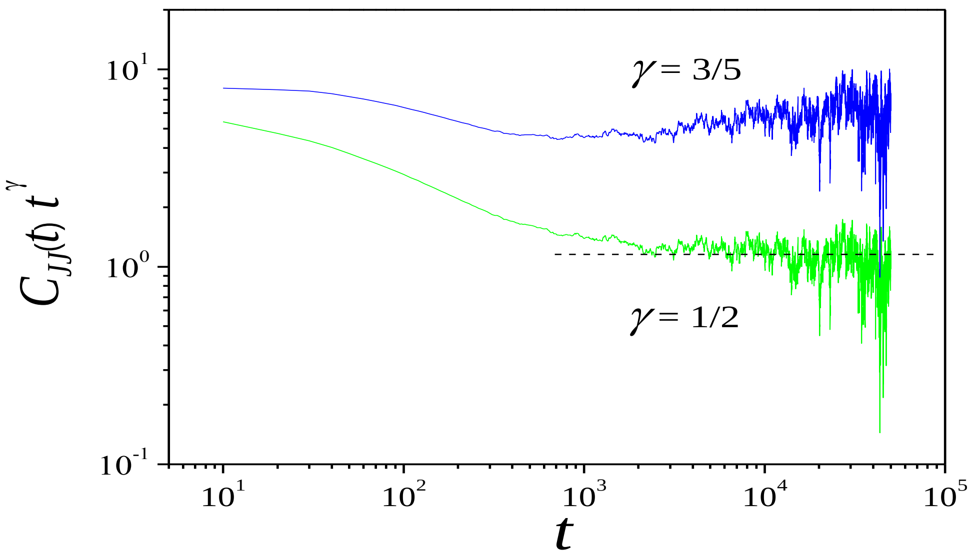
S. Chen et. al, Phys. Rev. E 87, 032153 (2014).

Models consistent to theoretical prediction

(2) The FPU$-\beta$ model

$$C(t) = < J(t)J(0) > \sim t^{-\gamma}$$

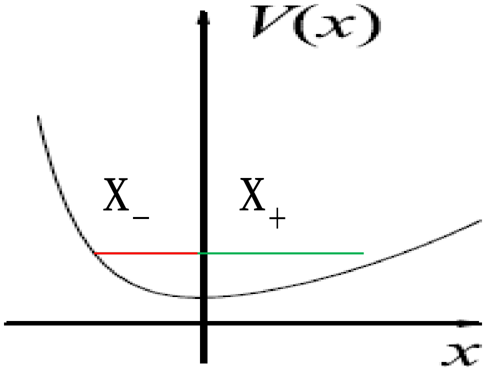
Simulation size	L	4096	8192	16384	32768	65536
FPU (t_0=1000)	γ	0.628	0.611	0.578	0.546	0.522
FPU (t_0=2000)	γ		0.565	0.583	0.532	0.518



L=131072

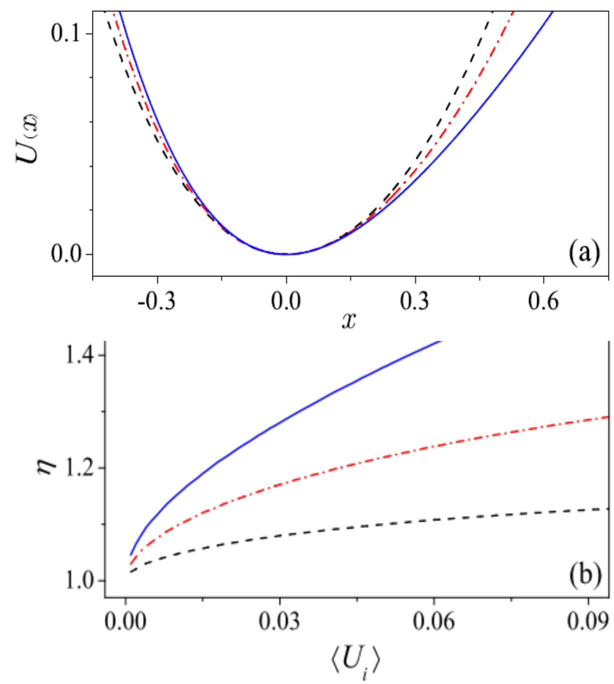
S. Chen et. al, Phys. Rev. E 87, 032153 (2013).

The asymmetry degree of potentials



$$\eta = \frac{x_+ - |x_-|}{x_+ + |x_-|} \; ? \quad \text{no!}$$

FPU-$\alpha\beta$ model: $V(x) = \frac{1}{2}x^2 - \frac{\alpha}{3}x^3 + \frac{1}{4}x^4$



Asymmetry degree:

$$\eta \equiv \frac{d}{d\langle U\rangle}(x_+ - |x_-|)$$

Corresponding to the thermal expansion rate

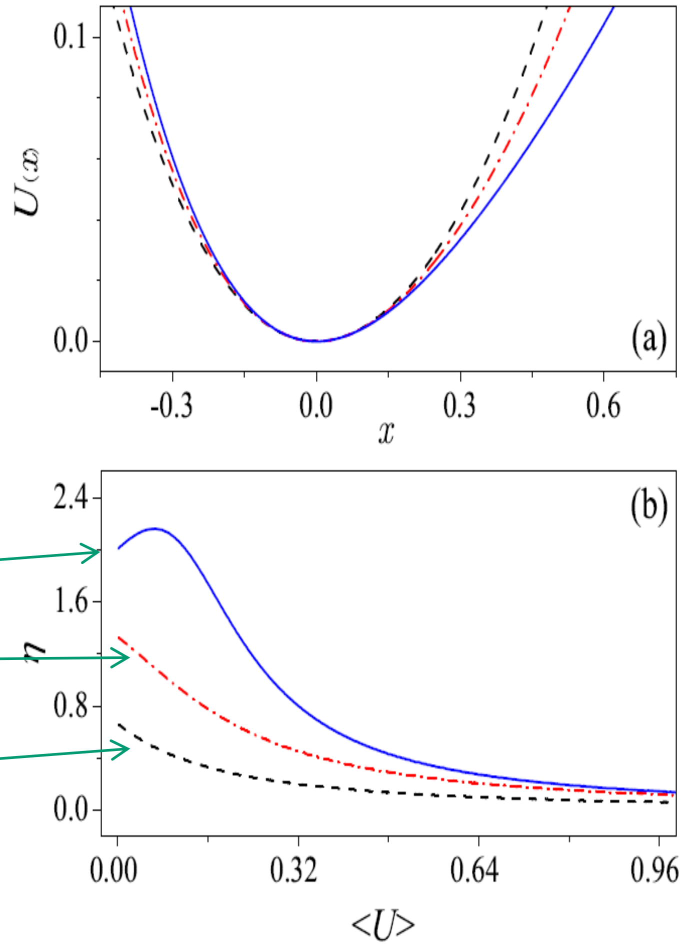
Correlation to the asymmetry degree

FPU-$\alpha\beta$ model: $$V(x) = \frac{1}{2}x^2 - \frac{\alpha}{3}x^3 + \frac{\beta}{4}x^4$$

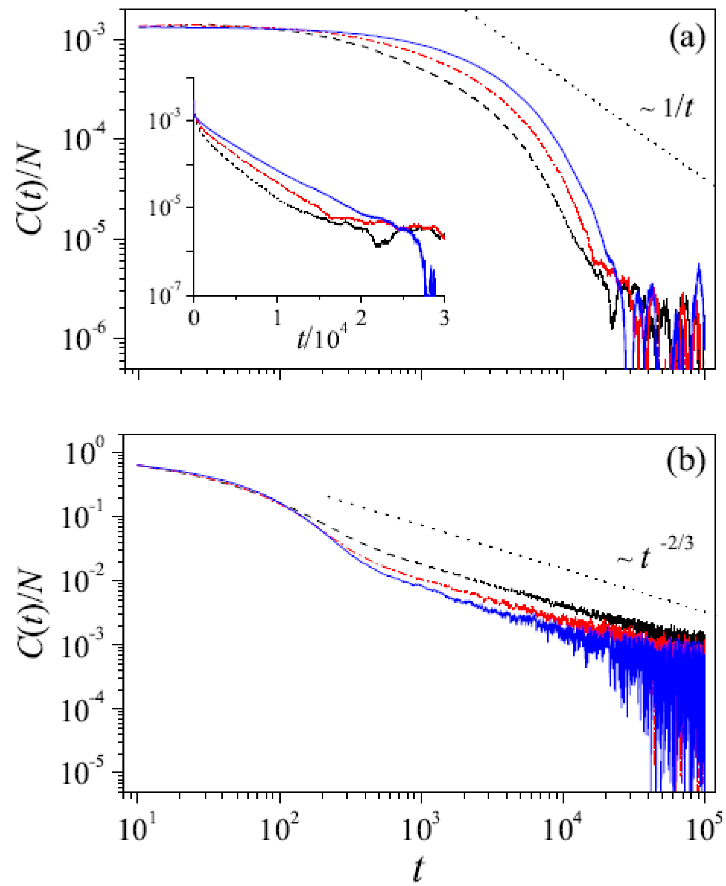
$\alpha = 1.5$

$\alpha = 1$

$\alpha = 0.5$



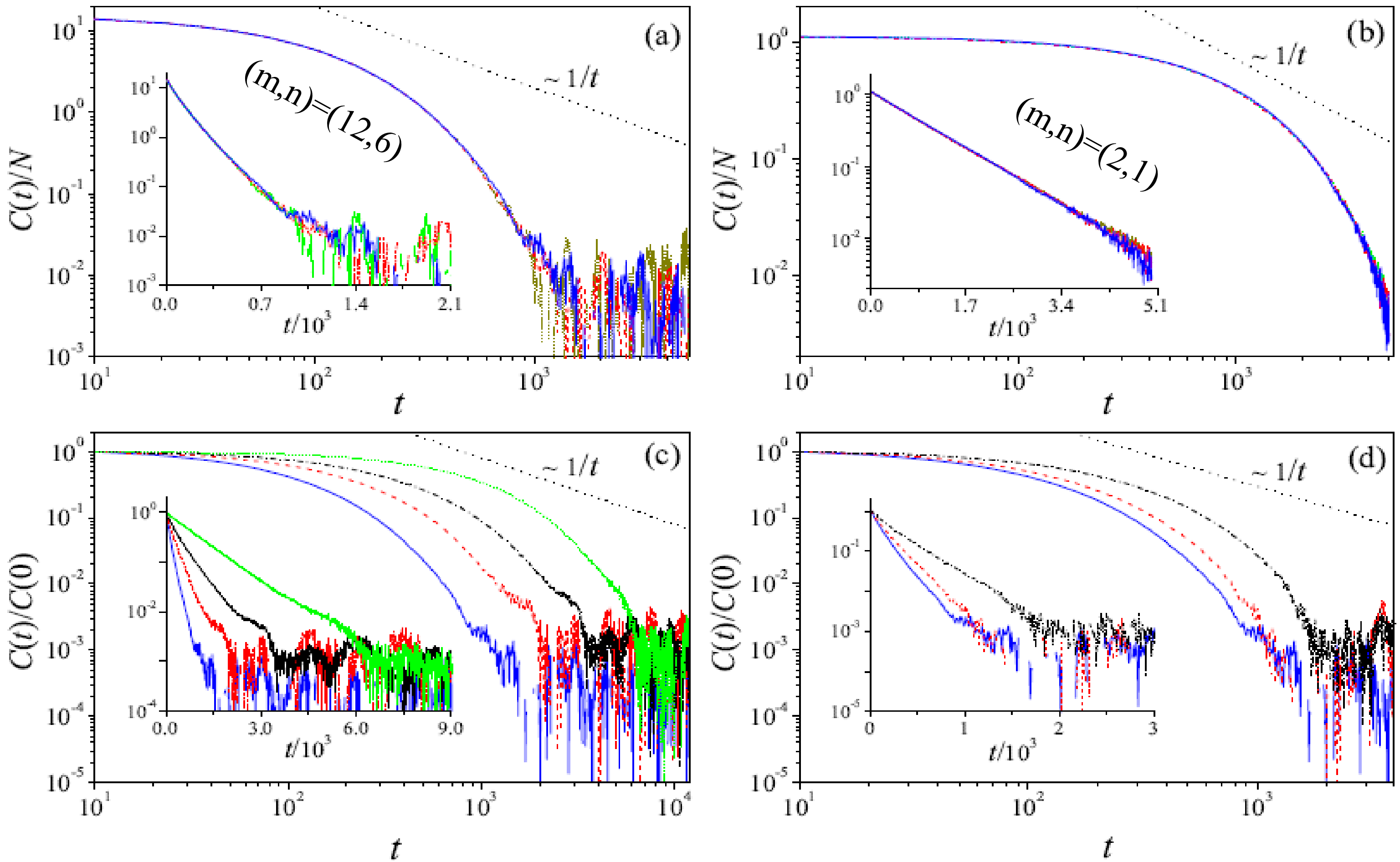
T=0.03



T=0.4

Models in contradiction with predictions

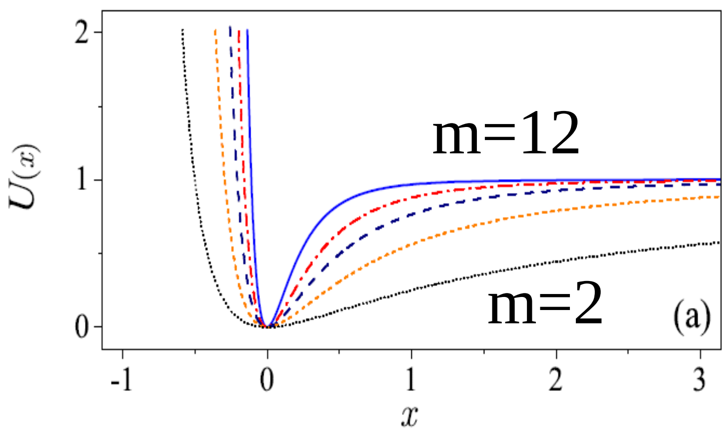
(b) L-J model: $$V(x) = [(\frac{x_c}{x+x_c})^m - 2(\frac{x_c}{x+x_c})^n + 1]$$

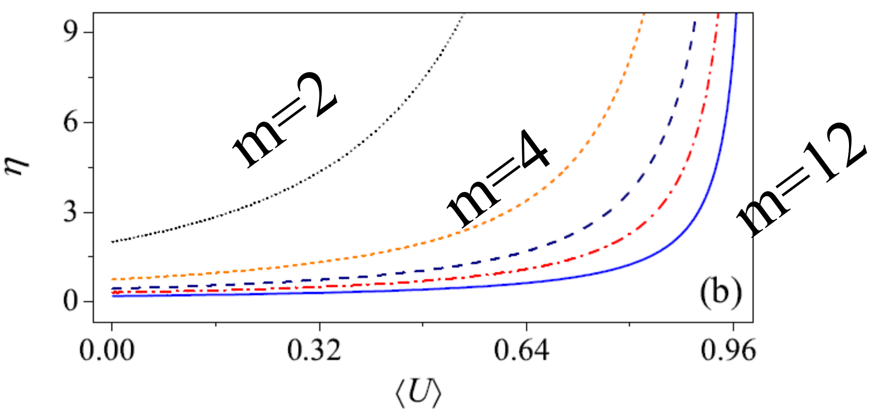


S. Chen Y. Zhang, J. Wang and H. Zhao,

Correlations with the asymmetry degree of potentials

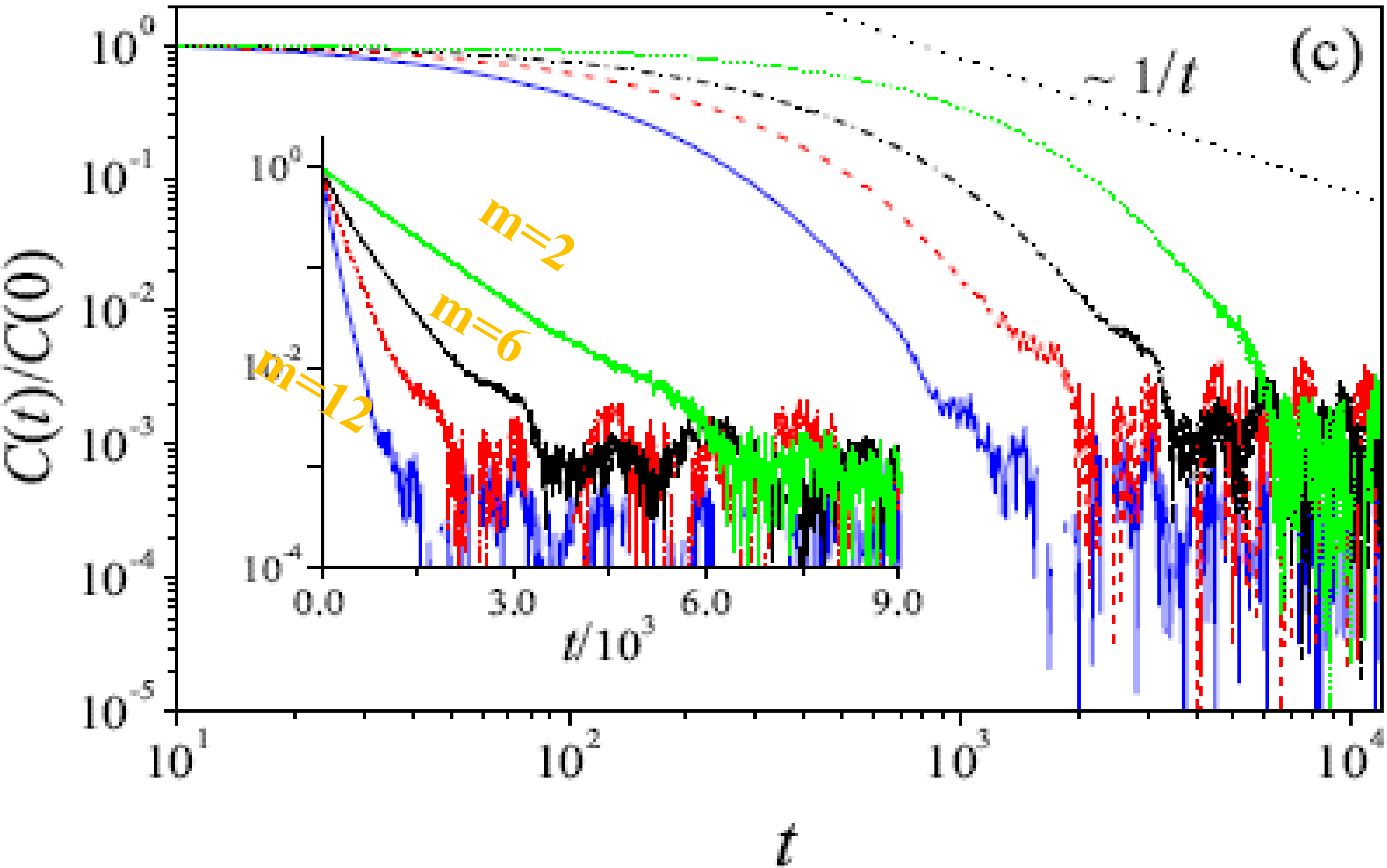
$$V(x) = [(\frac{x_c}{x+x_c})^m - 2(\frac{x_c}{x+x_c})^n + 1] \quad m=2,\ 4,\ 6,\ 8,\ 12$$





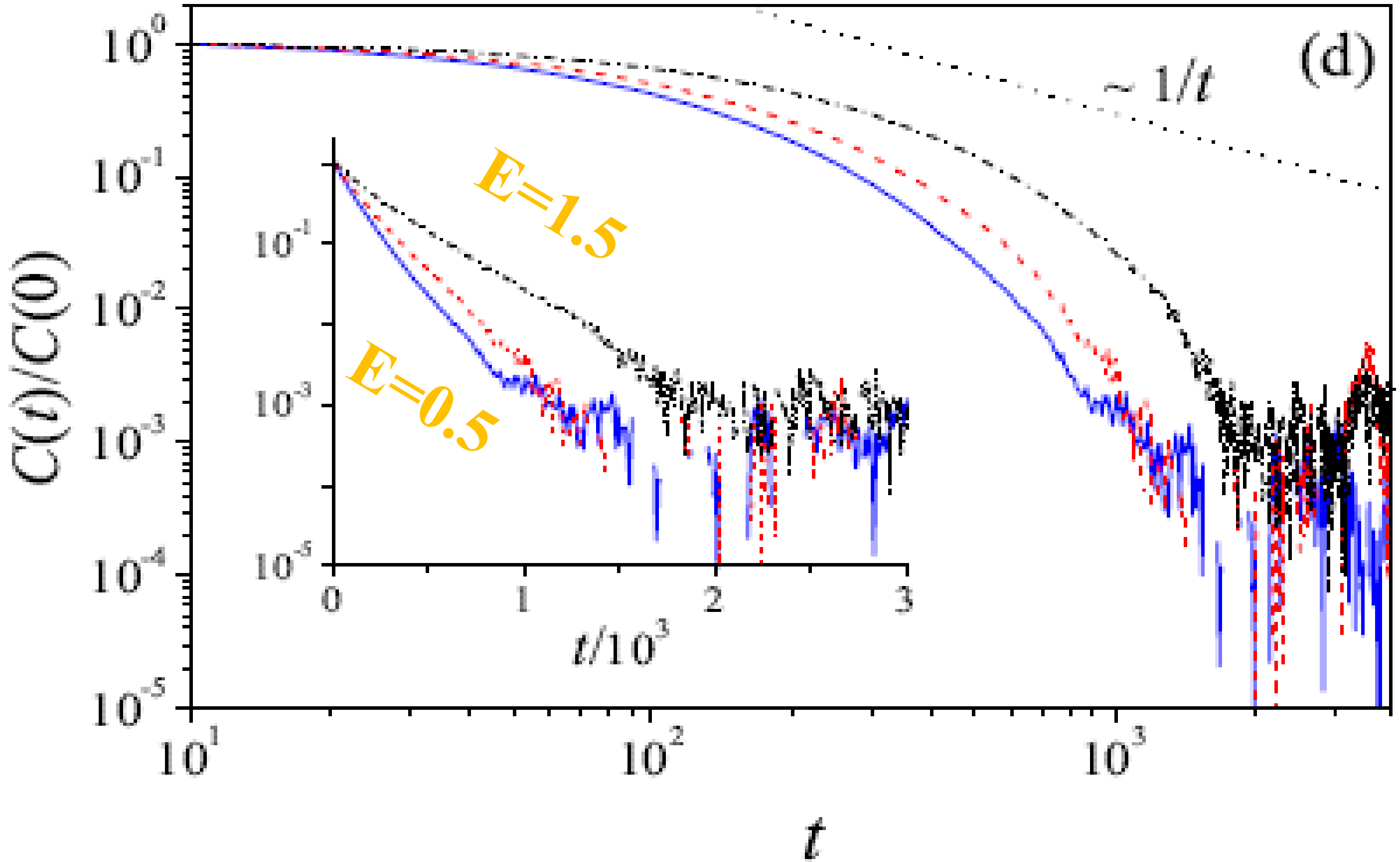
$\eta(2) > \eta(4) > \eta(6) > \eta(8) > \eta(12)$

Correlations with the asymmetry degree of potentials



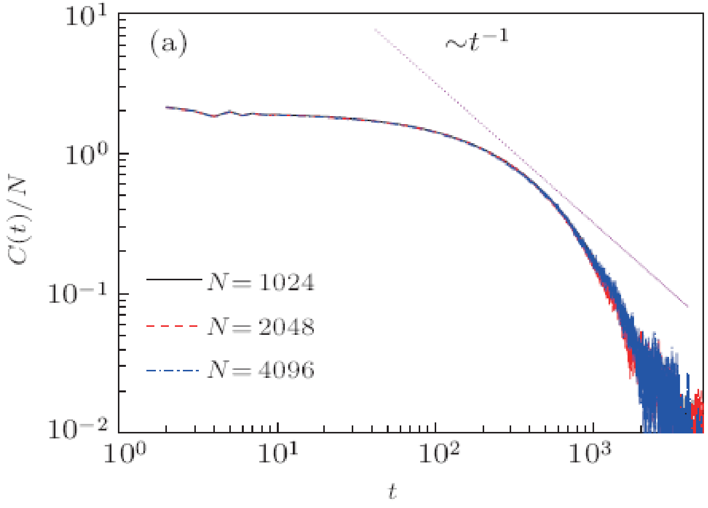
m=12,8,6,2 (from bottom to top) with N=16384 and E=0.5

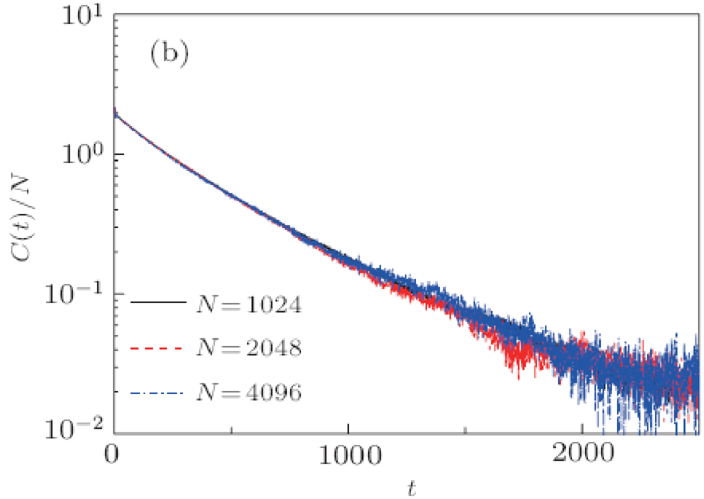
Correlations with the asymmetry degree of potentials

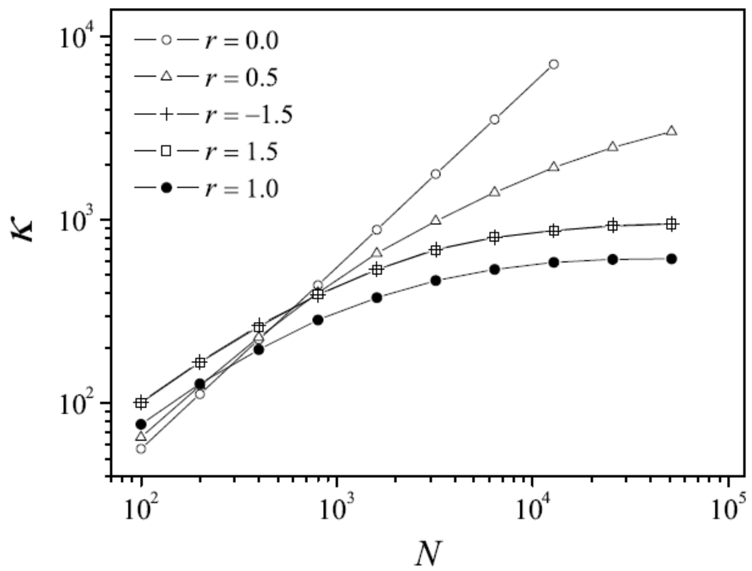


E=0.5, 1, 1.5 (from bottom to top) with N=16384 and m=12

(c) $$V(x) = \frac{1}{2}(x+r)^2 + e^{-rx}$$

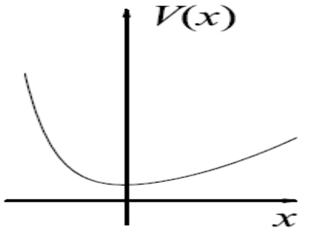


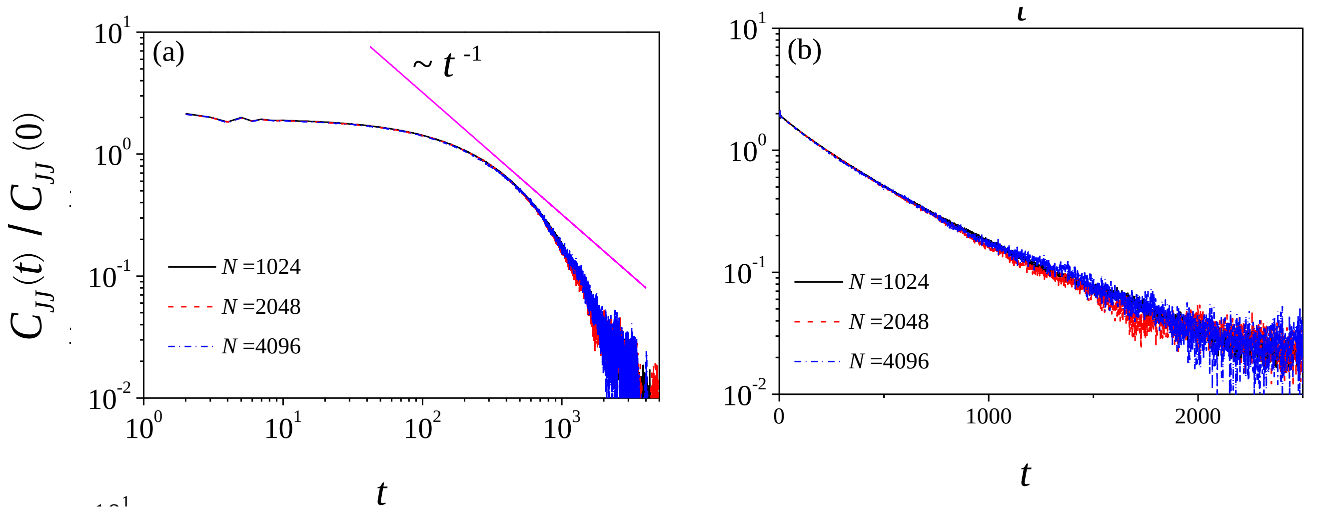




Y. Zhong Y. Zhang, J. Wang and H. Zhao, PRE 85, 060102(R) (2012)

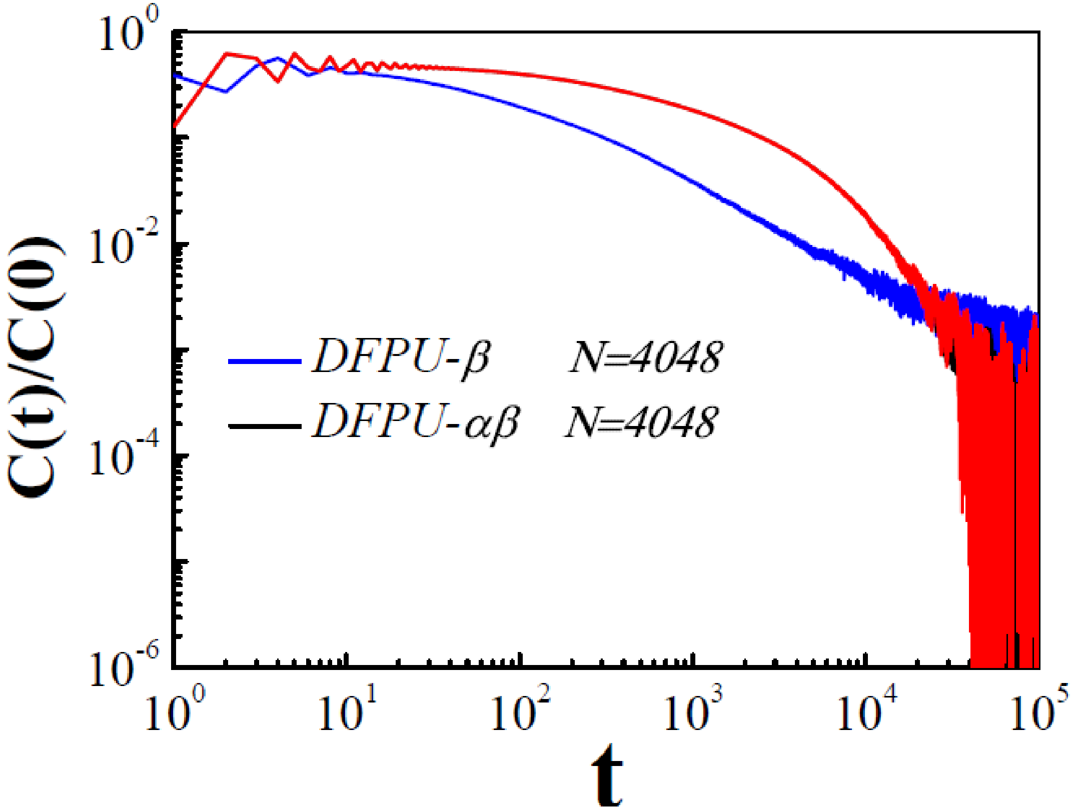
$$\text{(d)} \quad V(x) = \begin{cases} 0.5(1+r)x^2 & x<0 \\ 0.5(1-r)x^2 & otherwise \end{cases}$$





Y. Zhong, Y. Zhang, J. Wang and H. Zhao
Chin. Phys. B 22, 070505(2013)

$$(e): \text{Disordered FPU}-\alpha\beta \text{ model}: H = \sum \frac{p_i^2}{2m_i} + V(x_{i+1} - x_i)$$

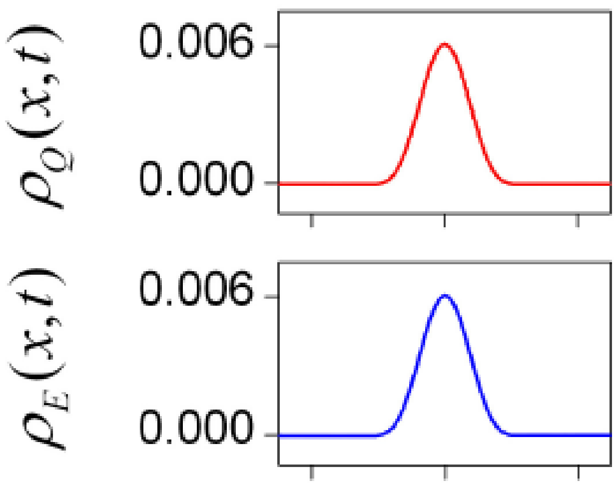


J. J. Wang, D.H. He, Y. Zhang, J. Wang and H. Zhao

Results for this model

heat

energy



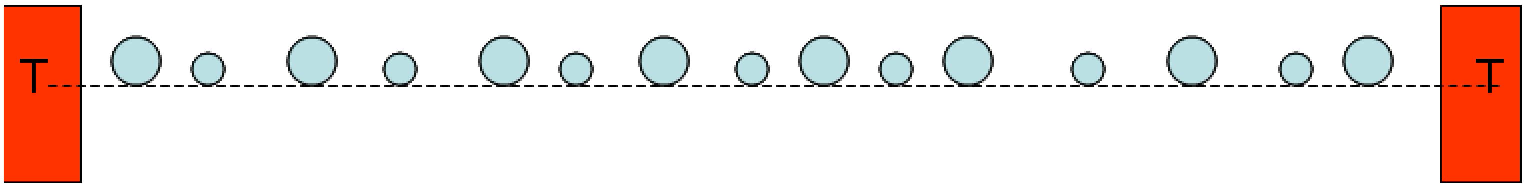
Normal diffusion

(t=300)

S. Chen, Y. Zhang, J. Wang and H. Zhao, Phys. Rev. E 87, 032153 (2013)

Example for

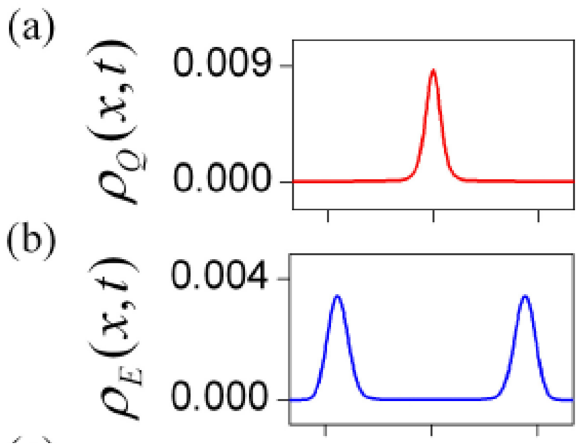
1D gas with alternating masses



Results for this model:

heat

energy



Supperdiffusion

Ballistic diffusion

$$< x^2(t) >_e \sim t^2$$

(t=300)

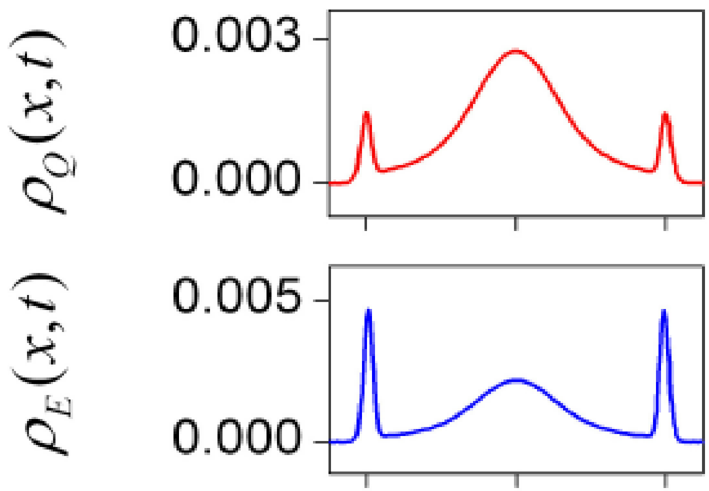
Example for

FPU- β lattice (momentum conserved, symmetric)

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + \frac{1}{2}\lambda(x_i - x_{i-1} - a)^2 + \frac{\beta}{4}(x_i - x_{i-1} - a)^4$$

heat

energy

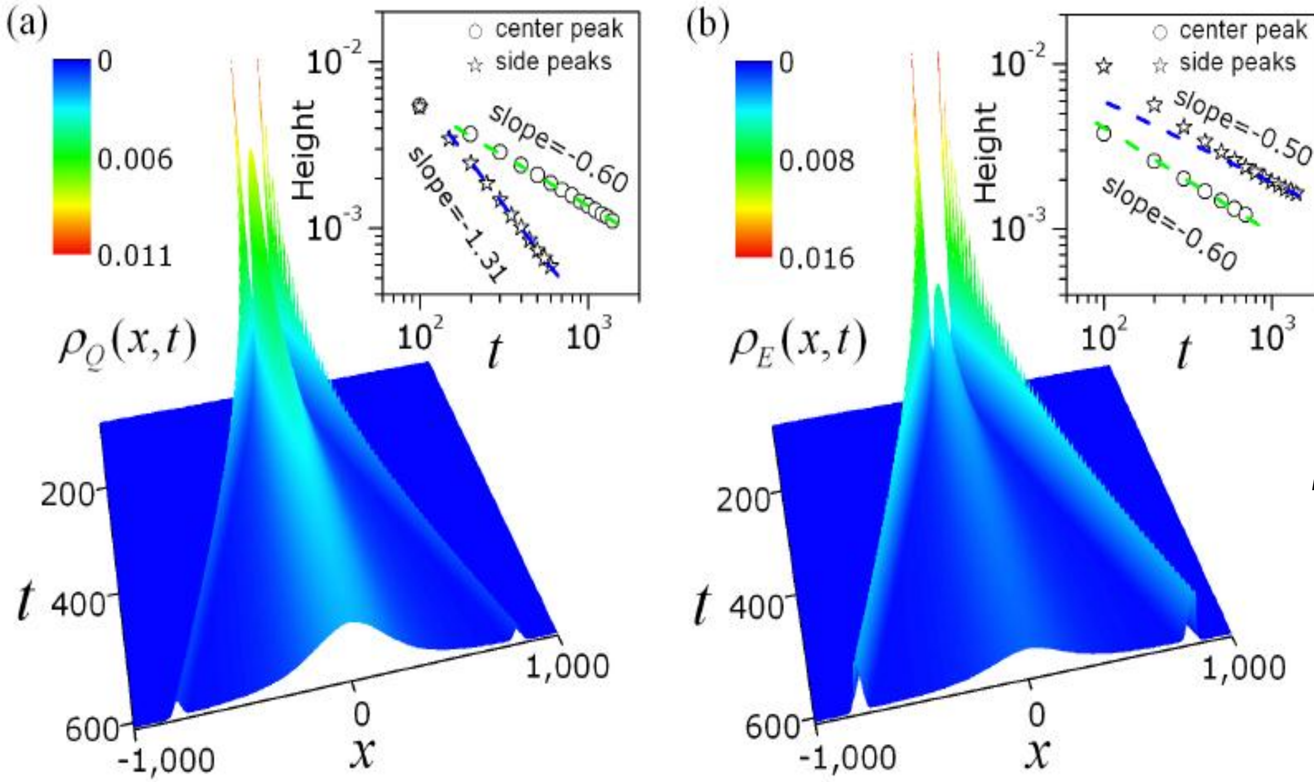


Supperdiffusion

Ballistic diffusion

(t=300)

$$h \sim t^{-1.33}$$



$$h \sim t^{-0.5}$$

$$< x^2(t) >_q \sim at^{1.2}$$

$$< x^2(t) >_e \sim at^{1.2} + bt^2 \sim t^2$$

Supperdiffusion

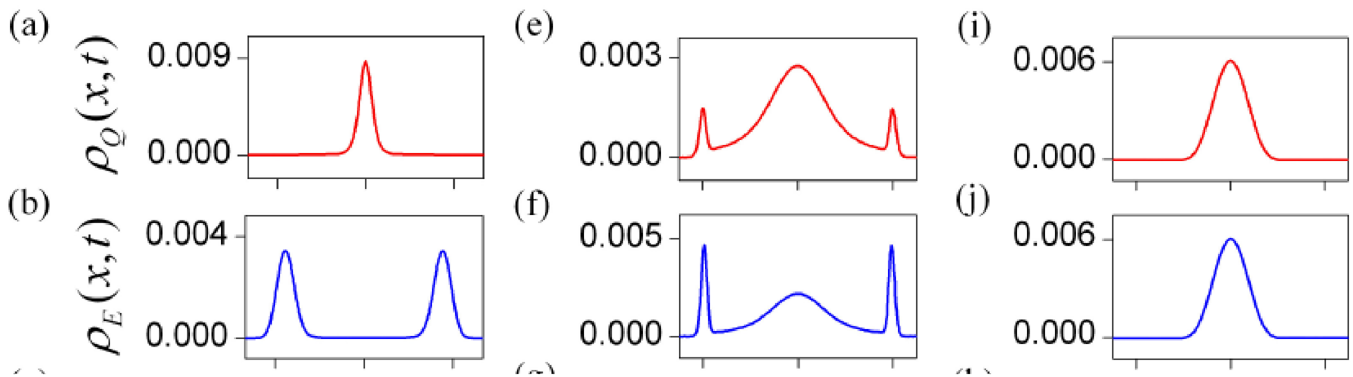
Ballistic diffusion

Results for the three 1D models

1d gas | FPU-β | φ^4 Lattice

Heat

Energy



(t=300)

S. Chen, Y. Zhang, J. Wang and H. Zhao, Phys. Rev. E 87, 032153 (2013)

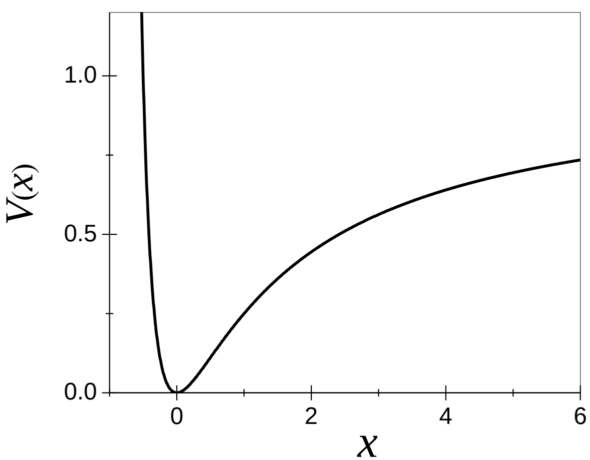
Example for

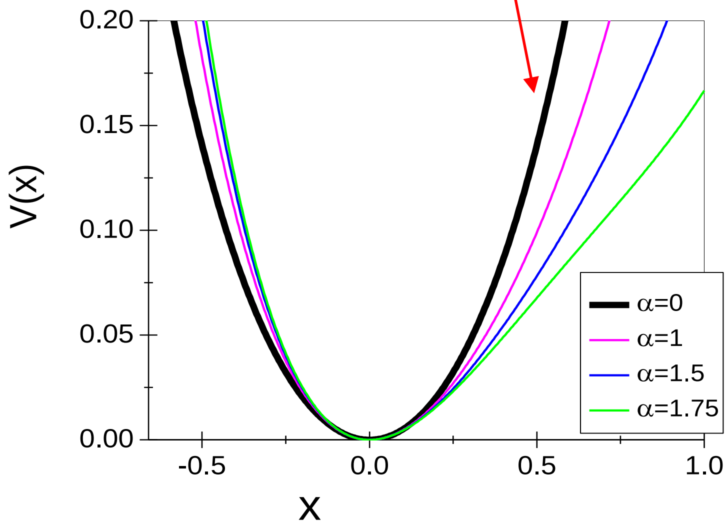
momentum conserved lattice models with asymmetric inter-particle interactions

$$V(x) = [(\frac{x_c}{x+x_c})^{12} - 2(\frac{x_c}{x+x_c})^{6} + 1] \quad \text{L-J model}$$

$$V(x) = \frac{1}{2}x^2 - \frac{\alpha}{3}x^3 + \frac{1}{4}x^4 \quad \text{FPU}-\alpha\beta \text{ model}$$

FPU-β model α=0

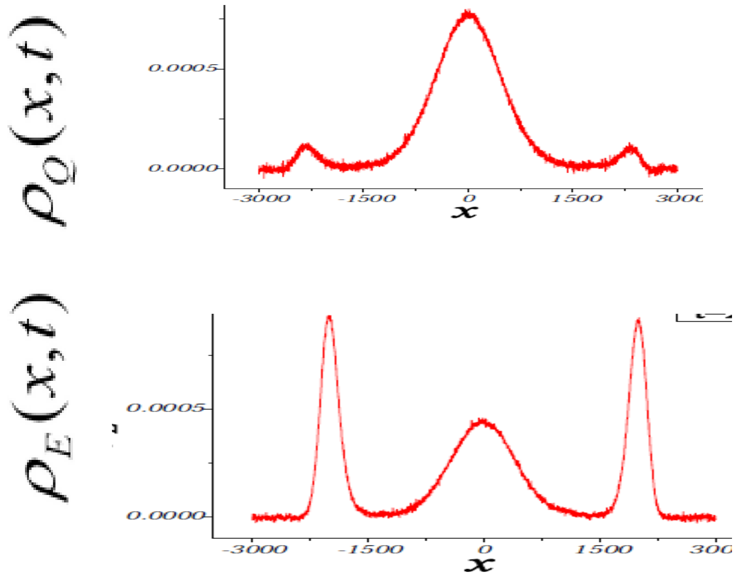




Results for the FPU $-\alpha\beta$ model

heat

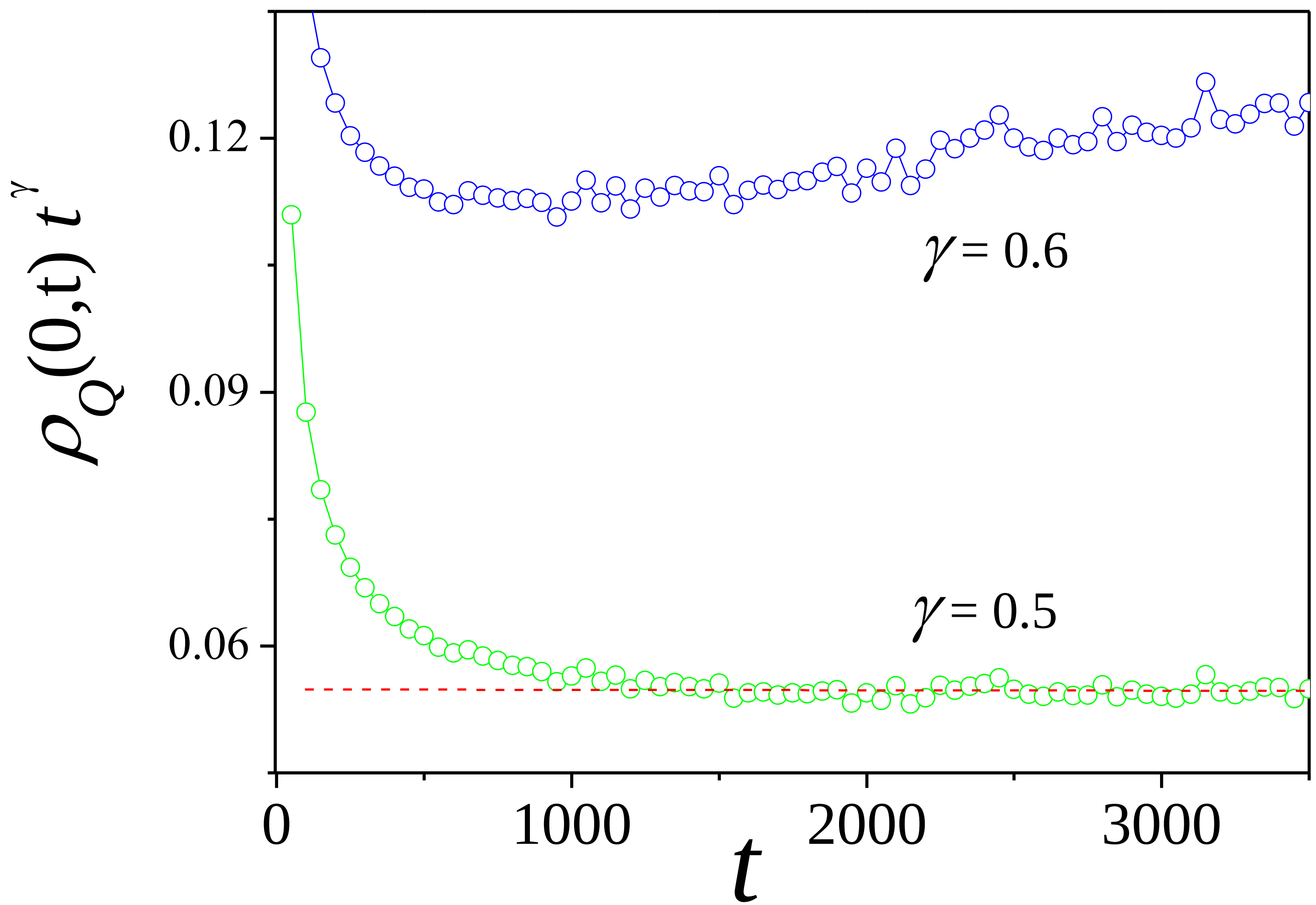
Normal diffusion



energy

Ballistic diffusion

(t=2500)



S.D. Chen et.al

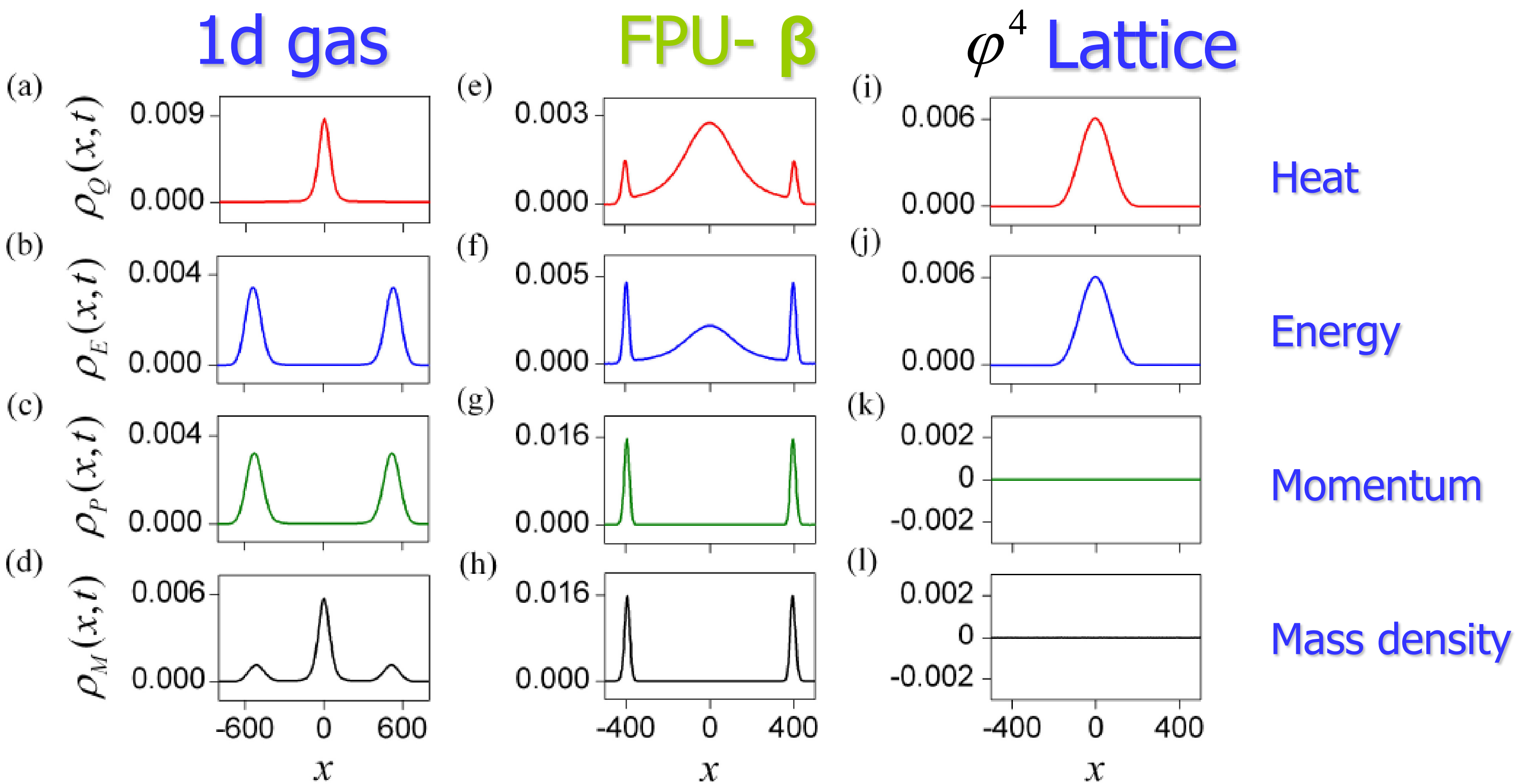
Scaling exponents of the heat and sound modes for several typical one-D models.

$$S(x,t) \sim t^{-\lambda} f_{KPZ}(t^{-\lambda}(\bar{x}))$$

	Heat mode	Sound mode
1D Gas	0.60	0.66
FPU-beta	0.60	0.50
FPU-alpha-beta	0.50	0.60
L-J	0.50	0.60

Distribution functions of heat, energy, momentum and mass density

(t=300)



S. Chen, Y. Zhang, J. Wang and H. Zhao, Phys. Rev. E 87, 032153 (2013)