Monogamy of correlations in a quantum world

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Bengaluru

Dec 2014

Outline

- What is monogamy?
- Local monotonicity independent of monogamy.
- Channel capacity of q channels can be strongly monogamous.
- Quantitative monogamy relations for qc
- All qc can be made monogamous.
- All qc monogamous for almost all states of moderately large systems.

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What is monogamy?

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If Alice n Bob share maximal qc, they can't share any qc with Charu.

Classical correlations r not monogamous

An equal mixture of

000 (three pink flowers) and 111 (three blue ones) violates monogamy maximally.

What is monogamy?

If Alice n Bob share maximal qc, they can't share any qc with Charu.

"Qualitative monogamy"

What is monogamy?

If Alice n Bob share maximal qc, they can't share any qc with Charu.

"Qualitative monogamy"

Ekert, PRL 1991

Bennett, Bernstein, Popescu, Schumacher, PRA 1996

What is monogamy?

If Alice n Bob share maximal qc, they can't share any qc with Charu.

Will refer to
entanglement measures (like ent of formation, log negativity, distillable entanglement)
and info-theoretic qc measures
(like quantum discord, quantum work deficit)
together as **"quantum correlations"**.

Quantitatively …

- Q is said to be monogamous for a quantum state r(ABC) if Q(AB) + Q(AC) <= Q(A:BC).

Coffman, Kundu, Wootters, PRA 2000

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"

$$C^2_{AB} + C^2_{AC} \leqslant C^2_{A(BC)}. \qquad (12)$$

Informally, Eq. (12) can be expressed as follows. Qubit A has a certain amount of entanglement with the pair BC. This amount bounds A's entanglement with qubits B and C taken individually, and the part of the entanglement that is devoted to qubit B (as measured by the squared concurrence) is not available to qubit C.

"

Coffman, Kundu, Wootters, PRA 2000

Quantitatively …

- Q is said to be monogamous for a quantum state r(ABC) if Q(AB) + Q(AC) <= Q(A:BC).

“

$$C_{AB}^2 + C_{AC}^2 \leqslant C_{A(BC)}^2. \qquad (12)$$

Info … ys. Qubit A
has … ir BC. This
am … nd C taken
ind … is devoted
to … nce) is not
ava …

”

Q(AB) + Q(AC) is bounded (above).

Coffman, Kundu, Wootters, PRA 2000

- All quantum correlations r qualitatively monogamous in d x d x d.

Didn’t prove this!

Holds becoz maximal quantum correlations occur only for pure states, for all known quantum correlations.

- All quantum correlations r qualitatively monogamous in d x d x d.

- Only some r quantitatively so.

Why consider monogamy?

Why consider monogamy?

- It is fundamentally quantum.

Classical correlations do not satisfy monogamy.

Why consider monogamy?

- It is fundamentally quantum.
- Can potentially lead to nonclassical applications.
- Useful in

  potential resolution of BH info paradox

  security considerations of q cryptography

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- Info-theoretic measures r also monotones under restricted LOCC classes.

- Are all LOCC monotones monogamous?

Locally accessible information

- {p_x, r_x(AB)} shared by Alice n Bob.

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- {p_x, r_x(AB)} shared by Alice n Bob.
- Task → to find x.
- I_M = mutual info between x and LOCC-based measurement results by Alice n Bob.
- Locally accessible info = I_M maximized over all LOCC-based measurement strategies by A&B.

Locally accessible information

- It is an LOCC monotone, like entanglement measures.

Locally accessible information

- It is an LOCC monotone, like entanglement measures.

- However, it violates monogamy maximally. It is not even qualitatively monogamous.

A. Sen(De), US, PRA 2012

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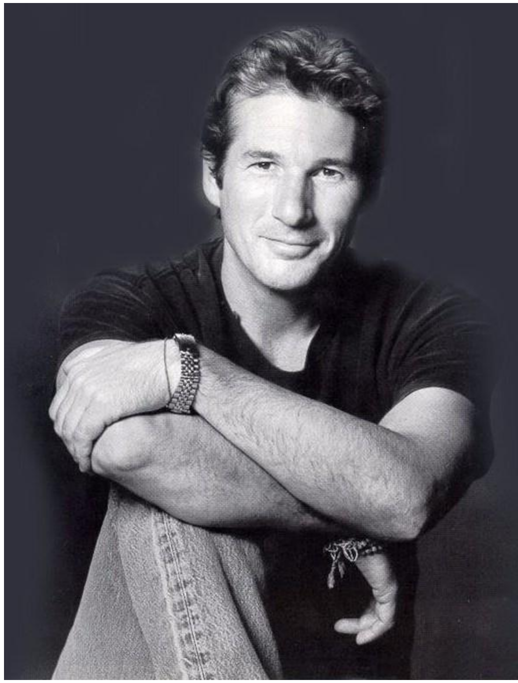
- Classical correlation known to violate monogamy maximally.
- Locally accessible info accesses info abt **classical** variable x in {p_x, r_x(AB)}.
- Is that reason for locally accessible info to violate monogamy?

Quantum dense coding



Alice

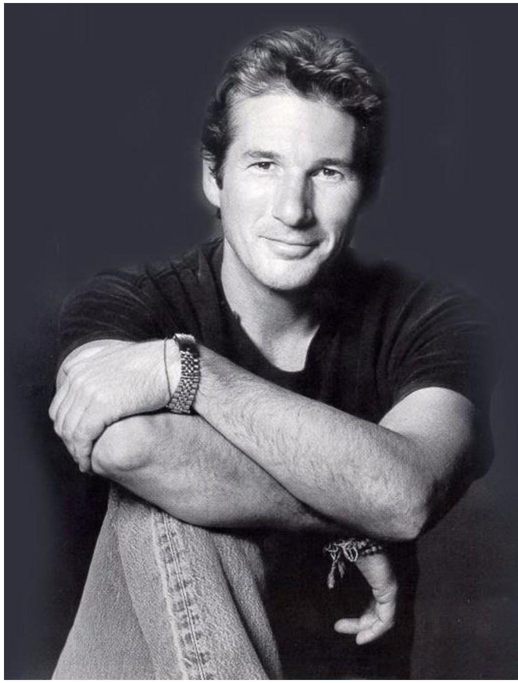
Sender

Bob

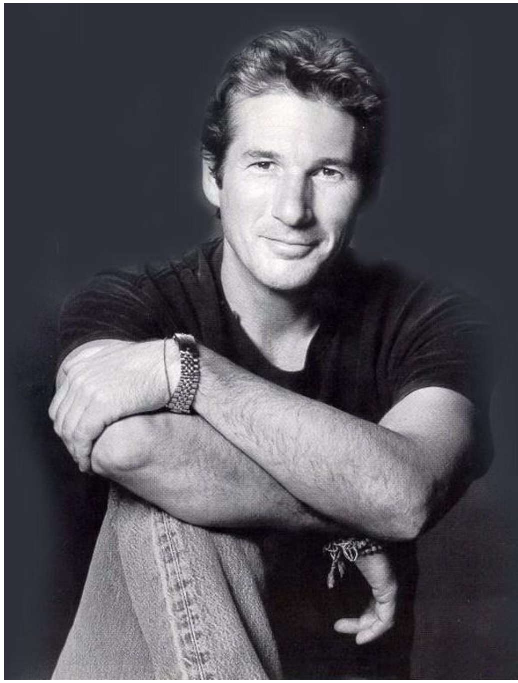
Receiver

Sender

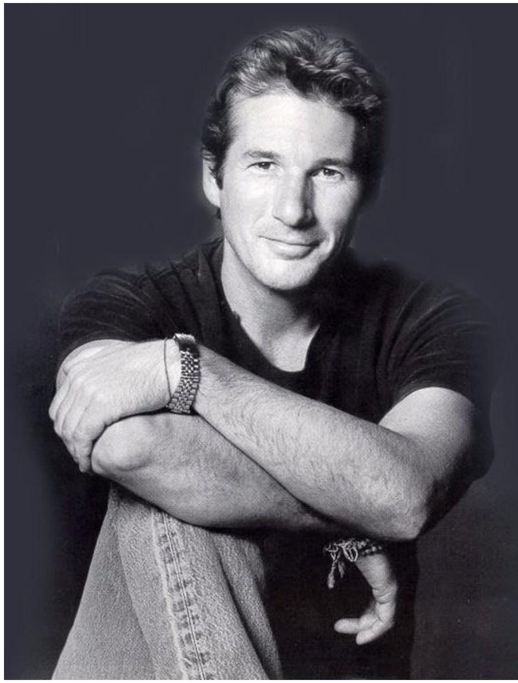
Receiver



Alice is in Hampi.
Bob is in Bengaluru.



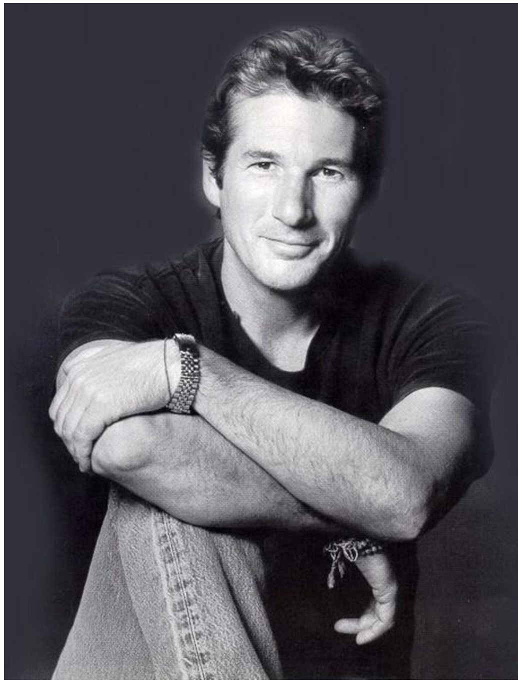
Alice wants to send info about weather in Hampi to Bob.



Alice wants to send info about weather in Hampi to Bob.

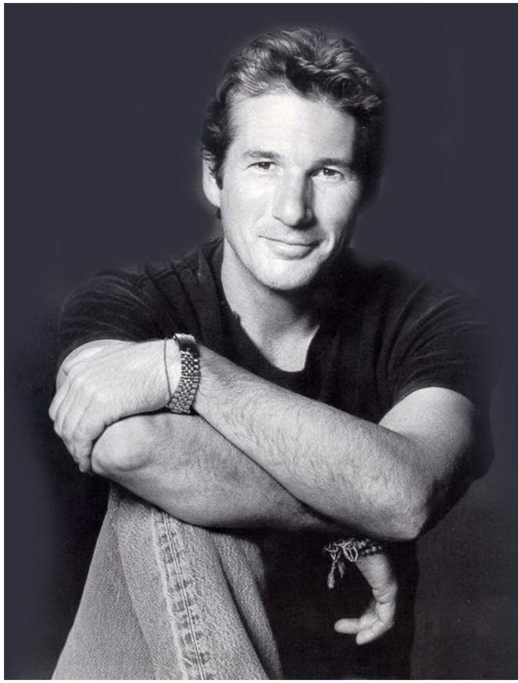
Sunny or not

Windy or not



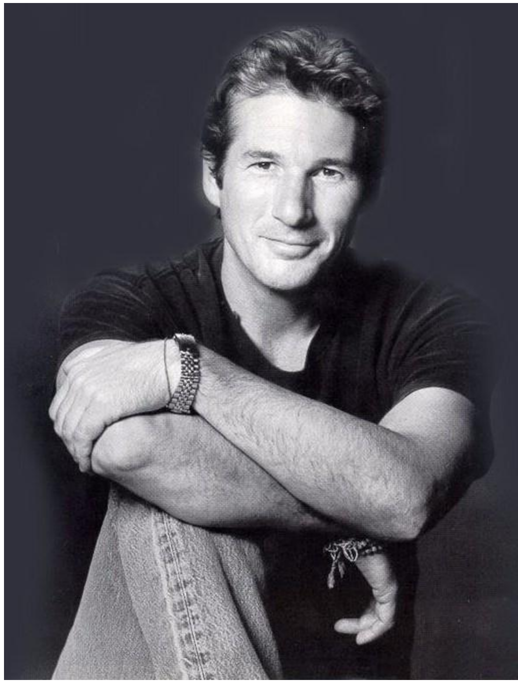
Alice wants to send info about weather in Hampi to Bob.

2 bits



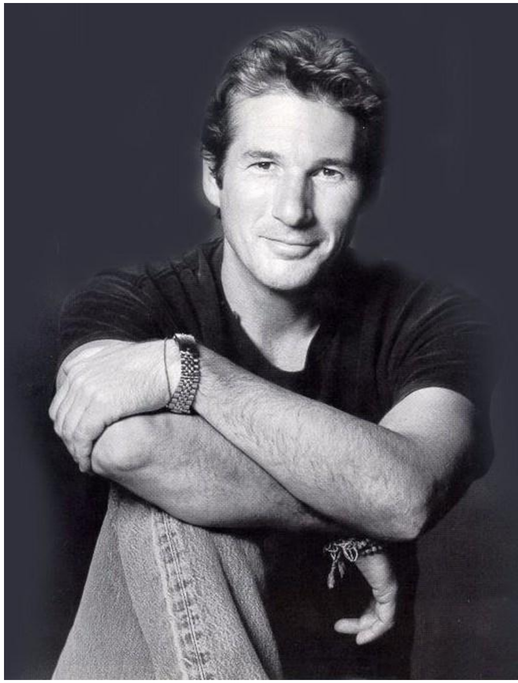
Alice wants to send info about weather in Hampi to Bob.

Can be sent by using …



Alice wants to send info about weather in Hampi to Bob.

Four balls of different colors.



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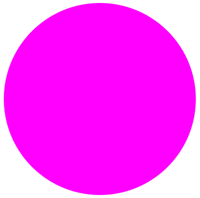
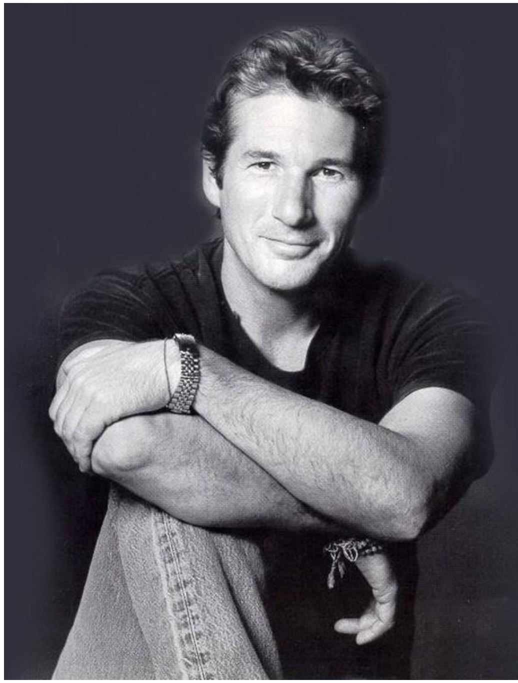


sunny and not windy



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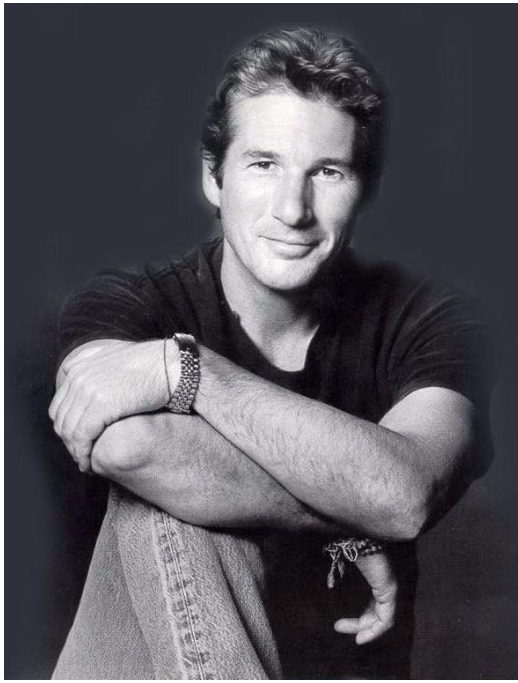
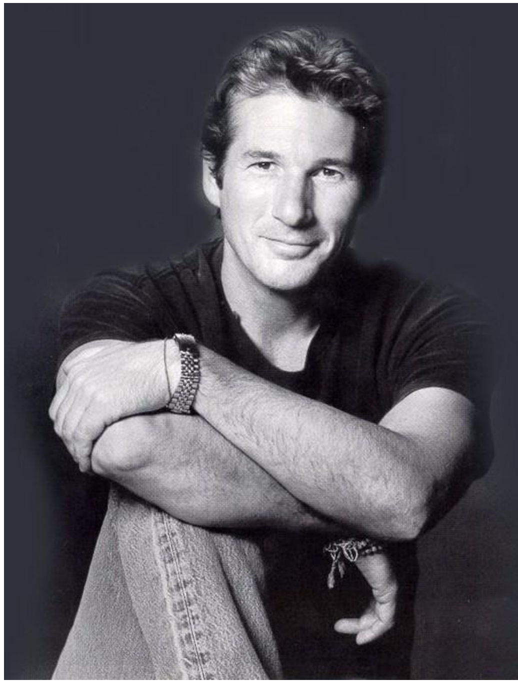


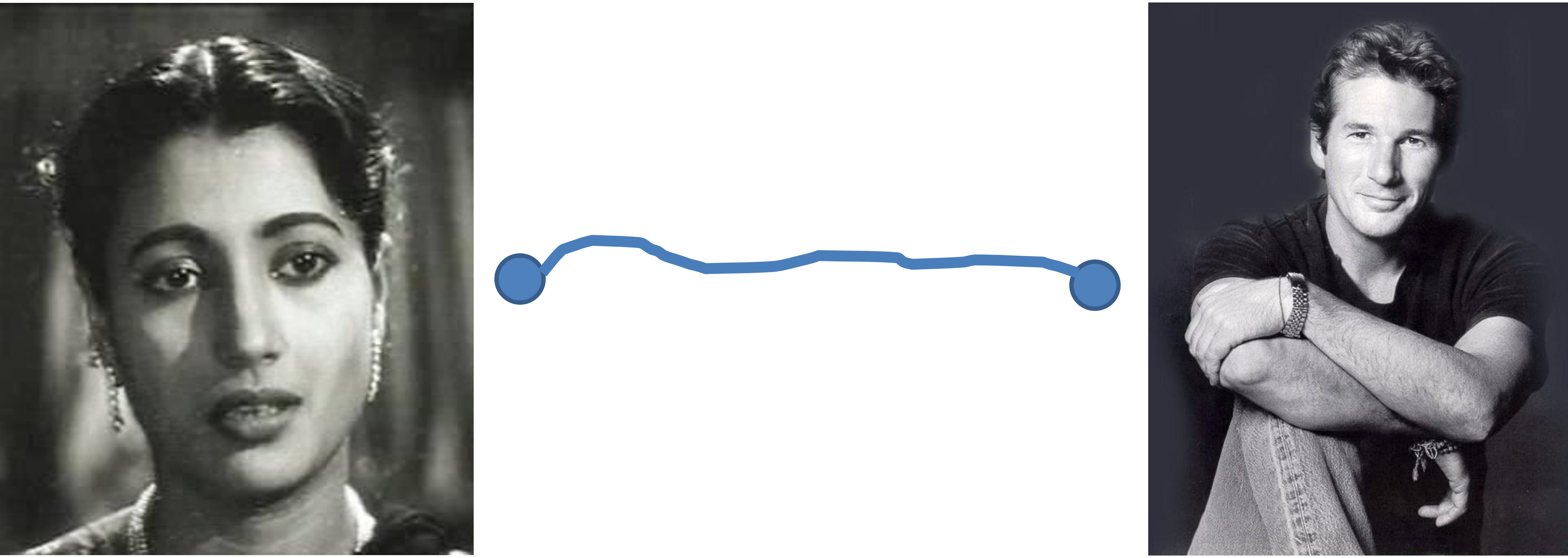
Alice wants to send info about weather in Hampi to Bob.

2 bits require 4 dim.



Alice wants to send info about weather in Hampi to Bob.

Using shared entanglement between Alice & Bob, …



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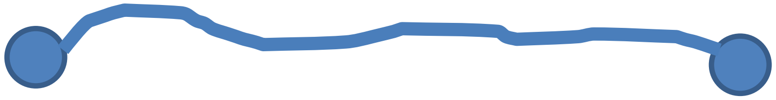
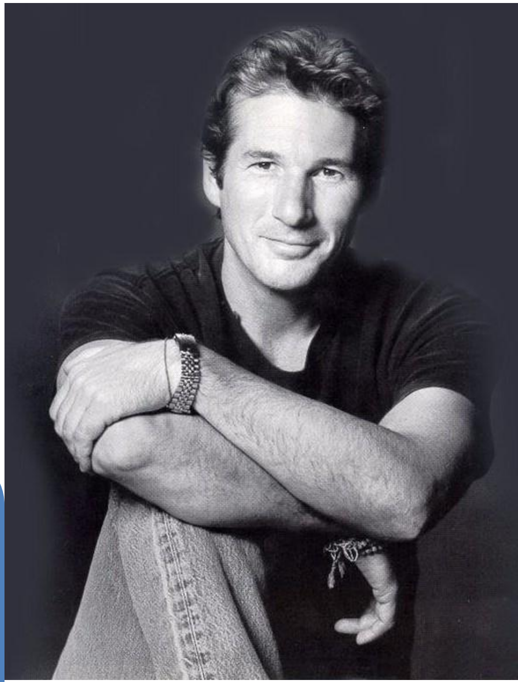


$$|\uparrow\rangle\otimes|\downarrow\rangle-|\downarrow\rangle\otimes|\uparrow\rangle$$



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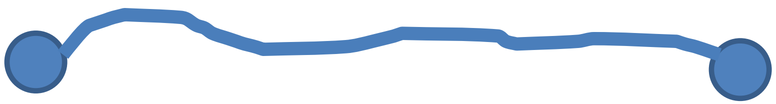
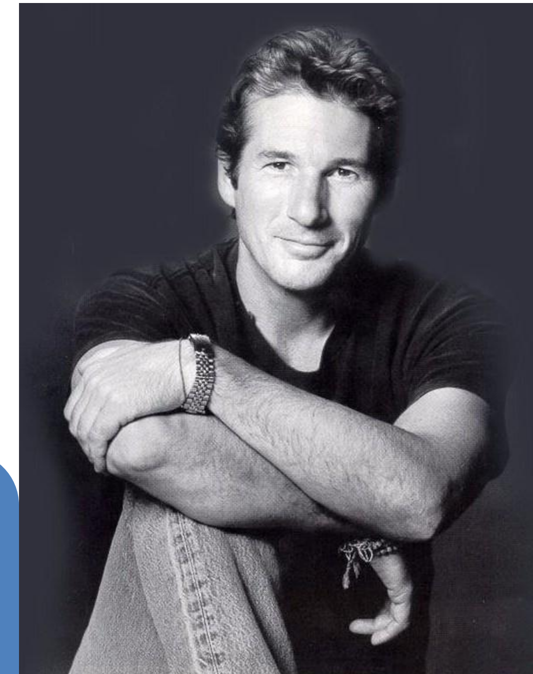


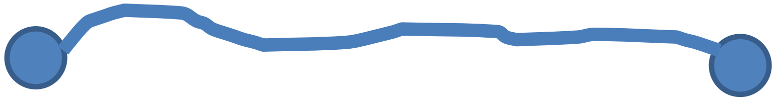


$$|\uparrow\rangle\otimes|\downarrow\rangle-|\downarrow\rangle\otimes|\uparrow\rangle$$

Alice wants to send info about weather in Hampi to Bob.

2 bits require 2 dim.

Quantum dense coding



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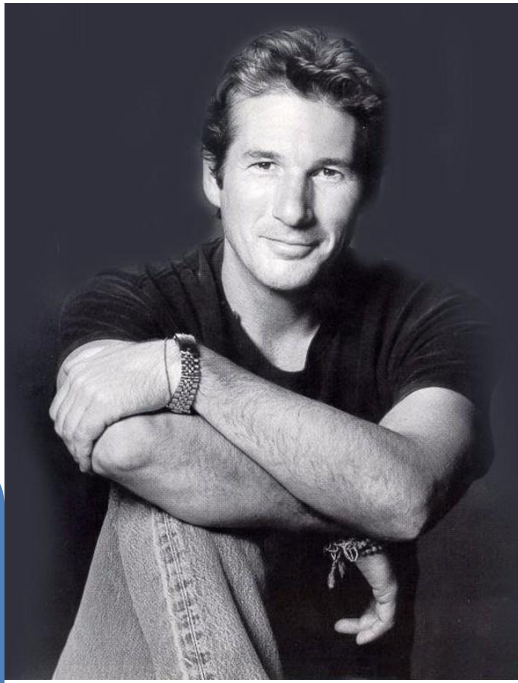

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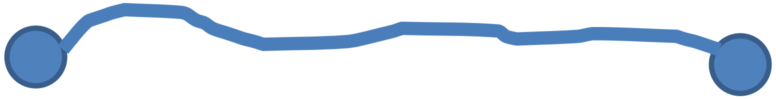

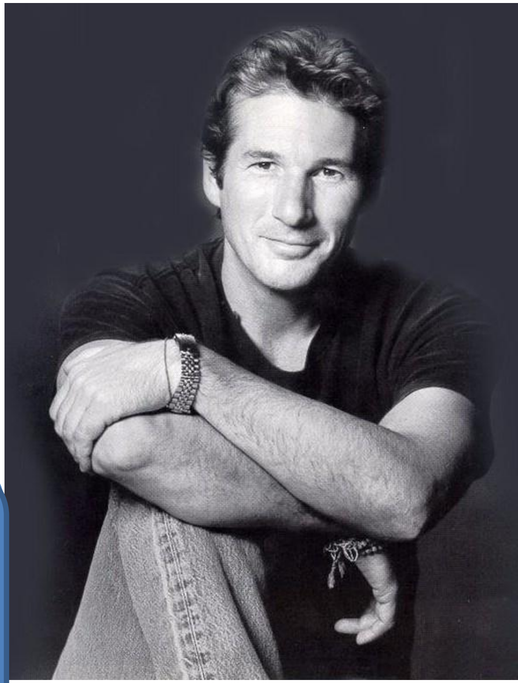

Bennett & Wiesner
1992

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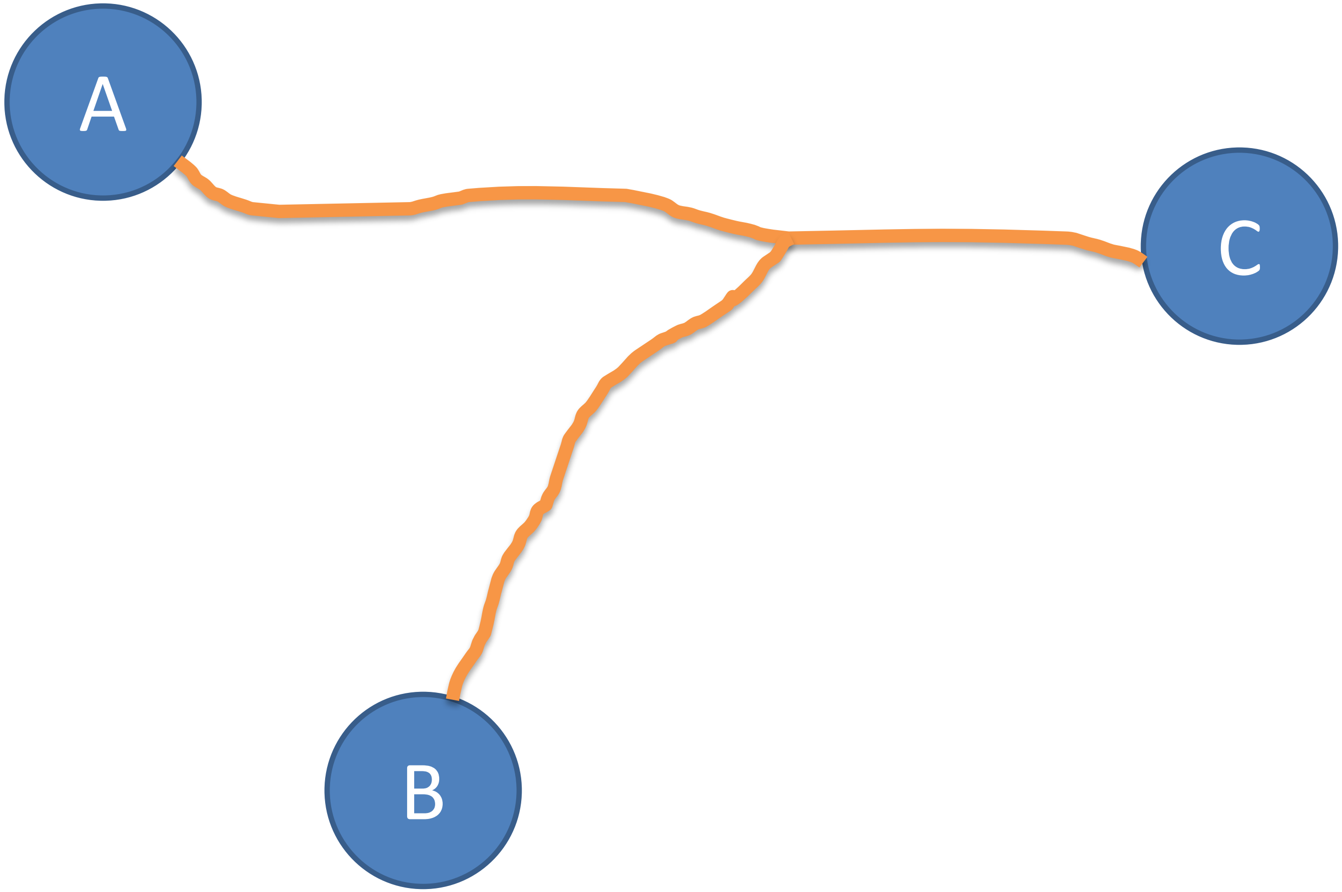
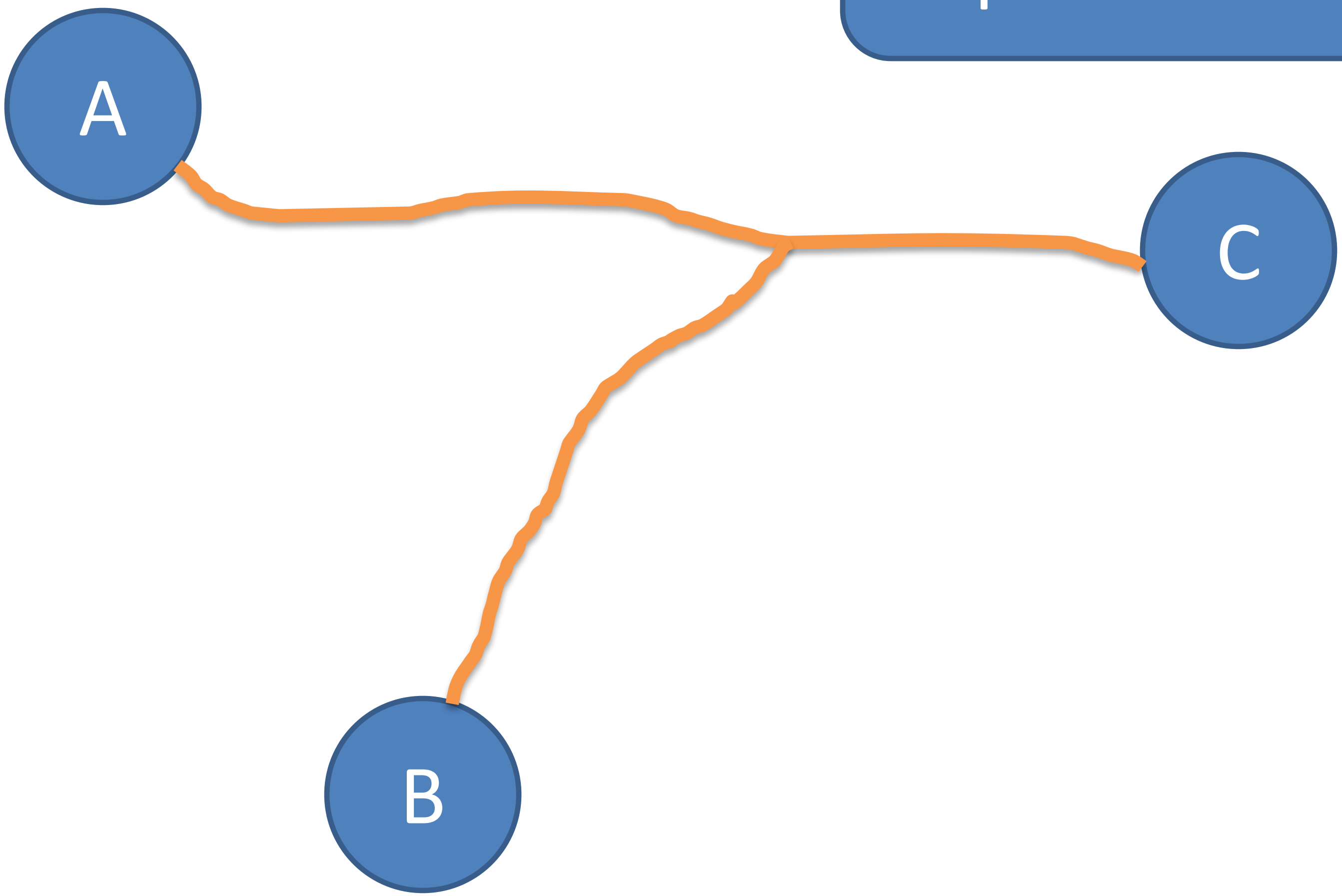
Distributed Q Dense Coding



Distributed Q Dense Coding

Towards a quantum internet



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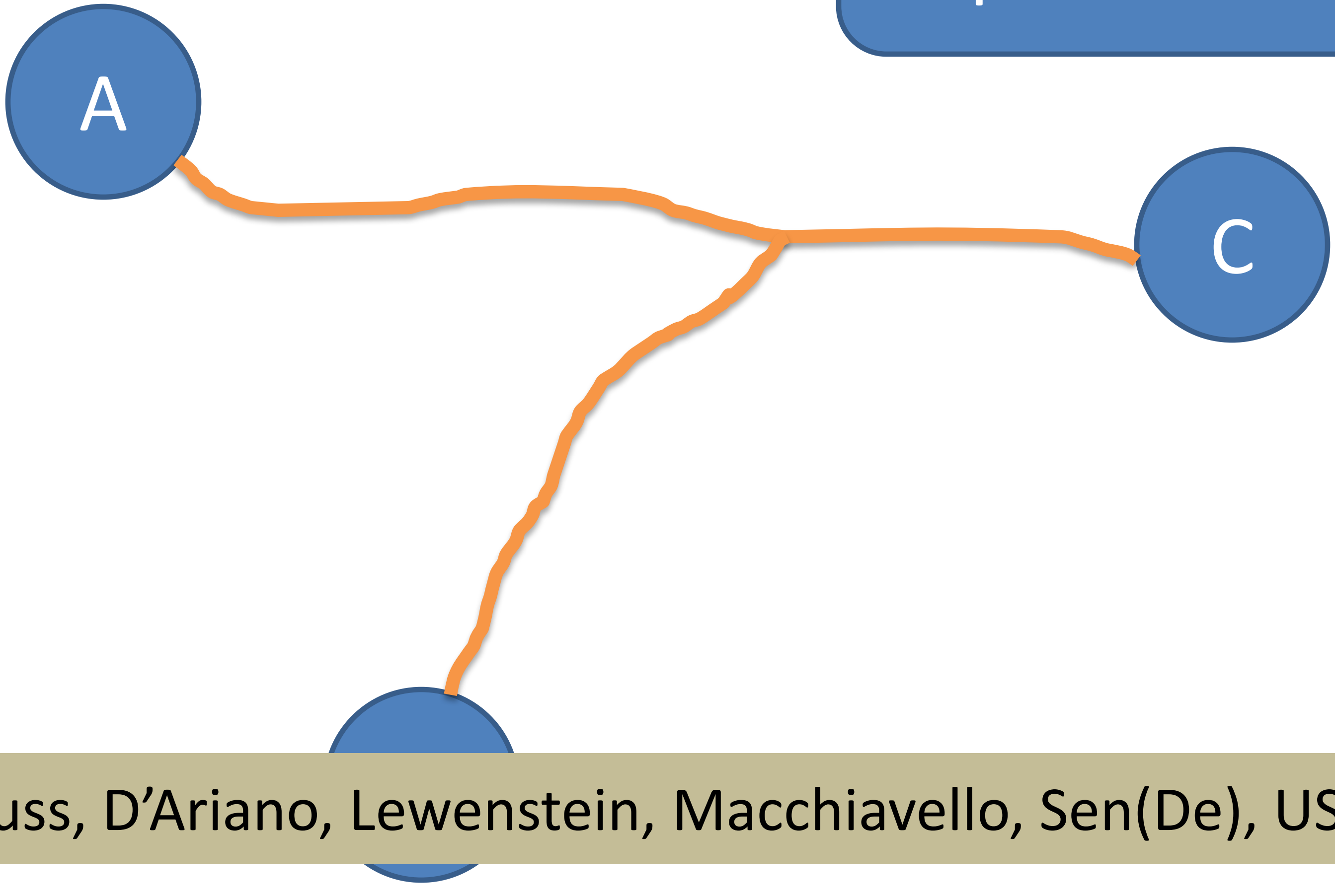

Bruss, D'Ariano, Lewenstein, Macchiavello, Sen(De), US, PRL 2004

Exclusion pple for Q Dense Coding

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Prabhu R, Pati, Sen(De), US, Phys. Rev. A 2013

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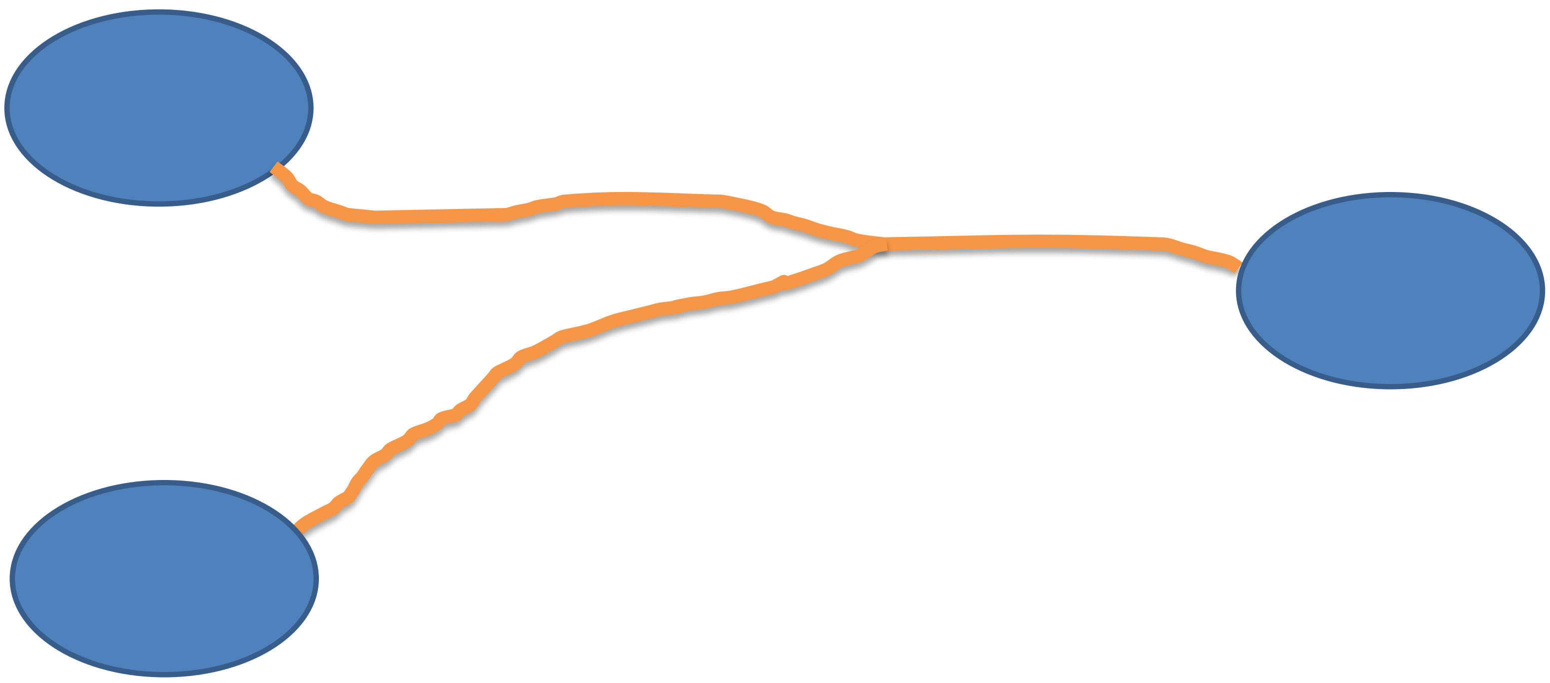

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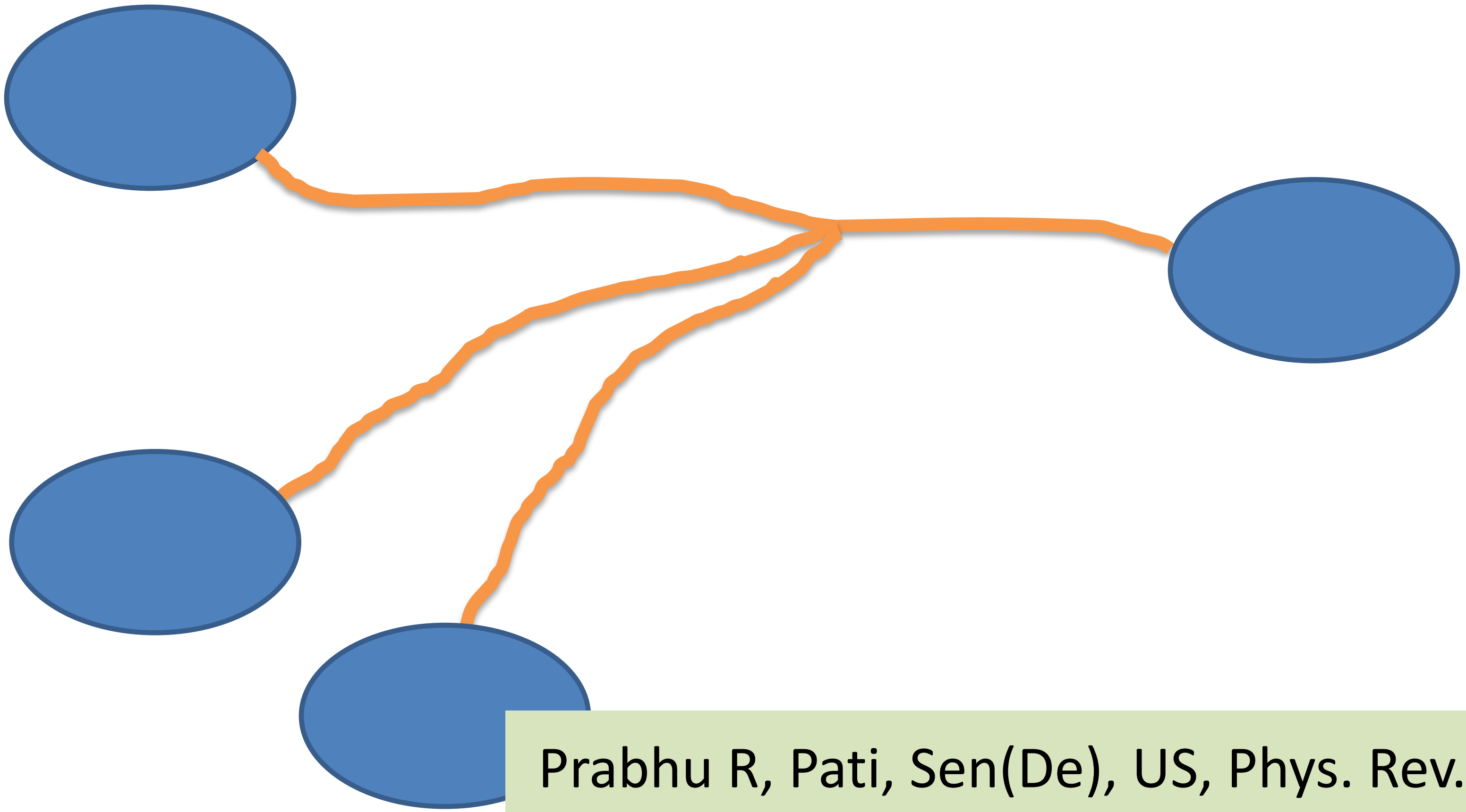

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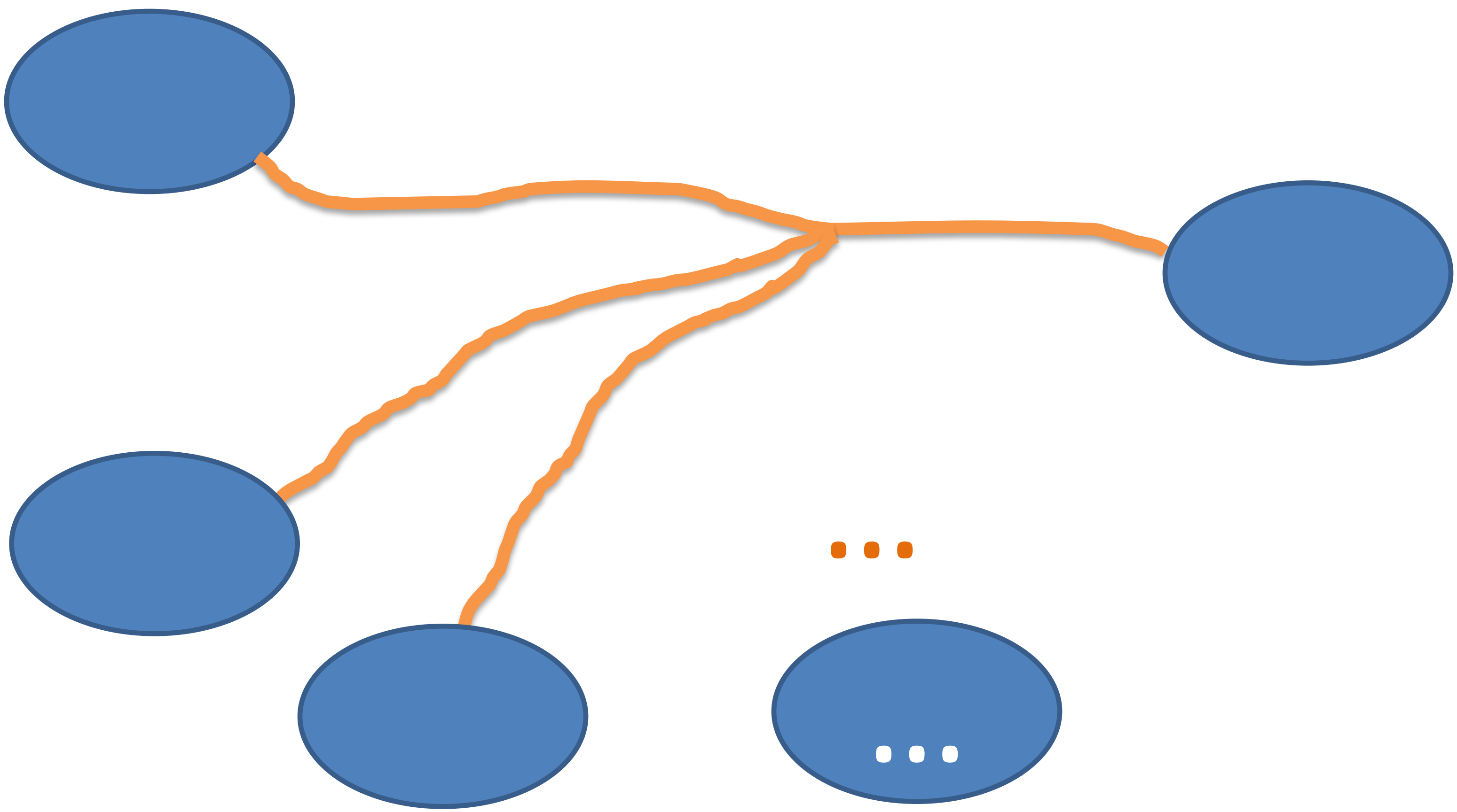

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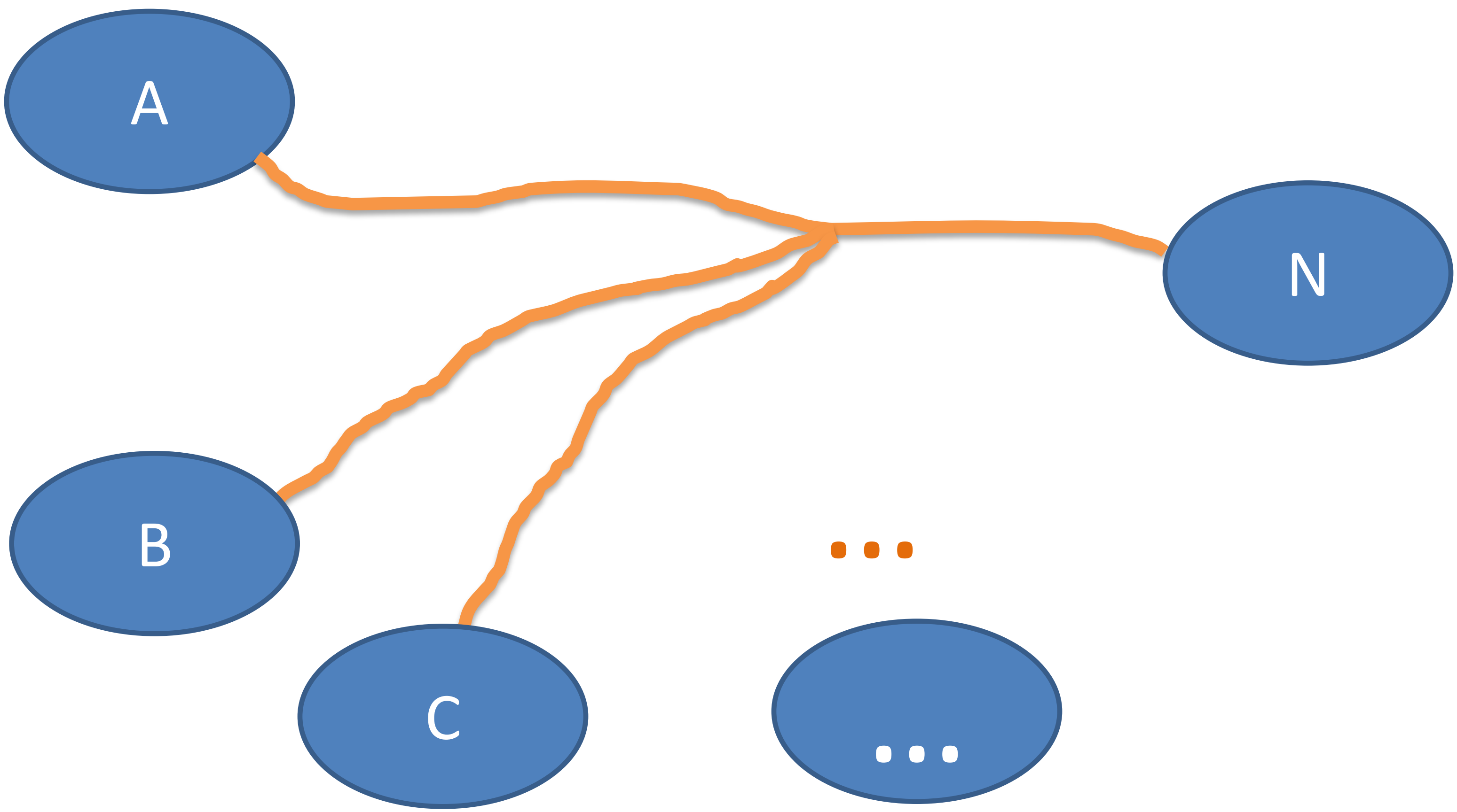

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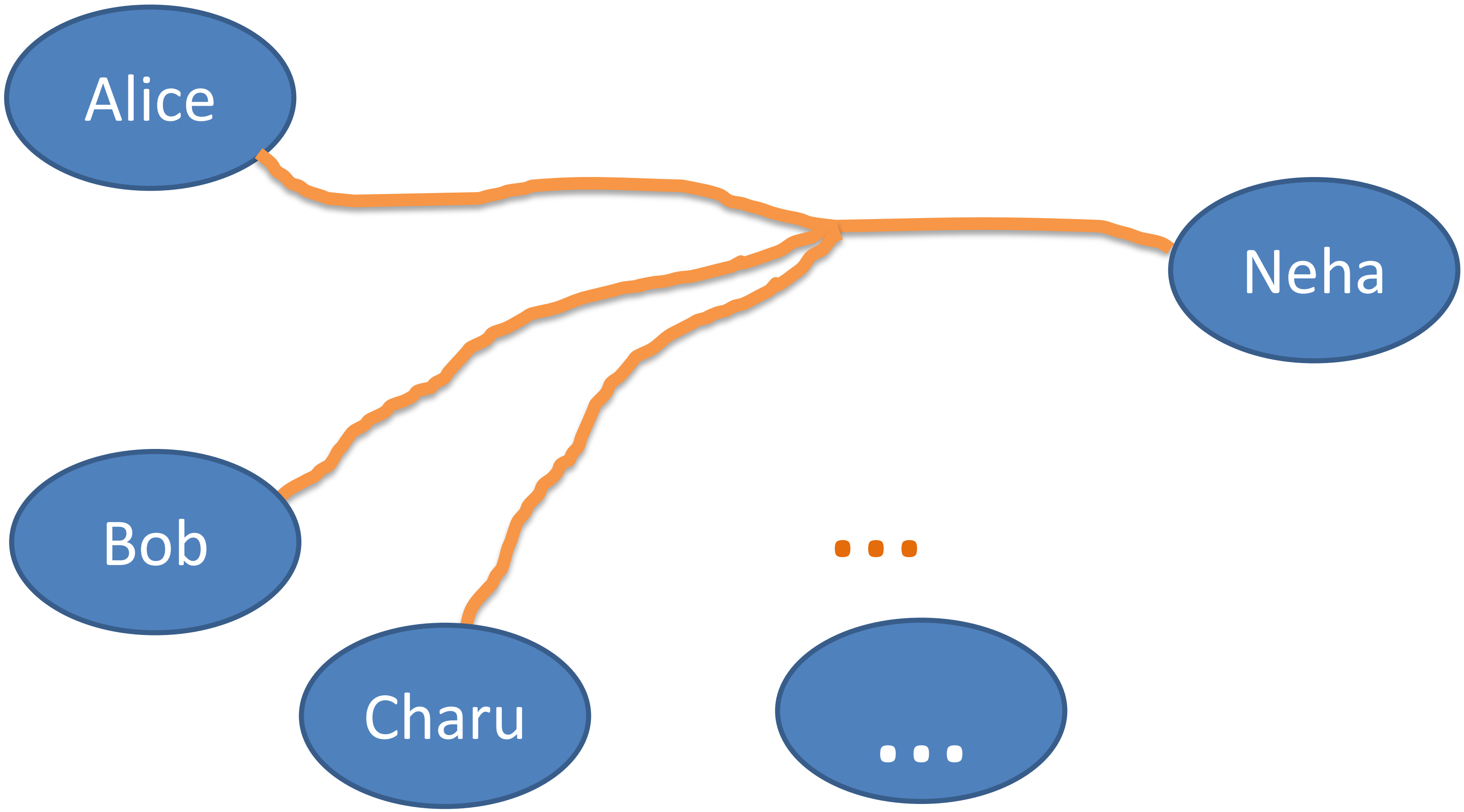

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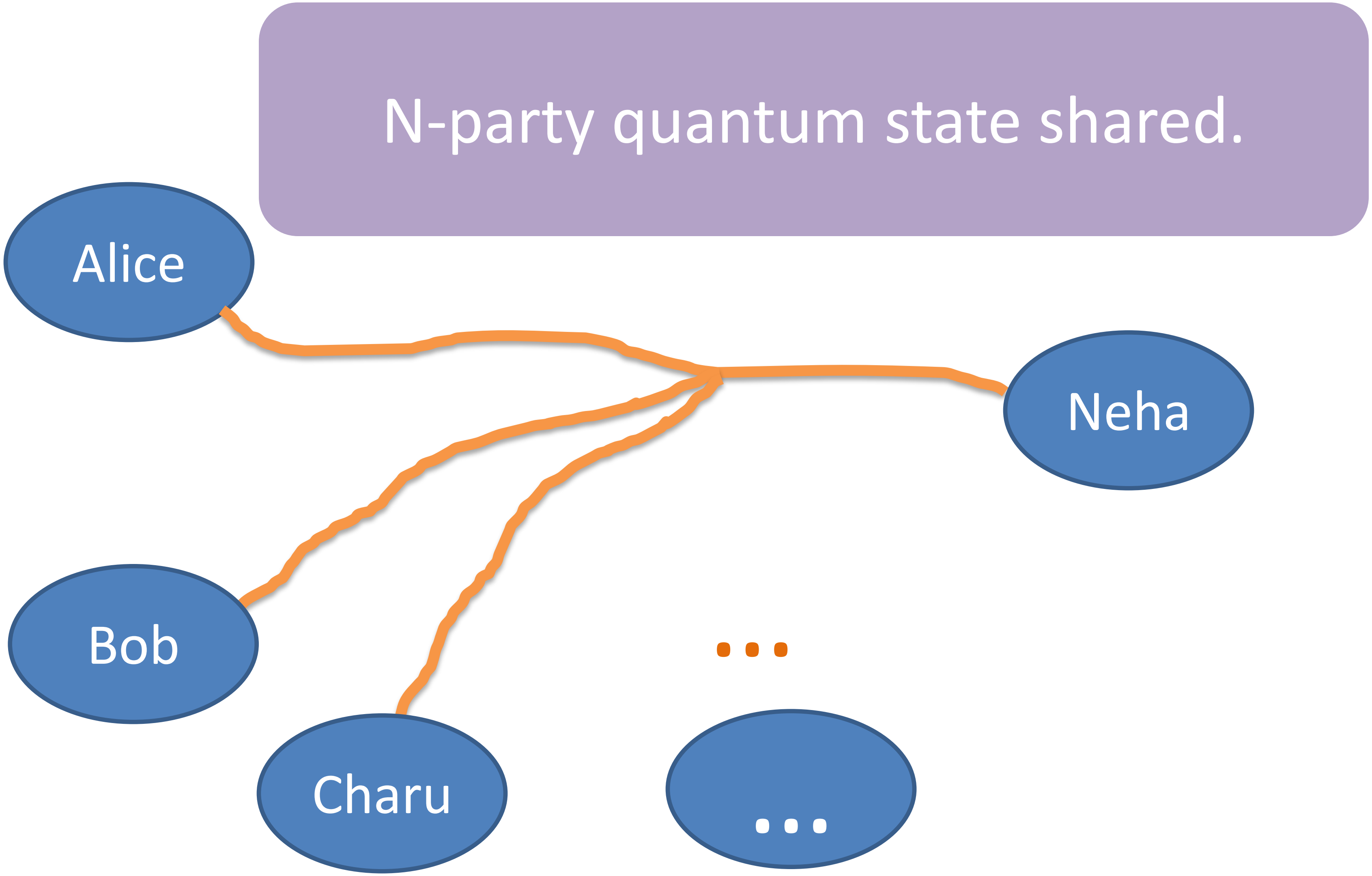
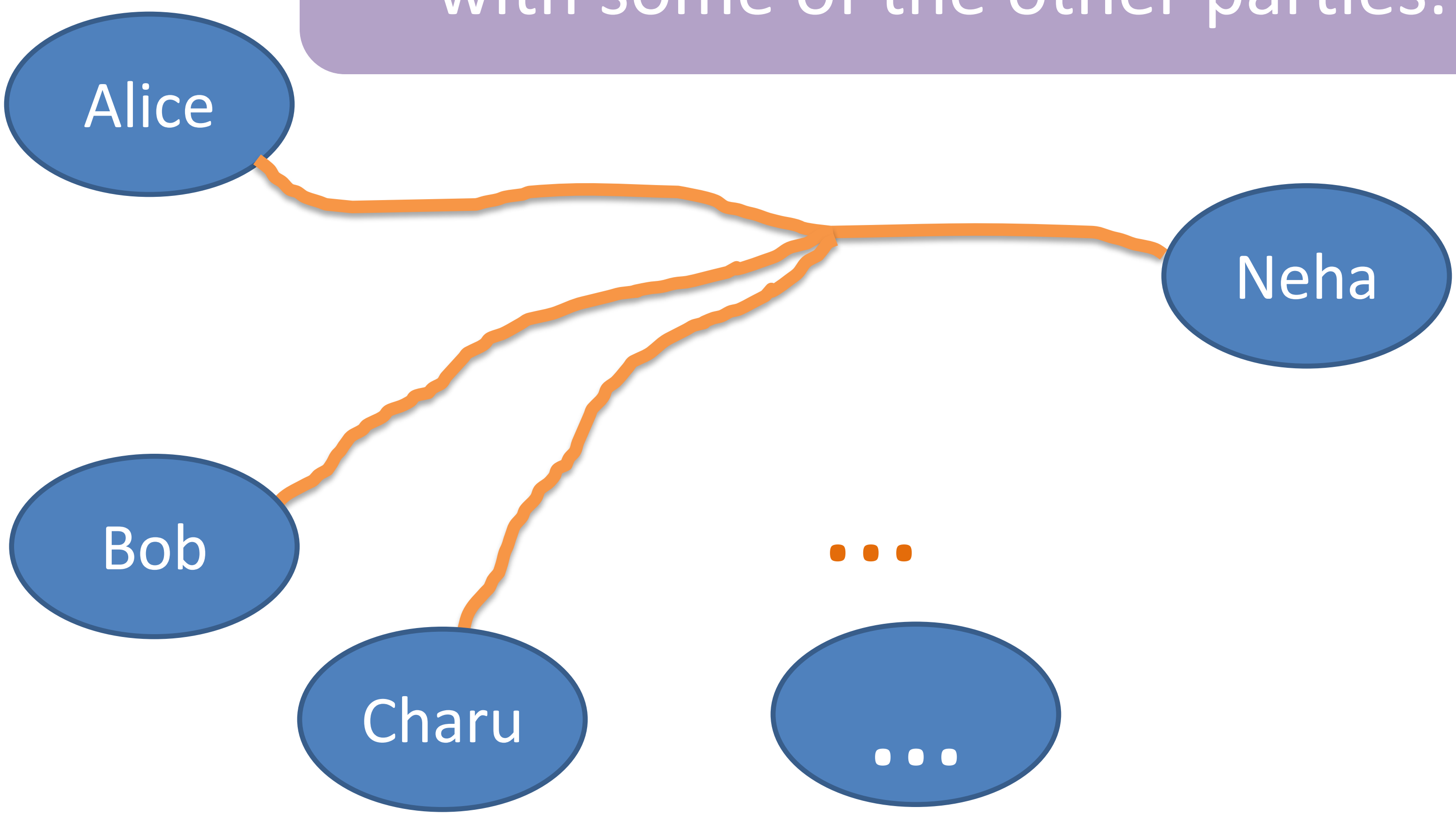


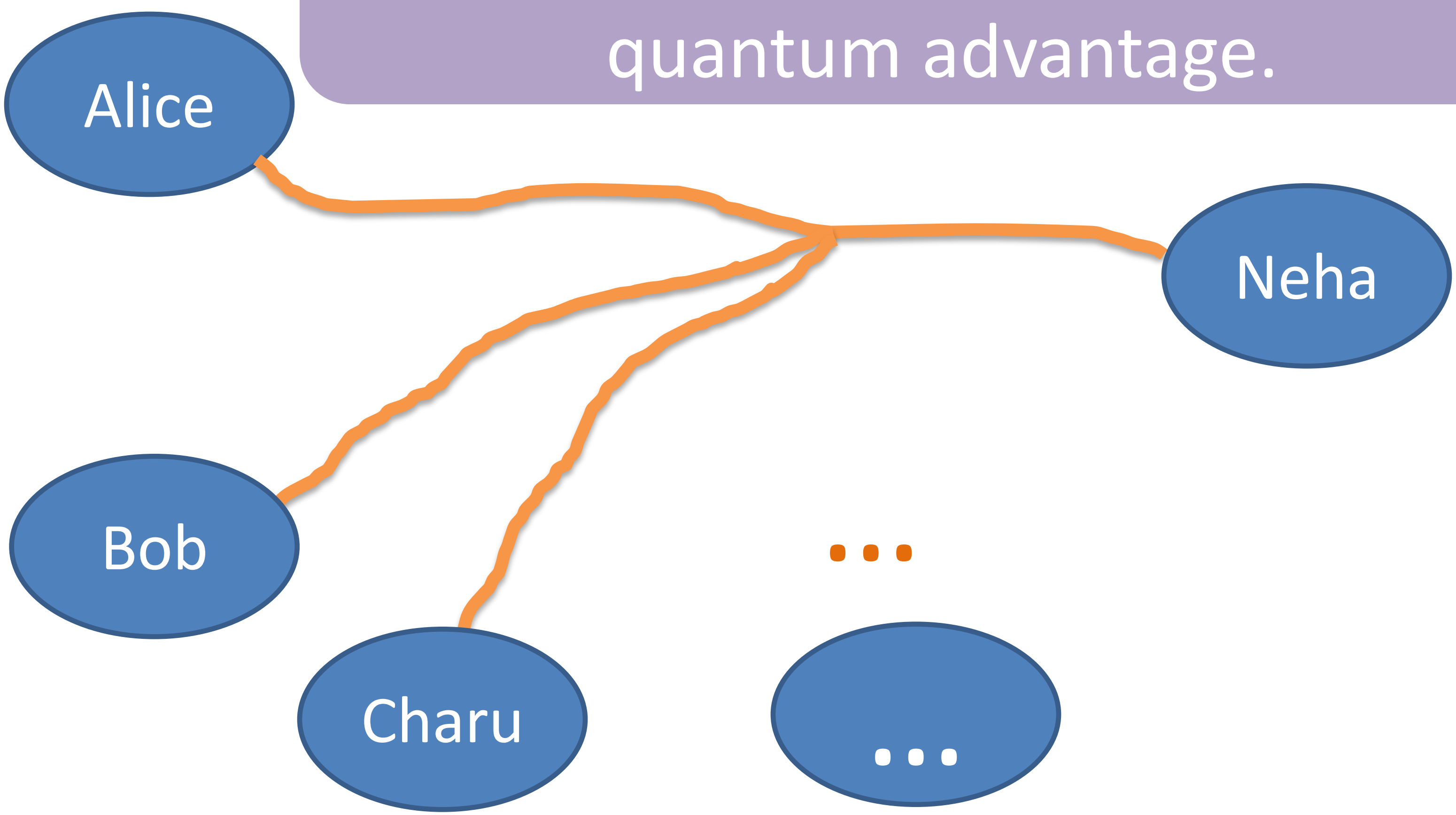
Exclusion pple for Q Dense Coding

Alice wishes to perform dense coding with some of the other parties.



Exclusion pple for Q Dense Coding

For every shared multiparty q state, **at most one** channel from Alice has a quantum advantage.



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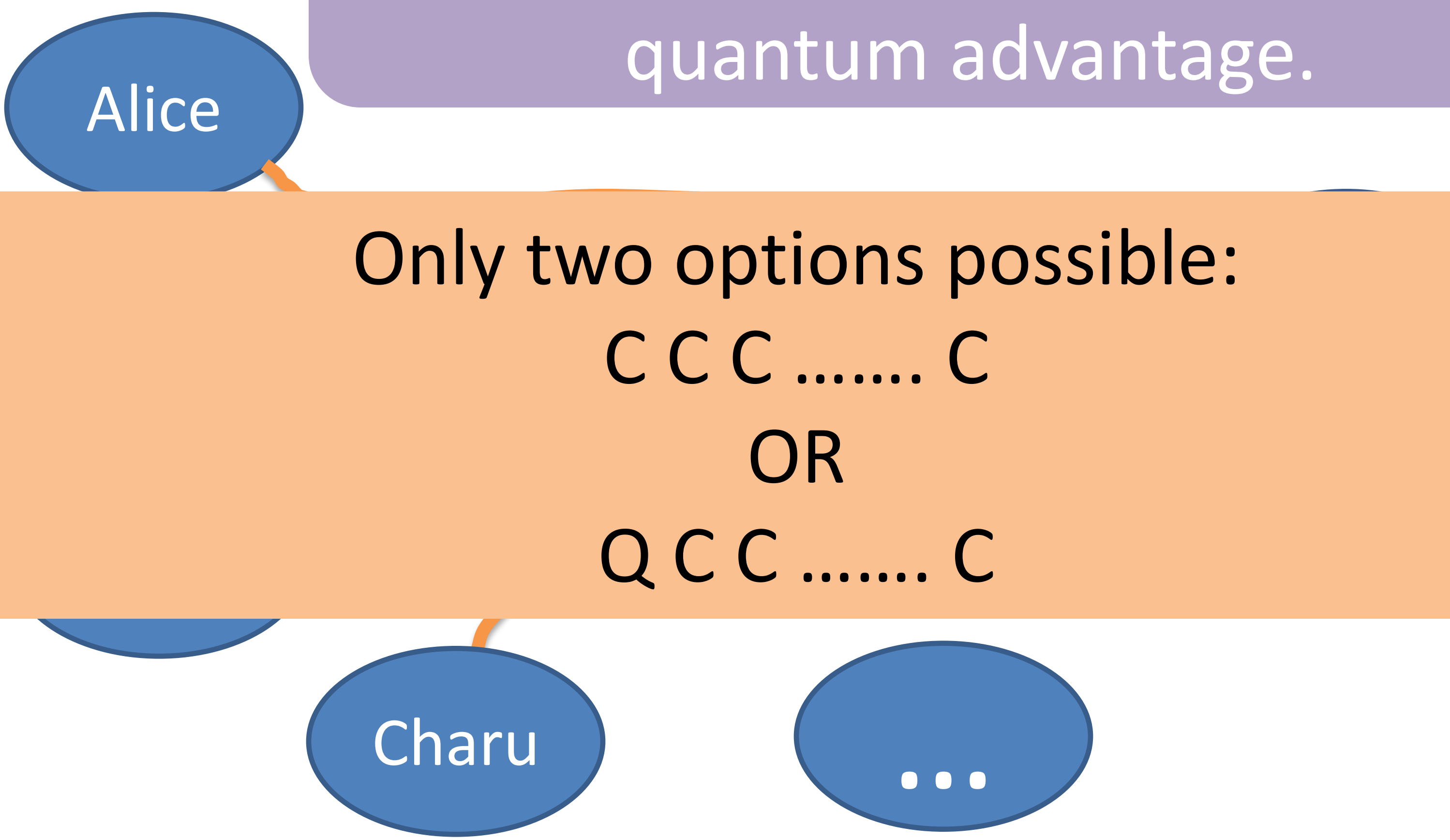

Only two options possible:

C C C ……. C

OR

Q C C ……. C

Exclusion pple for Q Dense Coding

Note that a **classical** capacity, albeit of a quantum channel, is shown to b strongly monogamous.

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Only two options possible:

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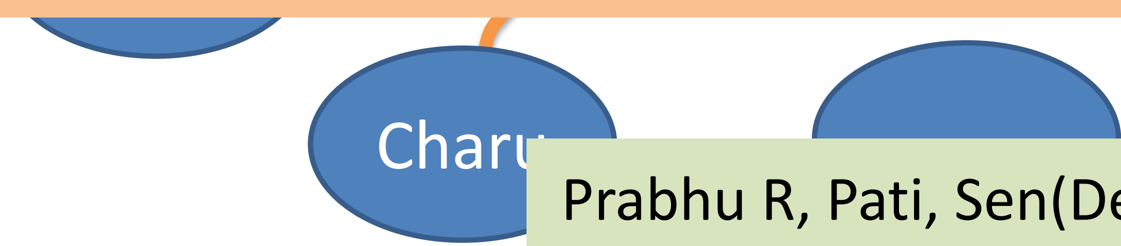

Prabhu R, Pati, Sen(De), US, Phys. Rev. A 2013

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Entanglement

- Quantum states of shared systems that can be created by LOCC are "separable" states.
- Others are "entangled".

Entanglement measures

- Quantum states of shared systems that can be created by LOCC are “separable” states.
- Others are “entangled”.

Can b quantified in many ways.
They r called “entanglement measures”.

“Limitations to sharing entanglement”

concurrence squared is quantitatively monogamous for arb n-qubit states.

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Ent of formation is not quantitatively monogamous even for 3-qubit pure states.

“Limitations to sharing entanglement”

Kim, Gour, Sanders, Contemp. Phys. 2012

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Quantum Discord

Circa 2001: Q discord and q work deficit defined.

Quantum Discord

Henderson, Vedral, JPhysA'01, Ollivier, Zurek, PRL'01
Horodeccy, Oppenheim, Sen(De), US, Synak, PRA'05

Quantum Discord

- Quantum correlation measure, defined indep of the entanglement-separability paradigm.

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Horodeccy, Oppenheim, Sen(De), US, Synak, PRA'05

Quantum Discord

- Quantum correlation measure, defined indep of the entanglement-separability paradigm.
- An info-theoretic measure.
- Can be nonzero for separable states.
- Potentially imp for understanding nonclassical phenomena in shared systems where entanglement is absent.

Henderson, Vedral, JPhysA'01, Ollivier, Zurek, PRL'01
Horodeccy, Oppenheim, Sen(De), US, Synak, PRA'05

Quantum Discord

- Qualitatively monogamous.
- But, violates the quantitative monogamy relation.

Prabhu R, Pati, Sen(De), US, Phys. Rev. A (R) 2012

Monogamy of Quantum Discord: GHZ vs W

- Two imp 3-qubit pure states: GHZ and W

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- W = 001 + 010 + 100

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- Generalized GHZ = a 000 + b 111
- Generalized W = a 001 + b 010 + c 100

Monogamy of Quantum Discord: GHZ vs W

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- GHZ = 000 + 111
- W = 001 + 010 + 100
- GHZ satisfies monogamy, while W violates.
- Generalized GHZ = a 000 + b 111
- Generalized W = a 001 + b 010 + c 100
- All gen GHZ satisfy, while all gen W violate.

Prabhu R, Pati, Sen(De), US, Phys. Rev. A (R) 2012

Monogamy of Quantum Discord: GHZ vs W

All three-qubit pure states = {GHZ-class} U {W-class}

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Monogamy of Quantum Discord: GHZ vs W

All three-qubit pure states = {GHZ-class} U {W-class}

All W-class states violate monogamy of q discord.

Prabhu R, Pati, Sen(De), US, PRA(R)'12; Giorgi PRA'11

Monogamy of Quantum Discord: GHZ vs W

All three-qubit pure states = {GHZ-class} U {W-class}

Monogamy of quantum discord can therefore act as an "witness for GHZ-class states".

All W-class states violate monogamy of q discord.

Prabhu R, Pati, Sen(De), US, PRA(R)'12; Giorgi PRA'11

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- Ent of formation is not monogamous. But, concurrence squared, which is a monotonic function of ent of formation, is.
- Discord is not monogamous. But discord squared is.

All qc can be made monogamous

- Ent of formation is not monogamous. But, concurrence squared, which is a monotonic function of ent of formation, is.
- Discord is not monogamous. But discord squared is.
- Theorem: This is generic.

Salini K, Prabhu R, A. Sen(De), US, Annals Phys (2013)

All qc can be made monogamous

More precisely,

Any quantum correlation measure, which does not increase under loss of a part of a local subsystem, can be made monogamous by considering an increasing function of the same.

Salini K, Prabhu R, A. Sen(De), US, Annals Phys (2013)

All qc can be made monogamous

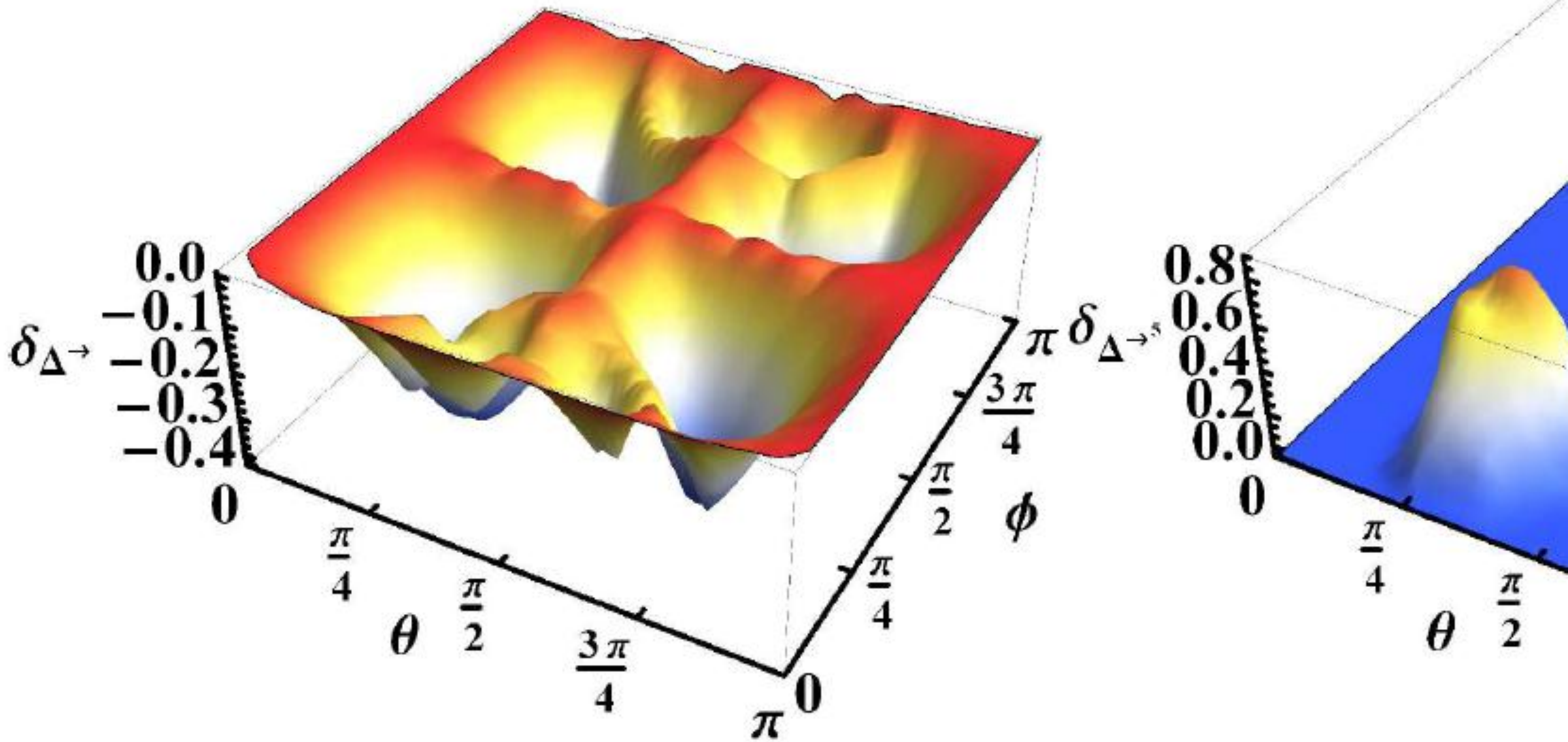
More precisely,

Any quantum correlation measure, which does not increase under loss of a part of a local subsystem, can be made monogamous by considering an increasing function of the same.

Note: The function of the measure is still a valid qc measure, and is reversible (so, no loss of data).

Salini K, Prabhu R, A. Sen(De), US, Annals Phys (2013)

Quantum work deficit for gen W states



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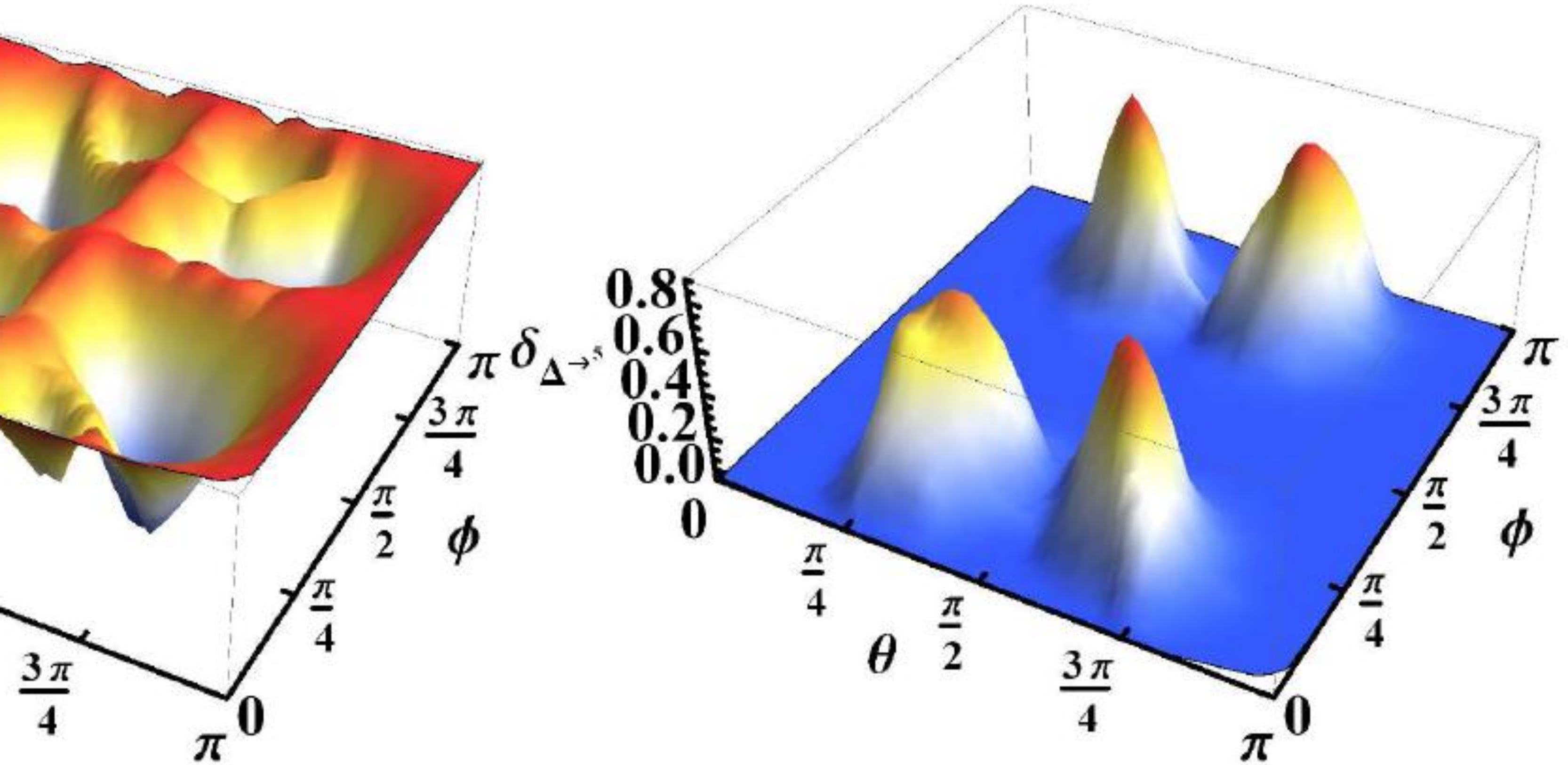

Salini K, Prabhu R, A. Sen(De), US, Annals Phys (2013)

Outline

- What is monogamy?
- Local monotonicity independent of monogamy.
- Channel capacity of q channels can be strongly monogamous.
- Quantitative monogamy relations for qc
- All qc can be made monogamous.
- All qc monogamous for almost all states of moderately large systems.

Monogamy in moderately large systems

Monogamy percentages for ent measures.

n	$\delta_{\mathcal{C}}$	$\delta_{\mathcal{E}}$	$\delta_{\mathcal{E}^2}$	$\delta_{\mathcal{N}}$	$\delta_{\mathcal{N}^2}$	$\delta_{E_{\mathcal{N}}}$	$\delta_{E_{\mathcal{N}}^2}$
3	60.2	93.3	100	91.186	100	68.916	100
4	99.6	100	100	99.995	100	99.665	100
5	100	100	100	100	100	100	100

10^5 states chosen Haar uniformly over all 3-qubit pure states.

Asutosh Kumar, Prabhu R, A. Sen(De), US, arXiv (2014)

Monogamy in moderately large systems

Monogamy percentages for info-theo qc measures.

n	$\delta_{\mathcal{D}}^{\rightarrow}$	$\delta_{\mathcal{D}^2}^{\rightarrow}$	$\delta_{\mathcal{D}}^{\leftarrow}$	$\delta_{\mathcal{D}^2}^{\leftarrow}$	$\delta_{\Delta}^{\rightarrow}$	$\delta_{\Delta^2}^{\rightarrow}$	$\delta_{\Delta}^{\leftarrow}$	$\delta_{\Delta^2}^{\leftarrow}$
3	90.5	100	93.28	100	56.29	88.10	57.77	89.56
4	99.997	100	99.99	100	94.27	99.99	97.63	100
5	100	100	100	100	99.98	100	99.99	100

10^5 states chosen Haar uniformly over all 3-qubit pure states.

Asutosh Kumar, Prabhu R, A. Sen(De), US, arXiv (2014)

Monogamy in moderately large systems

- Theorem: If ent of formation is monogamous for a certain state, all “good” ent measures are also so.

Asutosh Kumar, Prabhu R, A. Sen(De), US, arXiv (2014)

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Monogamy in moderately large systems

- Theorem: If ent of formation is monogamous for a certain state, all "good" ent measures are also so.
- A measure is "good" if it is less than ent of formation for all states, and equal to the local von Neumann entropy for pure states.
- Such measures include distillable entanglement, relative entropy of entanglement, etc.

Asutosh Kumar, Prabhu R, A. Sen(De), US, arXiv (2014)

Monogamy in moderately large systems

- Table shows ent of formation is monogamous for almost all pure states of 5 qubits & above.

Asutosh Kumar, Prabhu R, A. Sen(De), US, arXiv (2014)

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Asutosh Kumar, Prabhu R, A. Sen(De), US, arXiv (2014)

Monogamy in moderately large systems

- Table shows ent of formation is monogamous for almost all pure states of 5 qubits & above.
- Theorem then implies same for distillable ent & relative entropy of ent, which r analytically and numerically intractable.
- Statement not true for all states, as W states of arb # of qubits violate monogamy.

Asutosh Kumar, Prabhu R, A. Sen(De), US, arXiv (2014)

Summary

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- The terrain is intricate. Large deviations in behavior from one shared quantity to another.

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- The terrain is intricate. Large deviations in behavior from one shared quantity to another.
- However, general results that point to connecting themes are beginning to emerge.


QIC Group @ HRI, 2013


Thank you!

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Reference to previous work is incomplete!

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- Only some r quantitatively so.

- In particular,

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Ent of formation is not quantitatively monogamous even for 3-qubit pure states.

Outline

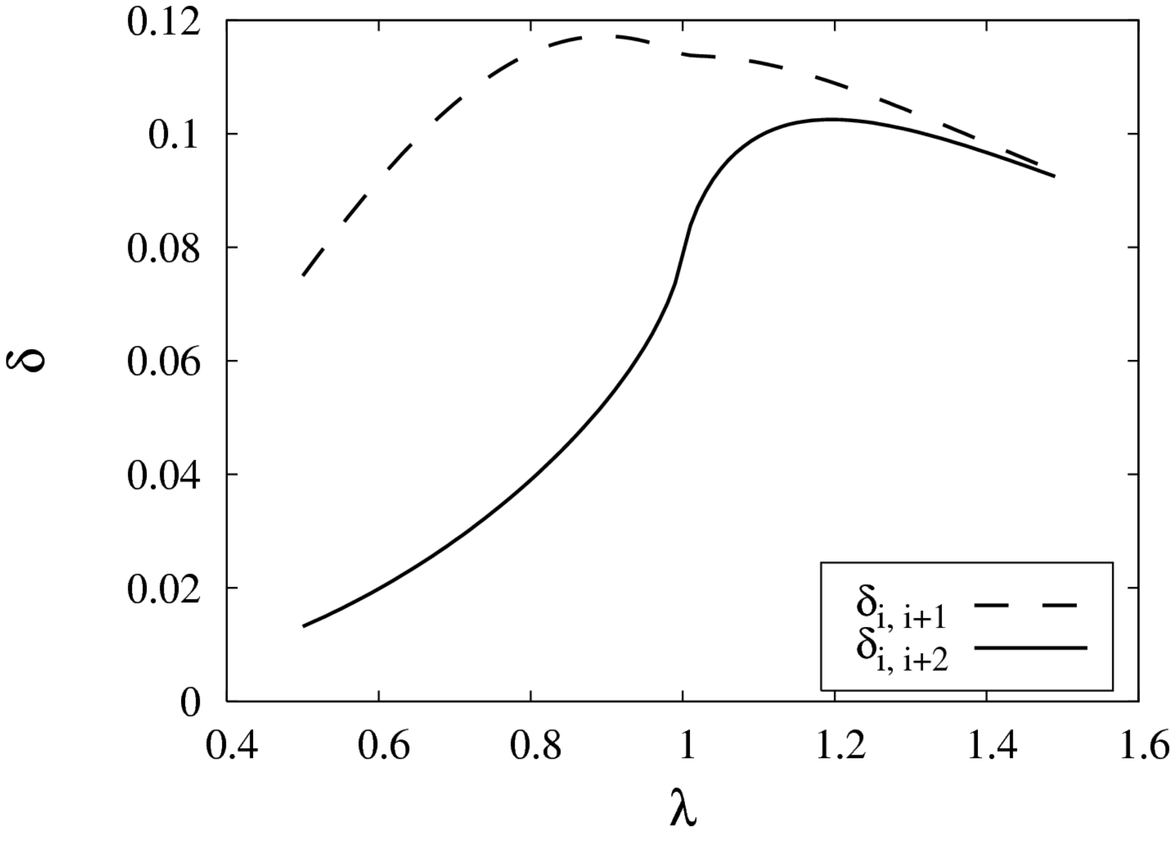
- Locally accessible information monotone, but strongly non-monogamous.
- DC channel capacity strongly monogamous: Exclusion pple.
- Quantum Discord qualitatively monogamous, but quantitatively not so: entire W class non-monogamous.
- Shared purity.
- All quantum correlations can be made monogamous.
- All quantum correlations monogamous for almost all states of moderately large systems.

- Entanglement very useful, but there’s phenomena beyond.

- Information-theoretic quantum correlation measures proposed.

- Discord and work-deficit have reproduced the explanations in a lot of phenomena.

Discord detects QPT



Dillenschneider, PRB'08

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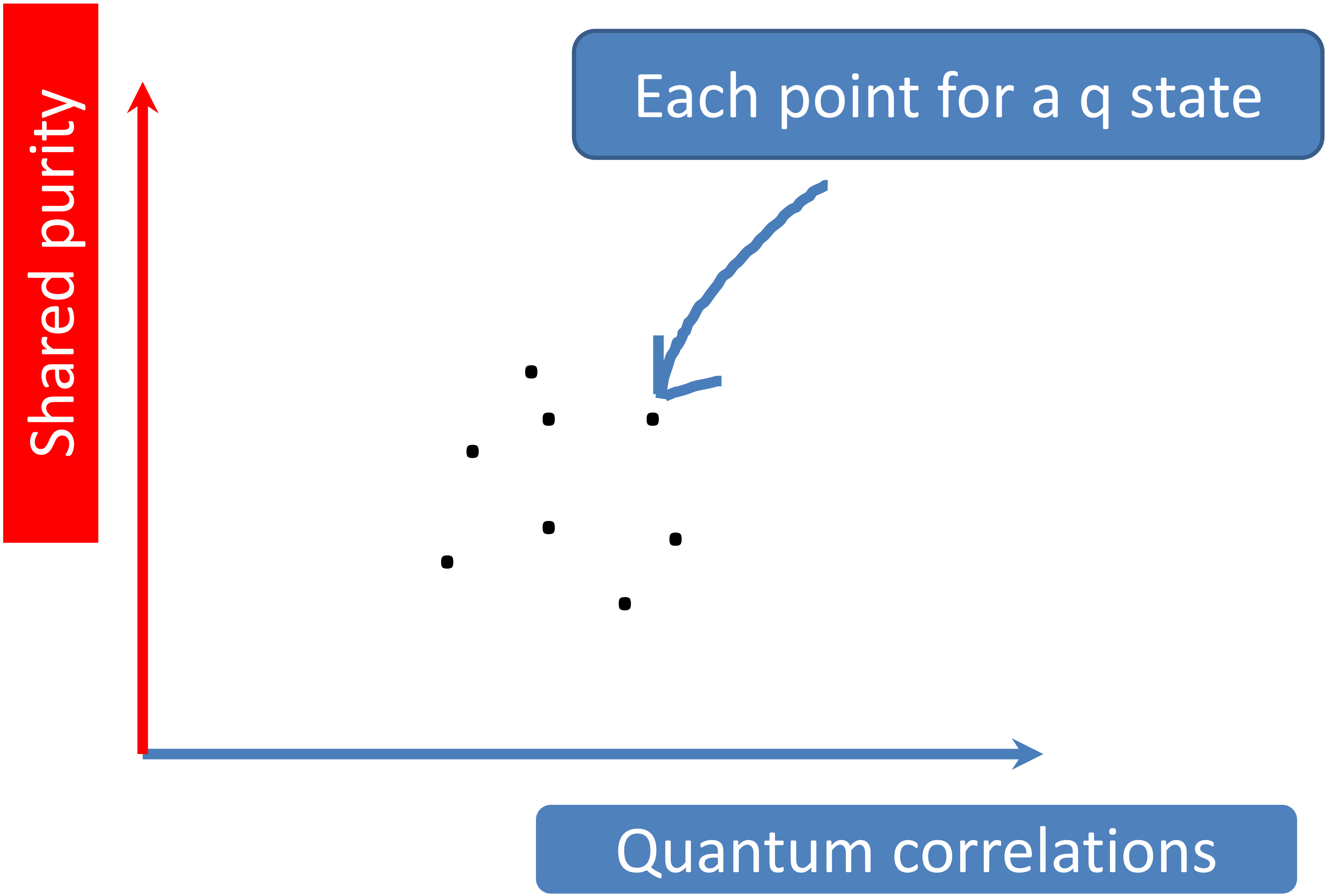
Eg. DQC1

Knill, Laflamme, PRL'98;
Datta, Shaji, Caves, PRL'08;
Dakic, Vedral, Brukner, PRL'10

A New Dimension



Purity

- “r” is a quantum state, possibly mixed.
- <a|r|a>, maximized over all |a>, is the “purity” of “r”.
- Well-defined for multiparty states. |a> is then an arb multiparty pure state.
- Maximal (unity) for pure “r”.

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For multi-party case, we term it “global purity”.

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Certainly,
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Often,
global purity > local purity

Shared Purity

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A Biswas, A Sen(De), US, arXiv (2013)

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Precisely,

shared purity = global purity – local purity.

A Biswas, A Sen(De), US, arXiv (2013)

Shared Purity

- Is it another quantum correlation?

No!
Why? →

A Biswas, A Sen(De), US, arXiv (2013)

- Can be nonzero for unentangled states.
- Can be zero for entangled states.

A Biswas, A Sen(De), US, arXiv (2013)

Is it quantum?

A Biswas, A Sen(De), US, arXiv (2013)

Is it quantum?

- Yes!
- Why?
- Because, it is qualitatively monogamous.

A Biswas, A Sen(De), US, arXiv (2013)