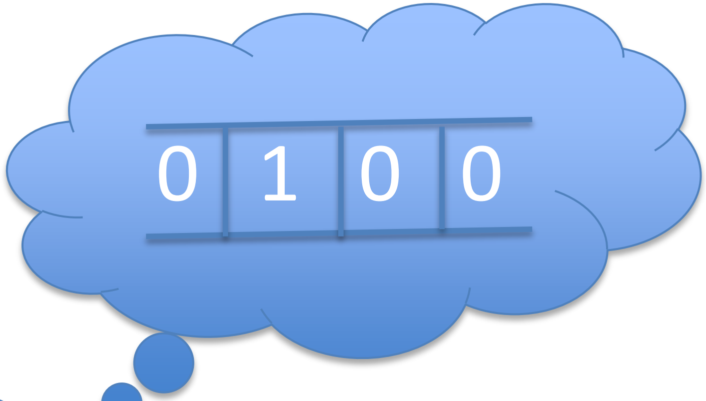


Modeling Maxwell's demon

D Mandal, UC Berkeley

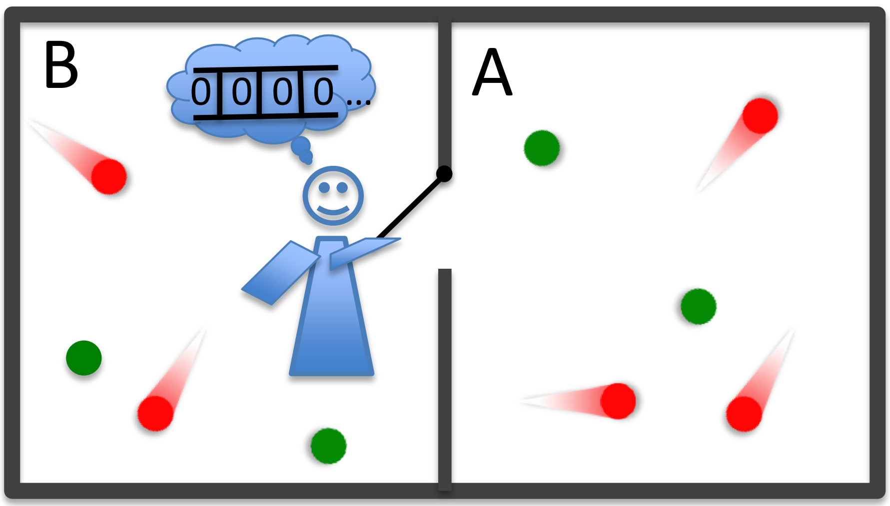
A curious observation

One liter of ordinary air has enough thermal energy to toss a 7 kilogram bowling ball 3 meters off the ground!

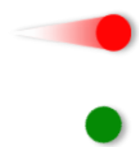
A solution to the energy crisis?

Maxwell’s demon

(born in 1867)



Before operation

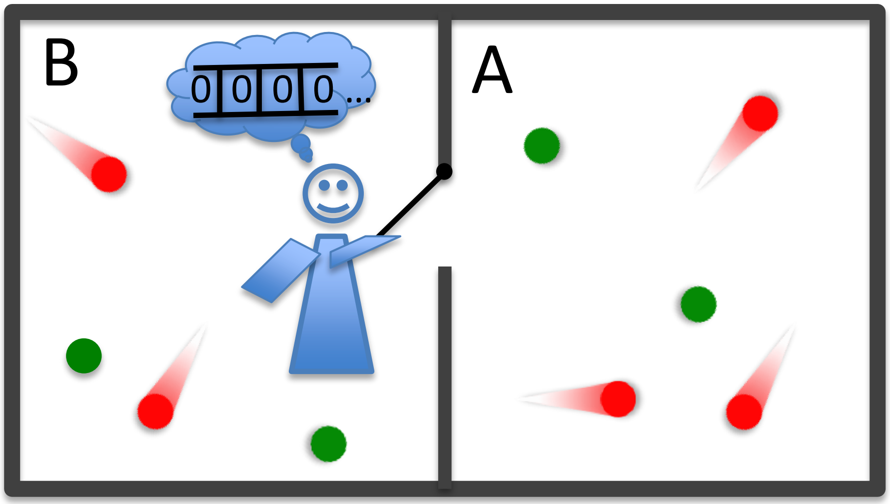


Fast

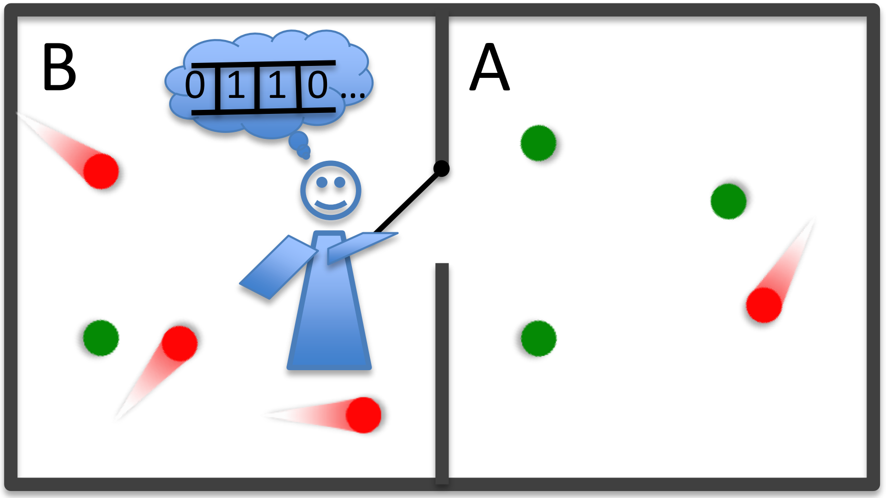
Slow

Maxwell's demon

(born in 1867)



Before operation



During operation

Violation of the second law!

No process is possible whose sole result is the extraction of energy from a single heat bath and its conversion into work.

Accepted resolution

Landauer's principle: A minimum of $kT \ln(2)$ amount of heat needs to be dissipated to erase one bit of information

Landauer (1961), Penrose (1970), and Bennett (1982)

Accepted resolution

Landauer's principle: A minimum of kT ln(2) amount of heat needs to be dissipated to erase one bit of information

Thermodynamics of information processing:

- Writing information increases entropy of memory
- Erasing information decreases entropy of memory

Landauer (1961), Penrose (1970), and Bennett (1982)

Goal

- An autonomous physical system, without intelligence or explicit thermodynamic force, that behaves like a demon

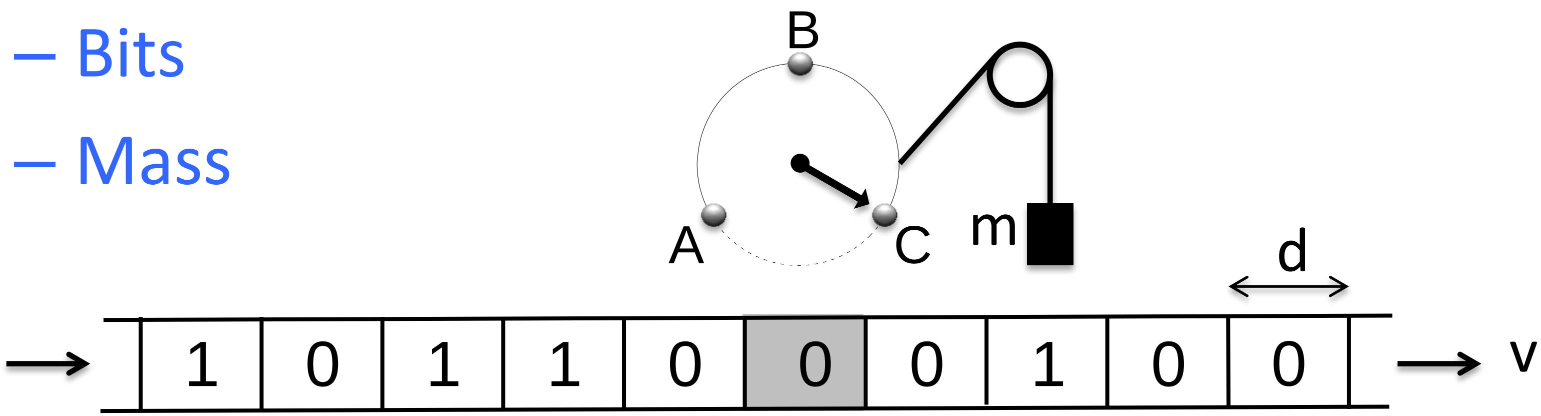
Goal

- An autonomous physical system, without intelligence or explicit thermodynamic force, that behaves like a demon
- Implications for the second law of thermodynamics

Model*: Overview

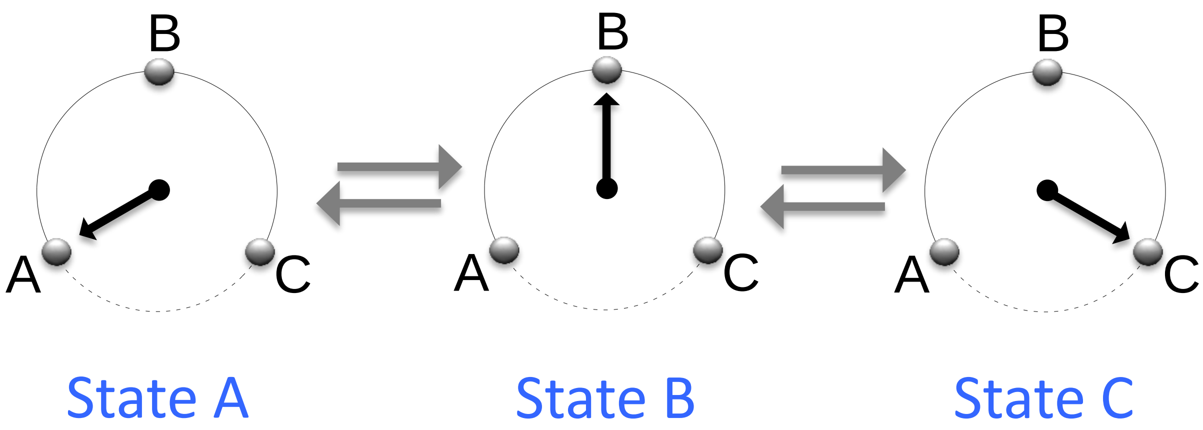
Three essential components, all immersed in a heat bath:

– Demon
– Bits
– Mass



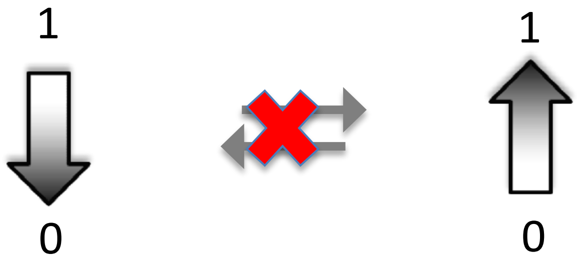
*Mandal and Jarzynski, PNAS 109, 11641 (2012)

Demon



No complete rotation

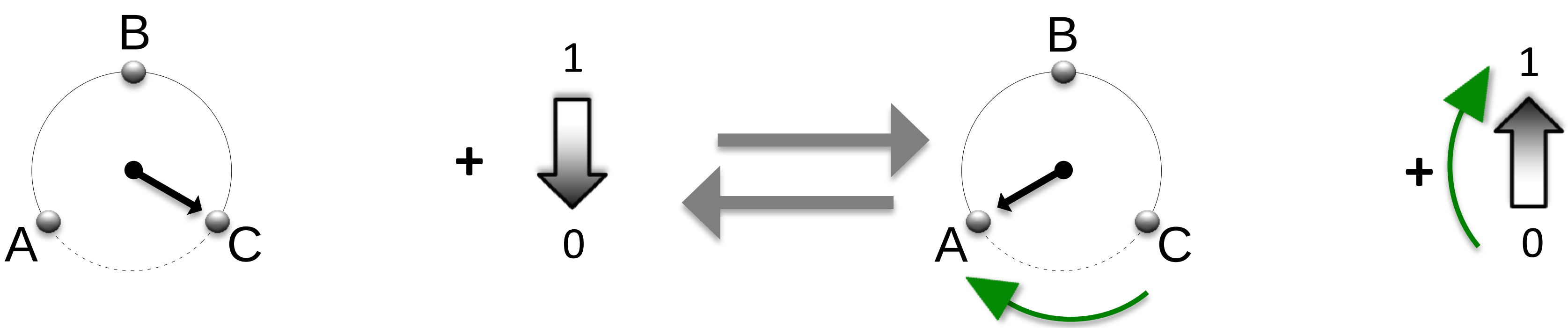
Bit



State 0 State 1

No intrinsic transitions

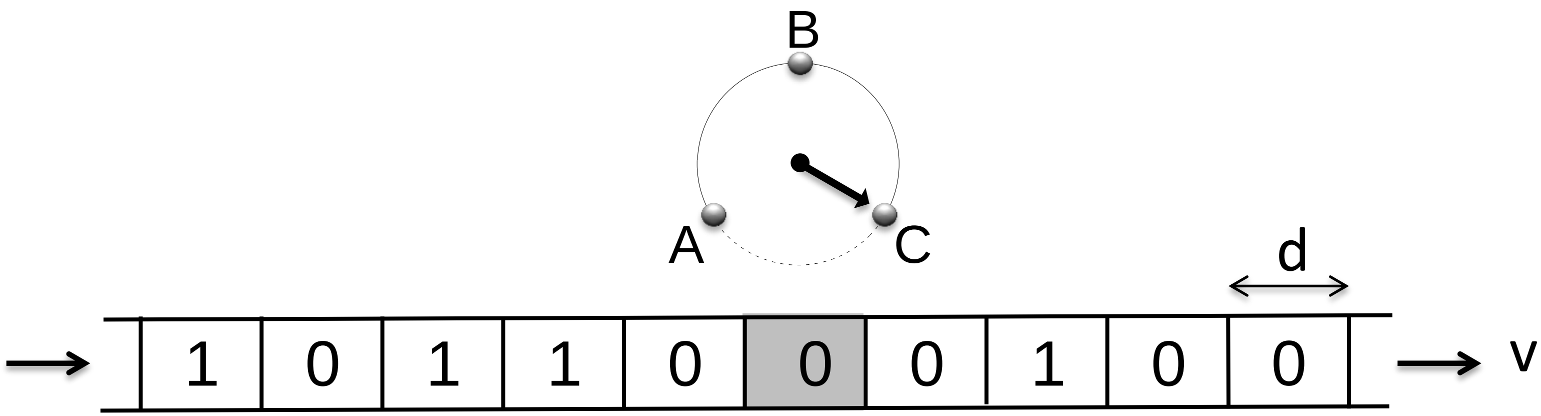
Demon + Bit



State C   State 0   State A   State 1

Demon + Stream of bits

- Stream of uncorrelated bits: $p_{0/1}$ = proportion of 0/1's
- The demon interacts with the nearest bit
- Interaction interval: $\tau = d/v$

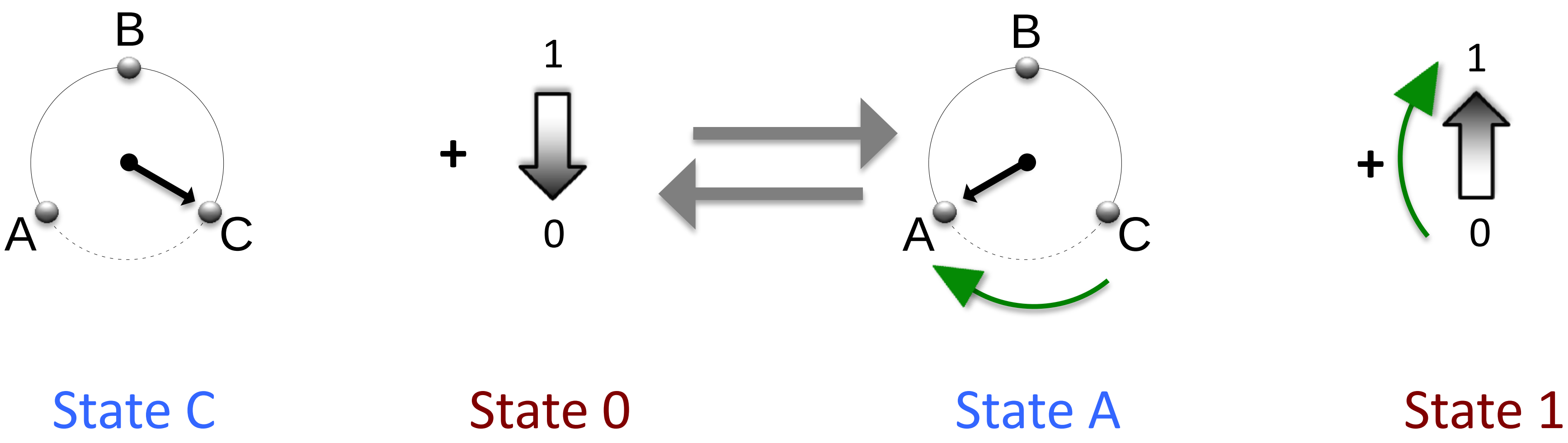


Claim

The demon rotates clockwise if all the incoming bits are in state 0

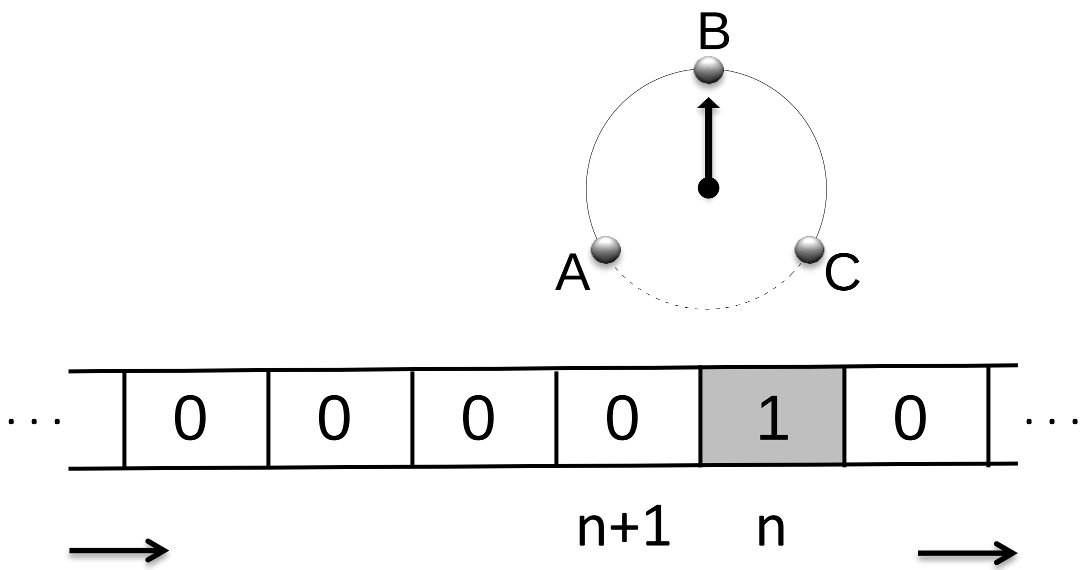
(Rectification of thermal fluctuations, using information)

Justification

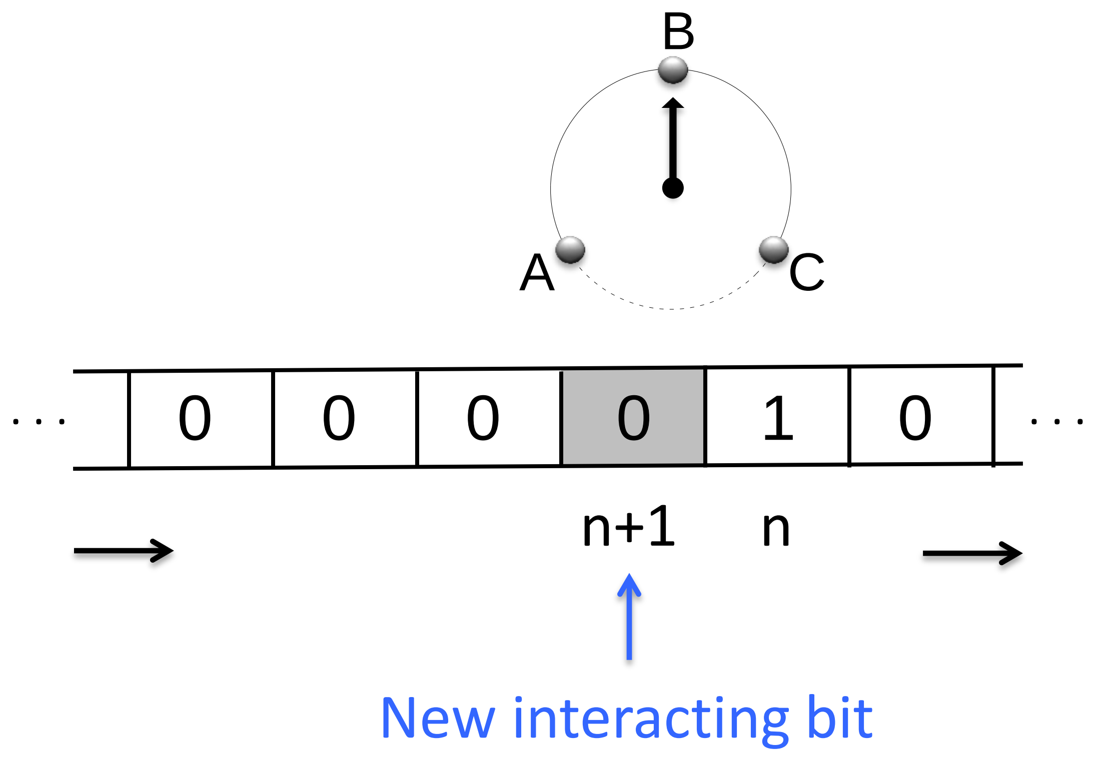


- If all incoming bits are in state 0
  - No full counter clockwise rotation
  - Sustained clockwise rotation on average
  - Thermal fluctuations are rectified!

Demonstration of rectification



Demonstration of rectification



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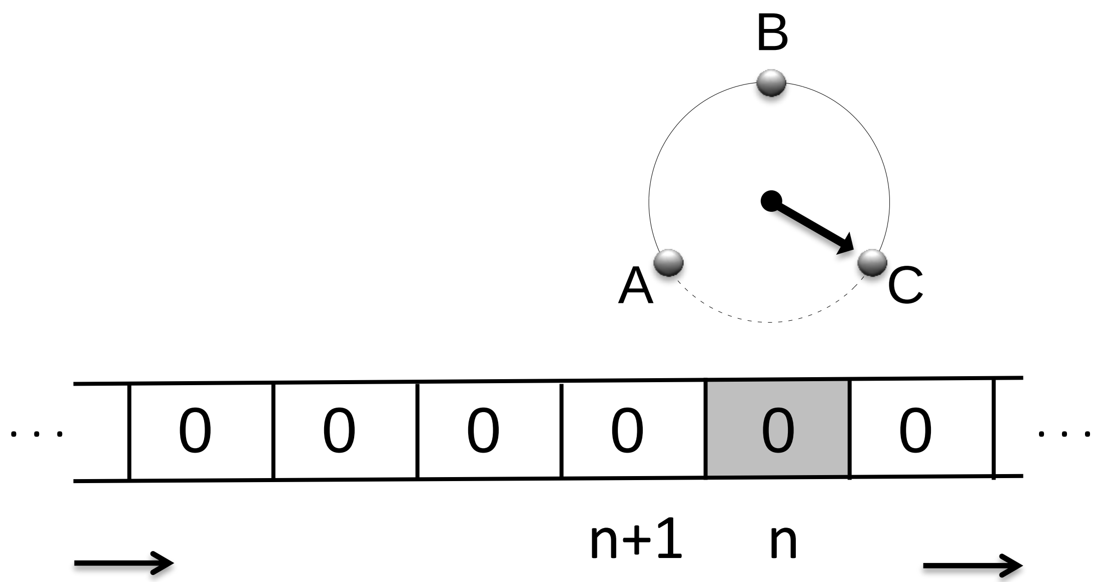

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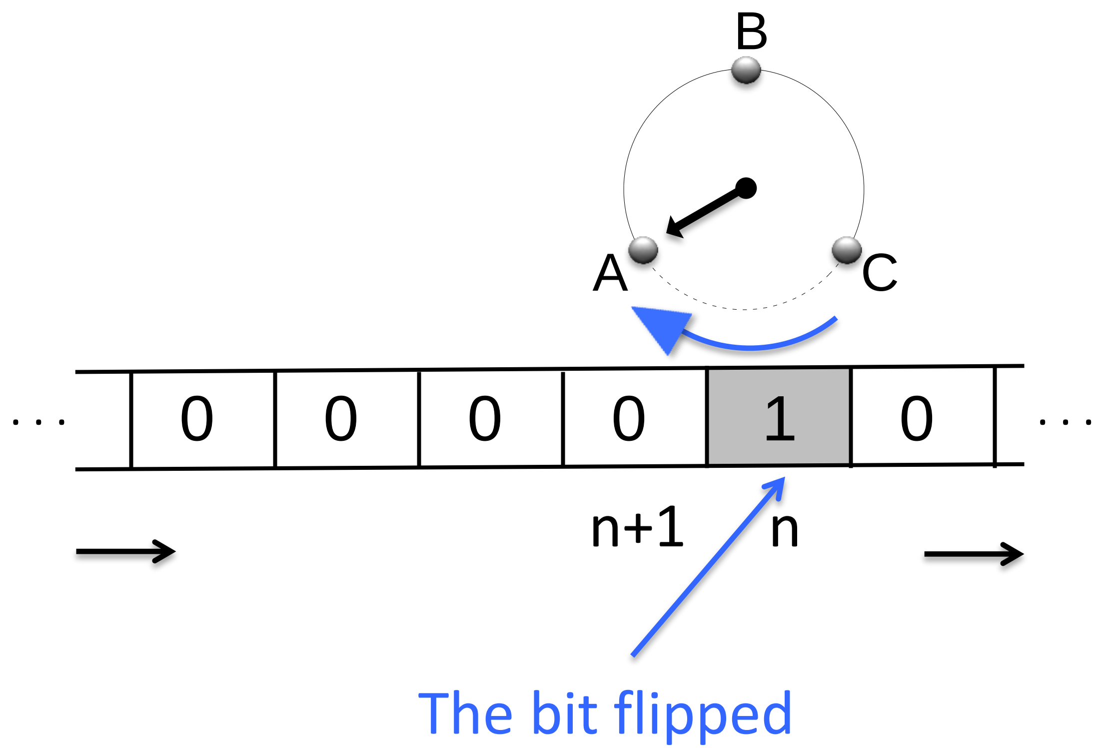
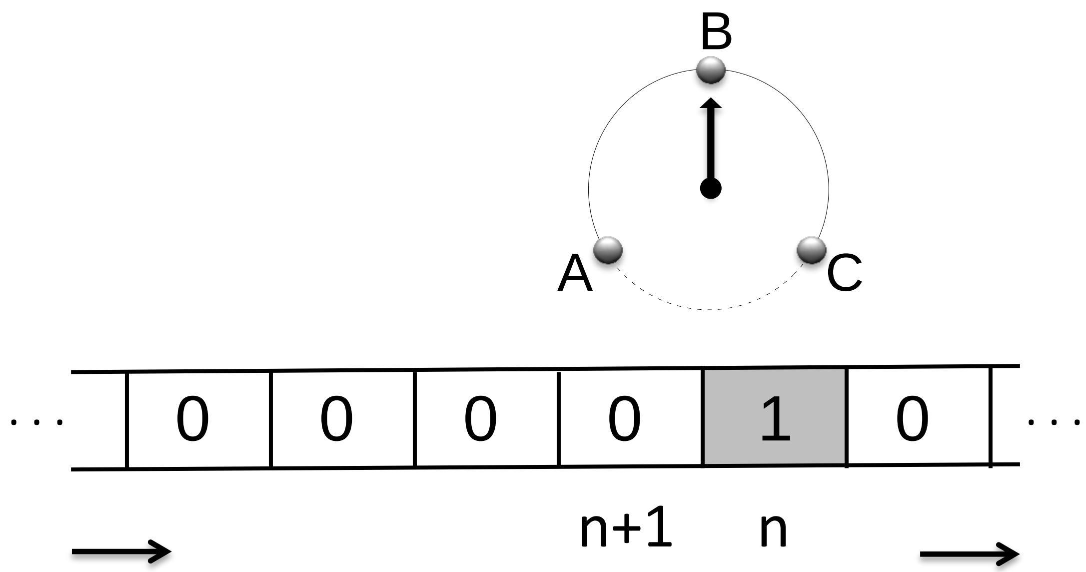
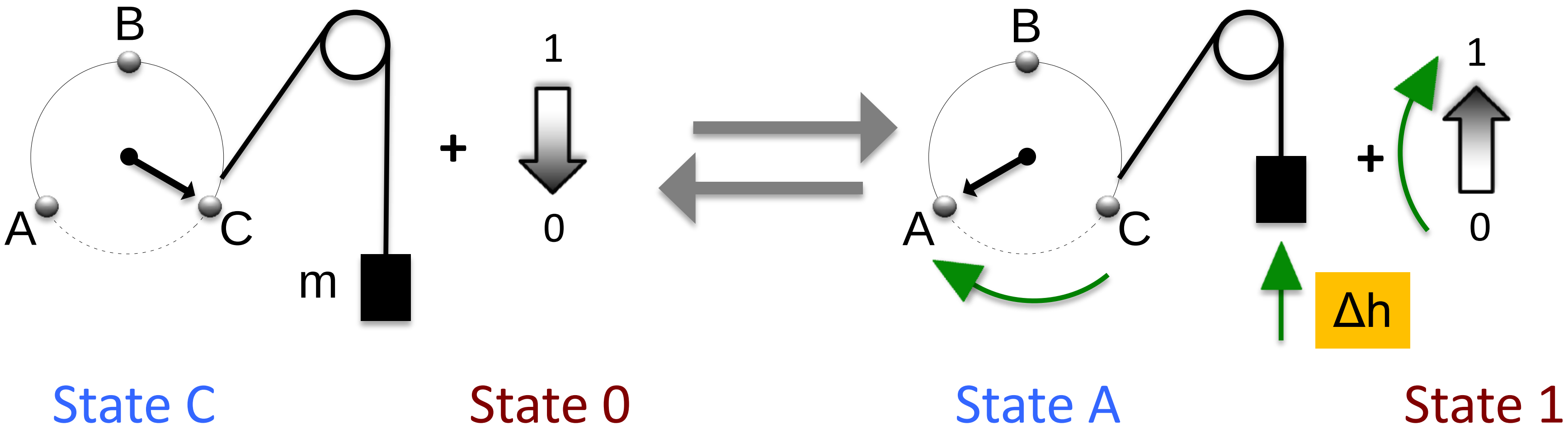


Demonstration of rectification

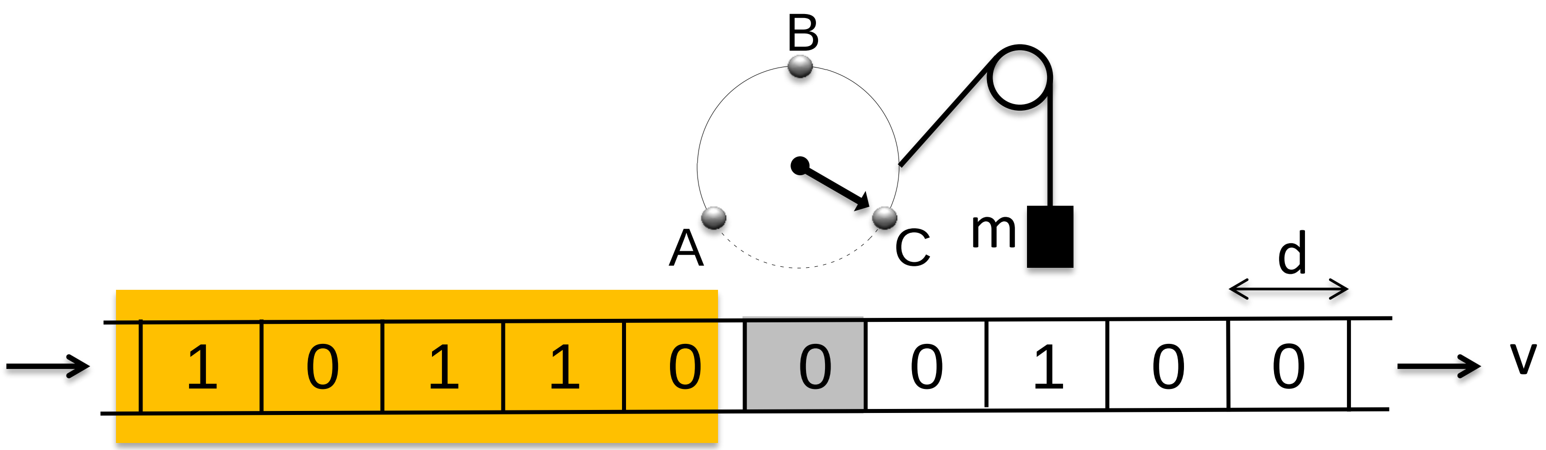


One complete clockwise rotation!

Demon + Bit + Mass



Complete setup



- Mixture of 0's and 1's
- $p_{0/1}$($p'_{0/1}$): Proportion of incoming (outgoing) bits in state 0/1

Information entropy

- Information / bit
  - $H_{in} = -p_0 \ln p_0 - p_1 \ln p_1$
  - $H_{out} = -p'_0 \ln p'_0 - p'_1 \ln p'_1$
  - $0 \leq H \leq \ln 2$

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- $H = \ln 2$: equal # of 0's and 1's (``full'')

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- $H = 0$: all bits either 0 or 1 (``blank”)
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- $\Delta S_B = k_B (H_{out} - H_{in})$: change in information ($\times k_B$)

Relevant variables

- (key) Parameters
  - $\delta = p_0 - p_1$: excess 0's of incoming bits
  - $\varepsilon = \tanh[m\, g\, \Delta h / 2\, k\, T]$: rescaled mass
  - $\tau = d/v$: interaction time / bit

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- Quantities of interest
  - $\Phi = p'_1 - p_1$: Avg. clockwise rotation / bit
  - $W = \Phi\ m\ g\ \Delta h$: Avg. work / bit
  - $\Delta S_B = H_{out} - H_{in}$: Change in information ($\times k_B$) / bit

Analytical results

- Φ: Avg. clockwise current per τ

$$\boxed{\Phi(\delta,\varepsilon;\tau)=\frac{\delta-\varepsilon}{2}\eta}$$

$$\eta=\left[1-\frac{1}{3}K(\tau)+\frac{\varepsilon\delta}{6}J(\tau,\varepsilon\delta)\right]\geq 0$$

$$K(t)=e^{-2t}\frac{\left(1+8a+4\sqrt{3}b\right)-\left(2+7a+4\sqrt{3}b\right)e^{-2t}}{3-\left(2+a\right)e^{-2t}}$$

$$J(t,ed)=\frac{\left(1-e^{-t}\right)\left[2e^{-2t}\left(a+\sqrt{3}b-1\right)\right]^{2}}{\left[3\left(1-ede^{-t}\right)-\left(1-ed\right)\left(2+a\right)e^{-2t}\right]\left[3-\left(2+a\right)e^{-2t}\right]}$$

$$b=\sinh\left(\sqrt{3}t\right)$$

$$a=\cosh\left(\sqrt{3}t\right)$$

Analytical results

- Φ: Avg. clockwise current per τ

$$\Phi(\delta,\varepsilon;\tau)=\frac{\delta-\varepsilon}{2}\eta$$

- W : Avg. work per τ = $\Phi kT\ln\left(\frac{1+\varepsilon}{1-\varepsilon}\right)$

Analytical results

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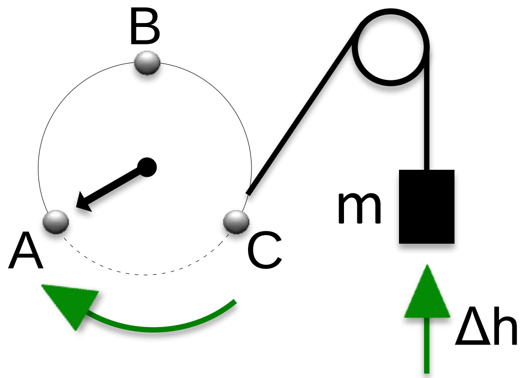
- W : Avg. work per τ = $\Phi kT\ln\left(\frac{1+\varepsilon}{1-\varepsilon}\right)$

- ΔS_B : Change in entropy per bit = $H(2\Phi-\delta)-H(\delta)$

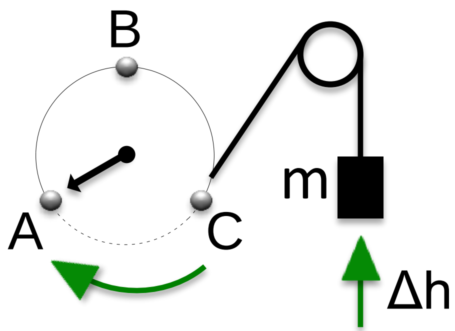
Competition between two forces: δ and ε

$$\Phi(\delta,\varepsilon;\tau)=\frac{\delta-\varepsilon}{2}\eta, \qquad \eta\geq 0$$



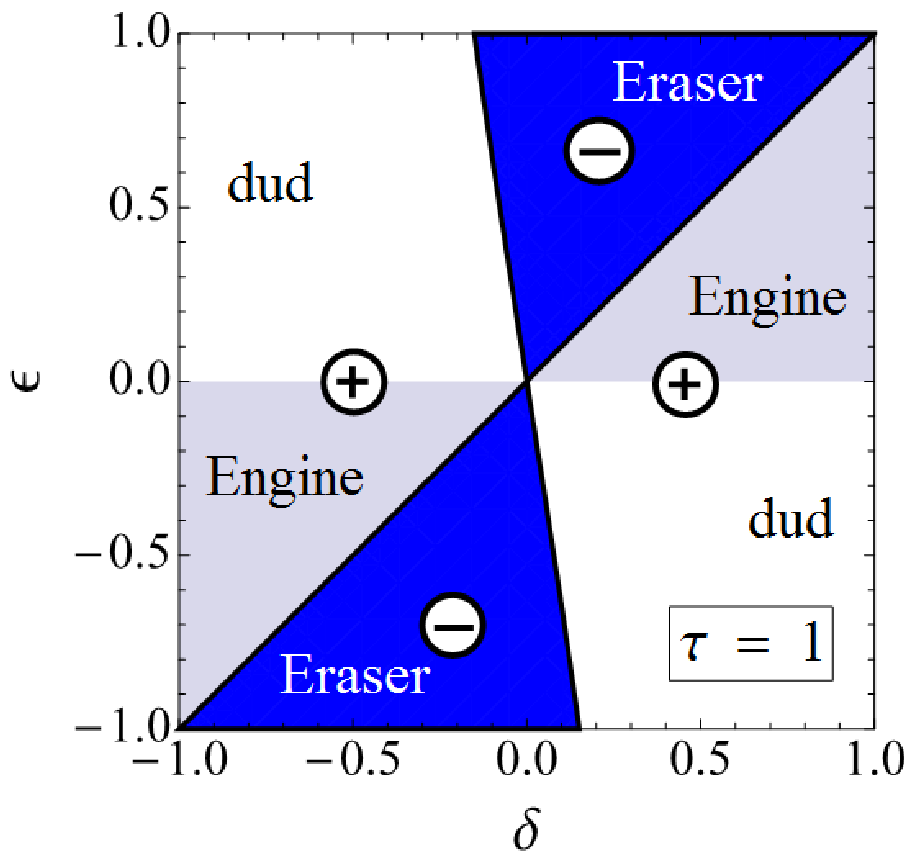
- δ (excess incoming 0's) favors CW rotation
- ε (gravitational force) favors CCW rotation

Non-equilibrium phase diagram

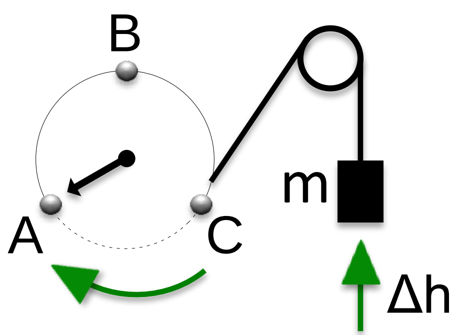


δ: CW

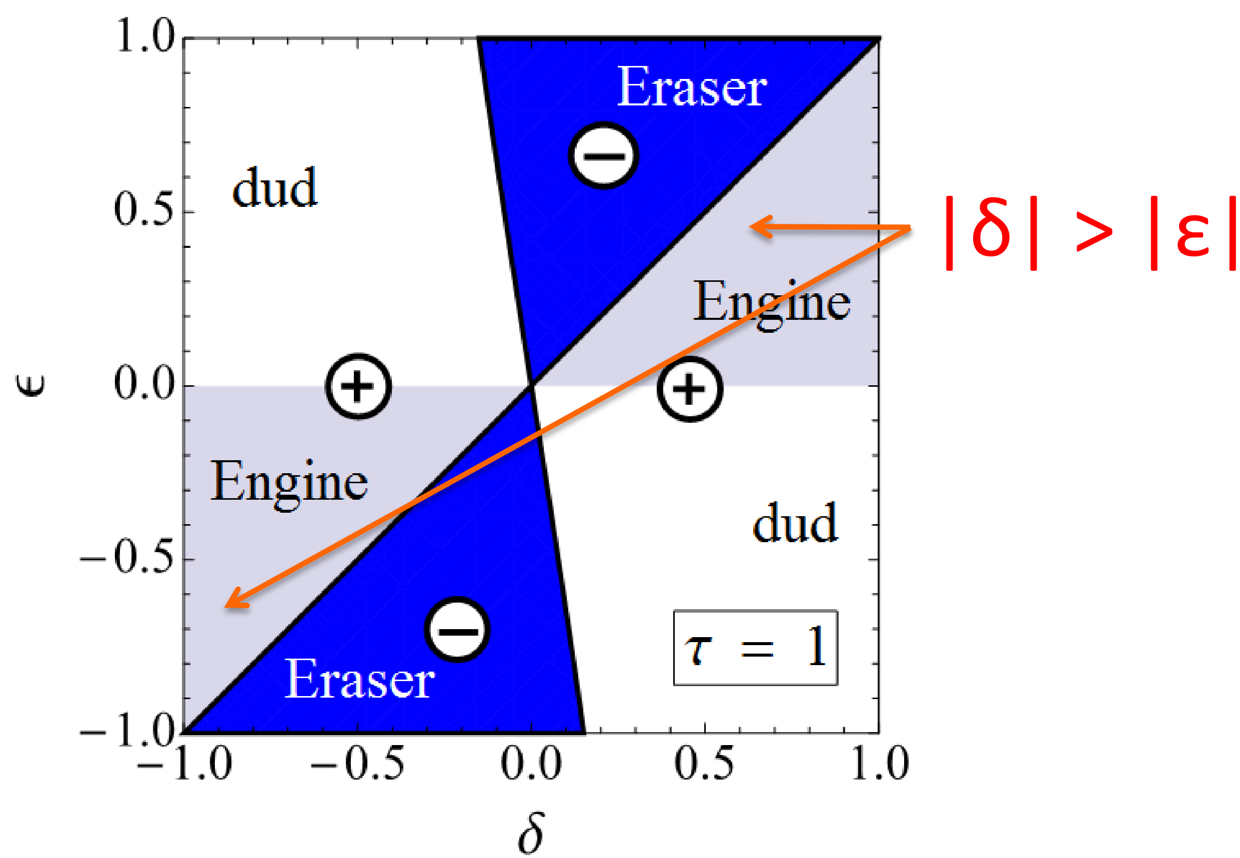
ε: CCW



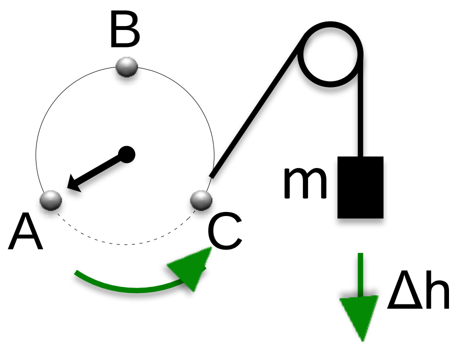
Non-equilibrium phase diagram – Engine ($W > 0$)



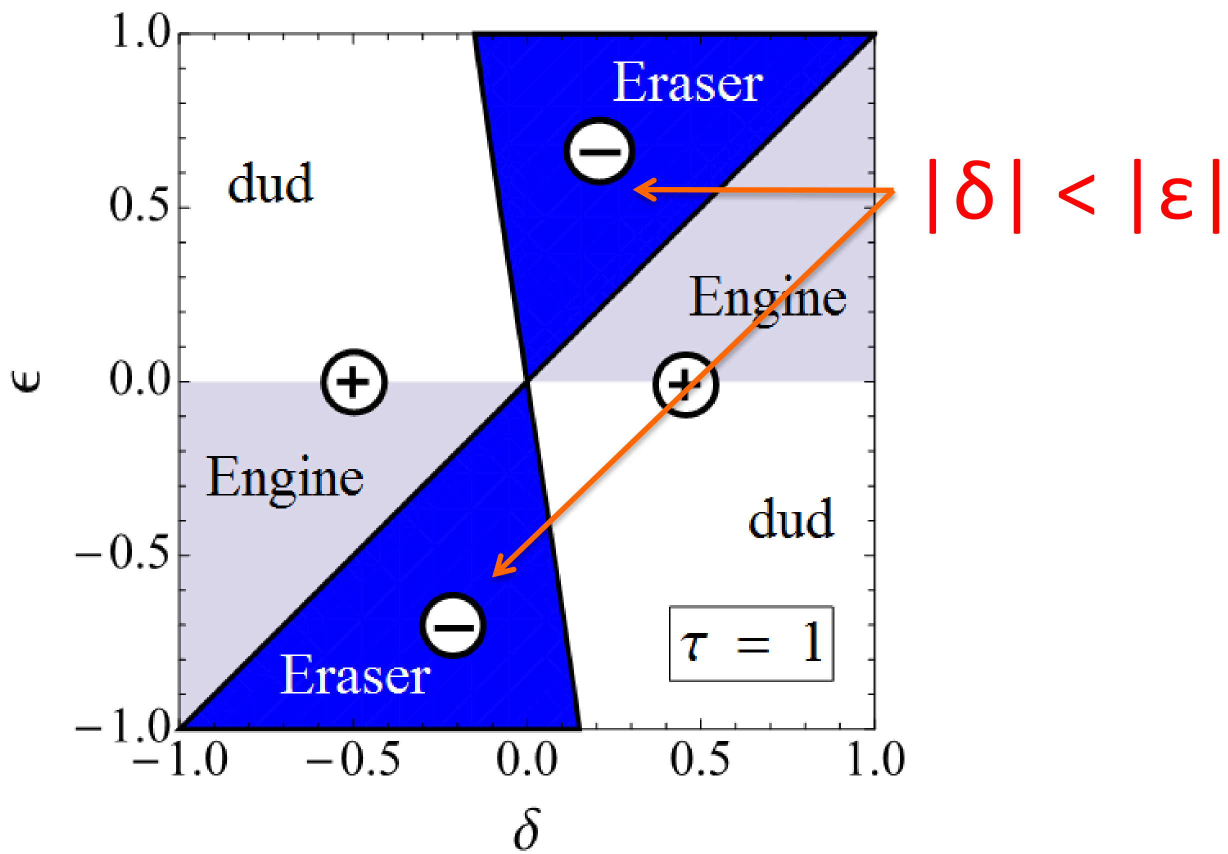
δ: CW
ε: CCW



Non-equilibrium phase diagram – Eraser ($\Delta S_B < 0$)



δ: CW
ε: CCW



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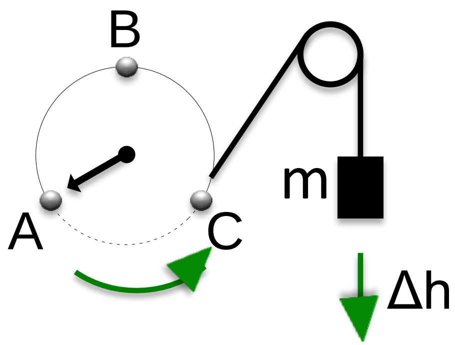
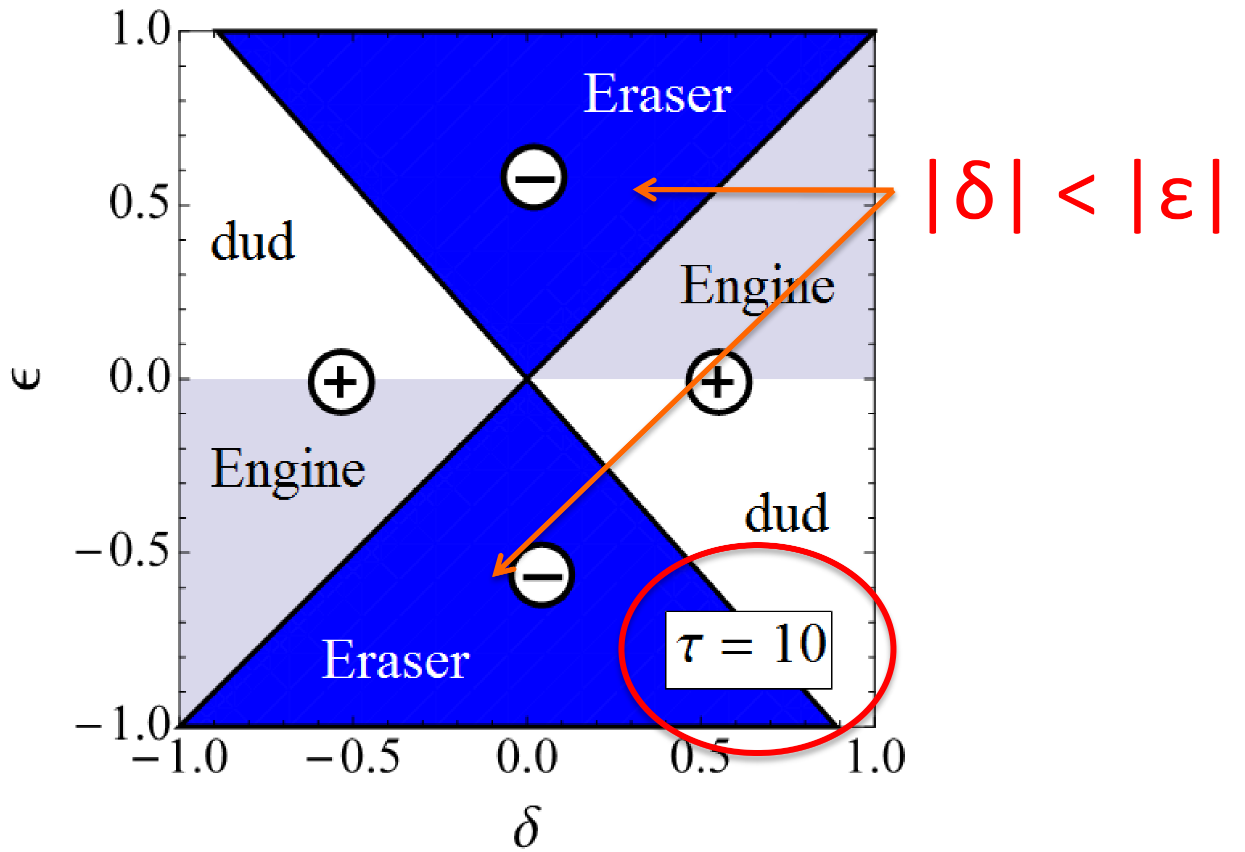


δ: CW
ε: CCW



Implications for the second law

- From dynamics: $\Delta S_B - W / k T \geq 0$

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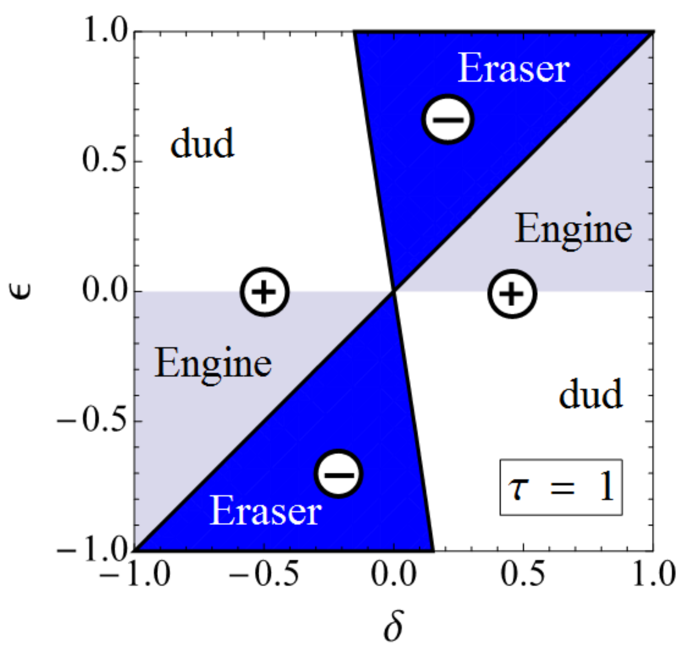


- Engine: $0 < W < kT\,\Delta S_B$
- Eraser: $0 > k\,T\,\Delta S_B > W$
- "Dud": $k\,T\,\Delta S_B > 0 > W$

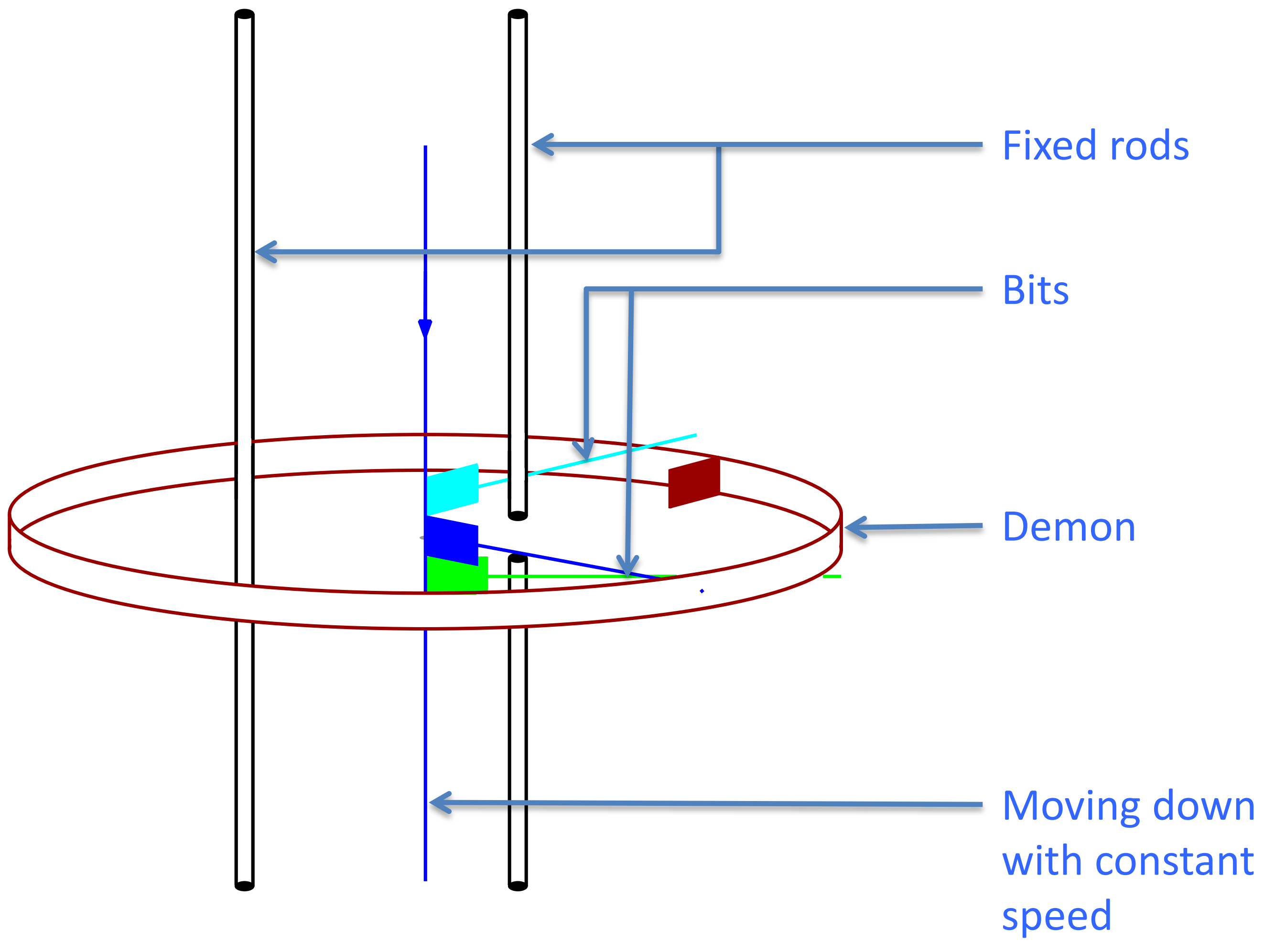
Not a real solution to the energy crisis!

About 200 million terabytes of data to heat a gram of water by just a single degree Celsius — an overwhelming demand for data storage!

Thank you!

Extra slides

Mechanical demon



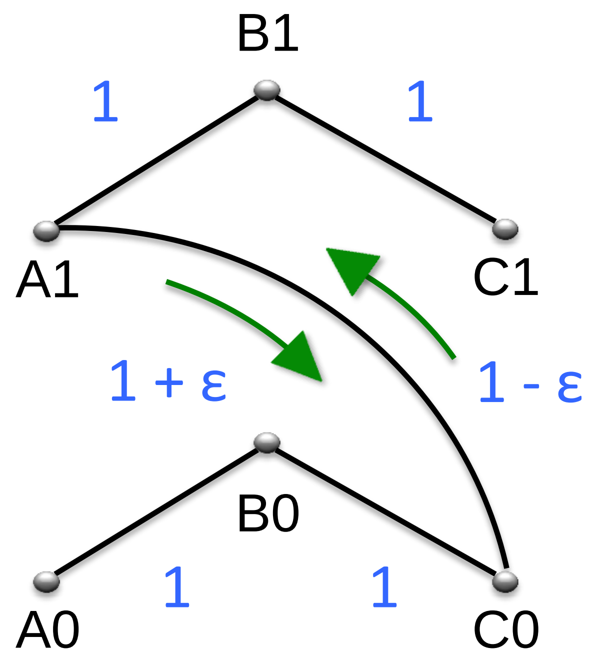
Joint dynamics

Rate Equation: $\frac{dp_i}{dt} = \sum_{j \neq i} \left(p_j R_{j \to i} - p_i R_{i \to j} \right), \quad i, j = A0, \ldots, C1$

$$R_{A1 \leftrightarrow B1} = R_{B1 \leftrightarrow C1} = 1$$

$$R_{C0 \to A1} = 1 - \epsilon, \quad R_{A1 \to C0} = 1 + \epsilon$$

$$R_{A0 \leftrightarrow B0} = R_{B0 \leftrightarrow C0} = 1$$



Key steps in the derivation of Φ

1. periodic steady state of the demon, $\mathbf{q}_{ps}$
2. Statistics of the outgoing bits in the periodic case, $(p'_{0,ps}, p'_{1,ps})^T$
3. $\Phi = p'_{1,ps} - p_1^*$

$^*p_0 = (1 + \delta)/2$, $p_1 = (1 - \delta)/2$

Step 1: Periodic steady state of the demon $\mathbf{p}_{D,ps}$

- t = 0: Joint state of the demon and the 1st bit is

$$\mathbf{p}_{DB}(0) = \mathbf{M}\,\mathbf{p}_{D}(0)$$

where $\mathbf{M} = (p0\,\mathbf{I}, p1\,\mathbf{I})^T$ and $\mathbf{I}$ is 3×3 identity matrix

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- $0 < t < \tau$: $\mathbf{p}_{DB}$ follows rate equation $d\mathbf{p}_{DB}/dt = \mathbf{R}\,\mathbf{p}_{DB}$

$$\text{solution: } \mathbf{p}_{DB}(\tau) = \mathrm{Exp}(\mathbf{R}\,\tau)\,\mathbf{p}_{DB}(0)$$

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where $\mathbf{P}_D = (\mathbf{I}, \mathbf{I})$ and $\mathbf{T}$ (3×3) $= \mathbf{P}_D\,\mathrm{Exp}(\mathbf{R}\,\tau)\,\mathbf{M}$

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- $T = n\,\tau$: State of the demon is $\mathbf{p}_D(n\,\tau) = \mathbf{T}^n\,\mathbf{p}_D(0)$

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- $T = n\, \tau$: State of the demon is $\mathbf{p}_D(n\, \tau) = \mathbf{T}^n\, \mathbf{p}_D(0)$
- For $n \gg 1$, $\mathbf{p}_D(n\, \tau) \approx \mathbf{p}_{D,ps}$ where $\mathbf{T}\, \mathbf{p}_{D,ps} = \mathbf{p}_{D,ps}$

Steps 2,3: Statistics of the outgoing bits and Φ

- Joint state of the demon and the interacting bit at the beginning of interaction is

$$\mathbf{M}\ \mathbf{p}_{D,ps}$$

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$$(p'_{0,ps},\ p'_{1,ps})^T = \mathbf{P}_B\ \mathrm{Exp}(\mathbf{R}\ \tau)\ \mathbf{M}\ \mathbf{p}_{D,ps}$$

where

$$\mathbf{P}_B = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

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