Math Circle Challenge

ICTS-RRI

December 4, 2022

Geodesic

Problem 1. *In the following diagram, how would you find the point P on the line segment AB so that $PR + PS$ is as small as possible? You can imagine that AR and BS are two vertical poles and signal transmitters are placed at R and S. The cost of sending a signal is directly proportional to the distance the signal has to travel. A person wants to place a receiver on the ground to receive signals from both the transmitters. Where should he/she place the receiver so that his total cost is as small as possible?*
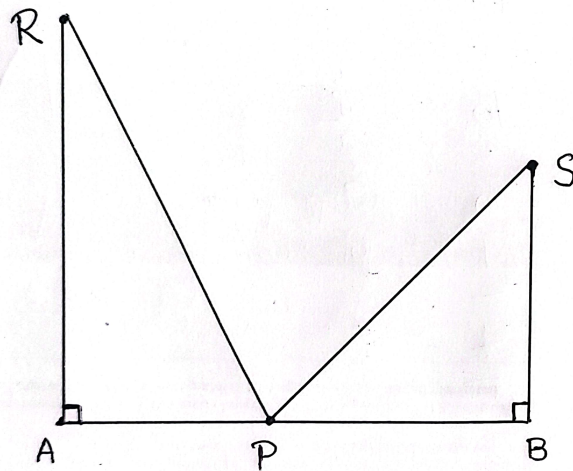

Figure 1: Figure shows the locations of two transmitters at R and S. P is the location of the receiver. The problem is to determine the optimum location of the receiver.

Problem 2. *A couple (let's call them Sharmila and Prakash) invite five couples over for dinner. When they meet, there are introductions and some of the people shake hands. Of course, one does not shakes one's own hand and that of his/her spouse. Sharmila notices that of the other people (excluding herself) in the gathering, no two people shake the same number of hands. How many hands does Prakash shake?*

Problem 3. *Imagine that 4 spiders are sitting on corners of a square of side length d all facing the centre of the square. Each spider wants to eat the spider to its right and moves towards it with uniform speed v. Will the spiders be able to catch their prey? If yes, how long will it take for them to meet? What will be the trajectory of each spider?*

Can you generalise this problem by replacing the square with a regular polygon? What happens as the number of sides gets larger without limit?

Problem 4.

Consider a cube each of whose edges is made of wire. Each wire has a resistance of 1 Ohm. What is the resistance between two diagonally opposite corners of the cube?

Problem 5. *Three fishermen (A,B and C) came back to shore with their catch of fish at 3:00 in the morning. Since it was dark, they left the catch in a pile on the beach and went to their houses to get some sleep and agreed to divide it equally in the morning. At 4:00 am, A woke up and decided to take his share. He found that there would be one extra fish, so he threw it back in the sea and took his share and went home. At 5:00 am, B woke up and did the same (not knowing that A had already taken his share): found one fish extra, threw it back in the sea, took his share and went home. At 6:00 am C did the same not knowing about A and B. How many fish did the fishermen catch? Can you find all the solutions?*

Problem 6. *10 cables were laid across the Zambezi river in a remote part of Africa. After laying the cables, the engineer realised that he had forgotten to label them, so he didn't know which of the 10 ends on one bank corresponded to the 10 ends on the other. He had at his disposal a multimeter to test continuity, some copper wire to connect the ends on each bank and a boat to cross the river with. As you may know, the Zambezi river has herds of hippos and these animals can be dangerous to crossing boats. It would be wise to minimise the number of river crossings.*

How would the engineer figure out the corresponding cables with a minimum number of crossings?