

# NONTRIVIAL QUANTUM OSCILLATION GEOMETRIC PHASE SHIFT IN A TRIVIAL BAND

---

Mandar M. Deshmukh  
DCMP and MS  
TIFR, Mumbai, India

*Datta et al. Science Advances 5, eaax6550 (2019).*

*Datta et al. Physical Review Letters 121, 056801 (2018),*

Support from Department of Atomic Energy and Department of Science and Technology of India

ICTS-TIFR Bengaluru  
23rd January 2020

[www.tifr.res.in/~nano](http://www.tifr.res.in/~nano)

## Collaborators

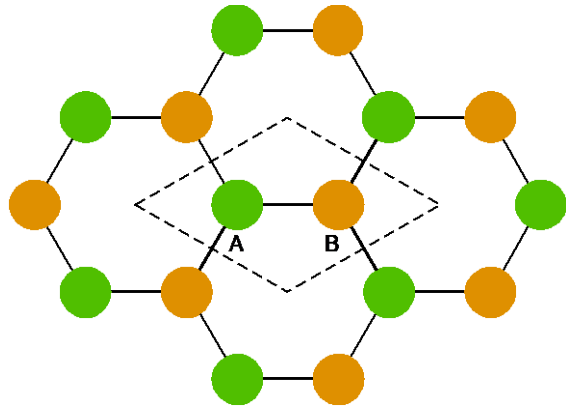
**Biswajit Datta**, Pratap Chandra Adak, Hitesh Agarwal  
Li-kun Shi (NTU Singapore), **Justin Song (NTU Singapore)**

**Kenji Watanabe (NIMS, Japan)**, **Takashi Taniguchi (NIMS Japan)**

## Outline

- Few layers of graphene – why are they interesting?
- Trilayer ABA graphene
- Measuring quantum oscillations
- Berry's phase in a multiband system

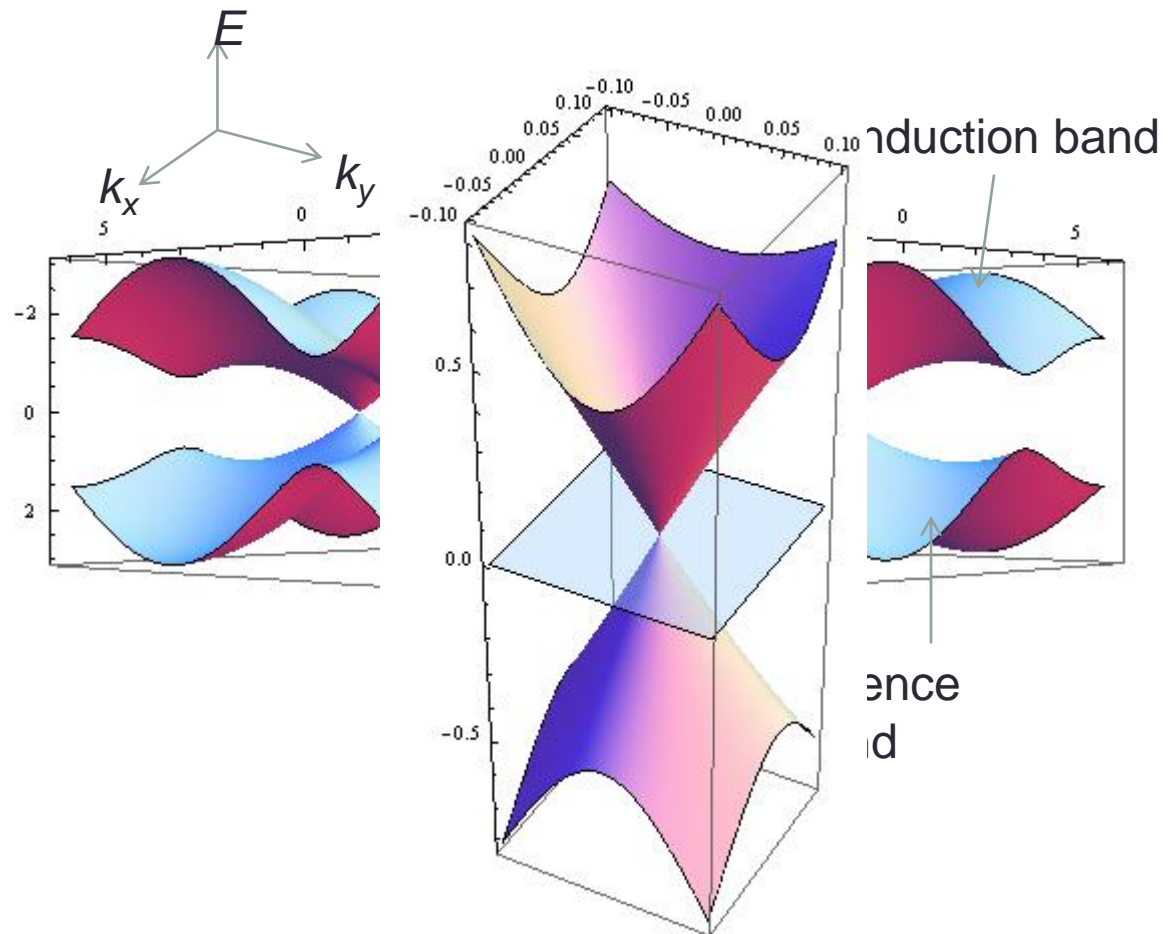
# Graphene basics



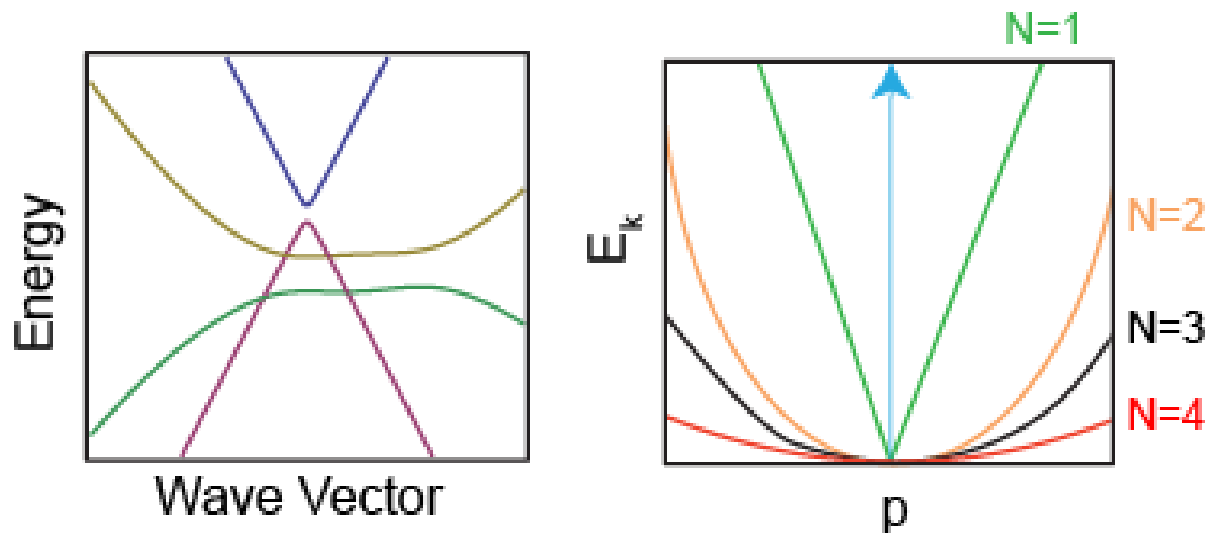
$$H_{\mathbf{K}} = v_F \boldsymbol{\sigma} \cdot \mathbf{p}$$

$$H_{\mathbf{K}'} = -H_{\mathbf{K}}$$

- Bravais lattice with two carbon atom basis



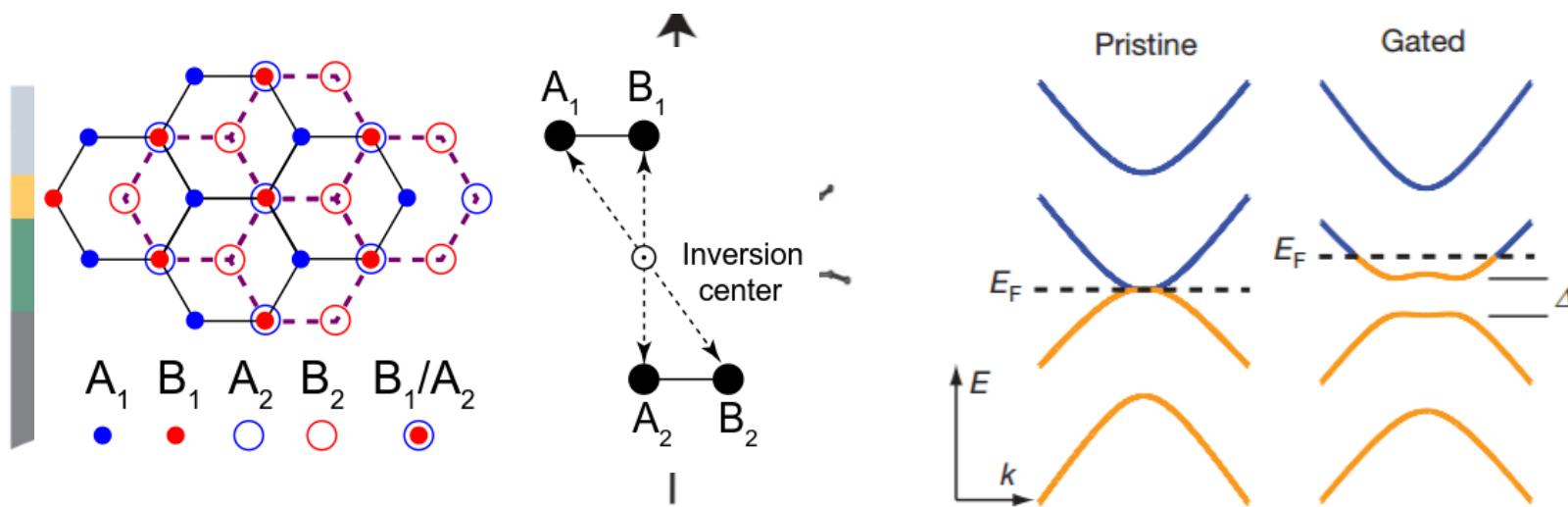
## Why few layers of graphene are interesting?



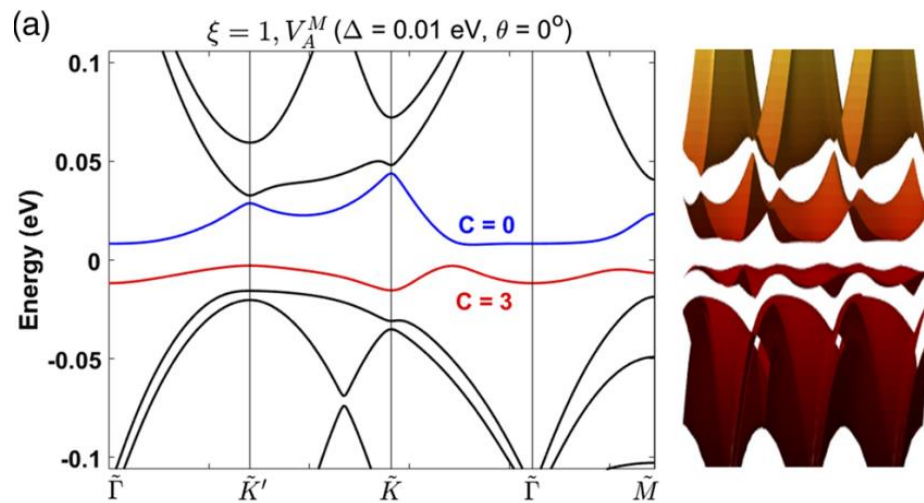
- Interactions become more important for multiple layers due to flatter bands
- Tunable symmetries
- Possibility of studying non-Abelian quantum Hall states

# Breaking inversion symmetry in bilayer graphene opens up a bandgap

Bilayer graphene



# Tuning topological properties



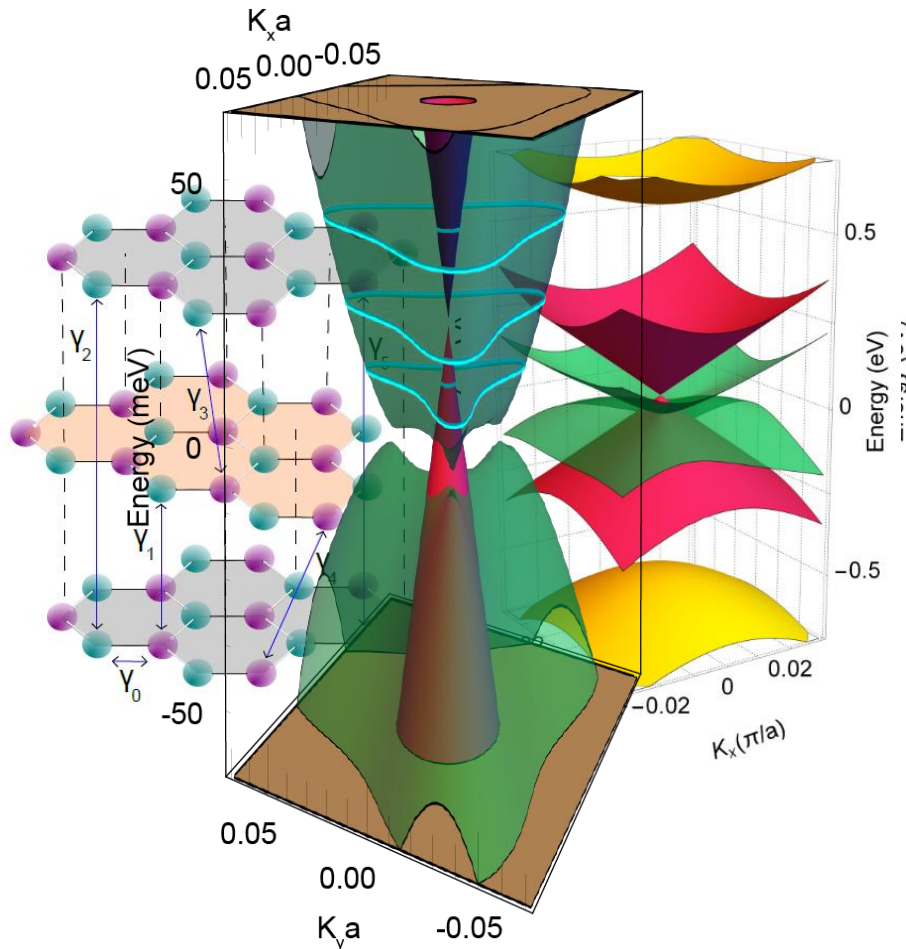
ABC trilayer, Chittari et al. PRL 122, 016401 (2019)

## Outline

- Few layers of graphene – why are they interesting?
- Trilayer ABA graphene
- Measuring quantum oscillations
- Berry's phase in a multiband system



# ABA-Trilayer Graphene: Crystal and Band Structure

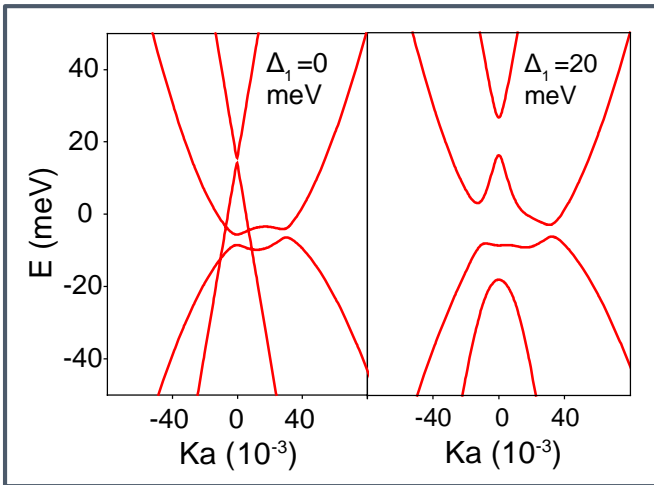


Monolayer graphene (MLG)-like  
and bilayer graphene (BLG)-like bands

Bands do not hybridize at the crossing  
points due to mirror symmetry protection

Some states are polarized in mirror  
symmetric and anti-symmetric basis

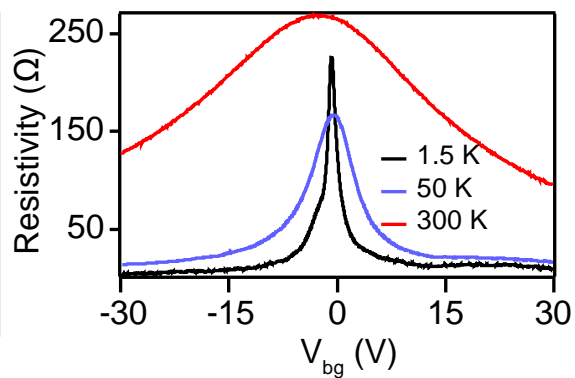
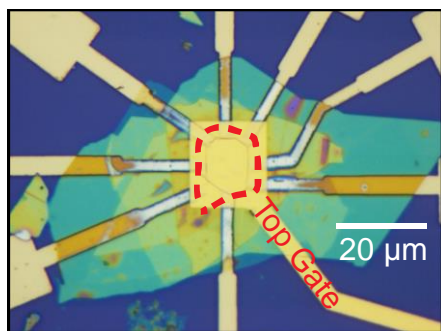
## Effect of Electric Field on the Band Structure



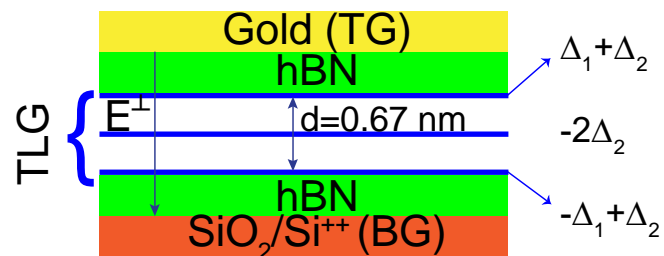
$$H_{\text{TLG}} = \begin{pmatrix} \frac{A_1 - A_3}{\sqrt{2}} & \frac{B_1 - B_3}{\sqrt{2}} & \frac{A_1 + A_3}{\sqrt{2}} & B_2 & A_2 & \frac{B_1 + B_3}{\sqrt{2}} & \frac{A_1 - A_3}{\sqrt{2}} \\ \Delta_2 - \frac{\gamma_2}{2} & v_0 \pi^\dagger & \Delta_1 & 0 & 0 & 0 & \frac{B_1 - B_3}{\sqrt{2}} \\ v_0 \pi & \Delta_2 + \delta - \frac{\gamma_5}{2} & 0 & 0 & 0 & \Delta_1 & \frac{A_1 + A_3}{\sqrt{2}} \\ \Delta_1 & 0 & \Delta_2 + \frac{\gamma_2}{2} & \sqrt{2} v_3 \pi & -\sqrt{2} v_4 \pi^\dagger & v_0 \pi^\dagger & B_2 \\ 0 & 0 & \sqrt{2} v_3 \pi^\dagger & -2\Delta_2 & v_0 \pi & -\sqrt{2} v_4 \pi & A_2 \\ 0 & 0 & -\sqrt{2} v_4 \pi & v_0 \pi^\dagger & \delta - 2\Delta_2 & \sqrt{2} \gamma_1 & \frac{B_1 + B_3}{\sqrt{2}} \\ 0 & \Delta_1 & v_0 \pi & -\sqrt{2} v_4 \pi^\dagger & \sqrt{2} \gamma_1 & \Delta_2 + \delta + \frac{\gamma_5}{2} & \frac{A_1 - A_3}{\sqrt{2}} \end{pmatrix}$$

- Electric field increases the band gap of MLG-like bands

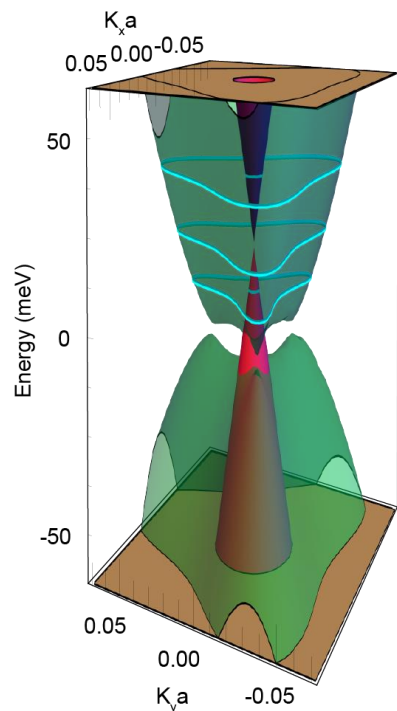
## Understanding our experimental system



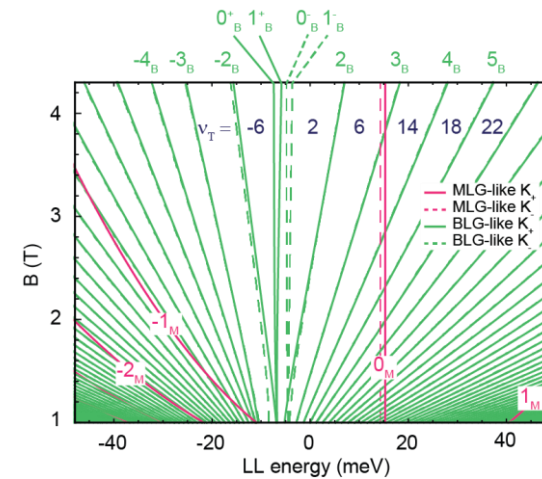
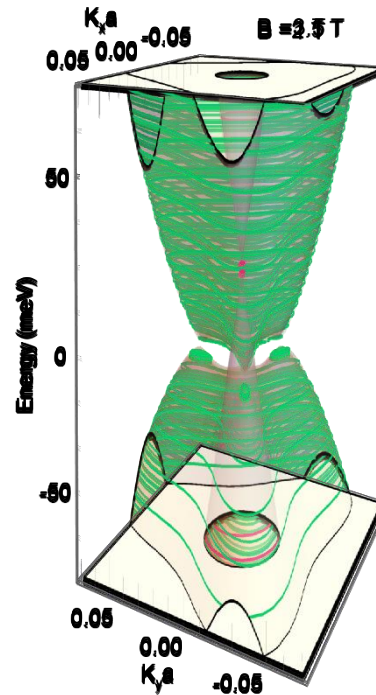
- Mobility  $\sim 800,000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$
- Mean free path  $\sim 10 \mu\text{m}$
- Average  $E^\perp = 2\Delta_1/d$  (e)



# Landau levels



Magnetic field



$$E_{MLG}(B, N) \sim \pm \sqrt{BN}, N = 0, 1, 2, \dots$$

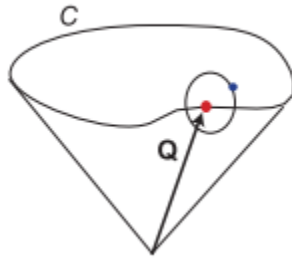
$$E_{BLG}(B, N) \sim \pm B \sqrt{N(N-1)}, N = 0, 1, 2, \dots$$

- Monolayer-like Landau Level gaps are much larger than bilayer-like bands
- Zeroth Monolayer-like Landau Level does not disperse with magnetic field

## Outline

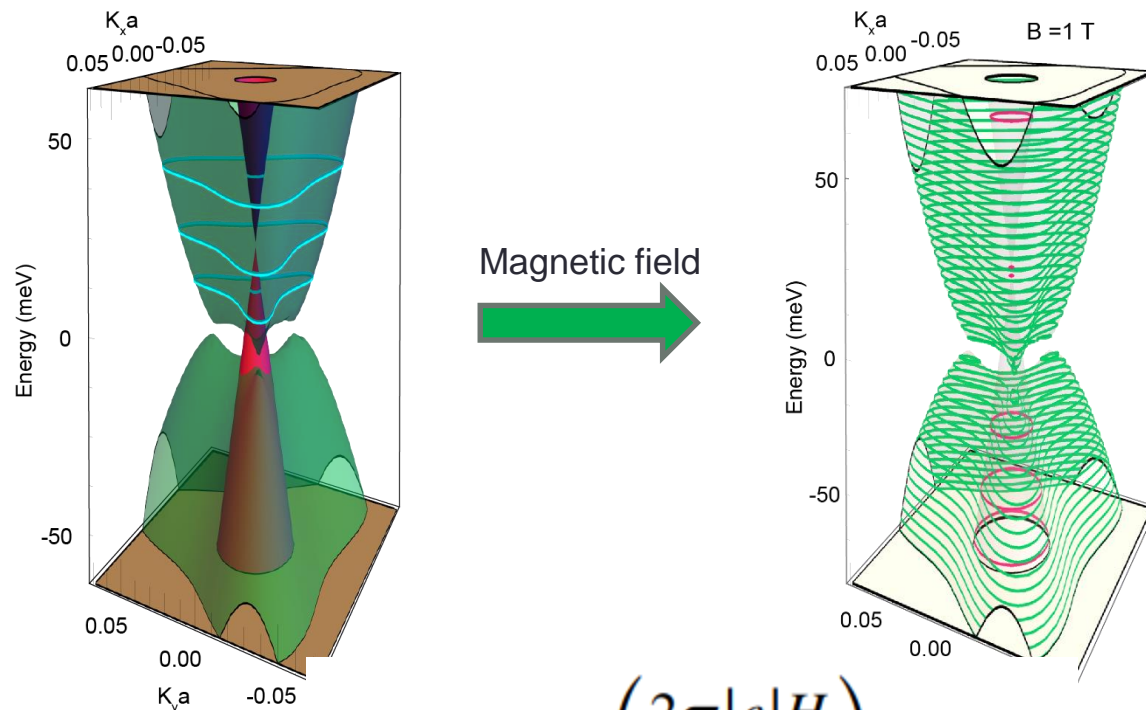
- Few layers of graphene – why are they interesting?
- Trilayer ABA graphene
- Breaking mirror symmetry
- Measuring quantum oscillations
- Berry's phase in a multiband system

## Berry's phase



- A version of this for optics proposed by Panchratnam
- The flux enclosed by the closed loop due to the Berry's curvature
- In a periodic lattice depends on symmetry of the lattice

# Semiclassical quantization and Berry's phase



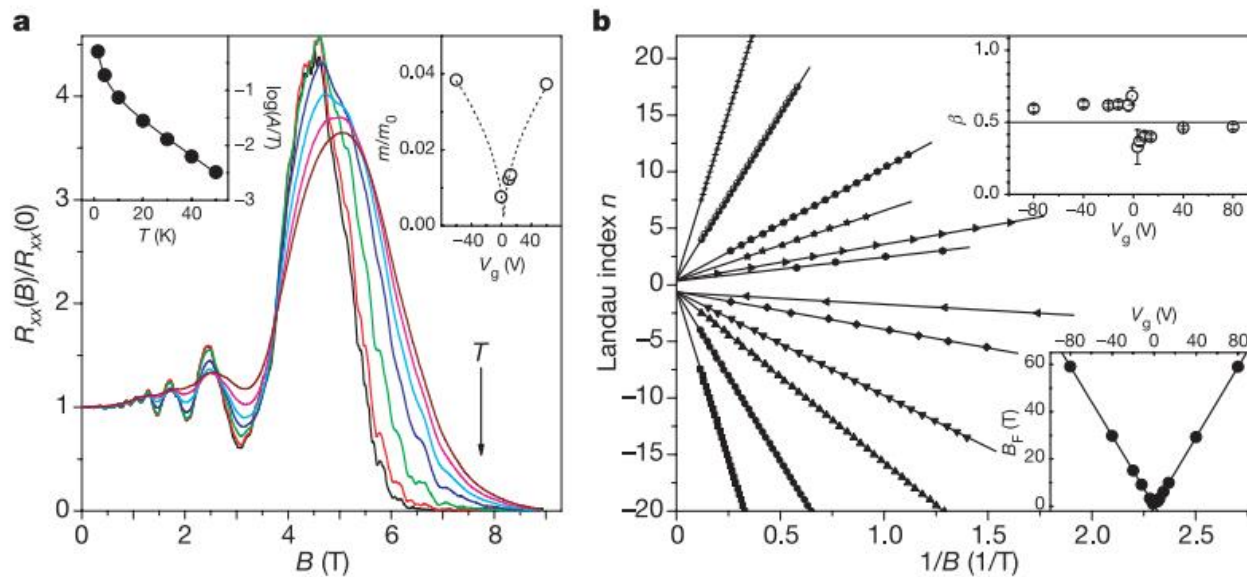
$$S(\varepsilon, k_z) = \left( \frac{2\pi|e|H}{\hbar c} \right) (n + \gamma)$$

$$\gamma - \frac{1}{2} = -\frac{1}{2\pi} \oint_{\Gamma} \Omega d\mathbf{k}$$

## SdH Oscillations

- Magnetoresistance oscillations reflecting density of states oscillations
- de Haas van Alphen can be used as it is essentially the same physics

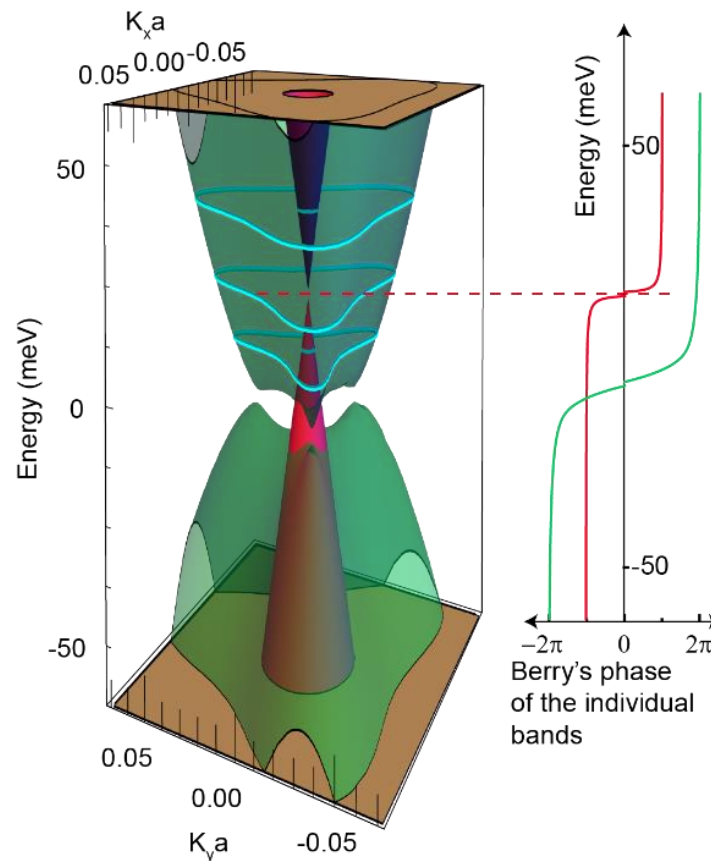
$$\Delta R_{xx} = R(B, T) \cos[2\pi(B_F/B + 1/2 + \beta)]$$



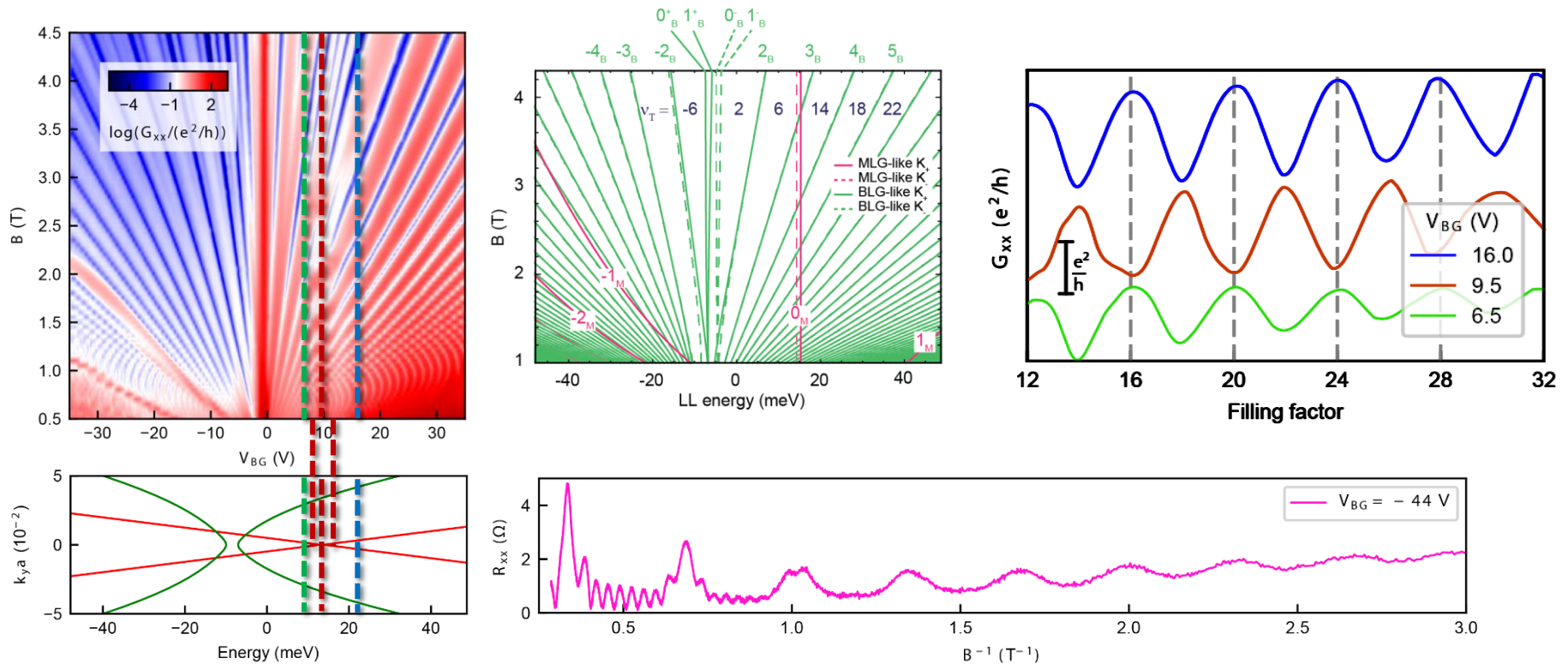


## Connection to trilayer physics

Gapped monolayer like levels and gapped bilayer like levels exist in trilayer ABA graphene

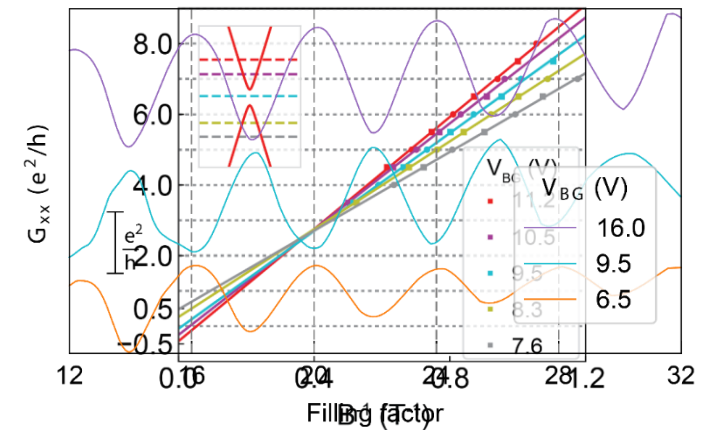
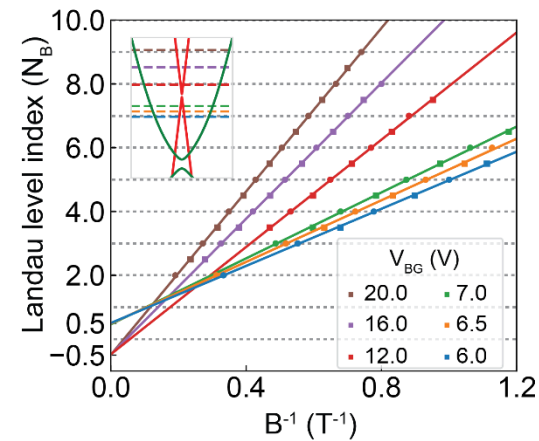
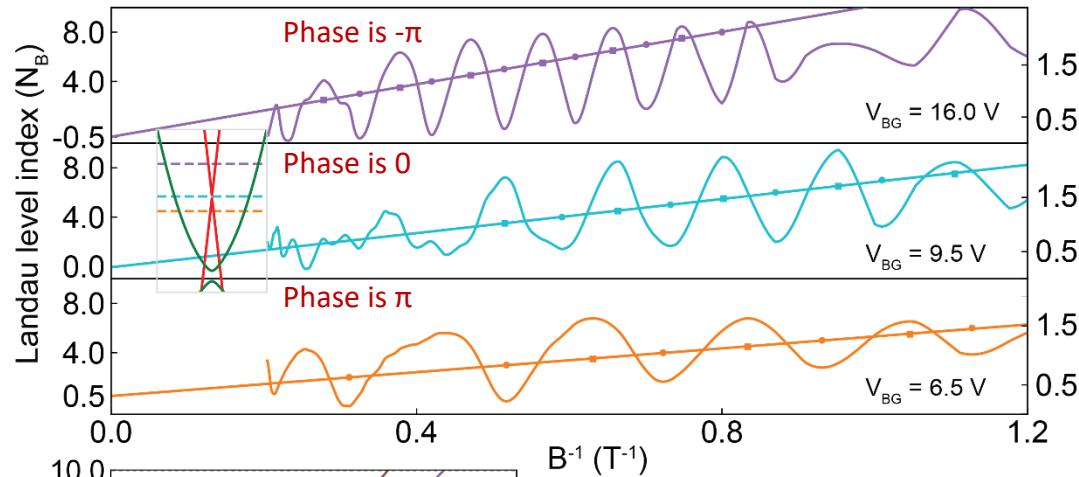


# Multiband magnetotransport



- Two SdH frequency results from two Fermi surfaces
- Phase of the BLG-like oscillations depend on the MLG-like band gap!
- Changing phase is not Berry's phase of the bilayer band

# Anomalous SdH phase shift measured in trivial band



- Phase changes continuously close to the band gap!

# How does one understand the anomalous SdH phase shift?

$$\Delta G_{XX} = G_M \cos \left[ 2\pi \left( \frac{B_{FM}}{B} + \gamma_M \right) \right] + G_B \cos \left[ 2\pi \left( \frac{B_{FB}}{B} + \gamma_B \right) \right]$$

$$B_{FM} = \frac{n_M h}{4e} \quad B_{FB} = \frac{n_B h}{4e}$$

$$\Delta G_{XX} \approx G_B \cos \left[ 2\pi \left( \frac{B_{FB}}{B} + \gamma_B \right) \right]$$

$$B_{FB} = \frac{n_B h}{4e} = \frac{(n_T - n_M) h}{4e} = B_{FT} - \frac{\nu_M B}{4}$$

$$\Delta G_{XX} \approx G_B \cos \left[ 2\pi \left( \frac{B_{FT}}{B} + \gamma_B - \frac{\nu_M}{4} \right) \right]$$

Below the gap  $\nu_M = -2$

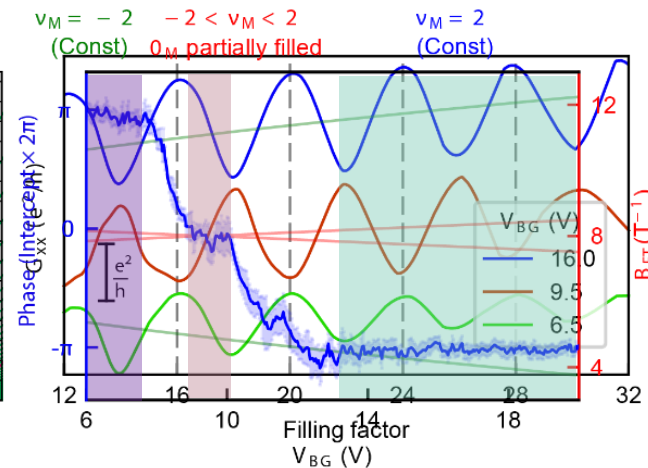
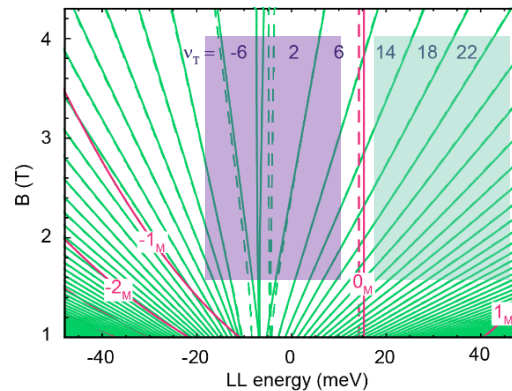
$$\Delta G_{XX} \approx G_B \cos \left[ 2\pi \left( \frac{B_{FT}}{B} + \gamma_B - \frac{1}{2} \right) \right]$$

In the gap  $\nu_M = 0$

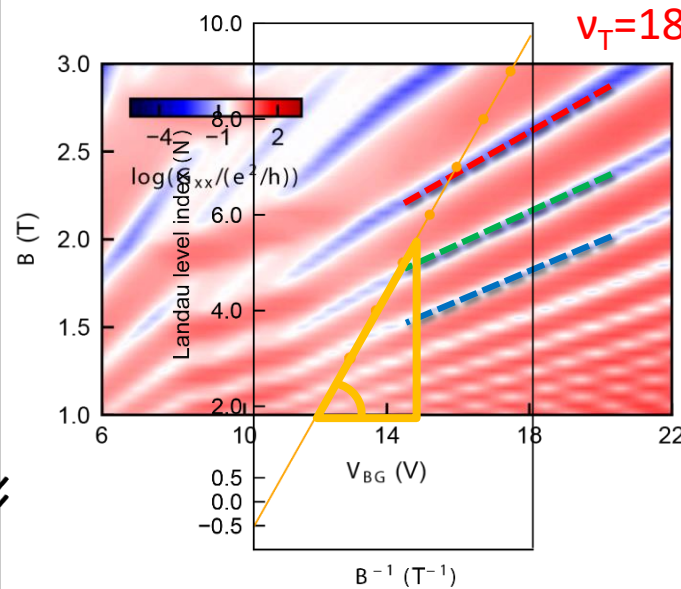
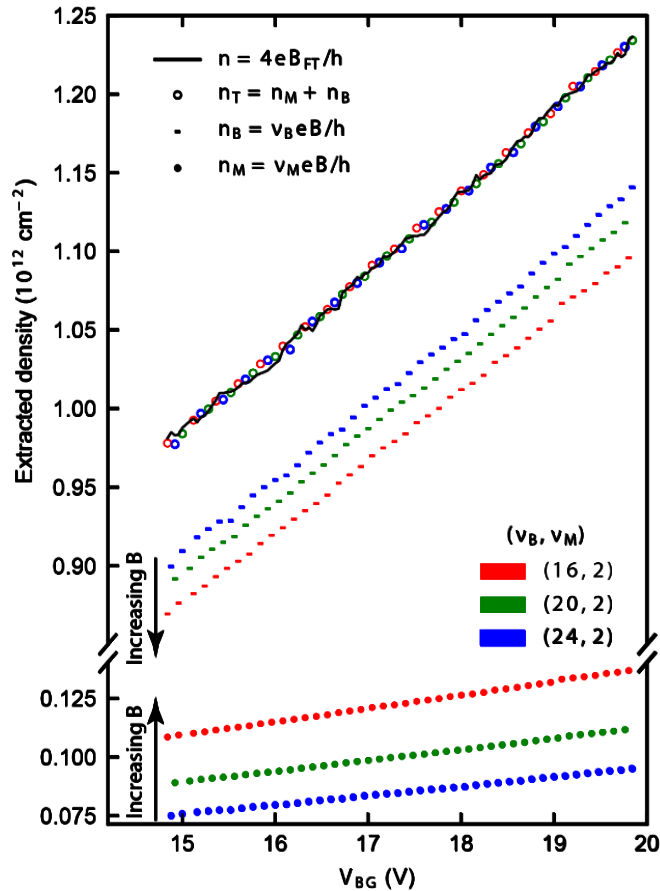
$$\Delta G_{XX} \approx G_B \cos \left[ 2\pi \left( \frac{B_{FB}}{B} + \gamma_B \right) \right]$$

Above the gap  $\nu_M = 2$

$$\Delta G_{XX} \approx G_B \cos \left[ 2\pi \left( \frac{B_{FT}}{B} + \gamma_B + \frac{1}{2} \right) \right]$$



# SdH frequency -- area of Fermi Surface and filling enforcement



$$N = \frac{B_{FT}}{B_N} + \frac{\Phi_B}{2\pi} - \frac{v_M}{4}$$

$$B_{FT} = \frac{n_T h}{4e}$$

Independent corroboration of the phase shift from the measurement of Fermi surface area

## Outline

- Few layers of graphene – why are they interesting?
- Trilayer ABA graphene
- Breaking mirror symmetry
- Measuring quantum oscillations
- Berry's phase in a multiband system