Chiral vortices in relativistic

Manavendra N Mahato

IIT Indore

Outline

Chiral effects in heavy ion collisions

Relativistic hydrodynamic equations

Solution by uplifting

Solution using Hopf fibration

Sphaleron solution

Other Applications

Summary

Chiral effects in RHIC

Topological gauge field configurations lead to local disparity between left and right chiral parities.

It leads to generation of electric field in presence of an external magnetic field.

Leads to a flow of current– known as chiral magnetic effect.

Observations

Chiral magnetic effect is a candidate model to explain parity violating effects in HIC

Charge separation is reflected as sine terms in fourier decomposition of azimuthal distribution

Correlations between their coefficients are deduced from experimental data

Azimuthal correlations

$$\frac{dN_{\pm}}{d\phi} \propto 1 + 2v_1 \cos(\phi - \Psi_{RP}) + 2v_2 \cos(2(\phi - \Psi_{RP})) +$$

$$+ \ldots + 2a_{\pm} \sin(\phi - \Psi_{RP}) + \ldots$$

- Correlator <a +a -> is deduced by measuring $\left\langle Cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \right\rangle$

- a± are parity violating.

Relativistic equations

$$\partial_\mu T^{\mu\nu} = F^{\mu\lambda} j_\lambda$$

$$\partial_\mu j^\mu = C E^\mu B_\mu$$

$$T^{\mu\nu} = h u^\mu u^\nu + P g^{\mu\nu} - \left(\varsigma - \frac{2}{3}\eta \right) P^{\mu\nu} \partial_\lambda u^\lambda$$

$$- \eta P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha u_\beta + \partial_\beta u_\alpha)$$

$$j^\mu = n u^\mu - \sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) + \sigma E^\mu + \xi \omega^\mu + \xi_B B^\mu$$

Solutions by uplifting

(Son, Rangamani, Ross, Thompson, JHEP 2009)

$$u^+ = \sqrt{\frac{1}{2}\frac{\rho_n}{\varepsilon_n + P_n}}$$

$$u^- = \frac{1}{3}\left(\frac{1}{u^+} + u^+ v^2\right)$$

$$u^i = u^+ v^i$$

$$P = P_n$$

$$\rho = 2\varepsilon_n + \rho_n$$

Taylor Green solution

$$v^1 = F(t)\sin x \cos y$$

$$v^2 = -F(t)\cos x \sin y$$

$$F(t) = e^{-2\nu t}$$

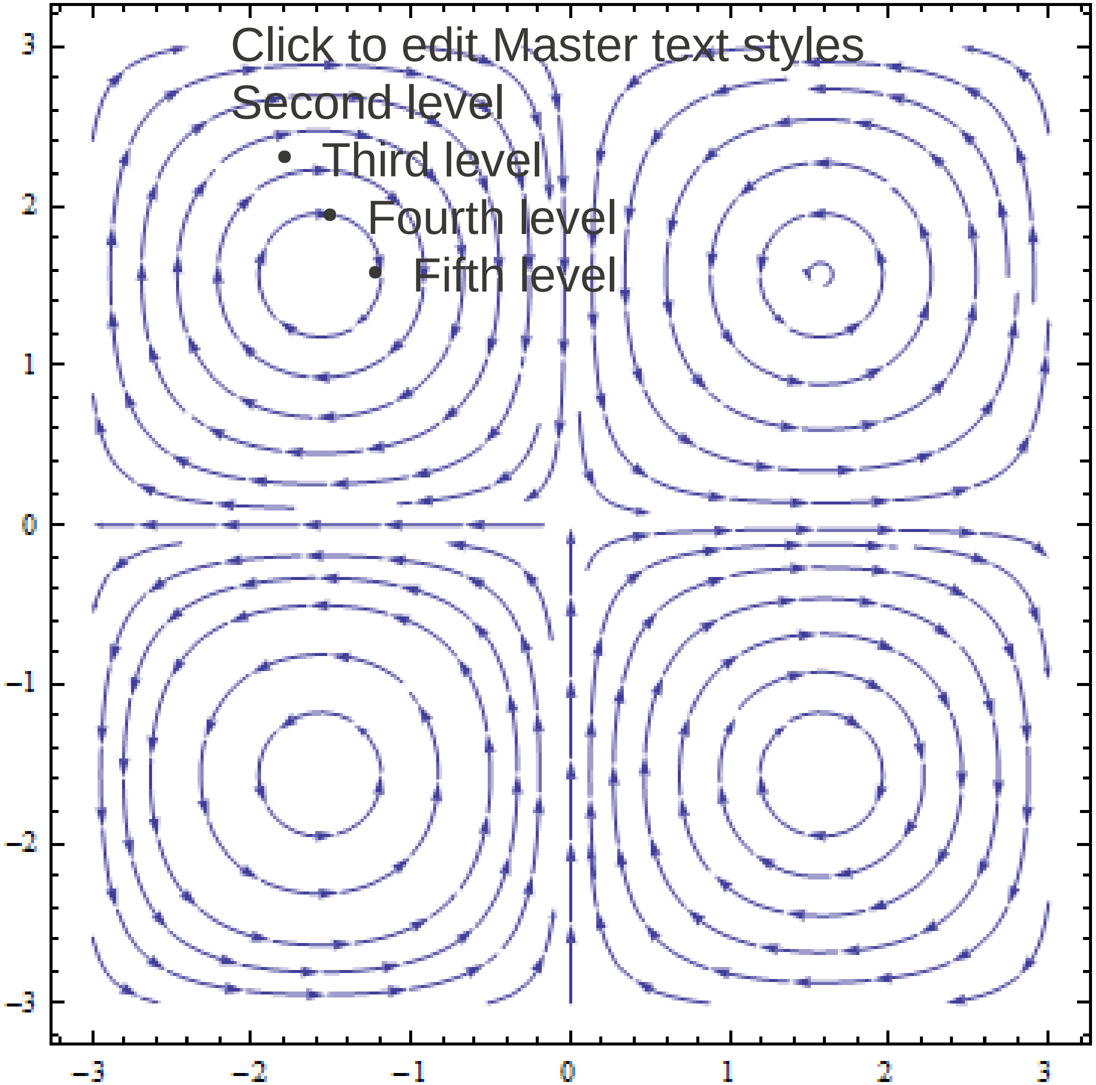
$$P_n = \frac{\rho_n}{4} F(t)(\cos 2x + \cos 2y)$$

Velocity profile

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level



Relativistic solution

$$\chi^{-2} = \frac{2\varepsilon_n}{\rho_n} + \frac{1}{2}(\cos 2x + \cos 2y)$$

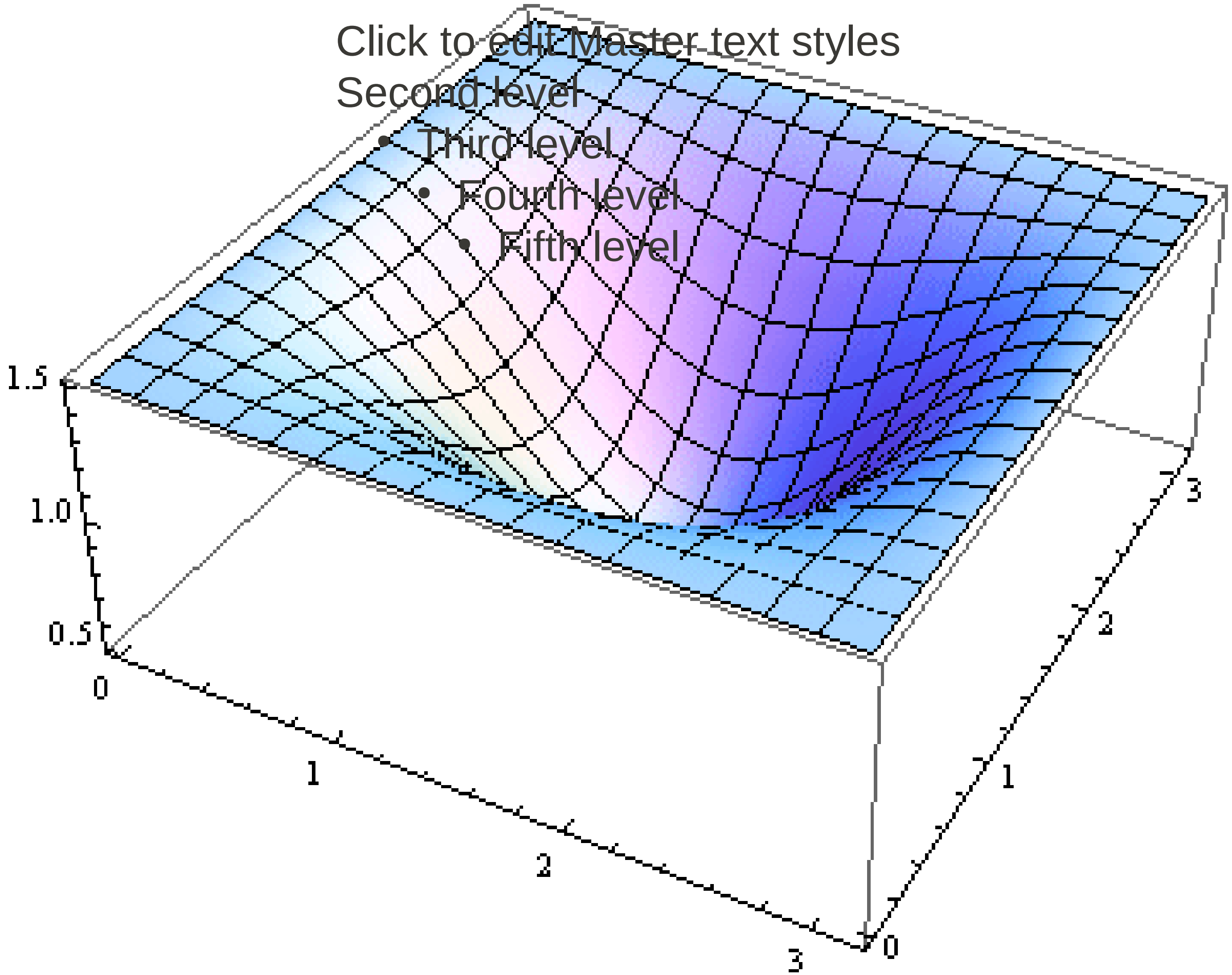
$$h = \frac{\rho_n}{\chi^2} \qquad u^\mu = \chi v^\mu$$

$$v^+ = 1 \qquad v^- = \frac{1}{2} + \frac{\varepsilon_n}{\rho_n} - \sin^2 x \sin^2 y$$

$$v^1 = \sin x \cos y \qquad v^2 = -\cos x \sin y$$

$$P = P_0 + \frac{\rho_n}{4}(\cos 2x + \cos 2y) \qquad n = \frac{\xi}{3}\chi \sin x \sin y$$

Velocity profile (v-)



Click to edit Master text styles

Second level

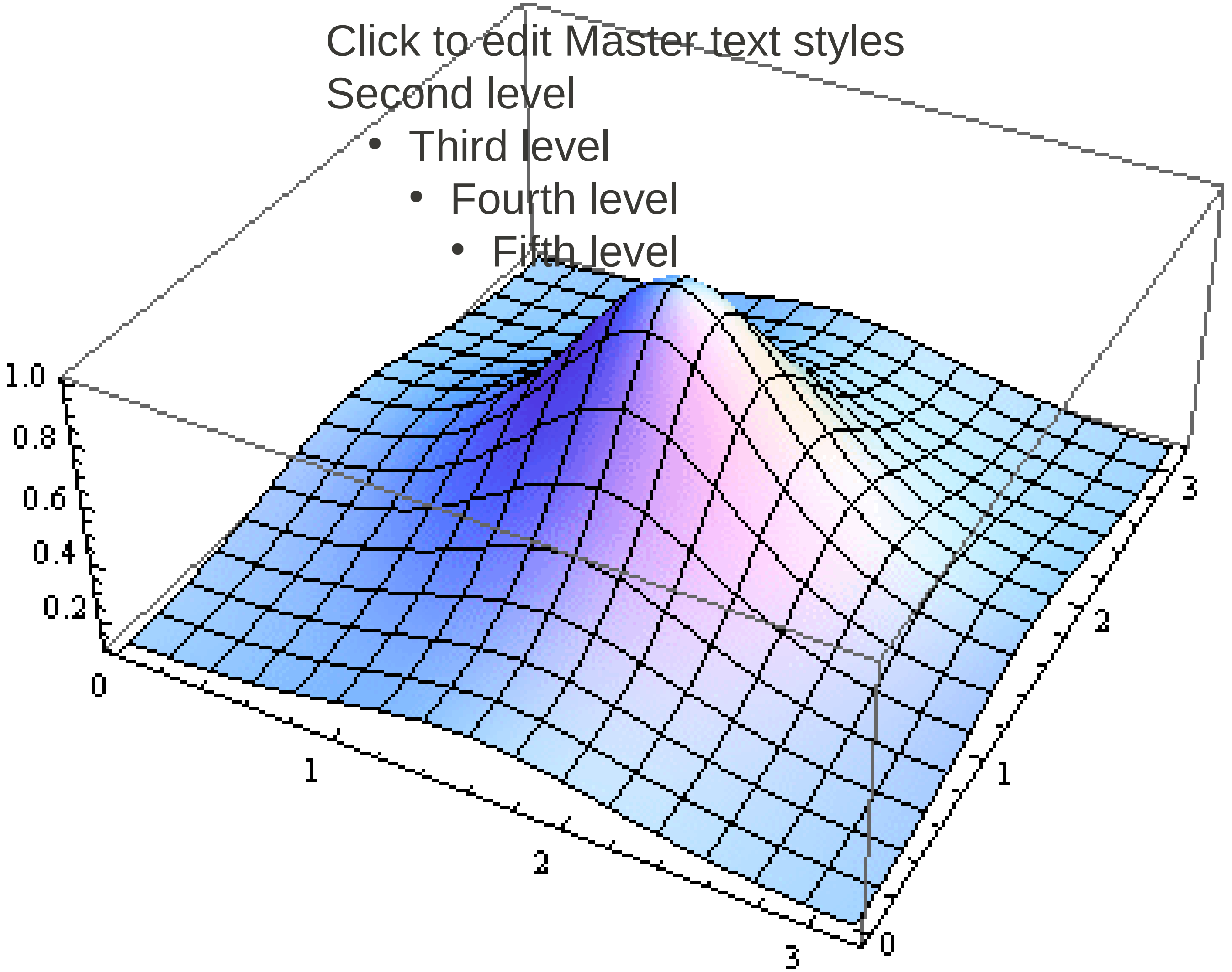
- Third level
  - Fourth level
    - Fifth level

Enthalpy profile

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

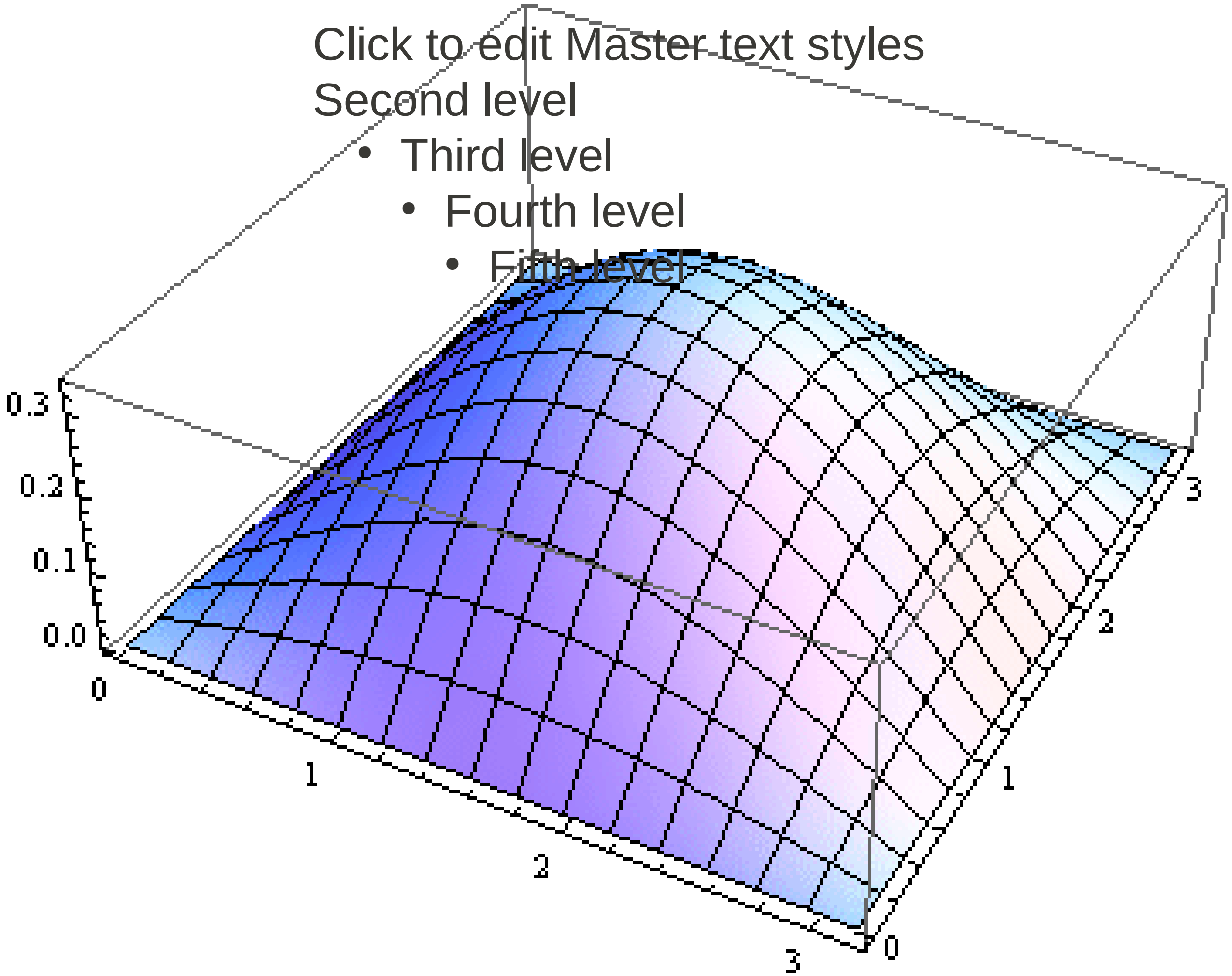


Axial density profile

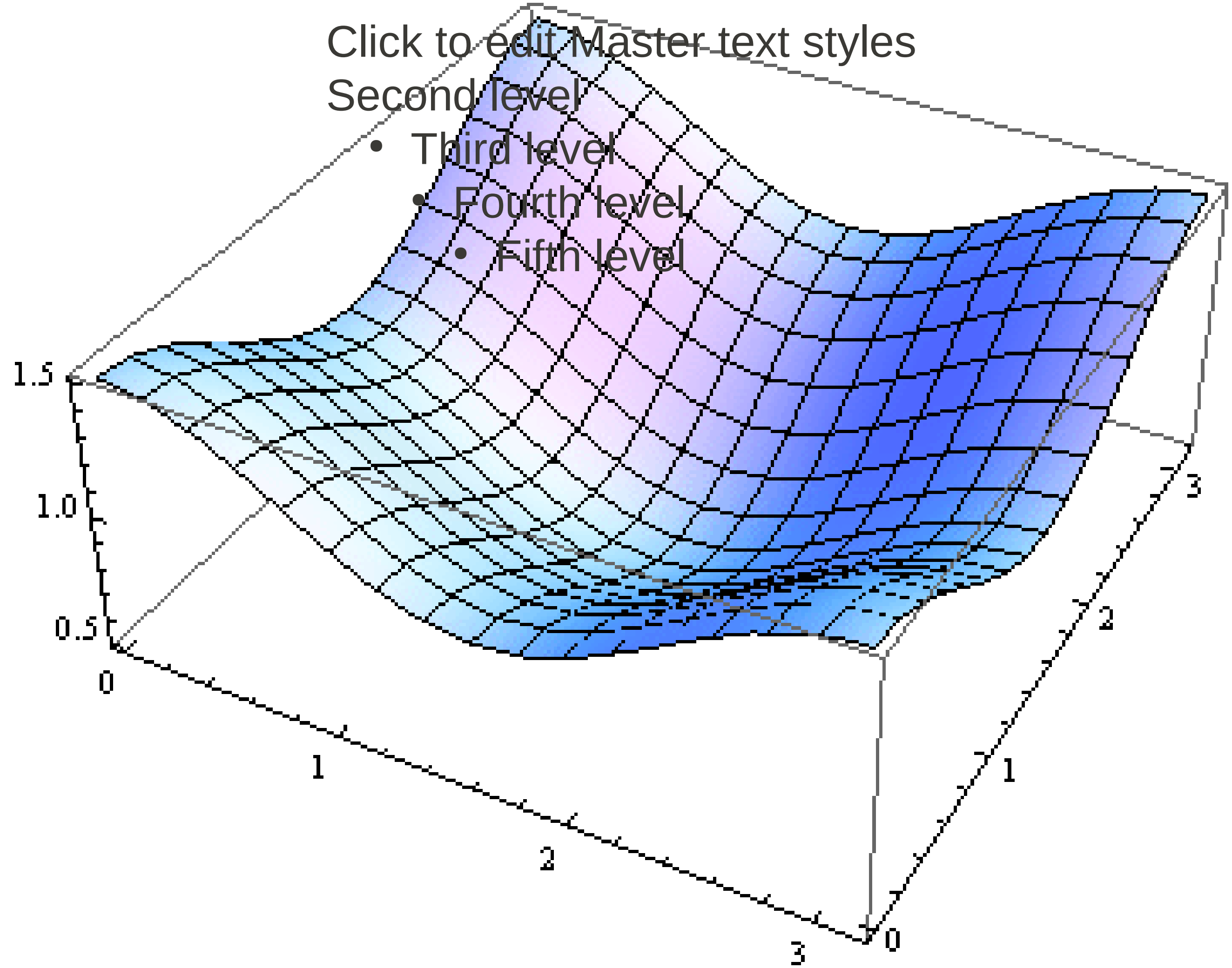
Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level



Pressure profile



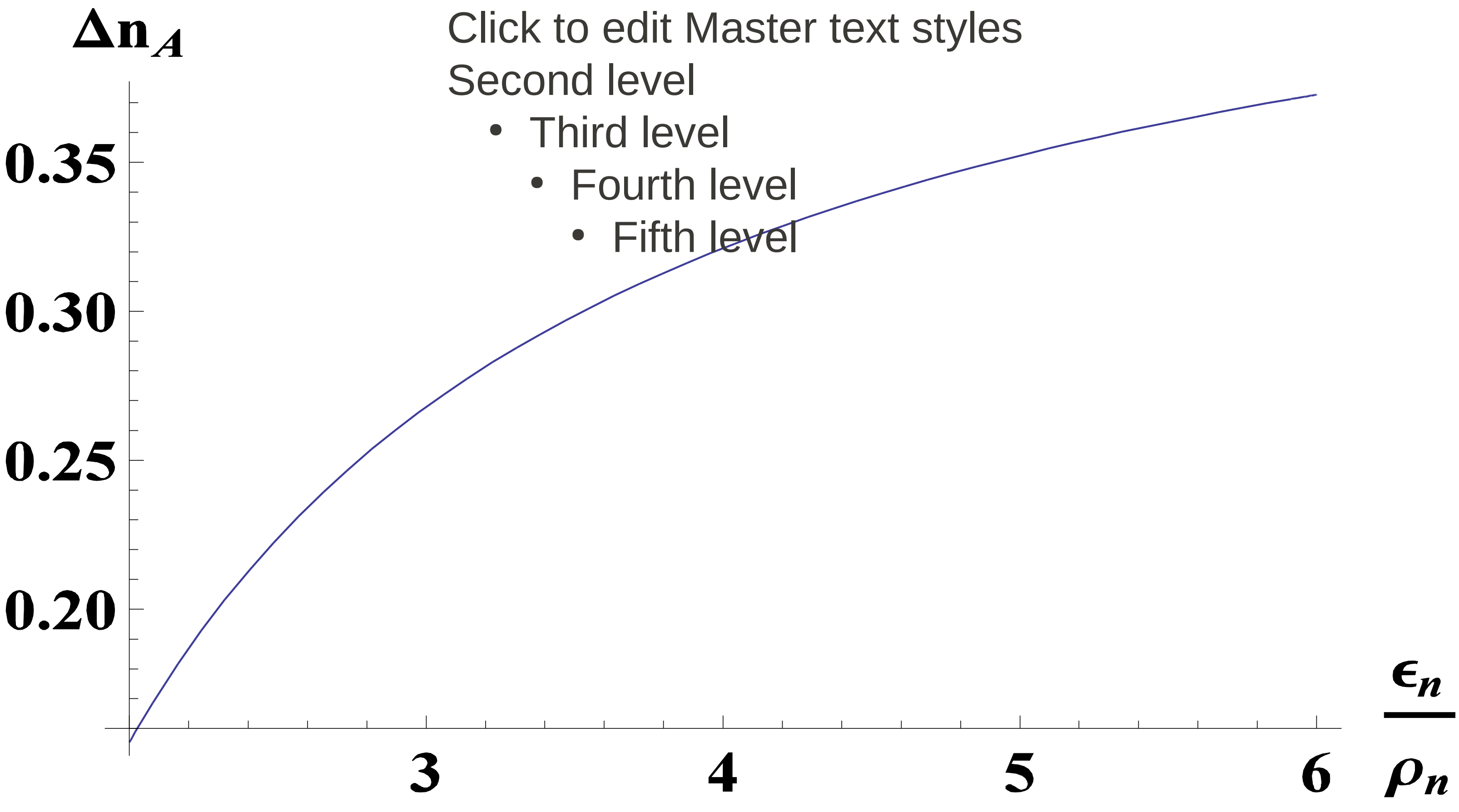
Click to edit Master text styles
Second level
- Third level
  - Fourth level
    - Fifth level

Axial charge

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level



Solutions using Hopf Fibration

$$A^1 = \frac{(xz - y)}{2r^2}$$

$$A^2 = \frac{(yz + x)}{2r^2}$$

$$A^3 = \frac{(z^2 + 1 - r/2)}{2r^2}$$

$$u^{\mu} = (v,\ fA^i) \qquad \overset{\sqcup}{A}^{\mu} = (\beta, \alpha A^i)$$

Using Hopf Fibration (Kamchatnov, 82)

Hopf map f: S3 → S2

$$f(z_0 = x_1 + ix_2, z_1 = x_3 + ix_4) = (2z_0 z_1^*, |z_0|^2 - |z_1|^2)$$

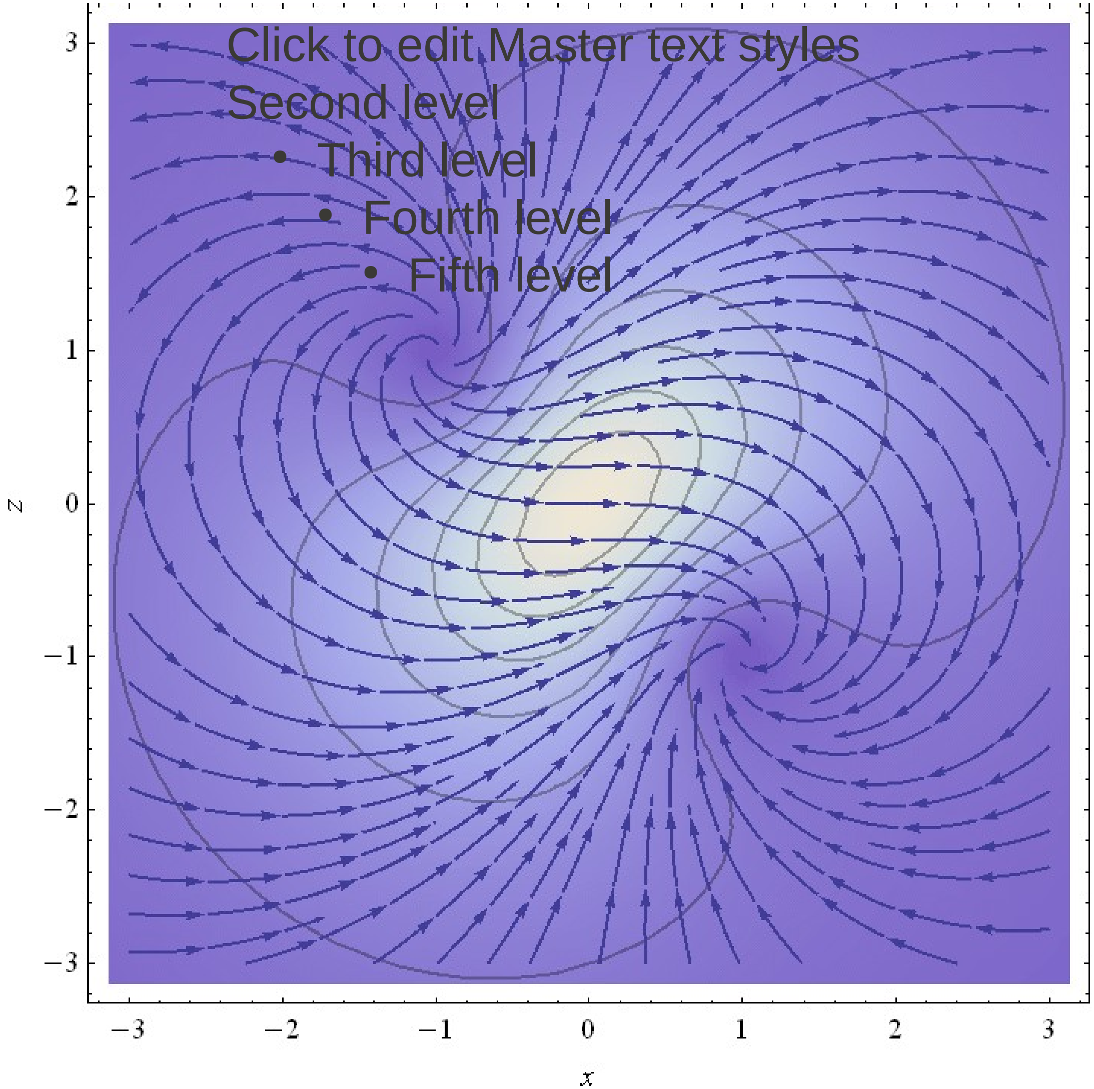
Pull back of volume form gives a 2-form on 3-sphere.

This 2-form is an exact differential.

Find the vector dual of corresponding one form

Project is on R3 stereographically to get Ai.

Velocity profile (y=-1)

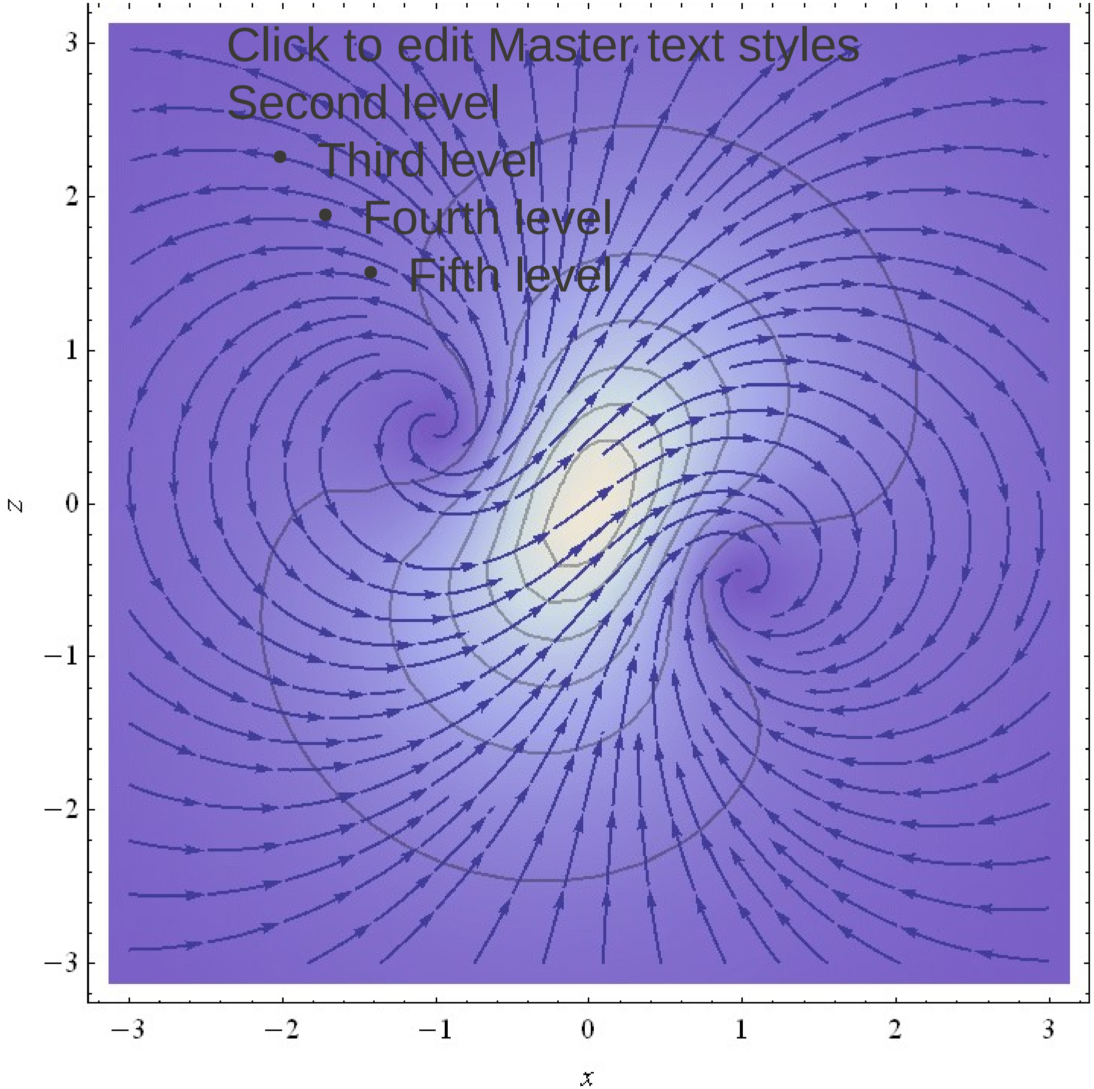


Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Velocity profile (y=-0.5)



Click to edit Master text styles

Second level

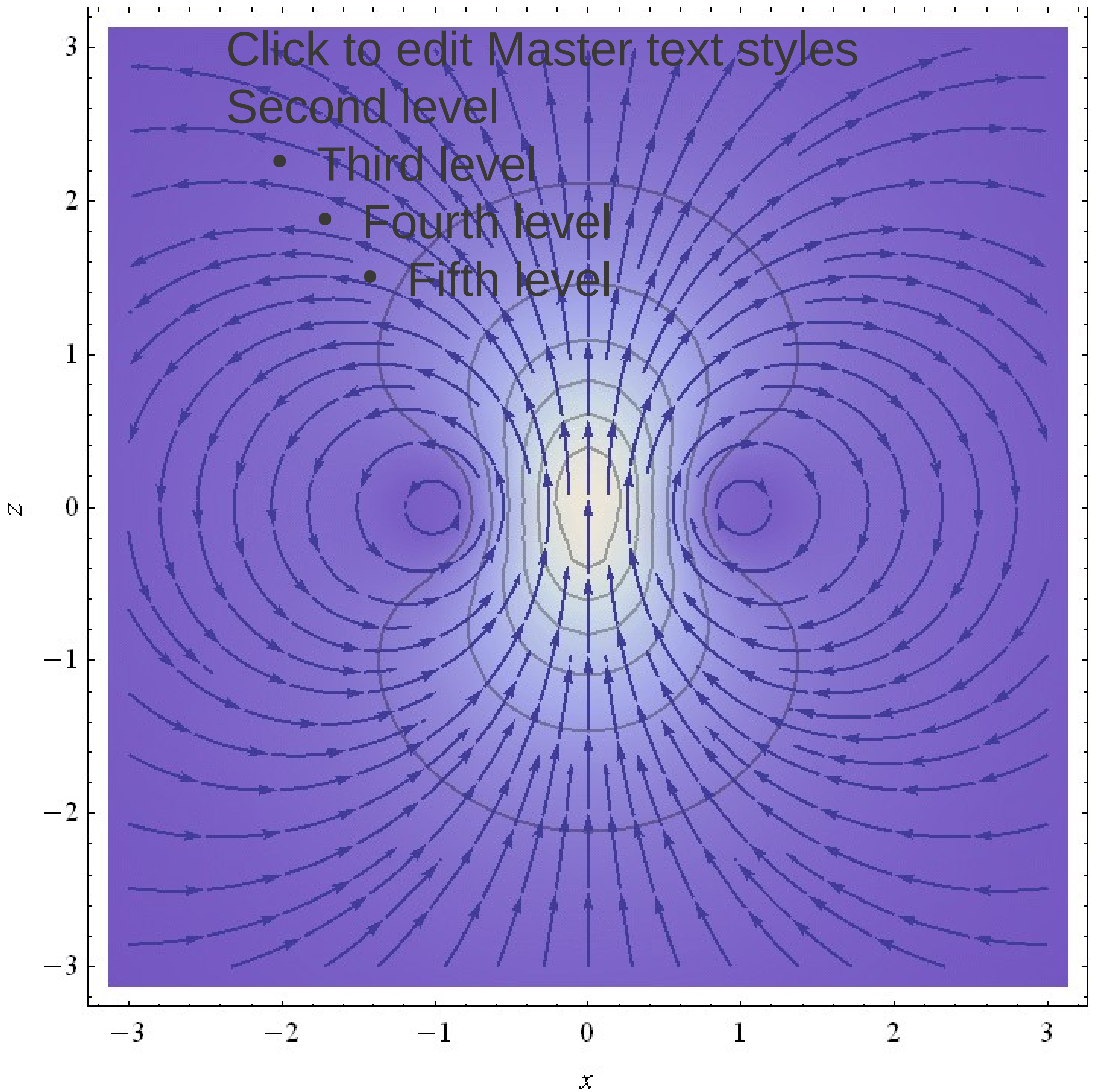
- Third level
  - Fourth level
    - Fifth level

Velocity profile (y=0)

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

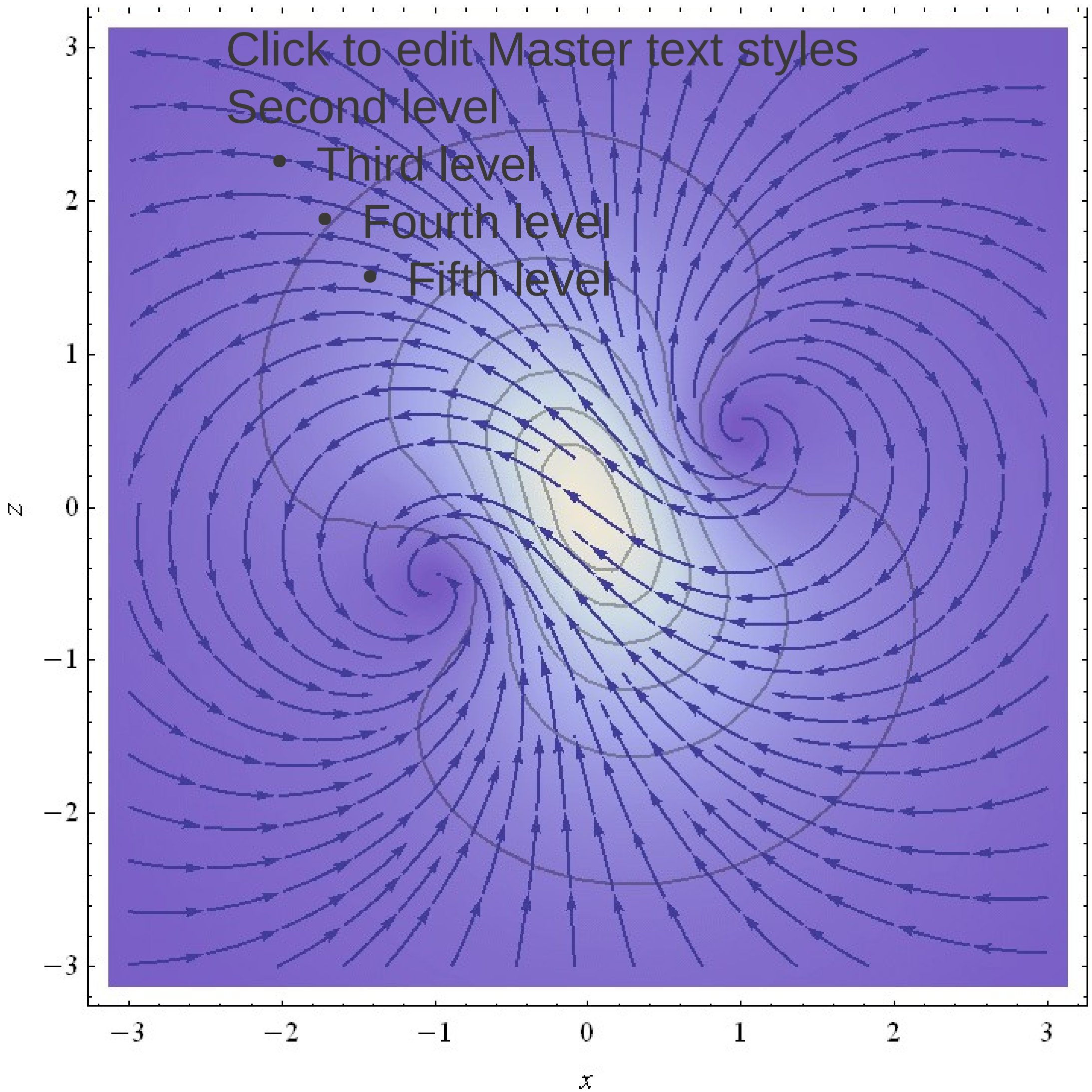


Velocity profile (y=0.5)

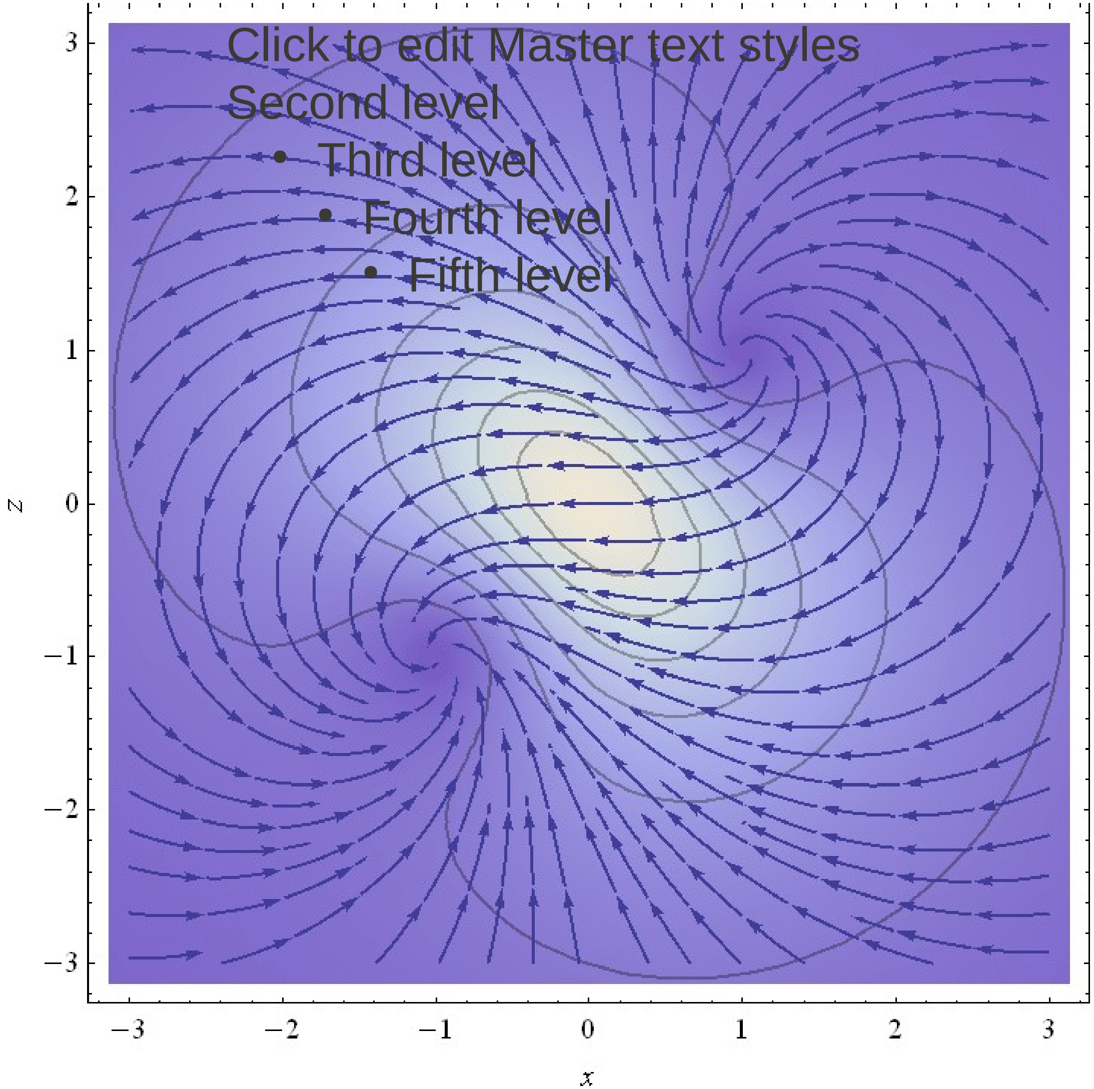
Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level



Velocity profile (y=1)



Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Using Hopf Fibration(Continued)

All the vector quantities in the relativistic equations can be expressed in terms of Ai and ai.

ai is (-y, x,1), anticlockwise moving vector field.

Hydrodynamic equations reduce to terms with scalar functions and their derivatives with all vector directions along Ai , ai ,zAi and zai only.

Choosing their coefficients to be zero gives us a set of pde s of scalar functions. Still non linear.

Simplify equations further by looking only for steady state solutions.

Sphaleron solution

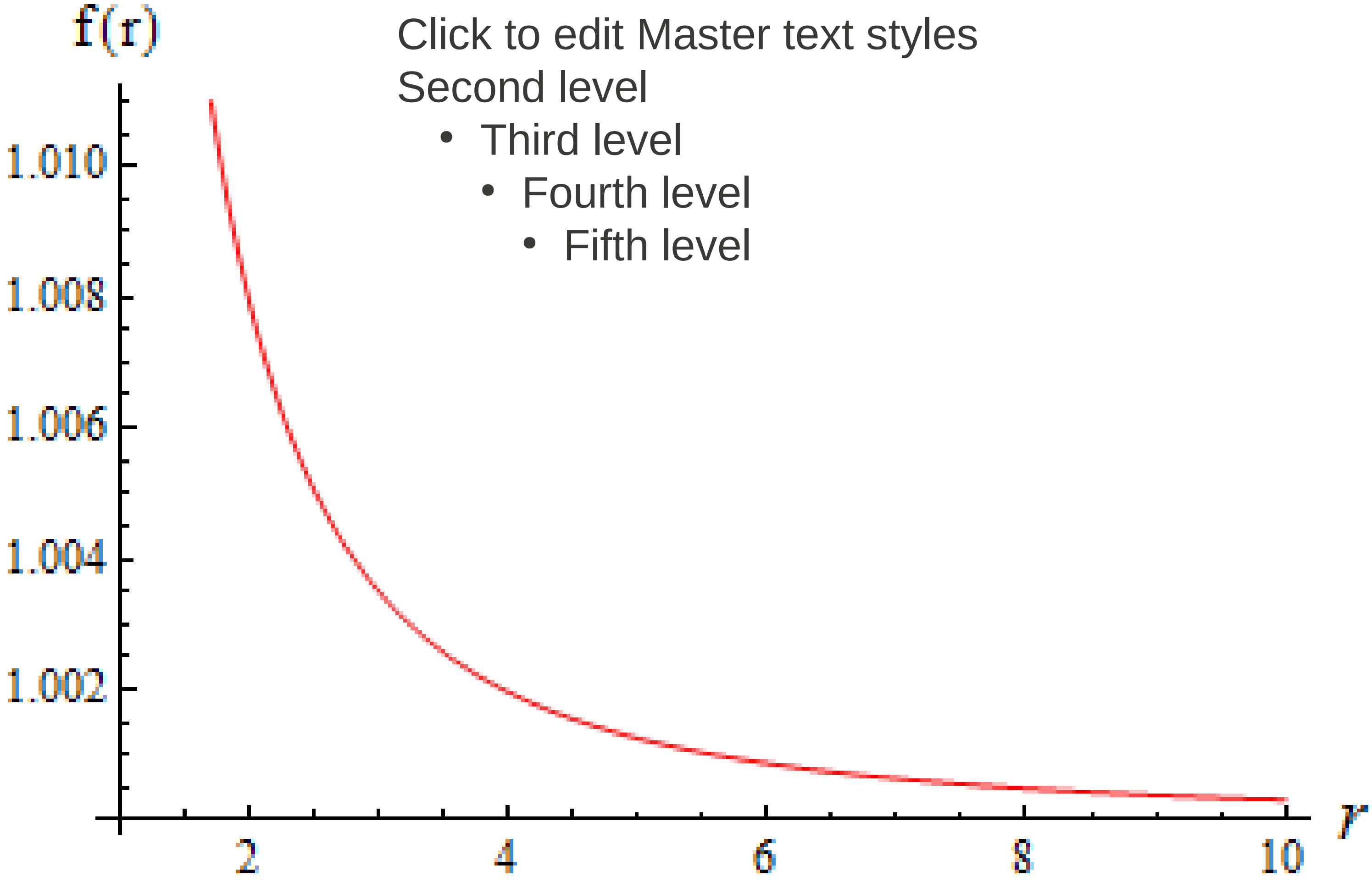
$$v = \lambda f\,,\beta = \lambda\alpha, \alpha = kf$$

$$f = \frac{4r}{\sqrt{16\lambda^2 r^2 - 1}} \qquad h = -\frac{2}{9}\lambda C k^3 \frac{f}{r}$$

$$n = \frac{2\lambda f}{3r}(Ck^2 + \xi + 2k\xi_B)$$

$$P = \frac{8}{3}\lambda k\left(\xi + 2k\xi_B + \frac{2}{3}Ck^2\right)\left(\tan^{-1}\frac{f}{4r} - \frac{f}{4r}\right)$$

Velocity profile

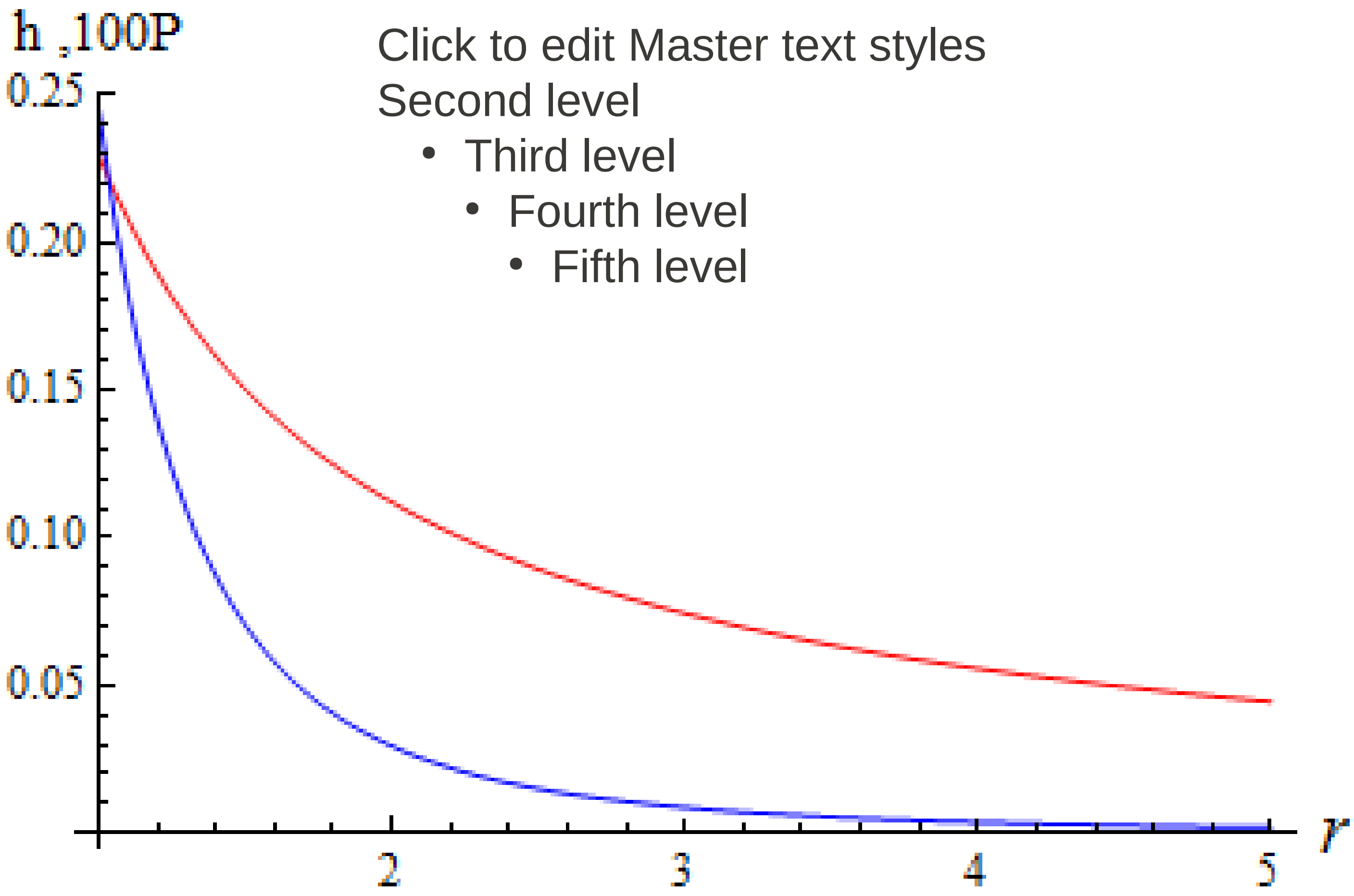


Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Enthalpy and Pressure

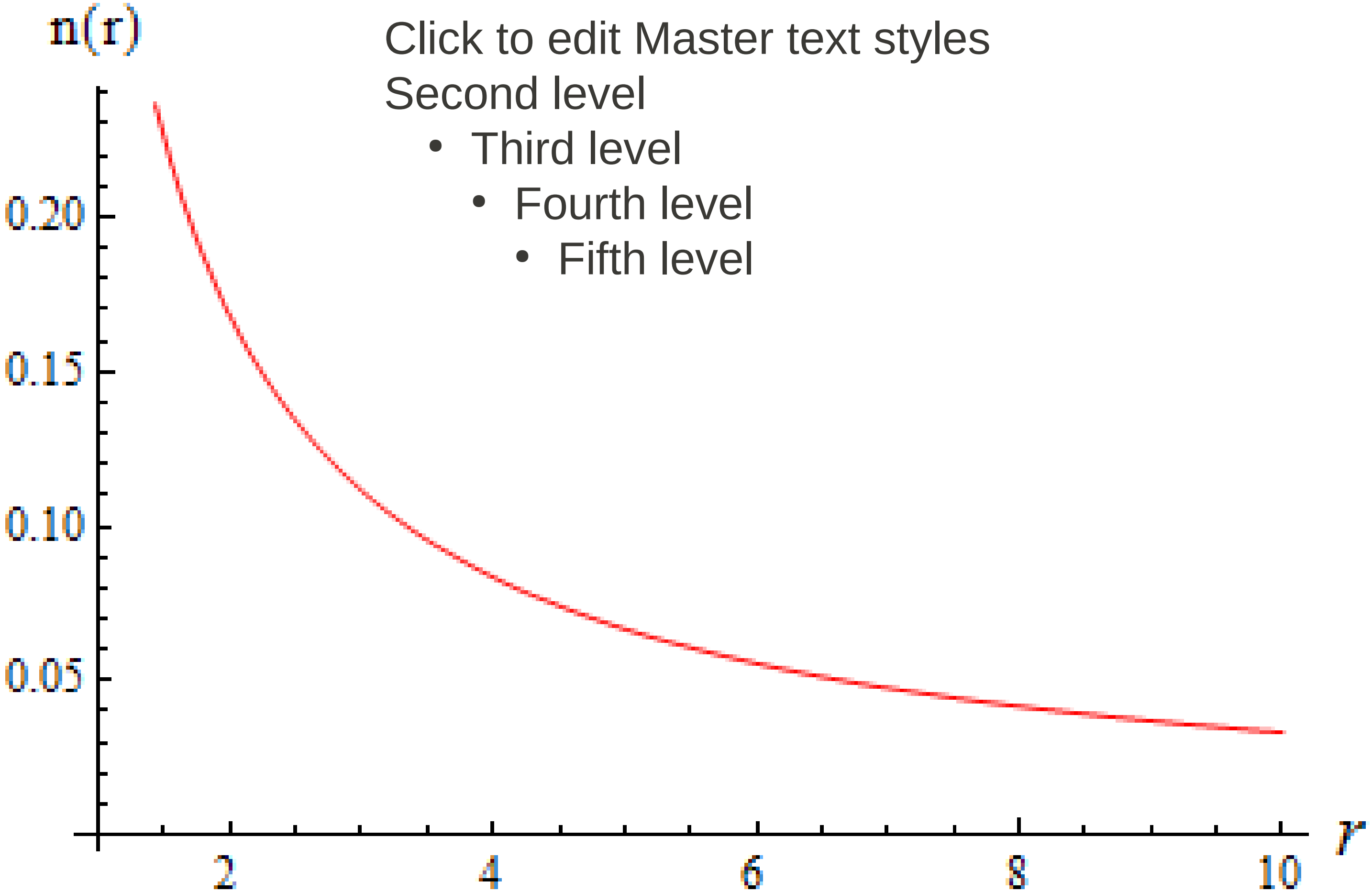


Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Axial density profile

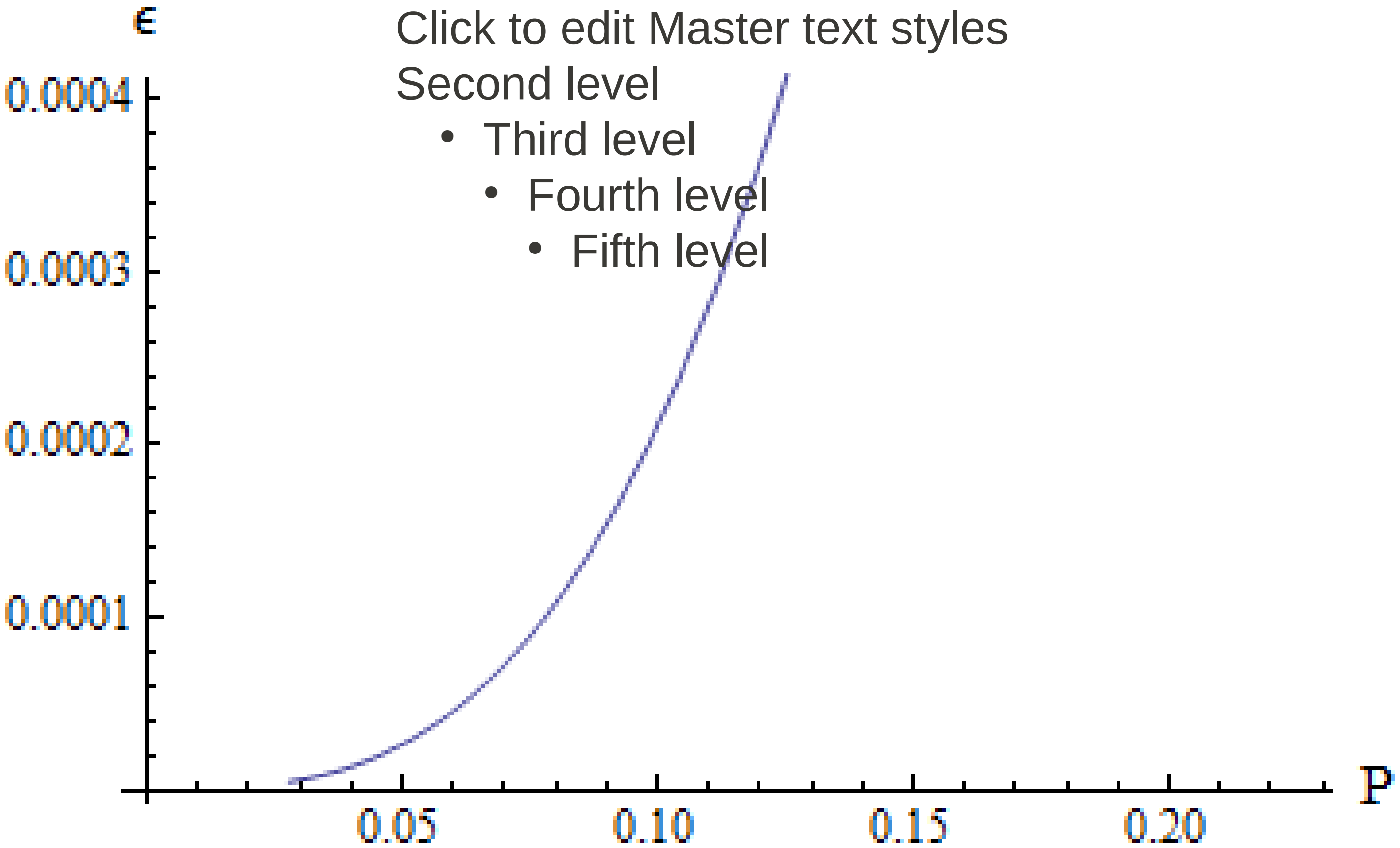


Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Equation of state



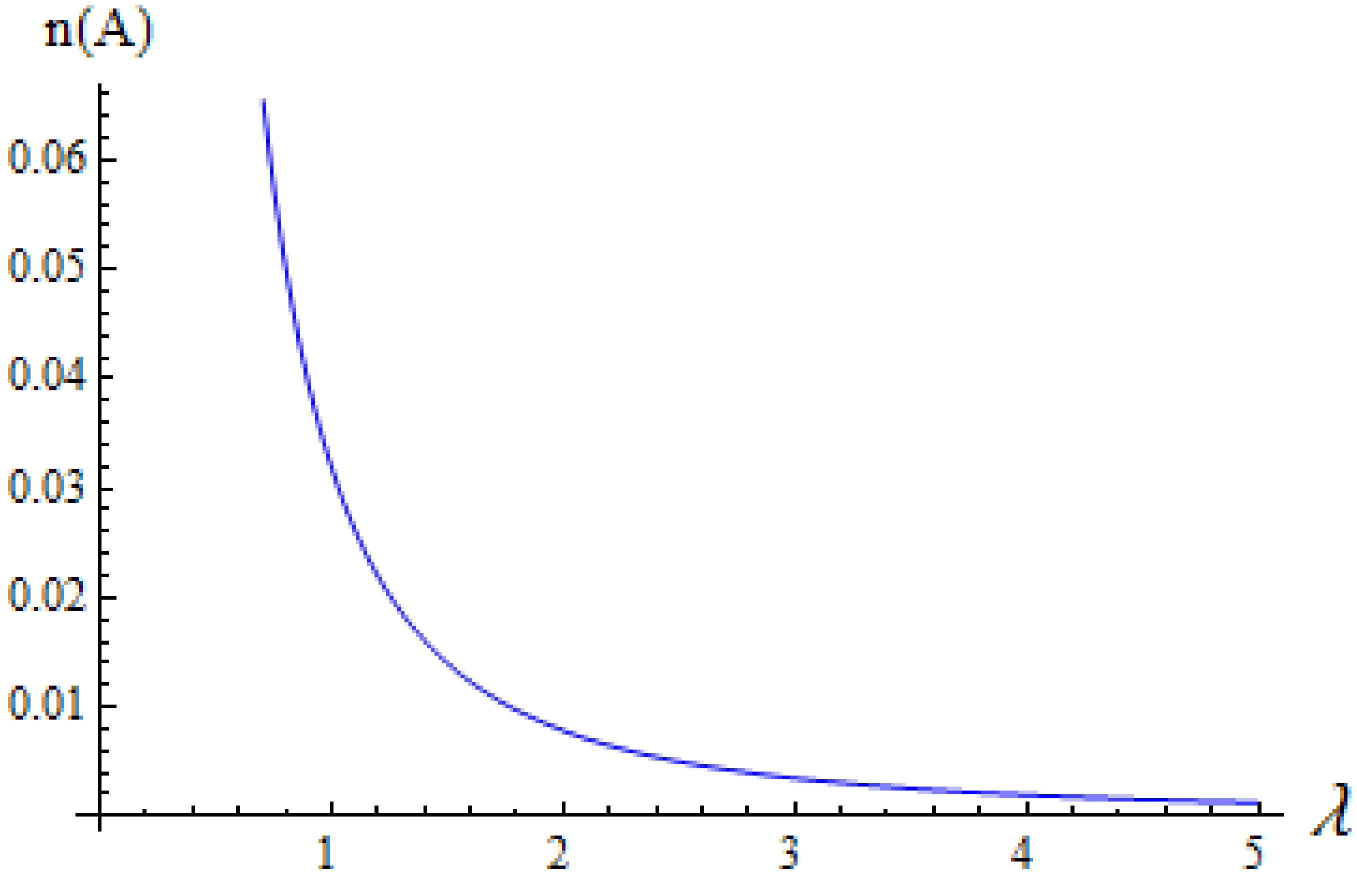
Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level

Chiral charge difference

$$\Delta n_A = -\frac{\pi^2(3Ck^2+\xi+2k\xi_B)}{3\sqrt{\lambda}}(\sqrt{4\lambda+1}+\sqrt{4\lambda-1}-4\sqrt{\lambda})$$



Application to other areas

Baryon asymmetry in early universe

Vortices in neutron stars

Chiral electric effects in superfluids

Berry phase configuration in metal crystals

Summary

Constructed few representative solutions to evaluate chiral charge separation out of topologically non trivial configurations.

Calculated their charge separation

The methods used to generate these solutions are more general and can be used to generate many different solutions suitable for different contexts.