Universal Laws of Thermodynamics

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MPQ

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Outline

1. Introduction

- *Thermodynamics – three laws*

2. Thermodynamics and Information

- *Maxwell's demon*
- *Landauer's principle – connecting heat with information*

3. Quantum thermodynamics: quantum information theoretic approach

4. Breakdown of thermodynamics in the presence of system-bath correlations

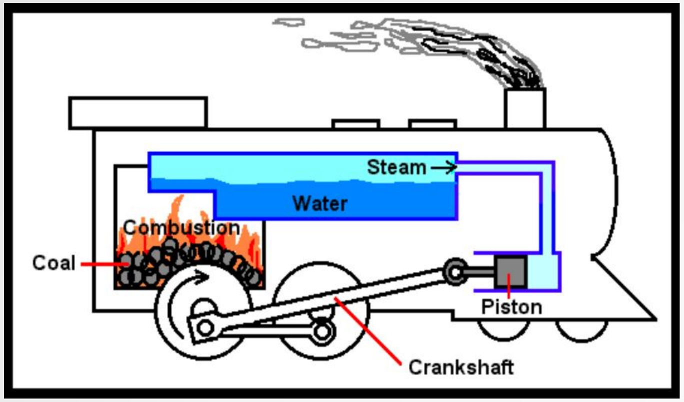
- *Violation Landauer's principle, TD laws and anomalous heat flow*

5. "Universal" remedy? – information theoretic approach

- *Redefinition of heat and assumptions*
- *Work extraction and mutual information*
- *Universal laws*

6. Outlooks

Thermodynamics – success story



1st wave (phenomenology)

Carnot (1824)

Joule (1843)

Kelvin (1849)

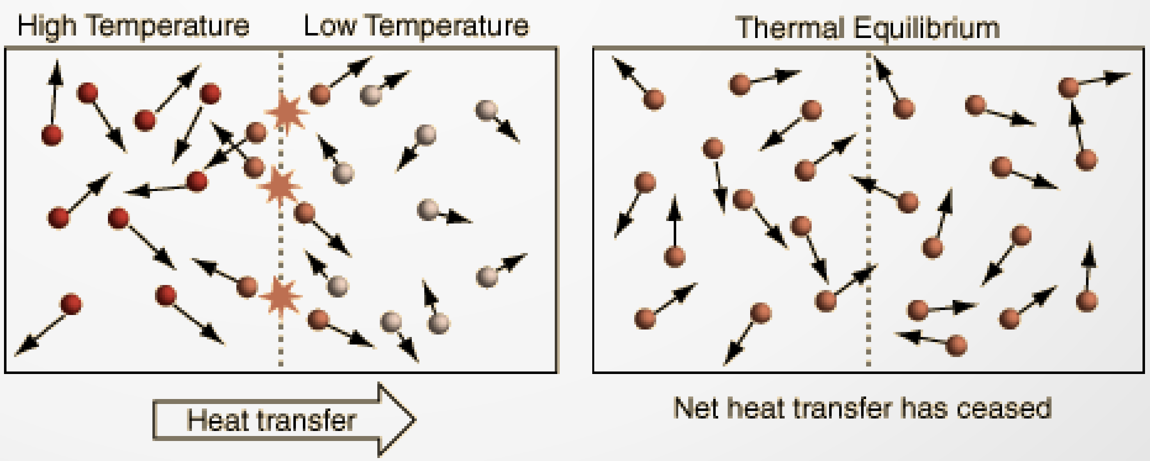
Clausius (1854)

2nd wave (microscopic description and statistical mechanics)

Maxwell (1871)

Boltzman (1875)

Gibbs (1876)

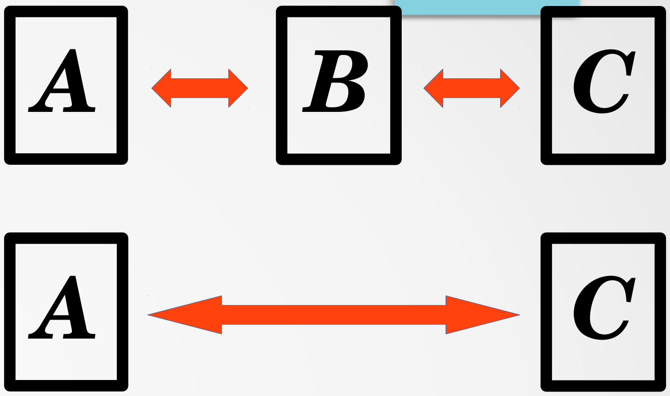


Thermodynamics → *quantum mechanics, relativity; black-hole...*

Thermodynamics – 0th and 1st laws

Zeroth Law:

- *Transitive property.*
- *Notion of thermal equilibrium as an equivalence relation.*
- *Temperature (T) labels the different equivalence classes.*



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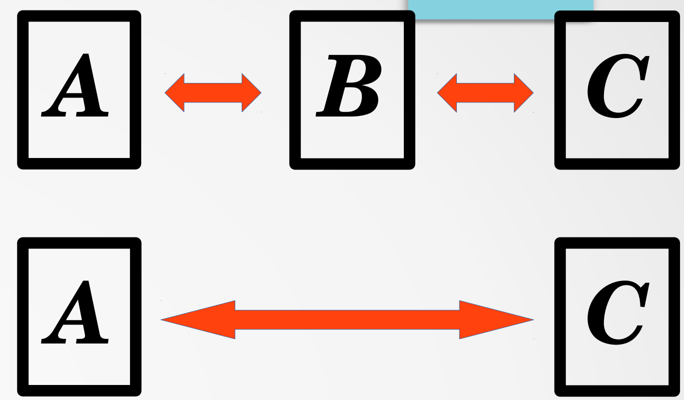
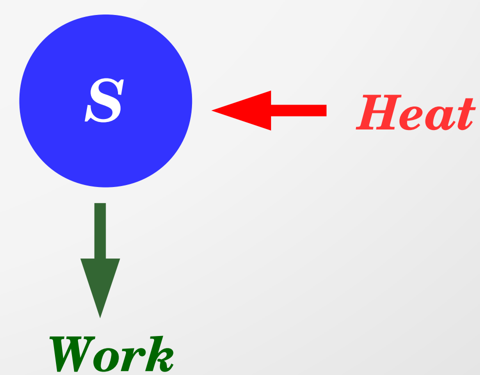


First Law:

Energy conservation; internal energy, heat and work

$$\Delta E_S = -W - Q$$

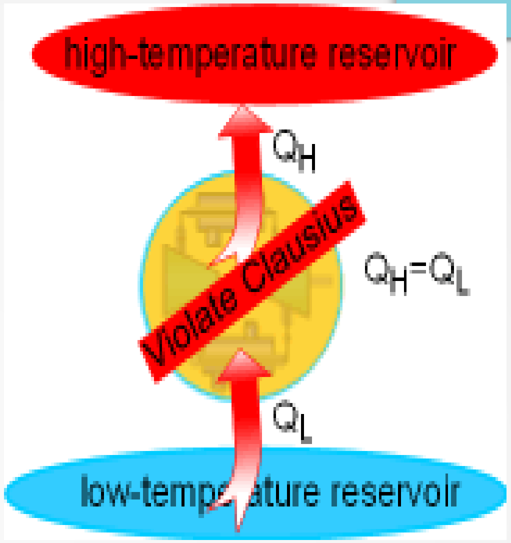


– *Q: heat added to the system*
– *W: work done by the system*

Thermodynamics – 2nd laws

Clausius statement:

No process is possible whose sole result is the transfer of heat from a cooler to a hotter body.



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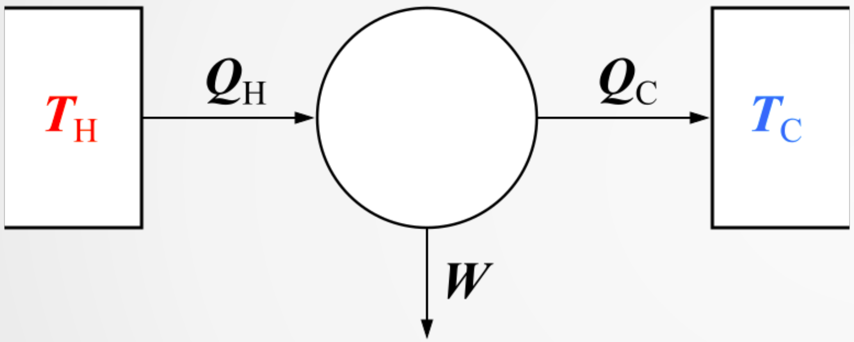

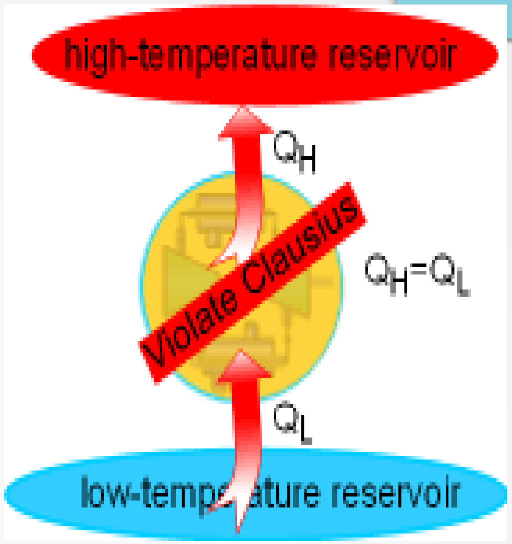

Kelvin-Planck Statement: $-Q_H > W$
No process is possible whose sole result is the absorption of heat from a reservoir and complete conversion of this heat into work.

Thermodynamics – 2nd laws

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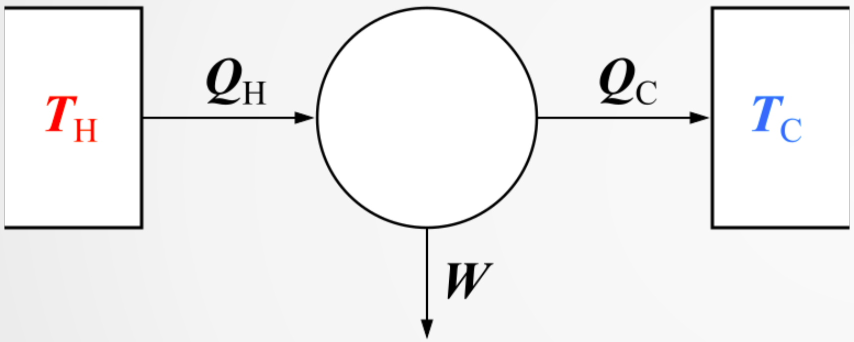

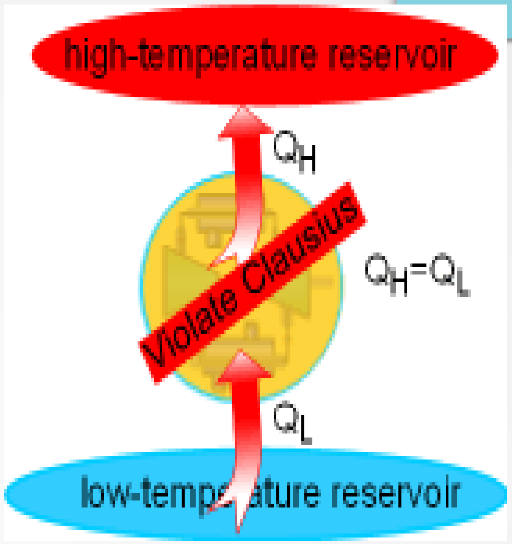

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No process is possible whose sole result is the absorption of heat from a reservoir and complete conversion of this heat into work.

Carnot statement:

$$\eta = \frac{W}{Q_H} \leqslant 1 - \frac{T_C}{T_H}$$

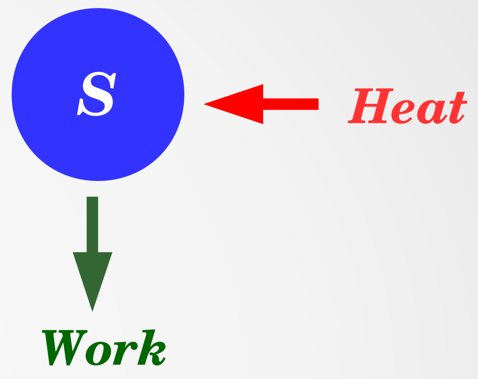
Work and heat

$\Delta E_S = -W - Q$ *Heat and work, path-dependent quantities*

WORK, done by system: $W \leqslant F_{fin} - F_{in}$

Helmholtz free energy: $F = E - T\,S$

Entropy: $S = -\sum_i p_i \ln p_i$



p_i *is the probability with which the system stays in* e_i

What about heat?

Work and heat

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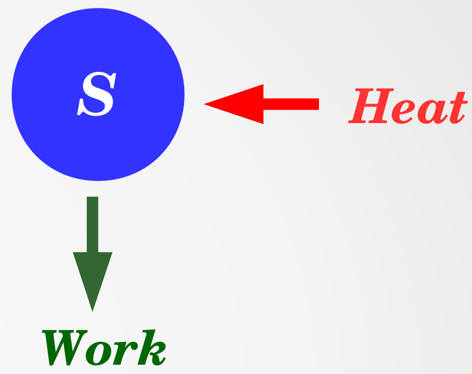

HEAT: the amount of energy flowing from one body to another, spontaneously due to their temperature difference, or by any means other than through work.

Work and heat

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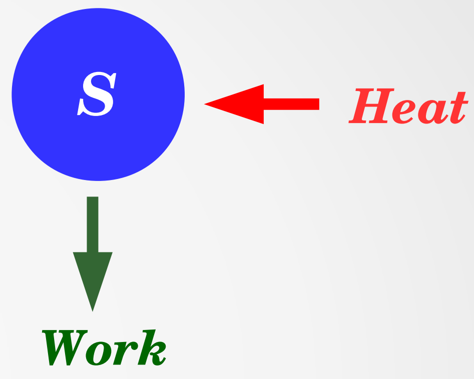

HEAT: *the amount of energy flowing from one body to another, spontaneously due to their temperature difference, or by any means other than through work.*

For a thermal bath, in Gibbsian form, minimizes free energy: $\gamma_B = \frac{e^{-H_B/T}}{Z}; \qquad Z = \mathrm{Tr}\, e^{-H_B/T}$

Temperature: T

Heat flow from the bath, to the system: $-Q = -\Delta E_B$

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- *Landauer's principle – "memory erasure causes heat dissipation"*

3. *Quantum thermodynamics: quantum information theoretic approach*

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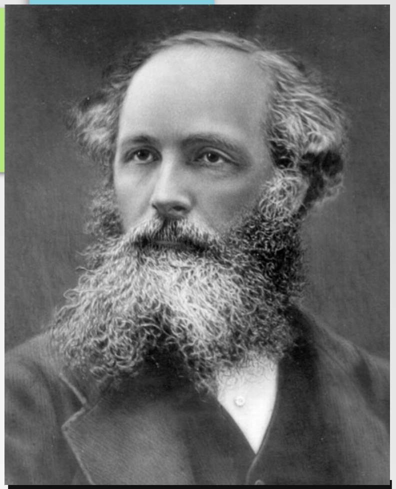
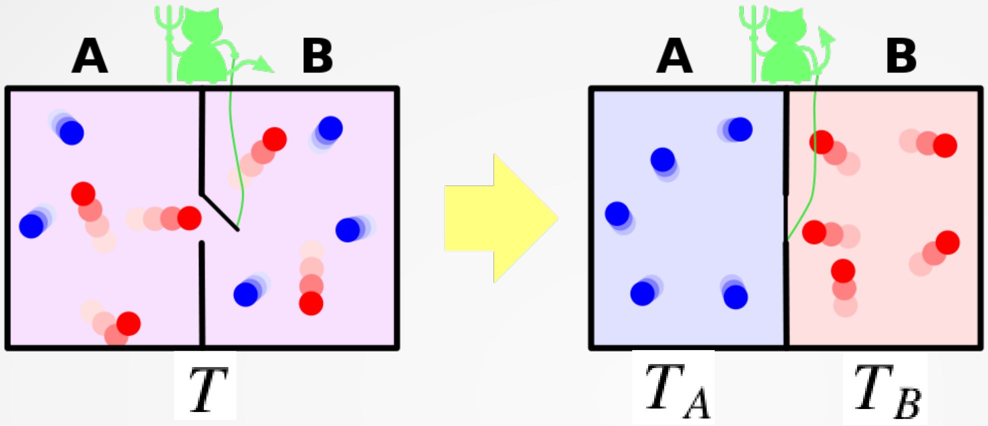
- *Universal laws*

6. *Outlooks*

Maxwell's demon



Gas in a partitioned box

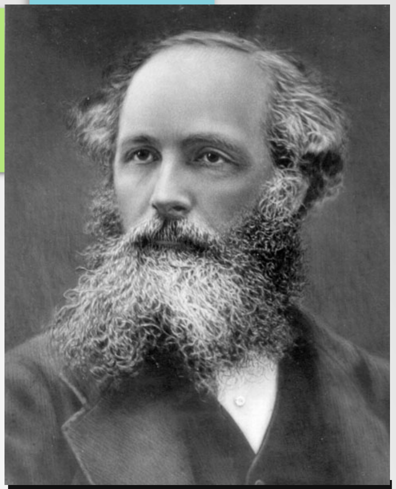


Information about particle (position/velocity) leads to

- *sorting temperature difference; decrease of entropy without work; could be used to run a heat engine and produce work.*
- *apparent violation of the second law*

What is missing?

Maxwell's demon



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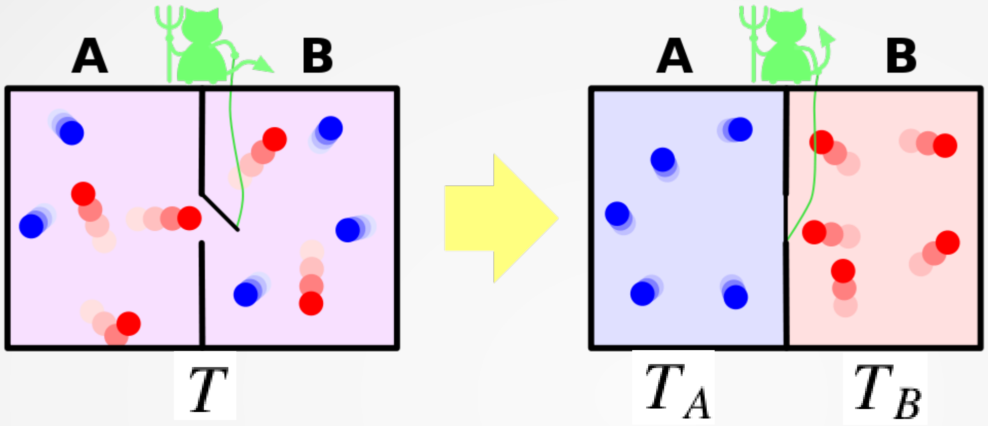

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Memory or a register:

$$|0\rangle\langle 0| \longrightarrow \rho = \sum_i p_i\, |x_i, v_i\rangle\langle x_i, v_i|$$

Heat, information and Landauer

Landauer's erasure principle:

An erasing operation is bound to be associated with a heat flow to the Environment. – Rolf Landauer (1961).

Erasing:

$$\rho \otimes \mathcal{B}(T) \to |0\rangle\langle 0| \otimes \mathcal{B}'(T)$$

Bound to heat-up the bath.

Information is physical!!

Quantitatively?

Heat, information and Landauer

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Erasing:

$$\rho \otimes \mathcal{B}(T) \rightarrow |0\rangle\langle 0| \otimes \mathcal{B}'(T)$$

Heat dissipation: $Q \geqslant T\, S(\rho), \quad S(\rho) = -\sum_i p_i \ln p_i$

Partial erasing: $\rho \longrightarrow \rho'$

$$Q \geqslant -T\, \Delta S(\rho)$$

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- *Universal laws*

6. Outlooks

Quantum thermodynamics (QTD)

Classical thermodynamics:

- *incoherent states:* *no superposition in different energy eigenstates*
- *number of particles* ⟶ *infinity*
- *bath-size* ⟶ *large*

What about quantum?

Quantum thermodynamics (QTD)

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QTD:

- *states with superpositions in different energy eigenstates*
- *inter-system correlations (even system-bath entanglement)*
- *number of particles* ⟶ *arbitrary*
- *bath-size* ⟶ *arbitrary*

How to address?

Quantum thermodynamics (QTD)

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QTD requires information theoretic approach:

- *Resource theory of QTD (bath-size* ⟶ *large)*
- *QTD from information conservation, for finite-bath (arXiv:1707.01750)*

PRL **111,** 250404 (2013) PHYSICAL REVIEW LETTERS week ending 20 DECEMBER 2013

Resource Theory of Quantum States Out of Thermal Equilibrium

Fernando G. S. L. Brandão,[1,2] Michał Horodecki,[3,4] Jonathan Oppenheim,[5] Joseph M. Renes,[6,7,*] and Robert W. Spekkens[8]

Thermal state (free states): $\gamma_B = e^{-\beta H_B}/\mathrm{Tr}\,(e^{-\beta H_B}); \quad \beta = 1/T$

Thermal operations (allowed / free operations):

$$\Lambda^T(\rho_S) = \mathrm{Tr}_B\left[U_{SB}\rho_S \otimes \gamma_B U_{SB}^\dagger\right]; \qquad [U_{SB}, H_S + H_B] = 0$$

Maps thermal states to thermal states: $\Lambda^T(\gamma_S) \in \{\gamma_S\}, \ \forall H_S$

System at thermal equilibrium –

$$\gamma_S = \frac{e^{-H_S/T}}{\mathrm{Tr}\,(e^{-H_S/T})}$$

- *Any a-thermal state has a non-zero resource*
- *Resource, that is useful* ⟶ *work potential*
- *How to quantify?*
- *What are the properties it should satisfy?*
  – monotone
- *Resource extraction and state transformation*

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A resource measure, monotone

$$M(\rho_S) = D(\rho_S \,\|\, \gamma_S) = \mathrm{Tr}\,(\rho_S \ln \rho_S - \rho_S \ln \gamma_S)$$

$$\Lambda^T(\rho_S) = \sigma_S \implies M(\rho_S) \geqslant M(\sigma_S)$$

Relative entropy, an information theoretic (distance-like) measure

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$$T\, M(\rho_S) = F(\rho_S) - F(\gamma_S)$$

Helmholtz free energy: $F(\rho_S) = E(\rho_S) - T\, S(\rho_S)$

Resource theory QTD

Asymptotic transformation, or in classical limit

$$n \longrightarrow \infty$$

Work extraction

$$\Lambda^T \left(\rho_S^{\otimes n} \otimes |0\rangle\langle 0|_{battery}^{\otimes n} \right) = (\rho'_S)^{\otimes n} \otimes |W\rangle\langle W|_{battery}^{\otimes n}$$

$$W_{battery}^{max} = TM(\rho_S) = F(\rho_S) - F(\gamma_S)$$

State transformation?

Resource theory QTD

Asymptotic transformation, or in classical limit

$$n \longrightarrow \infty$$

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State transformation: $\rho_S \to \sigma_S; \qquad \Lambda^T\left(\rho_S^{\otimes n}\right) \approx \sigma_S^{\otimes m}$

$$\lim_{n\to\infty}\left(\frac{m}{n}\right) = \frac{M(\rho_S)}{M(\sigma_S)} = \frac{D(\rho_S \parallel \gamma_S)}{D(\sigma_S \parallel \gamma_S)}$$

$$D(\rho_S \parallel \gamma_S) = D(\sigma_S \parallel \gamma_S)$$

$$E(\rho_S) \neq E(\sigma_S), \; S(\rho_S) \neq S(\sigma_S)$$

→ *Asymptotic convertibility*

Resource theory QTD

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What about finite-copy (one-shot) transformations, or in quantum limit?

$$n \ll \infty$$

nature COMMUNICATIONS

ARTICLE

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DOI: 10.1038/ncomms3059

Fundamental limitations for quantum and nanoscale thermodynamics

Michał Horodecki[1,*] & Jonathan Oppenheim[2,3,*]

One has to consider Renyi entropies as the measure of information, Renyi relative entropies to quantify free energies (!!), in place of von Neumann ones

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ARTICLE

Received 27 Mar 2013 | Accepted 28 May 2013 | Published 26 Jun 2013

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Fundamental limitations for quantum and nanoscale thermodynamics

PNAS

The second laws of quantum thermodynamics

Fernando Brandão[a,1], Michał Horodecki[b], Nelly Ng[c], Jonathan Oppenheim[c,d,2], and Stephanie Wehner[c,e]

[a]Department of Computer Science, University College London, London WC1E 6BT, United Kingdom; [b]Instytut Fizyki Teoretycznej i Astrofizyki, University of Gdansk, 80-952 Gdansk, Poland; [c]Centre for Quantum Technologies, National University of Singapore, 117543 Singapore; [d]Department of Physics & Astronomy, University College London, London WC1E 6BT, United Kingdom; and [e]School of Computing, National University of Singapore, 117417 Singapore

Edited by Peter W. Shor, Massachusetts Institute of Technology, Cambridge, MA, and approved January 12, 2015 (received for review June 26, 2014)

The second law of thermodynamics places constraints on state transformations. It applies to systems composed of many particles, however, we are seeing that one can formulate laws of thermodynamics when only a small number of particles are interacting

Here, we show that this is not the case in the microscopic regime, and we therefore needs to talk about "how cyclic" a process is when stating the second law. We also derive in this work, a zeroth law of thermodynamics, which is stronger than the ordinary zeroth law.

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Fundamental limitations for quantum and nanoscale th

PNAS

The second laws

Fernando Brandão[a,1], Michał Horodec

[a]Department of Computer Science, University C
of Gdansk, 80-952 Gdansk, Poland; [c]Centre for
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OPEN

Description of quantum coherence in thermodynamic processes requires constraints beyond free energy

Matteo Lostaglio[1], David Jennings[1] & Terry Rudolph[1]

So far, thermodynamics consider

absence of initial (system-bath) correlations

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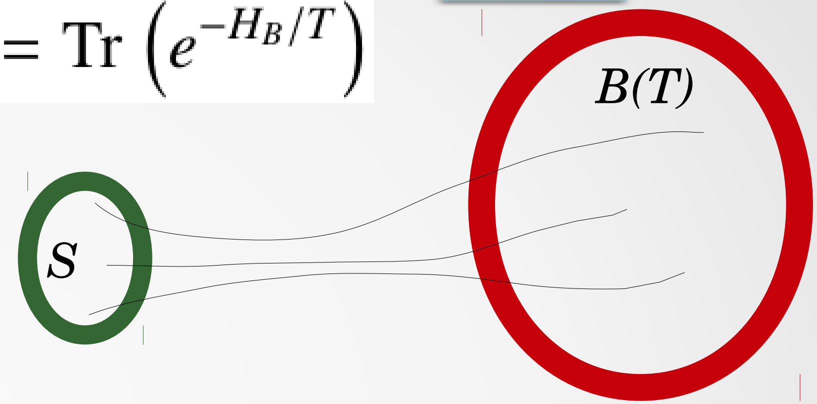
Presence of system-bath correlations

$$\rho_{SB} \longrightarrow \mathrm{Tr}_S(\rho_{SB}) = \rho_B = e^{-H_B/T}/Z, \quad Z = \mathrm{Tr}\left(e^{-H_B/T}\right)$$



What kinds of correlations it could have?

Presence of system-bath correlations

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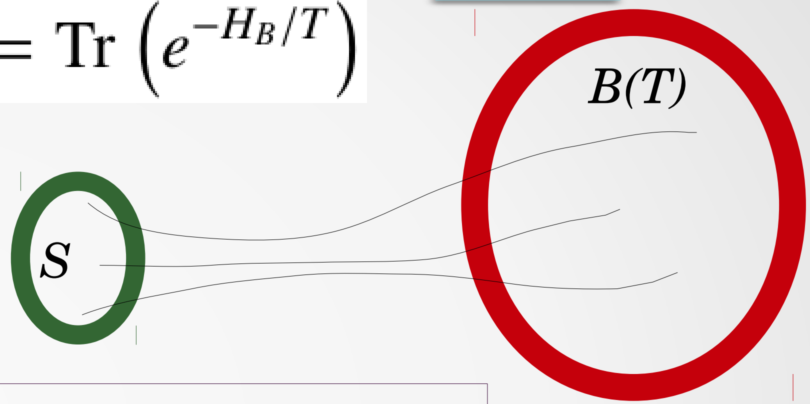

Classical: $$\rho_{SB} = \sum_i p_i \, |i\rangle\langle i|_S \otimes |i\rangle\langle i|_B$$

Quantum:

Separable $$\rho_{SB} = \sum_i p_i \, \rho_S^i \otimes \rho_B^i$$

Entanglement

$$\rho_{SB} \neq \sum_i p_i \, \rho_S^i \otimes \rho_B^i$$

an example:

$$|\psi\rangle_{SB} = \sum_i \sqrt{p_i}\, |i\rangle_S \otimes |i\rangle_B$$

How to quantify them?

Presence of system-bath correlations

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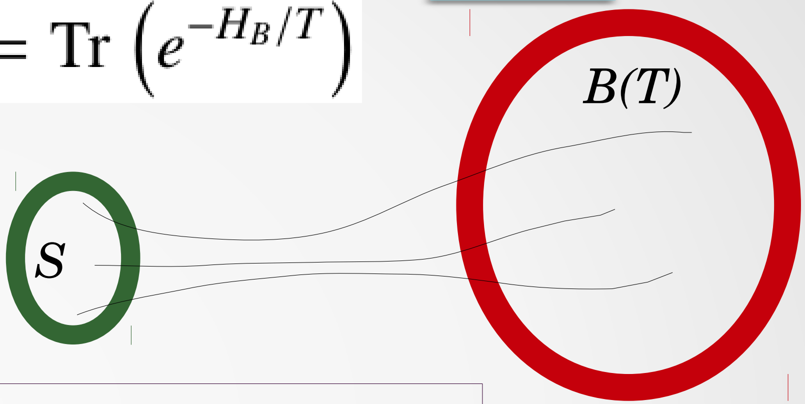

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an example:

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A measure of correlation: *mutual information*

$$\mathcal{I}(S:B) = D(\rho_{SB} \,\|\, \rho_S \otimes \rho_B) = \mathcal{S}_S + \mathcal{S}_B - \mathcal{S}_{SB}$$

Violations of Landauer's principle

Hamiltonians unchanged

Free energy: $F(\rho_S) = E(\rho_S) - T\,\mathcal{S}(\rho_S)$

$$\rho^c_{SB} = \sum_i p_i\,|i\rangle\langle i|_S \otimes |i\rangle\langle i|_B \xrightarrow{U^c_{SB}} \rho'_{SB} = |0\rangle\langle 0|_S \otimes \sum_i p_i\,|i\rangle\langle i|_B$$

$$|\psi\rangle^e_{SB} = \sum_i \sqrt{p_i}\,|i\rangle_S \otimes |i\rangle_B \xrightarrow{U^e_{SB}} |\psi'\rangle_{SB} = |0\rangle_S \otimes \sum_i \sqrt{p_i}\,|i\rangle_B$$

$$\mathrm{Tr}_B(\rho^c_{SB}) = \mathrm{Tr}_B(|\psi\rangle^e_{SB}) = \rho_S = \sum_i p_i\,|i\rangle\langle i|_S \longrightarrow |0\rangle\langle 0|, \quad \Delta E_B = Q = 0$$

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Hamiltonians unchanged

Free energy: $F(\rho_S) = E(\rho_S) - T\,\mathcal{S}(\rho_S)$

$$\rho^c_{SB} = \sum_i p_i\,|i\rangle\langle i|_S \otimes |i\rangle\langle i|_B \xrightarrow{U^c_{SB}} \rho'_{SB} = |0\rangle\langle 0|_S \otimes \sum_i p_i\,|i\rangle\langle i|_B$$

$$|\psi\rangle^e_{SB} = \sum_i \sqrt{p_i}\,|i\rangle_S \otimes |i\rangle_B \xrightarrow{U^e_{SB}} |\psi'\rangle_{SB} = |0\rangle_S \otimes \sum_i \sqrt{p_i}\,|i\rangle_B$$

$$\mathrm{Tr}_B(\rho^c_{SB}) = \mathrm{Tr}_B(|\psi\rangle^e_{SB}) = \rho_S = \sum_i p_i\,|i\rangle\langle i|_S \longrightarrow |0\rangle\langle 0|,\quad \Delta E_B = Q = 0$$

No heat flow, in case of classical correlation!!

Work performed on bath, in case of entanglement!!

$$Q \ngeq T\,\mathcal{S}(\rho_S)$$

!!

Violations of thermodynamical laws

Hamiltonians unchanged

Free energy: $F(\rho_S) = E(\rho_S) - T\,\mathcal{S}(\rho_S)$

$$\rho^c_{SB} = \sum_i p_i\, |i\rangle\langle i|_S \otimes |i\rangle\langle i|_B \xrightarrow{U^c_{SB}} \rho'_{SB} = |\phi\rangle\langle\phi|_S \otimes \sum_i p_i\, |i\rangle\langle i|_B$$

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where $|\phi\rangle_S = \sum_i \sqrt{p_i}\, |i\rangle_S, \;\; \Delta E_S = \Delta E_B = 0, \;\; F(|\phi\rangle\langle\phi|_S) > F(\rho_S)$

Note, initial system $\mathrm{Tr}_B(\rho^c_{SB}) = \mathrm{Tr}_B(|\psi\rangle^e_{SB}) = \rho_S = \sum_i p_i\, |i\rangle\langle i|_S$

Work done on the system!

Violations of thermodynamical laws

Hamiltonians unchanged

Free energy: $F(\rho_S) = E(\rho_S) - T\,\mathcal{S}(\rho_S)$

$$\rho^c_{SB} = \sum_i p_i\,|i\rangle\langle i|_S \otimes |i\rangle\langle i|_B \xrightarrow{U^c_{SB}} \rho'_{SB} = |\phi\rangle\langle\phi|_S \otimes \sum_i p_i\,|i\rangle\langle i|_B$$

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where $|\phi\rangle_S = \sum_i \sqrt{p_i}\,|i\rangle_S, \quad \Delta E_S = \Delta E_B = 0, \quad F(|\phi\rangle\langle\phi|_S) > F(\rho_S)$

First law violated: $\Delta E_S \neq -W - Q$

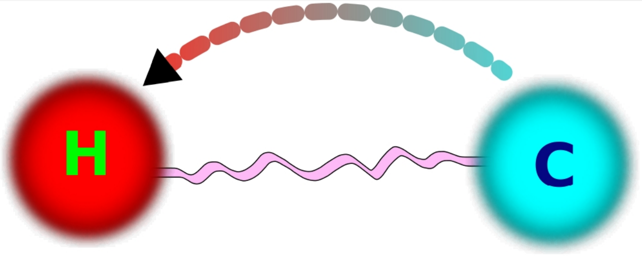
Second law (Kelvin-Planck statement) violated: $-W \nless -Q$

!!

Anomalous heat flow

Consider an energy conserving unitary…

$$\rho_{HC} \xrightarrow{U_{HC}} \rho'_{HC}$$



With $Q_H = \Delta E_H = -Q_C; \quad \mathcal{I}(H:C) = \mathcal{S}_H + \mathcal{S}_C - \mathcal{S}_{HC}$

$$\Delta F_H = \Delta E_H - T_H\, \Delta\mathcal{S}_H \geqslant 0$$
$$\Delta F_C = \Delta E_C - T_C\, \Delta\mathcal{S}_C \geqslant 0$$

$$\Rightarrow \quad Q_H\left(\frac{1}{T_H} - \frac{1}{T_C}\right) \geqslant \Delta\mathcal{I}(H:C)$$

Note $\Delta\mathcal{I}(H:C) = \Delta\mathcal{S}_H + \Delta\mathcal{S}_C$

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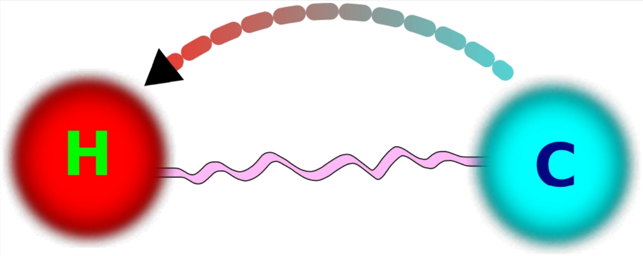

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$$\Rightarrow \quad Q_H \left(\frac{1}{T_H} - \frac{1}{T_C} \right) \geqslant \Delta \mathcal{I}(H:C)$$

$$\rho_{HC} = \rho_H \otimes \rho_C \quad \rightarrow \quad \Delta \mathcal{I}(H:C) \geqslant 0$$

$$\rho_{HC} \neq \rho_H \otimes \rho_C \quad \rightarrow \quad \Delta \mathcal{I}(H:C) < 0$$!!

Initial correlation could lead to an anomalous heat flow!!

Phys. Rev. E 81, 061130 (2010)

Violations of zeroth law

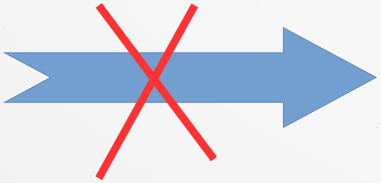
Zeroth Law:

- *Notion of thermal equilibrium as an equivalence relation.*
- *Temperature (T) labels the different equivalence classes.*
- *Transitive property.*

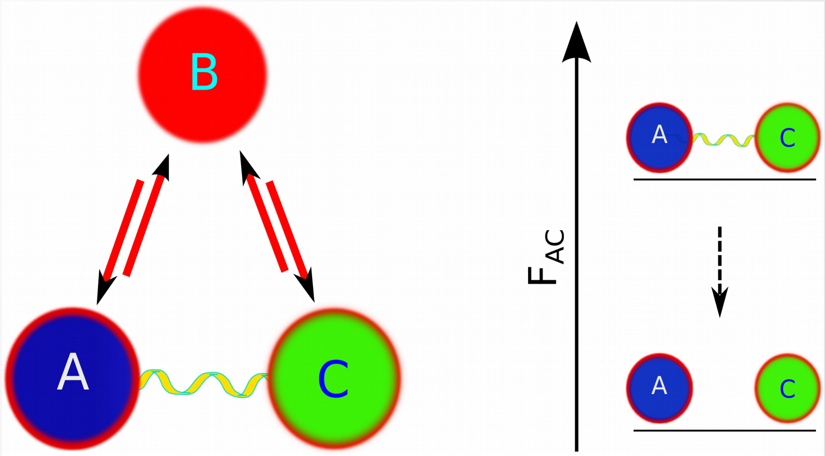
Thermal equilibrium → *global minimum in free energy*

$A \rightleftharpoons B$

$B \rightleftharpoons C$



$A \rightleftharpoons C$



$$\rho_{AC} \longrightarrow \rho_A \otimes \rho_C, \quad F(\rho) = E(\rho) - T\,S(\rho)$$

$$S(\rho_{AC}) \leqslant S(\rho_A \otimes \rho_C) \Longrightarrow F(\rho_{AC}) \geqslant F(\rho_A \otimes \rho_C)$$

!!

Outline

1. Introduction
- *Thermodynamics – the four laws*

2. Thermodynamics and Information
- *Maxwell's demon*
- *Rolf Landauer's principle – "memory erasure causes heat dissipation"*

3. Quantum thermodynamics: quantum information theoretic approach

4. Breakdown of thermodynamics in the presence of system-bath correlations
- *Violation Landauer's principle, TD laws and anomalous heat flow*

5. "Universal" remedy? – information theoretic approach
- *Redefinition of heat and assumptions*
- *Work extraction and mutual information*
- *Universal laws*

6. Outlooks

Definition of heat?

A thermal equilibrium state $\gamma_B = \frac{e^{-H_B/T}}{Z}; \qquad Z = \mathrm{Tr}\, e^{-H_B/T}$

Heat flow to a system $-Q = -\Delta E_B$

What if... $\rho_S \otimes \gamma_B \xrightarrow{\mathbb{I}\otimes U_B} \rho_S \otimes \gamma'_B$

To make it inaccessible, bath has to increase its entropy. Otherwise, it could be accessible in the form of work!!

Its not heat, rather work!

Definition of heat?

A thermal equilibrium state $\gamma_B = \frac{e^{-H_B/T}}{Z}; \qquad Z = \mathrm{Tr}\, e^{-H_B/T}$

Heat flow to a system $-Q = -\Delta E_B$

What if… $\rho_S \otimes \gamma_B \xrightarrow{\mathbb{I}\otimes U_B} \rho_S \otimes \gamma_B'$

Correct definition of heat,
flow to the bath: $\gamma_B \longrightarrow \gamma_B'$

$$\begin{aligned} Q &= \Delta E_B - \Delta F_B \\ &= T\, \Delta S_B \end{aligned}$$

Definition of heat?

A thermal equilibrium state $\gamma_B = \frac{e^{-H_B/T}}{Z}; \qquad Z = \mathrm{Tr}\, e^{-H_B/T}$

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Correct definition of heat,
flow to the bath: $\gamma_B \longrightarrow \gamma_B'$

$$Q = \Delta E_B - \Delta F_B$$
$$= T\, \Delta \mathcal{S}_B$$

Entropy-preserving operations $\rho \xrightarrow{\Lambda^{ep}} \sigma \quad \Rightarrow \quad \mathcal{S}(\rho) = \mathcal{S}(\sigma)$

Applicable for asymptotic transformations

Universal Landauer's principle

Erasing: $\rho_{SB} \xrightarrow{\Lambda^{ep}} |0\rangle\langle 0|_S \otimes \rho'_B$

One has to erase information in the system, taking into account the one in correlation!

Universal Landauer's principle

Erasing: $\rho_{SB} \xrightarrow{\Lambda^{ep}} |0\rangle\langle 0|_S \otimes \rho'_B$

$$\boxed{Q = T\, \mathcal{S}(S|B)}$$

Conditional entropy $\mathcal{S}(S|B) = \mathcal{S}_{SB} - \mathcal{S}_B$

Can be negative!!!! A measure of "QUANTUM" information

For pure states, its a measure of entanglement.

Quantifies information stored in (system + correlation)

Universal Landauer's principle

Erasing: $\rho_{SB} \xrightarrow{\Lambda^{ep}} |0\rangle\langle 0|_S \otimes \rho'_B$

$$Q = T\,\mathcal{S}(S|B)$$

Conditional entropy $\mathcal{S}(S|B) = \mathcal{S}_{SB} - \mathcal{S}_B$

For partial erasing $\rho_{SB} \xrightarrow{\Lambda^{ep}} \rho'_{SB}$

$$Q = -T\,\Delta\mathcal{S}(S|B)$$

Note $-\Delta\mathcal{S}(S|B) = \Delta\mathcal{S}_B$

Universal Landauer's principle

Erasing: $\rho_{SB} \xrightarrow{\Lambda^{ep}} |0\rangle\langle 0|_S \otimes \rho'_B$

$$Q = T\,\mathcal{S}(S|B)$$

Conditional entropy $\mathcal{S}(S|B) = \mathcal{S}_{SB} - \mathcal{S}_B$

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$$\rho^c_{SB} = \sum_i p_i\, |i\rangle\langle i|_S \otimes |i\rangle\langle i|_B \xrightarrow{U^c_{SB}} \rho'_{SB} = |0\rangle\langle 0|_S \otimes \sum_i p_i\, |i\rangle\langle i|_B$$

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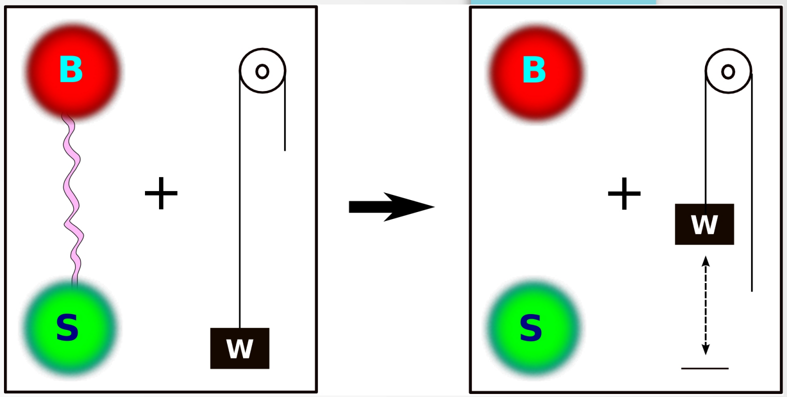
For entanglement: bath cools down!!

Work from correlations

Correlation stores work-potential in ρ_{SB}

$$W_{cor} = T\,\mathcal{I}(S:B)$$

where $\mathcal{I}(S:B) = \mathcal{S}_S + \mathcal{S}_B - \mathcal{S}_{SB}$



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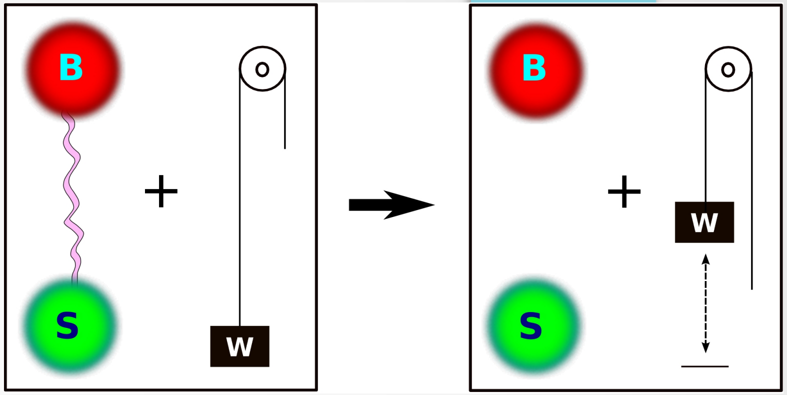

How to extract?

Step 1: *attach an ancillary system with trivial Hamiltonian H=0*

$$\tau_A = \left(\frac{\mathbb{I}_2}{2}\right)^{\otimes \mathcal{I}(S:B)}$$

Step 2: *apply entropy-preserving operation, as*

$$\tau_A \otimes \rho_{SB} \xrightarrow{\Lambda^{ASB}} |0\rangle\langle 0|_A \otimes \rho_S \otimes \rho_B$$

Step 3: *extract work, at T*

$$|0\rangle\langle 0|_A \to \tau_A; \qquad -\Delta F_A = W_{cor} = T\,\mathcal{I}(S:B)$$

Universal Helmholtz free energy

Work potential in the system alone

$$F_S = E_S - T\mathcal{S}_S$$

Work potential in system + correlation

$$\begin{aligned}\mathcal{F} &= F_S + W_{cor} = F_S + T\,\mathcal{I}(S:B)\\ &= E_S - T\,\mathcal{S}(S|B)\end{aligned}$$

Universal Helmholtz free energy

Work potential in the system alone

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Work potential in system + correlation

$$\mathcal{F} = F_S + W_{cor} = F_S + T\,\mathcal{I}(S:B)$$
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$$\rho^c_{SB} = \sum_i p_i\,|i\rangle\langle i|_S \otimes |i\rangle\langle i|_B;\quad \rho_S = \sum_i p_i\,|i\rangle\langle i|_S$$

$$|\psi\rangle^e_{SB} = \sum_i \sqrt{p_i}\,|i\rangle_S \otimes |i\rangle_B;\quad \rho_S = \sum_i p_i\,|i\rangle\langle i|_S$$

Universal first law

Given a transformation $\rho_{SB} \longrightarrow \rho'_{SB}$*, the distribution of the change in the system's internal energy into work and heat satisfies*

$$\Delta E_S = -(W_{S+cor} - W_B) - (W_B + Q),$$

where the heat dissipated to the bath is given by

$$Q = -T\,\Delta\mathcal{S}(S|B),$$

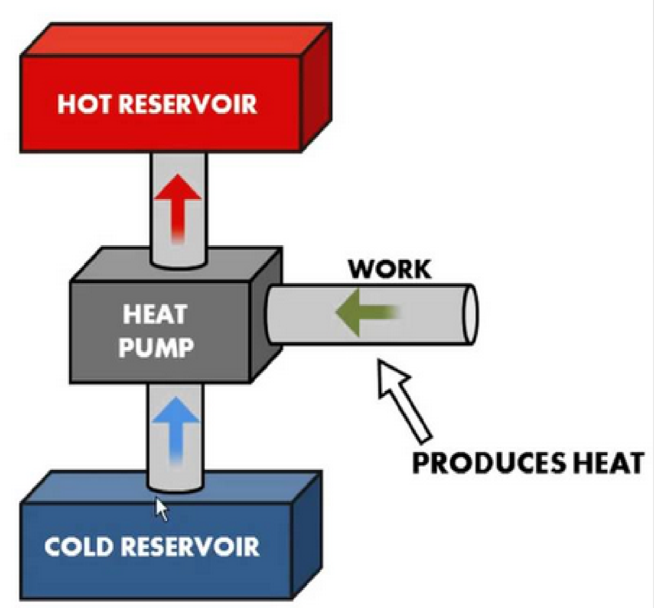
the maximum extractable work from the system and correlation is

$$W_{S+cor} = -\Delta\mathcal{F} = -\left(\Delta E_S - T\,\Delta\mathcal{S}(S|B)\right),$$

and the work performed on the bath is $W_B = \Delta E_B - kT\,\Delta\mathcal{S}_B$.

Anomalous heat flow as refrigeration; Carnot

Energy preserving operations $\rho_{HC} \xrightarrow{\Lambda^{ep}} \rho'_{HC}$



$$\Delta E_H - T_H \Delta \mathcal{S}_H \geqslant 0$$
$$\Delta E_C - T_C \Delta \mathcal{S}_C \geqslant 0$$

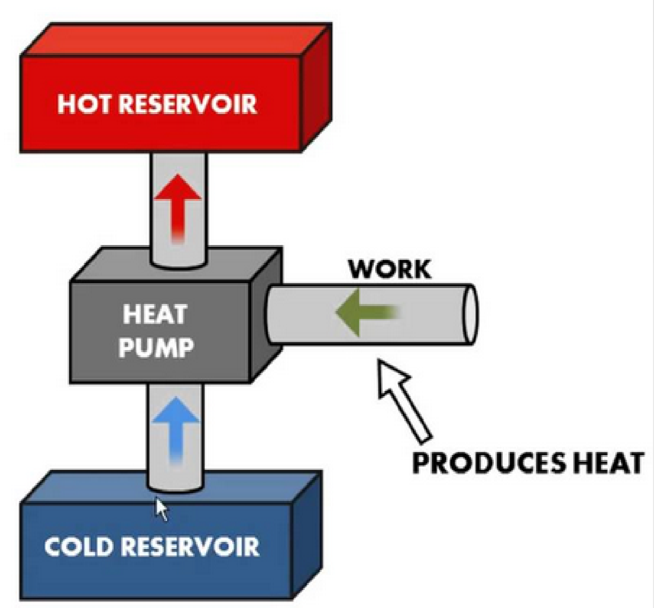
$$T_H \Delta \mathcal{S}_H + T_C \Delta \mathcal{S}_C \leqslant 0$$
$$\Rightarrow T_C \Delta \mathcal{S}_C (T_H - T_C) \leqslant T_C \, T_H \Delta \mathcal{I}(H:C)$$
$$\Rightarrow \mathcal{Q}_C (T_H - T_C) \leqslant T_C \, W_{cor}(T_H)$$

Note, change in correlation $\Delta \mathcal{I}(H:C) = \Delta \mathcal{S}_H + \Delta \mathcal{S}_C$

Anomalous heat flow as refrigeration; Carnot

Energy preserving operations

$$\rho_{HC} \xrightarrow{\Lambda^{ep}} \rho'_{HC}$$



$$\Delta E_H - T_H \Delta S_H \geqslant 0$$
$$\Delta E_C - T_C \Delta S_C \geqslant 0$$

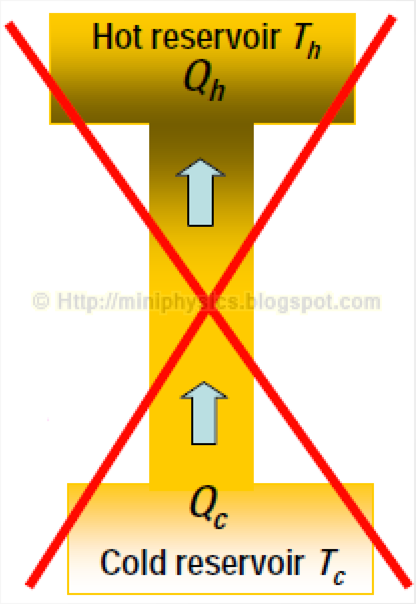
$$T_H \Delta S_H + T_C \Delta S_C \leqslant 0$$
$$\Rightarrow T_C \Delta S_C (T_H - T_C) \leqslant T_C \, T_H \Delta \mathcal{I}(H:C)$$
$$\Rightarrow Q_C (T_H - T_C) \leqslant T_C \, W_{cor}(T_H)$$

Coefficient of performance, for refrigeration (driven by correlation):

$$\eta_{cop} = \frac{Q_C}{W_{cor}(T_H)} \leqslant \frac{T_C}{T_H - T_C}$$

Universal Clausius statement

No process is possible whose sole result is the transfer of heat from a cooler to a hotter body, where the work potential stored in the correlations does not decrease.



Such heat flow could occur, if

$$\Delta \mathcal{I}(H:C) < 0$$

Universal Kelvin-Planck statement

No process is possible whose sole result is the absorption of heat (as per new definition) from a reservoir and complete conversion of it into work (both in system and form of correlation).

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$$\rho^c_{SB} = \sum_i p_i\, |i\rangle\langle i|_S \otimes |i\rangle\langle i|_B \xrightarrow{U^c_{SB}} |\phi\rangle\langle\phi|_S \otimes \sum_i p_i\, |i\rangle\langle i|_B$$

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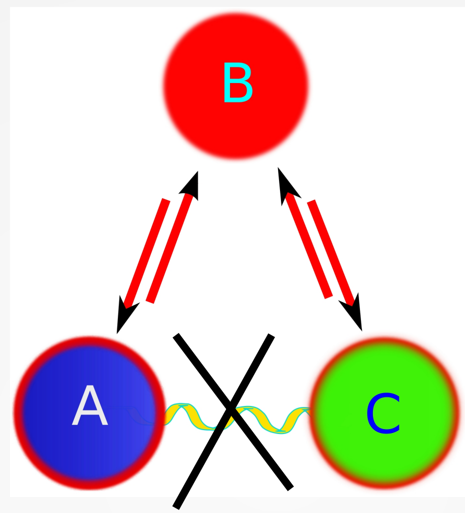
$$|\phi\rangle_S = \sum_i \sqrt{p_i}\, |i\rangle_S ,$$

$$\Delta E_S = 0$$

Universal zeroth law



Universal zeroth law:

A collection $\{\rho_X\}_X$ of states is said to be in mutual thermal equilibrium with each other, at a certain temperature T, if and only if no work can be extracted from any of their combinations under entropy preserving operations.

So far…

- *Laws of TD breakdowns in presence of correlations, even in presence classical correlations*
- *Universal Landauer's principle: connecting heat with "QUANTUM" information, using conditional entropy*
- *Redefined heat based on universal Landauer's principle.*
- *Quantified work potentials stored in correlations*
- *Universalize laws of TD, incorporating such potentials*
- *Anomalous heat flow, as refrigeration*

Acknowledgements

Arnau Riera

Maciej Lewenstein

Andreas Winter

Manabendra Nath Bera, Arnau Riera, Maciej Lewenstein, Andreas Winter,

Universal Laws of Thermodynamics,

ArXiv:1612.04779 (2016),

accepted in NATURE COMMUNICATIONS

Next...

- *One-shot thermodynamics (in preparation)*
- *Thermodynamics with finite-bath, or bath-temperature independent thermodynamics (arXiv:1707.01750). Featured in NATURE 551, 20–22, (02 November 2017).*
- *Quantum heat-engines*
- *Quantum batteries (in preparation)*

"I think of my lifetime in physics as divided into three periods:

In the first period ...I was convinced that

EVERYTHING IS PARTICLE,

I call my second period

EVERYTHING IS FIELD,

now I have new vision, namely that

EVERYTHING IS INFORMATION."

---Wheeler

Thank you