

Black holes and membranes: From jets to violation of cosmic censorship

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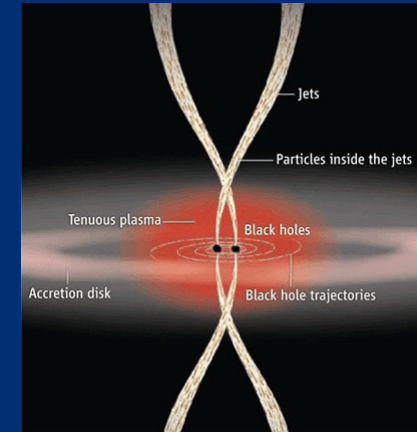
Outline:

2 fronts for black holes and ‘membranes’

- Astro:

Jets from binary black holes
(a new twist on an old story)

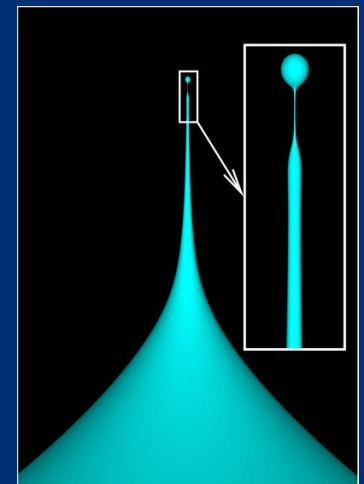
[Science V329, 927 (2010), PRD V82, 044045 (2010), PNAS 2011]



- Black hole stability in higher dims:

Violating cosmic censorship and then some
(resolution to a few old questions)

[PRL V105, 101102 (2010)]



*In both cases, strong gravity/high dynamics require full GR (at least)
→ Numerical simulations needed.*

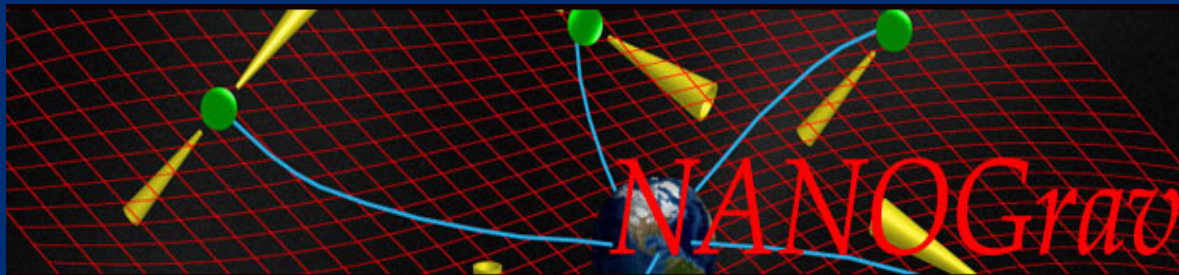
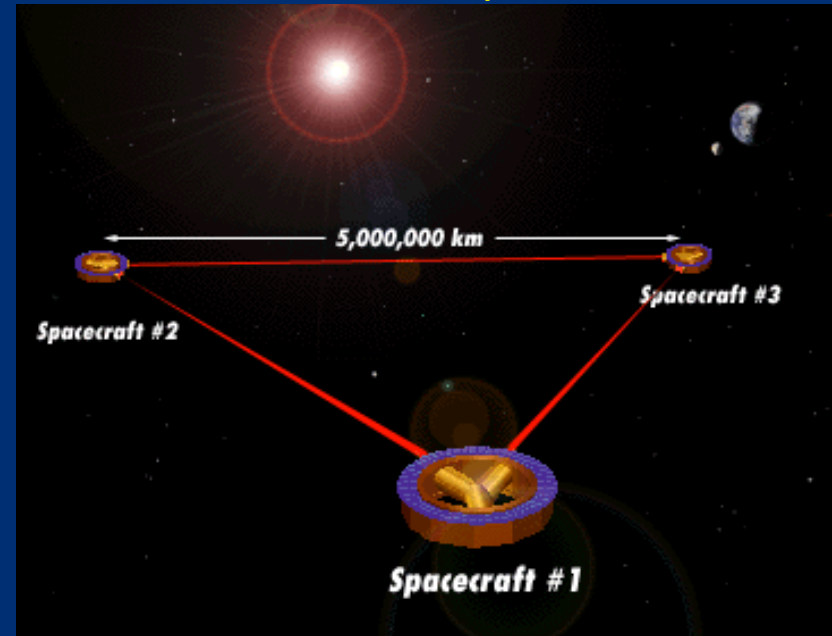
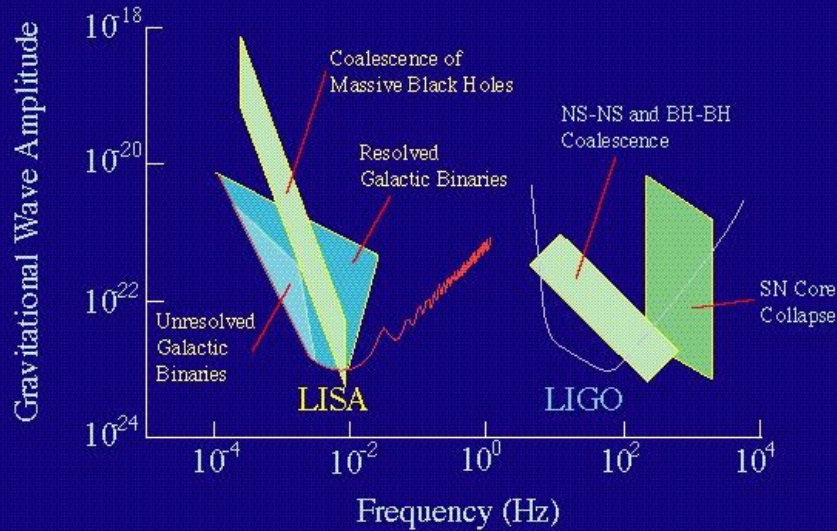
Membranes, then and now

- Membrane paradigm (80s): [Damour,Thorne,McDonnald,Price]
 - Dynamics of a “stretched horizon” is governed by the Navier-Stokes equations for a relativistic fluid with very low shear-viscosity $\sigma=1/16\pi$
 - Electrodynamics behavior of “stretched horizon” understood as a (poor) conductor with resistance $R = 1/4\pi$
- Recently, within the context of AdS/CFT [Bhattacharyya et al.;Bredberg et.al.] and also AF [Emparan et al]
 - ‘Formal’ expansion, recasting Einstein Eqns as Navier-Stokes eqns + ‘extrinsic’ eqns governing given timelike surfaces [for a particular fluid]

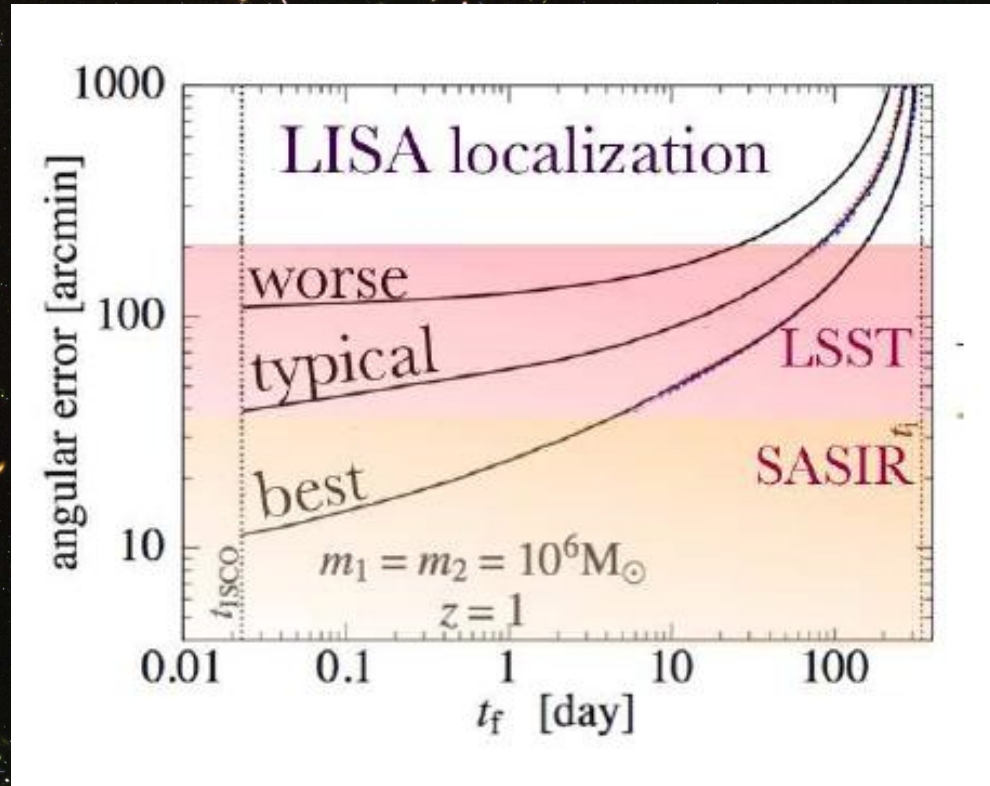
Analogies, if trusted can be used to obtain a natural handling of some problems and perhaps speculate/guess interesting behaviors

GWs and 2(?) detectors... LISA & PTA

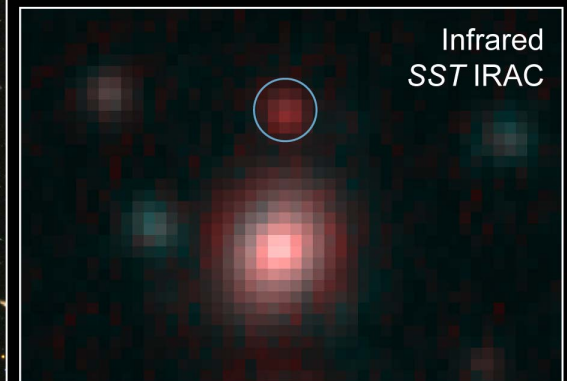
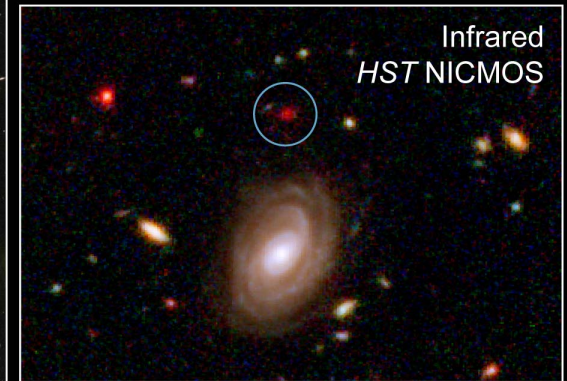
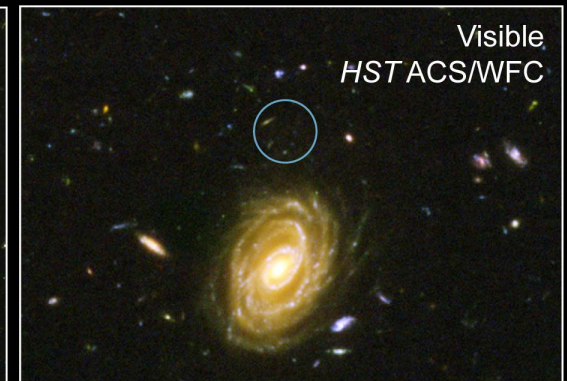
(see talks by: Thorne, Schutz, Whitcomb)



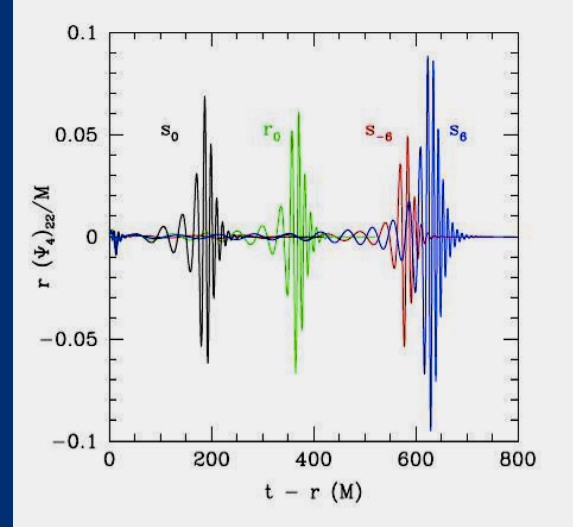
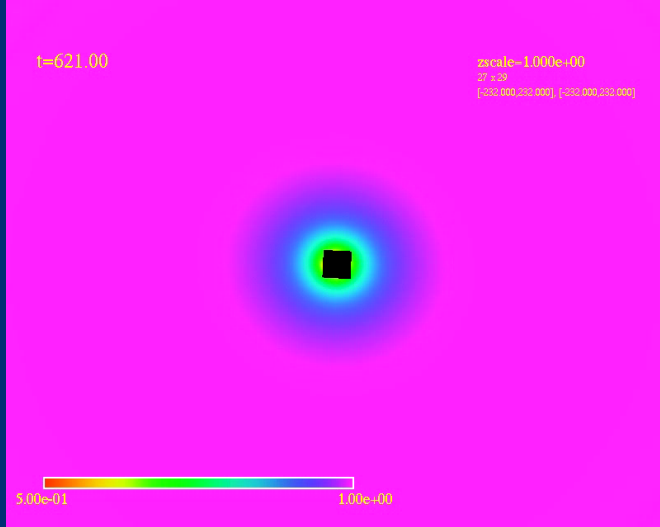
BBH waveforms for both these options are under control
-- in the process, interesting physics dug out (eg. energetics, recoils, etc)



(arcmin² << deg²)



Distant Galaxy in the Hubble Ultra Deep Field • HUDF-JD2
Hubble Space Telescope ■ ACS/ WFC

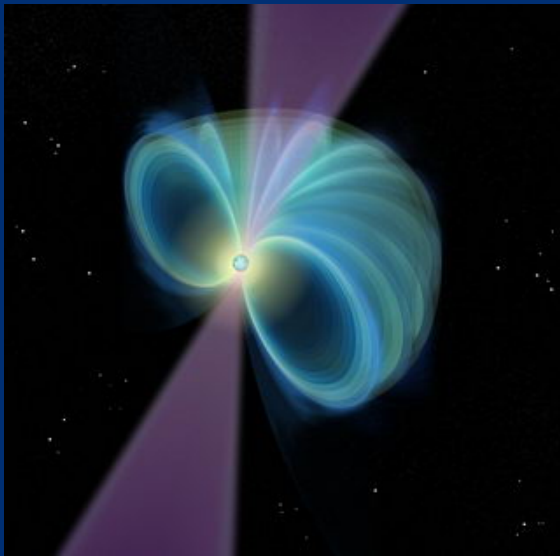


- Can we get bhs to do more? (recall $L_{\text{GW}} \sim 10^{23} L_{\text{sun}}$)
 - Can the merger dynamics give rise to observable electromagnetic counterparts?
 - Going deeper into the physics of different systems
 - Aid in the detection (localization) of the source
 - Confronting gravitational and electromagnetic based knowledge. i.e. Luminosity distance vs redshift diagram.
 - *lots of activity in the past few years on this front, still in early stages though.
 - Need to understand what can produce the counterparts

[work with Palenzuela, Liebling, also Neilsen, Anderson, Hirschmann, Thompson, Hanna]

Where to we go from here?

- Need to find what's the right model
 - microphysics, EM fields, what's outside compact objects? (This is tough, but nature doesn't care what we think...)
- So... let's start simple... consider a star and its dipole... Pulsars radiate.... Dipole radiation?



$$L \sim B^2 \Omega^4 R^6 \sin(x)^2$$

*(but this isn't right,
 $L \sim B^2 \Omega^4 R^6 [1 + \sin(x)^2]$)*

Spitkovsky 2006

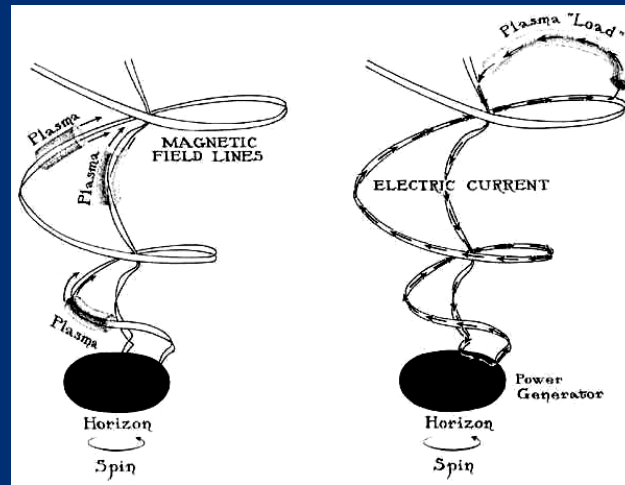
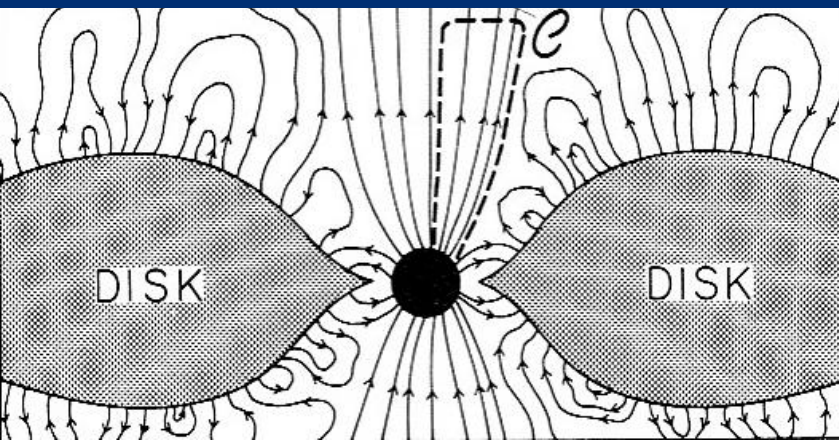
Binary black holes as blenders.

A new spin on an old story

How does the curvature/dynamics influence EM fields?

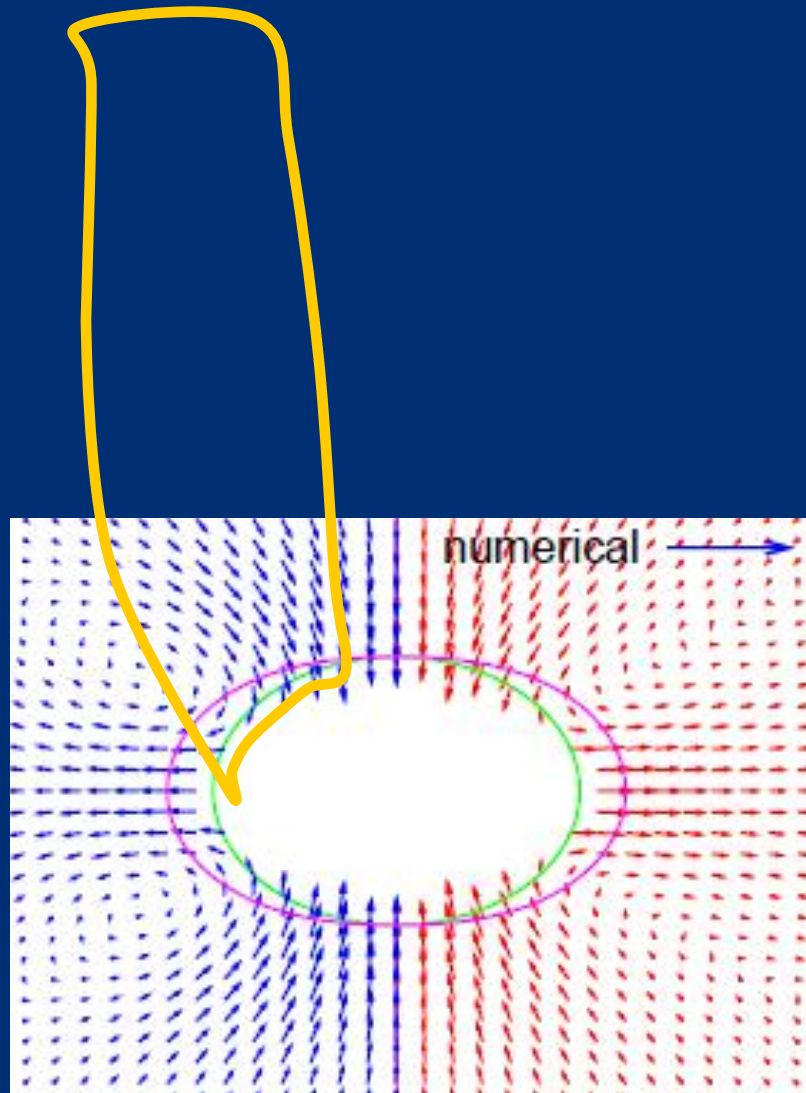


- Blandford-Znajek. Emission mechanism for Kerr bh's surrounded by magnetic fields (anchored by an accretion disk)
- Stray charges accelerate \rightarrow photons \rightarrow pair production \rightarrow cascade. BH becomes surrounded by a tenuous conducting plasma with little inertia
- Blandford-Znajek: BH acquires an induced charge distribution, bh rotation provides an EMF with $V \sim B a \rightarrow L \sim (Ba)^2$



[Goldreich-Julian,
Blandford-Znajek]

Basic picture from the membrane paradigm



BH: (poor) conductor

Battery: Black hole's rotation

Plasma to close the circuit

Far load: to dissipate energy

$$L \sim B^2 a^2$$

However, this is just a picture, does it hold? Need full solution to compare against

Approach: Force-free electrodynamics

$$\nabla_a T^{ab} = 0 \quad \rightarrow \quad \nabla_a T^{ab}_{(\text{fluid})} = -\nabla_a T^{ab}_{(\text{em})} = -F^{ab} J_a$$

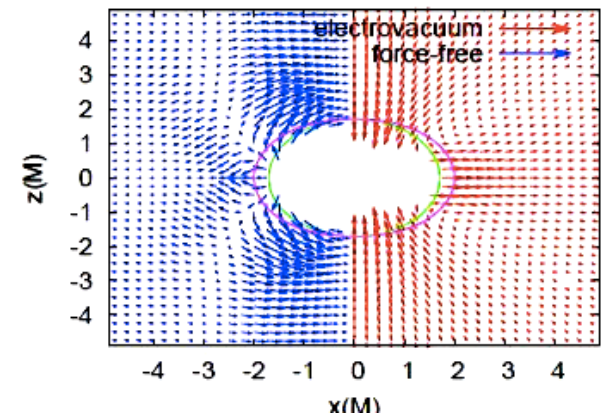
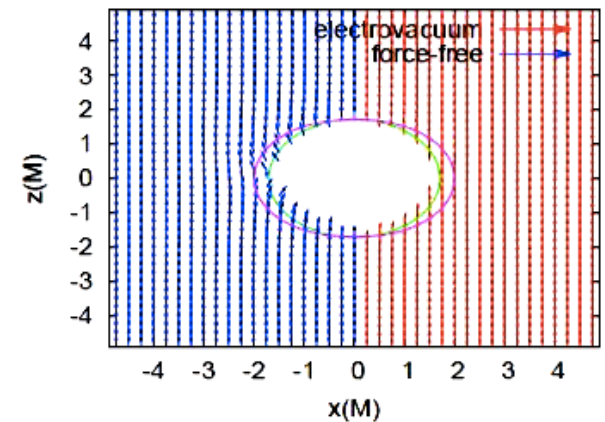
$$\text{if } \rho, P \ll B^2 \quad \text{then} \quad \nabla_a T^{ab}_{(\text{fluid})} \ll F^{ab} J_a \approx 0$$

$$q \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0 \quad \rightarrow \quad \mathbf{E} \cdot \mathbf{B} = 0, \quad \text{and non-zero } \mathbf{J}$$

System can thus be studied in an “effective way”

- plasma supplies charges/currents and enforce $\mathbf{E} \cdot \mathbf{B} = 0$
- furthermore, fields can carry charged particles, and establish a circuit

*Need to solve this problem, what can we expect that is interesting/relevant?
-- and, can we guess what to expect?*



- **IF** analogy can be pushed further, there is little special about BH's rotation, any relative motion of conductor wrt ambient magnetic field would give and EMF
- SMBH merger will give such scenarios
 - Prior to merger, 2 bhs orbital motion inside the circumbinary disk region
 - After merger final BH rotates, but also might have a velocity due to recoil
- Can this intuition be confirmed? And connection further exploited?

• we knew. $L \sim B^2 a^2$ in the aligned

case [finer version

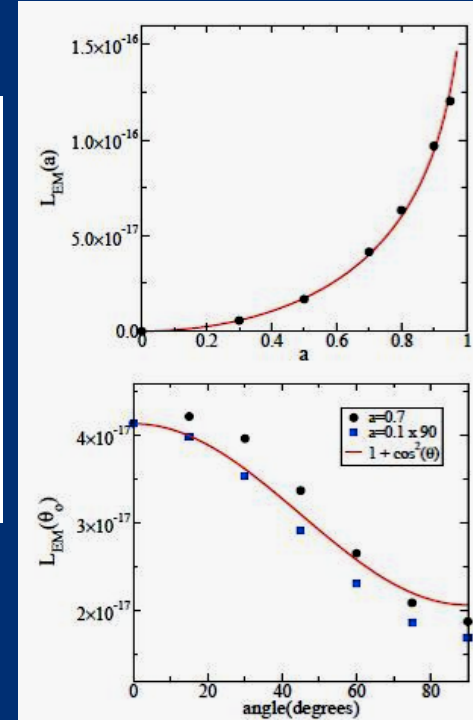
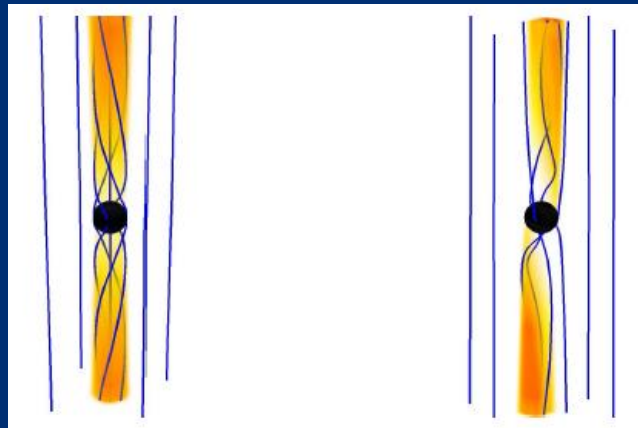
Tchechovskoy, Narayan, McKinney 2010].

• For misaligned case?

- Poynting flux still there, along B

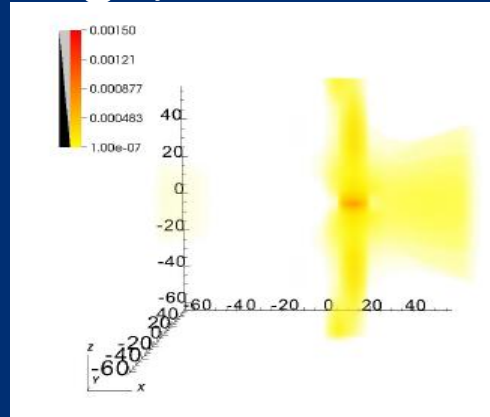
- $L \sim B^2 a^2 (1 + \cos^2)$

(can be predicted using Damour 74 + mp!)



What if it moves?

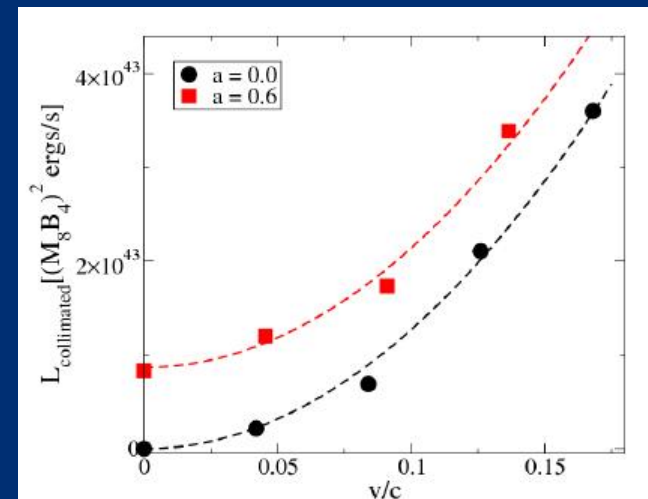
- E.g. after black holes merge, individual black holes prior to merger.



- Where from? From membrane paradigm \rightarrow BH is a conductor. If moving through a B field, induce $E \sim v \times B$
 $\rightarrow EMF=V \sim (vB)$; $L \sim V^2$

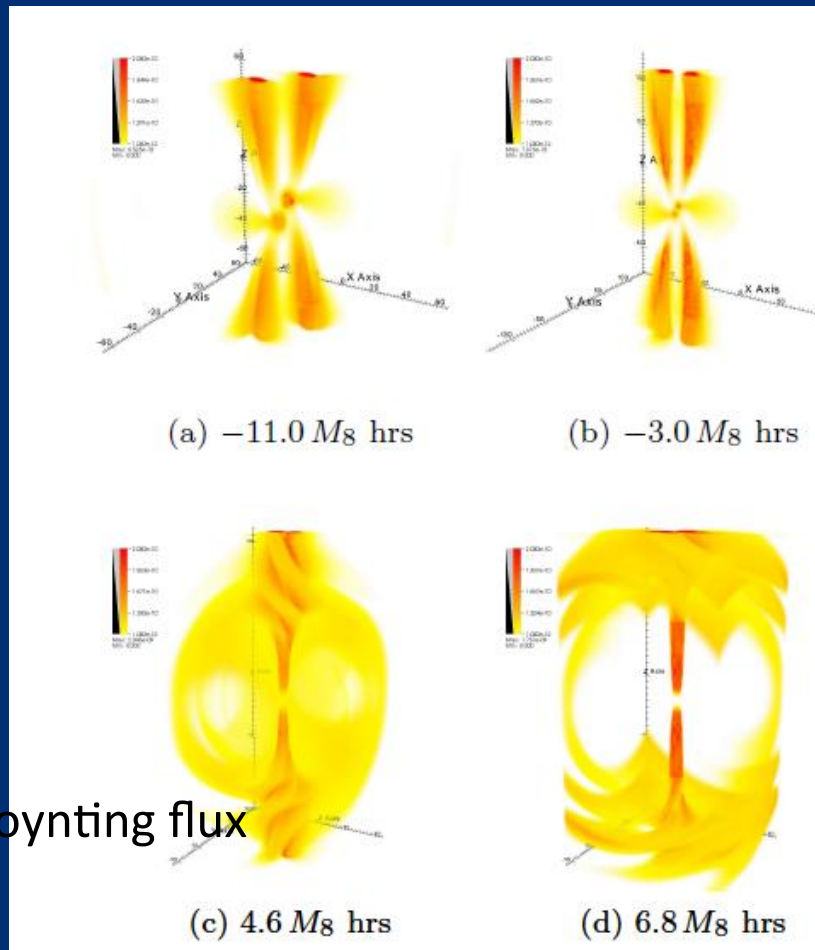
- Thus, $L \sim B^2 v^2$

(Can be predicted using theory of satellite propulsion Drell,Foley,Rudderman 65!)

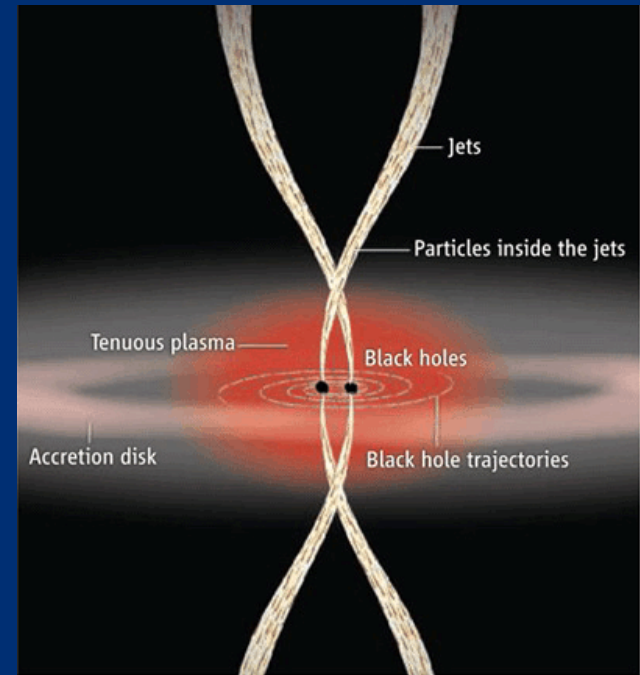


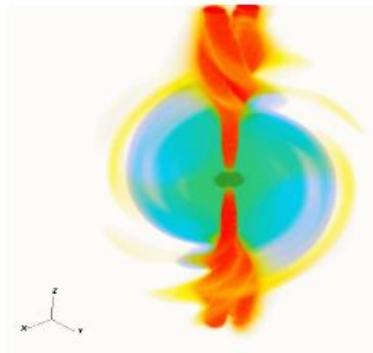
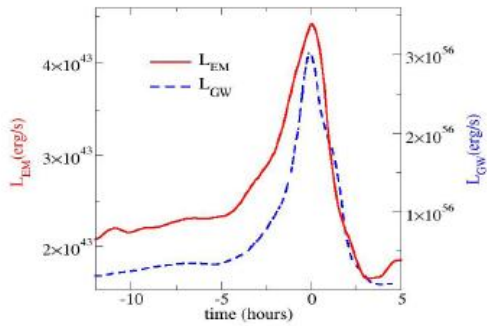
Onto the binary case

- Orbit \rightarrow Black holes move through B. Hall effect analogue.
- As in previous cases, 'circuit' can be established due to charge separation
- Thus, expect Poynting flux through orbiting stages. Also contribution from standard BZ .



Poynting flux





Putting all together:

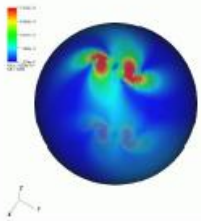
$$L \sim (1 [a/0.6]^2 + 100 v^2) 10^{43} \text{ ergs } [M_8 B_4]^2$$

- * EM flux acts as a “spacetime tracer”
- * Can exploit ‘standard’ BBH results to predict much of the EM flux behavior

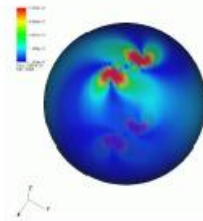
Multimessenger? : LISA & PTA for gravity waves

EM observations? For $10^4 G$, $10^8 M_\odot$ flux $\sim 10^{43-44}$ ergs. IF Poynting flux energy efficiently transferred to observable emissions, interesting pre/post merger observations possible; to $z=1$?

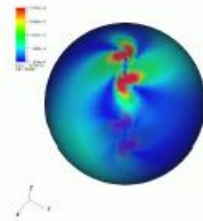
How about Ligo/Virgo sources?



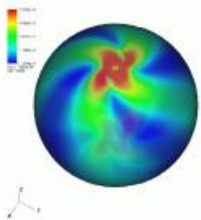
(a) $-8.2 M_8$ hrs



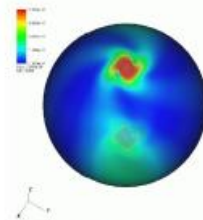
(b) $-5.5 M_8$ hrs



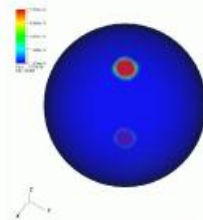
(c) $-3.0 M_8$ hrs



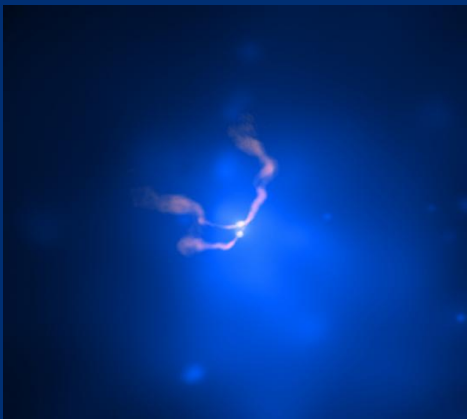
(d) $2.0 M_8$ hrs

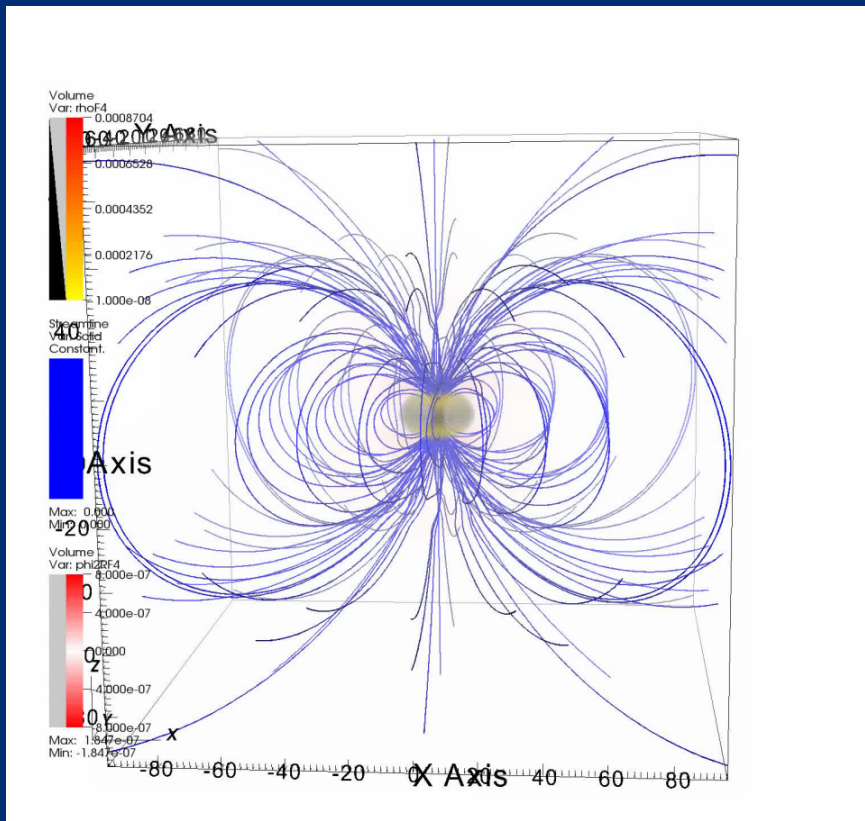


(e) $4.6 M_8$ hrs



(f) $6.8 M_8$ hrs

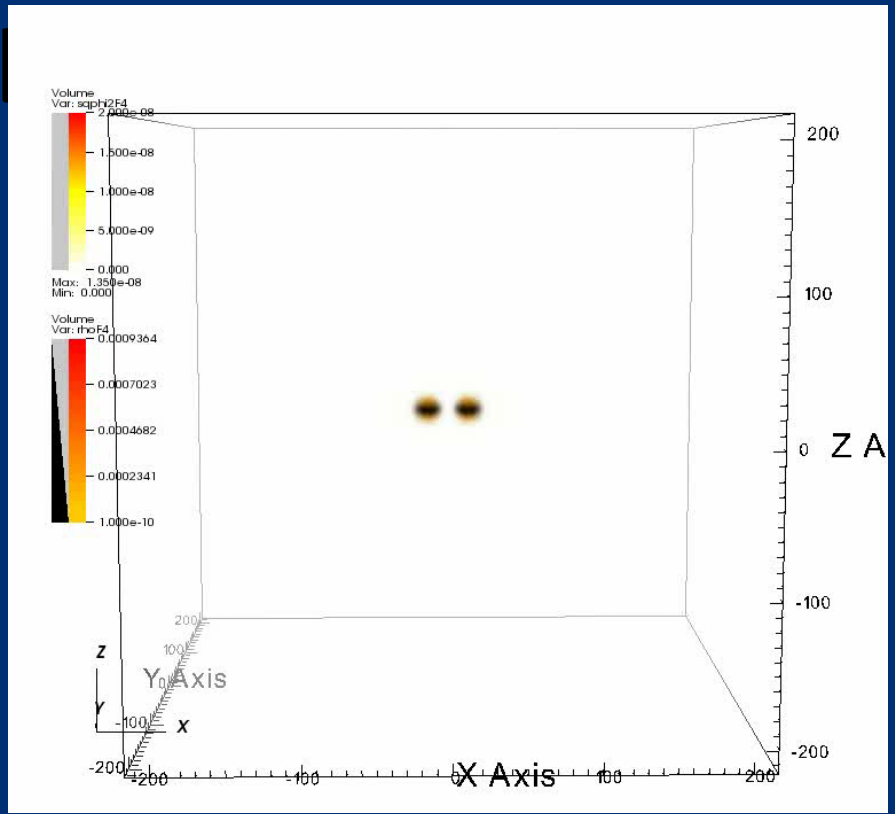




Single NS collapsing:

- Cons of flux $B \sim 1/R^2$
- Cons of ang mom, $\Omega \sim 1/R^2$
- Pulsar spindown $\rightarrow L \sim 1/R^6$

(see talks by Shibata, Bhattacharya)



Binary NS

- Magnetosphere interactions: field direction \rightarrow reconnections prior to merger
- B strength enhancement by collision
- unavoidable collapse to a BH
- field reordering in disk, wind on disk, etc

Stability of (higher dimensional) BHs

[work with F. Pretorius]

if cosmic censorship holds \rightarrow an observer does not need quantum gravity...

If unstable \rightarrow naked singularity? \rightarrow violation of cosmic censorship. Is it generic?

--in 4D: examples are either non-generic or requiring 'unphysical' matter

- At the gravity level alone, $D=4$ is special, but how special?
 - Beyond behavior of orbits and no “Kerr-bound”, there are richer geometries in higher dimensional Ricci-flat Lorentzian manifolds, in particular the zoo of “black objects” - black spheres, rings, strings, saturns, drops, ...
- If string theory is providing the correct path to a consistent theory of nature valid at Planck scales, the universe is fundamentally higher dimensional
- Lots of examples on (holographic) correspondences of string theory (AdS/CFT in particular) to describe many aspects of conventional non-gravitational 4D physical processes in terms of higher dimensional gravity
 - superconductors, superfluidity, quark-gluon plasmas, etc.
 - interestingly, the gravitational dual to these processes studied to date involves *black holes*
(*see talks by Horowitz-Minwalla*)

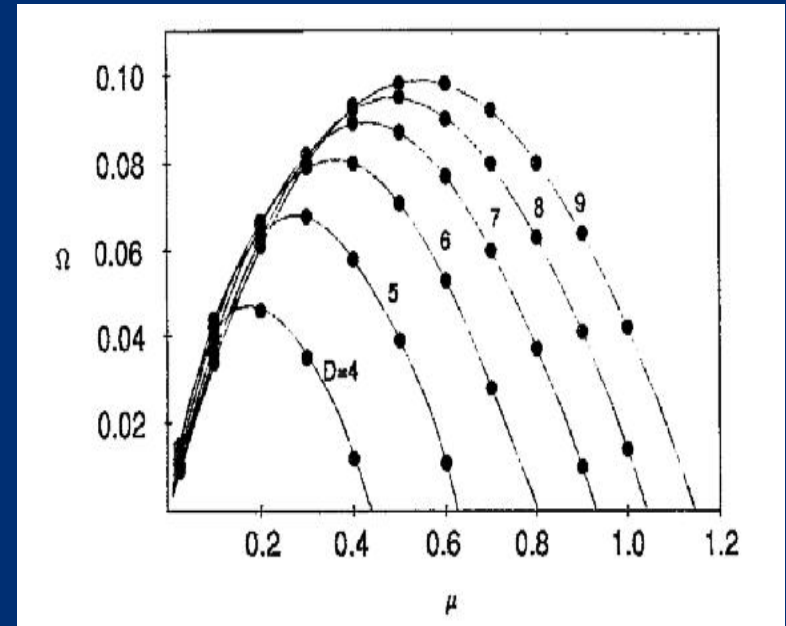
BHs in higher dims: come Black strings

- 1.- Contain singularities
- 2.- Ruled by null-rays
- 3.- Non-unique even in spherical symmetry



Stability? (Gregory-Laflamme 93)

- Black string perturbations admit exponential growth for $L > L_c$ ($L_c \sim 14.3m$)



End-state of the instability?

- Entropic argument:

- above a similar critical wavelength L_c the total *area/length* of a sequence of 5D hyper-spherical black holes, each a distance L_c apart, is *greater* than a 5D black string with the *same* total mass/length ($M=mL$):

$$\frac{A_{BH}}{A_{BS}} = \sqrt{\frac{8}{27\pi} \frac{L}{m}} \Rightarrow L_c = \frac{27\pi}{8} m \approx 10.6m$$

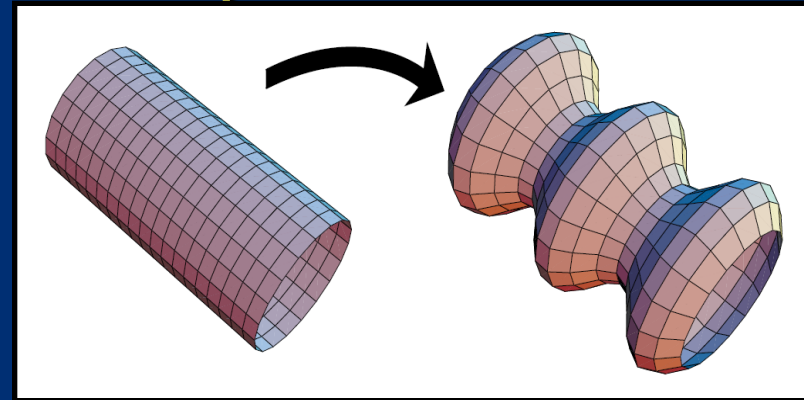


Image from:
R. Gregory and R. Laflamme,
Nucl.Phys.B428 (1994)

Conjecture: Black strings will bifurcate $\parallel \dashrightarrow \}} \dashrightarrow \bigcirc \dashrightarrow \bigcirc$

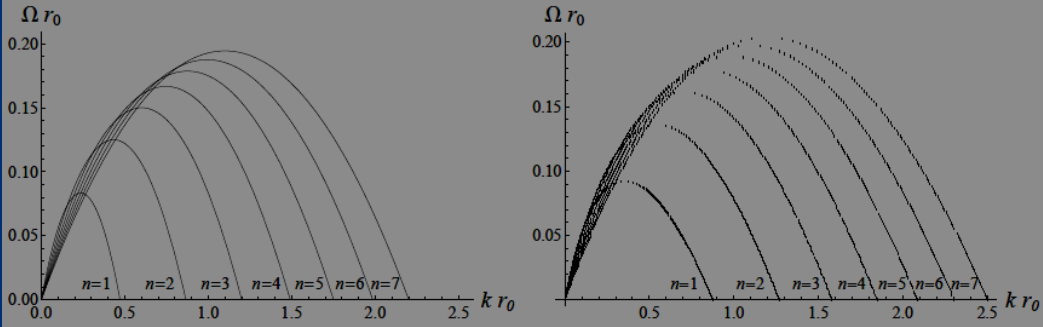
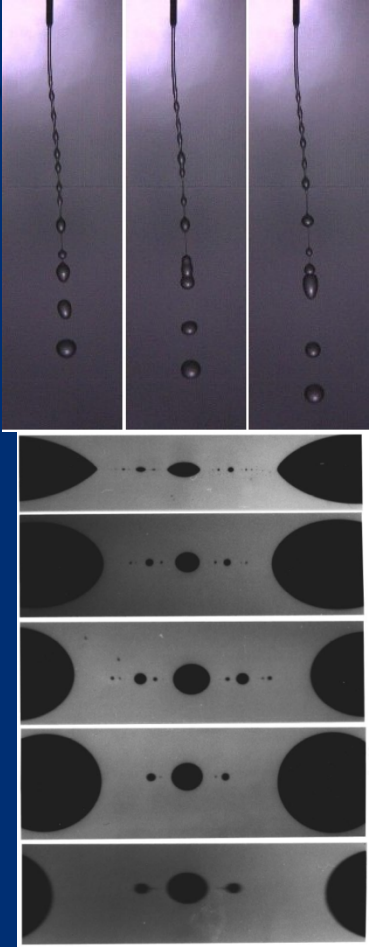
- But, this cannot happen without the appearance of a naked singularity (the “no-bifurcation” theorems still hold in 5D) \rightarrow a *generic* example of cosmic censorship violation in higher dimensional gravity

What is the end-state of the system?

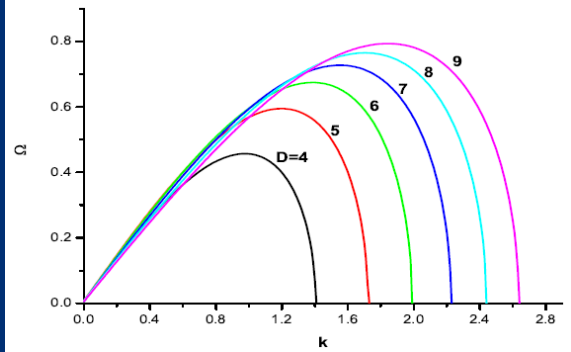
- Horowitz and Maeda [*PRL 87, 131301 (2001)*] proved that black string horizons cannot shrink to zero cross-sectional radius in *finite affine time* of the generators of the horizon
 - based on this, they conjectured the end-state would be a new, static, non-uniform solution with the same topology as the black string
 - this spurred a search for such solutions; a couple were found [*S. S. Gubser, CQG. 19, 4825 (2002)*, *T. Wiseman, CQG. 20, 1137 (2003)*, *E. Sorkin, PRD74:104027 (2006)*]; however, these solutions have less entropy (area) than the uniform black string, so could not be the end-state of the GL instability
- Also, other more radical conjectures (e.g. spacetime collapses on itself)
- 2003... Numerical attempt. Didn't resolve it

What in the world is the end-state of the instability?

- Further (anecdotal) evidence in favor of the pinch-off scenario gathered in the form of various correspondences between equations governing viscous hydrodynamics and horizon dynamics
 - the membrane paradigm [Damour; Thorne, Price, Macdonald, Eds. (1986)] → Navier-Stokes equations.
 - Cardoso and Dias [PRL 96 (2006)] (right figure) showed that the spectrum of unstable modes of a cylindrical flow of fluid with surface tension, subject to the Rayleigh-Plateau instability, was quantitatively similar to that of black strings
 - more recently developed frameworks [Bhattacharya et al., JHEP 02 (2008), R. Emparan et al. JHEP 03 (2010)] established similar relationships; [J. Camps et al., arxiv:1003.3636 (2010)] (left figures) used the “black folds” approach to re-derive the Gregory-Laflamme spectrum of modes to leading order



unstable sound waves in effective black string fluid (left) compared to GL modes (right)



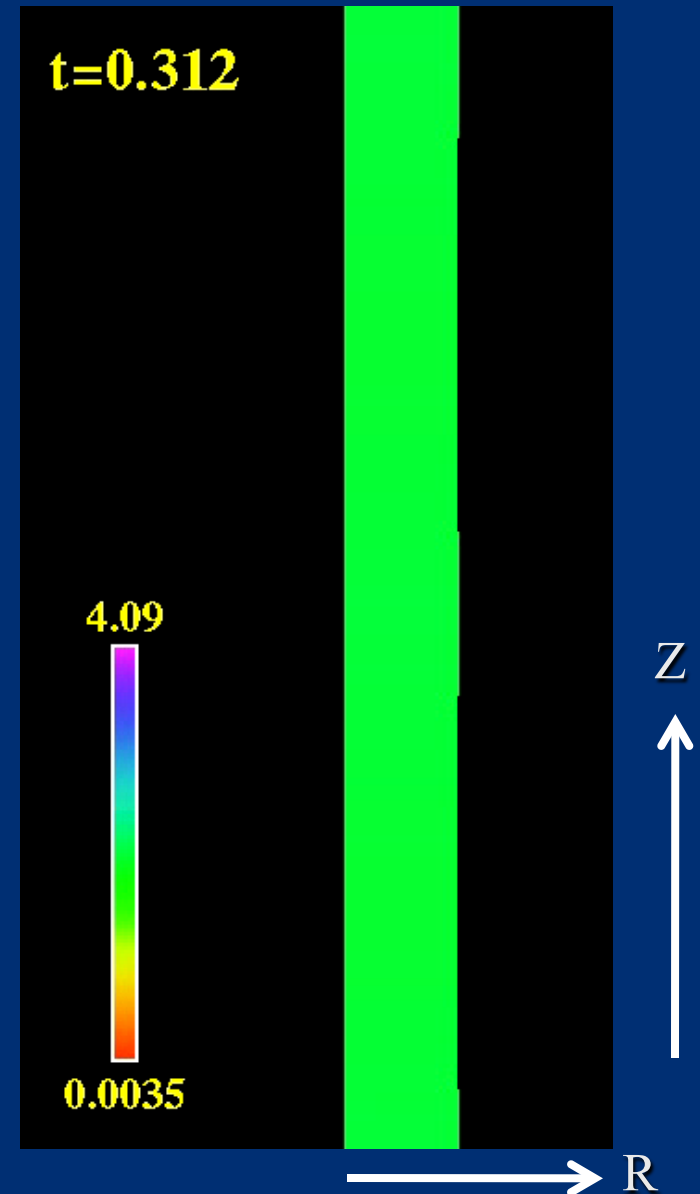
Rayleigh-Plateau analogue

Sealing the fate.....

- New numerical study...
- map the geometric 1D shape of each $t=x=y=constant$ slice of the apparent horizon to a flat (R,Z) Euclidean space; i.e. in parametric form

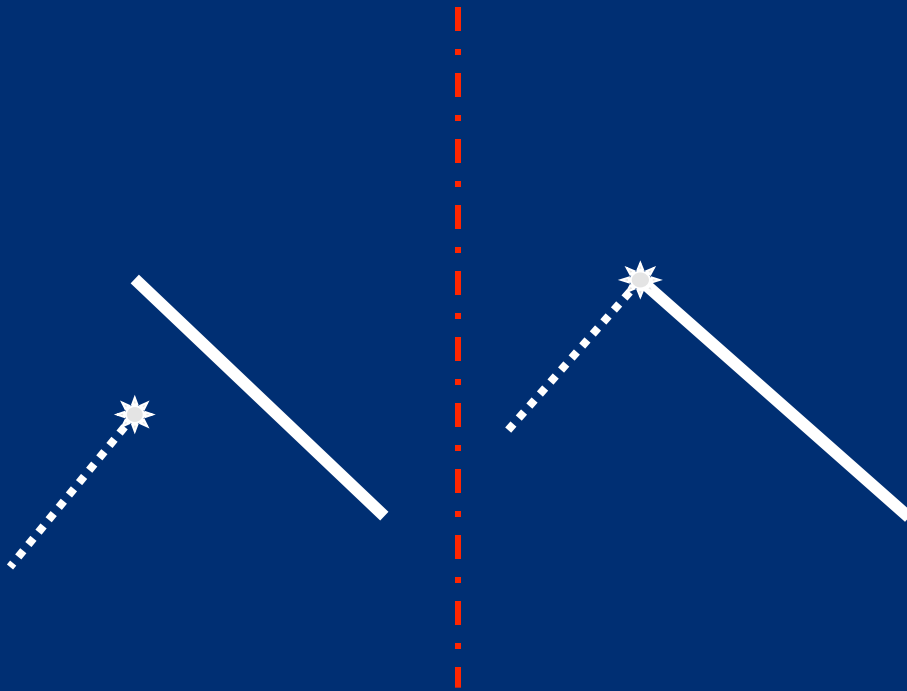
$$(R, Z) = (R(\xi), Z(\xi))$$

- $R(\xi)$ is the areal radius of that point on the horizon, and $Z(\xi)$ is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
- the movie shows this curve spun around $R=0$ to form a surface for visual aid
- color is mapped to R
- (note that time is “slowing down” !)

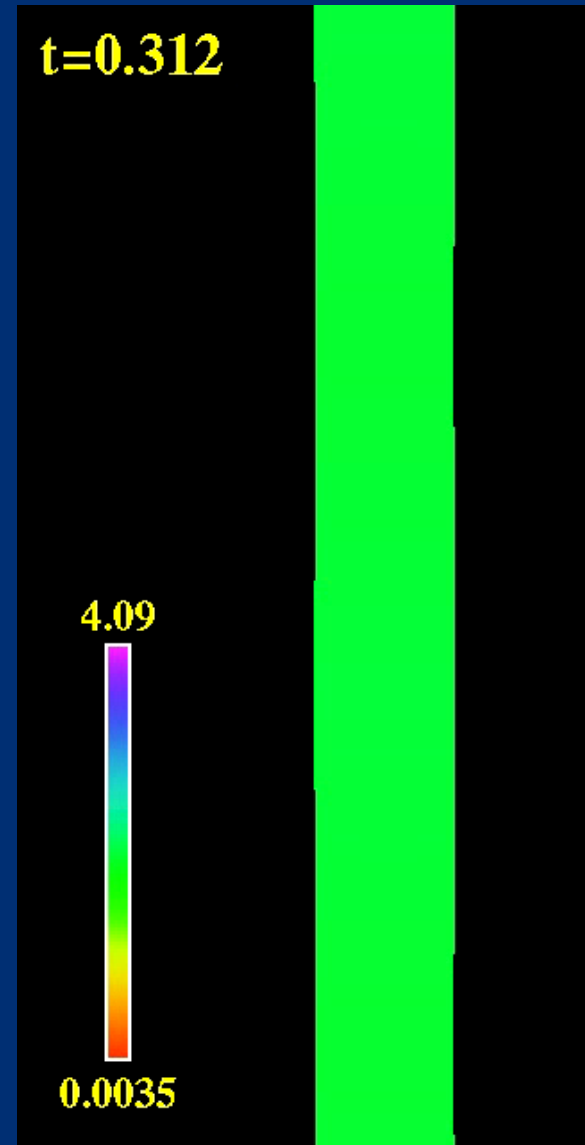


zooming in, affine time

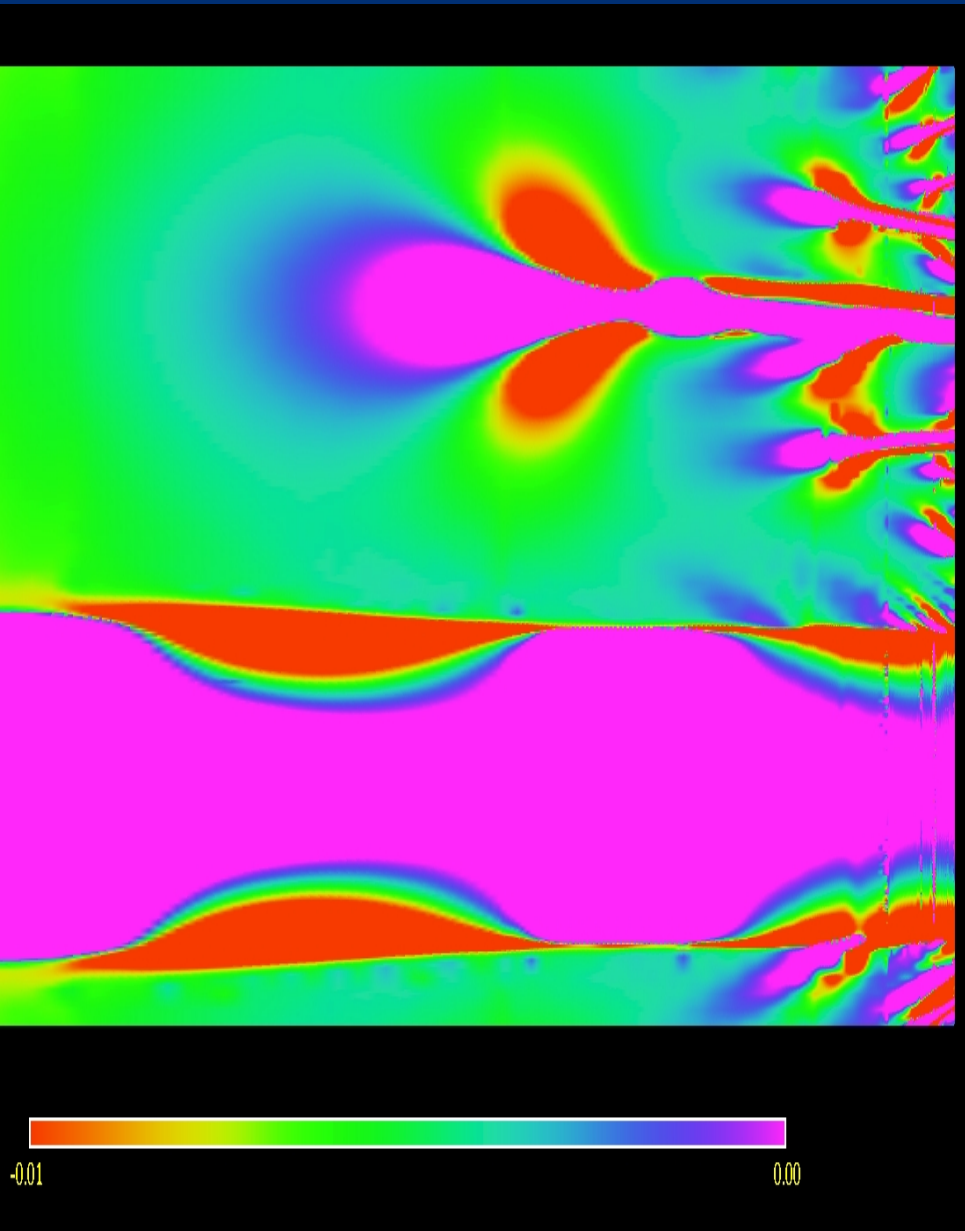
- *NO tension with Horowitz-Maeda result*
affine time \rightarrow infinity
 $\lambda \sim m e^{40}$ (at $t \sim 165m$ already)



What is it? ...let's check some more...



AH behavior..



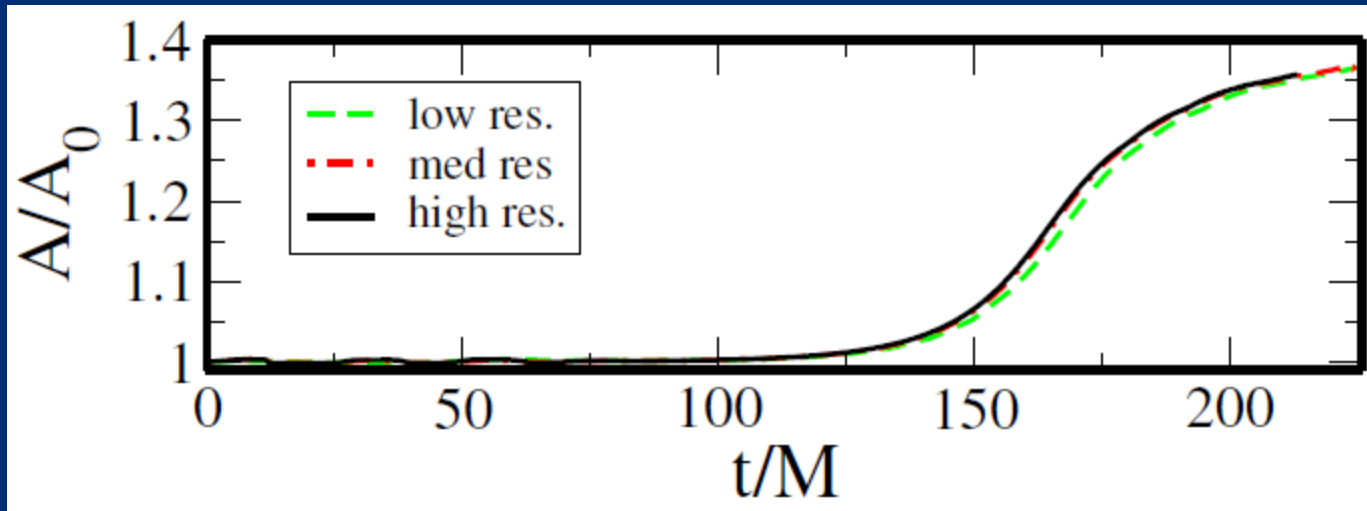
Hausdorff dimension?

$$L(t) = (t_c - t)^{(1-d)}$$

$$d \sim 1.05$$

(i.e. simple fractal structure)

Apparent Horizon Area



- Resolutions chosen via specification of the maximum estimated truncation error τ , from τ_0 , $\tau_0/8$ to $\tau_0/64$ (“low” to “high”)
- For this configuration, ignoring the (small amount of) energy from the initial perturbation, a sequence of spherical black holes (one per period) with the same energy as the initial string has an area

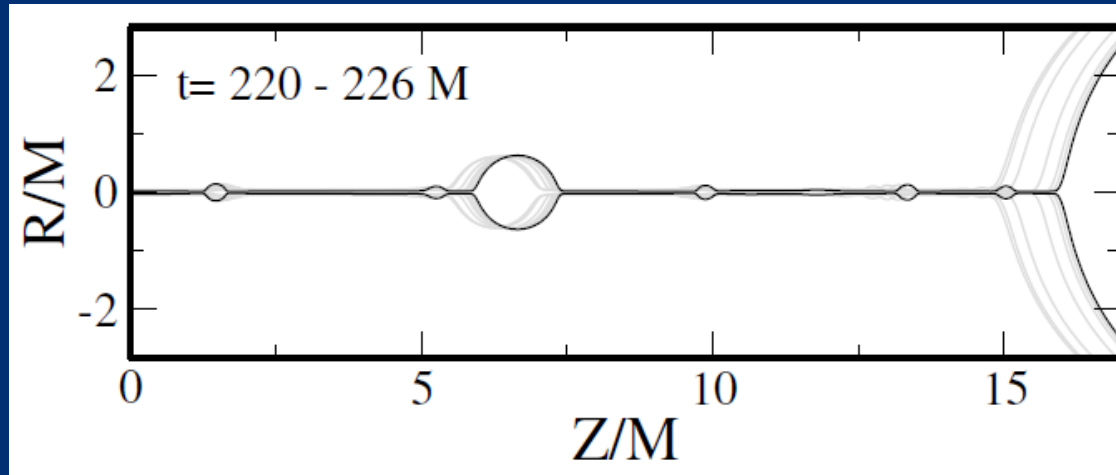
$$A_{BH} / A_{BS} = 1.374$$

The lowest resolution simulation, which has run the longest in physical time, has reached a value

$$A / A_0 (t = t_{end}) \approx 1.369 \pm 0.005$$

- This is consistent with the argument that the dynamics of the instability is such as to saturate the entropy of the spacetime. “No mass is left behind”

Apparent Horizon Dynamics



- At late times the horizon certainly *looks* like it can be described as a sequence of spherical black holes connected by string segments; to quantify this a bit, we evaluate the following curvature invariants on the horizon:

$$I = R_{abcd} R^{abcd}; \quad J = R_{abcd} R^{cdef} R_{ef}{}^{ab}$$

and construct the following dimensionless scalars

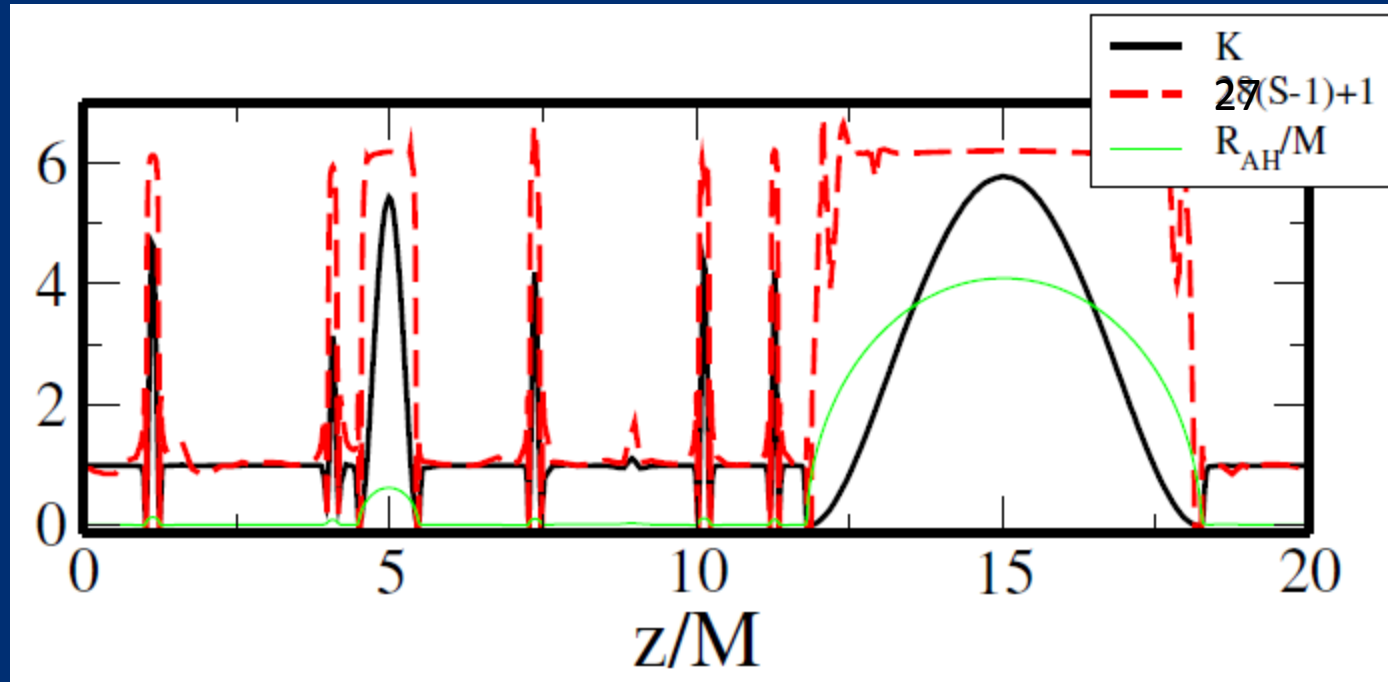
$$K = IR_{AH}^4 / 12; \quad S = 12J^2 I^{-3}$$

which evaluate to the following for the exact black sphere/black string solutions

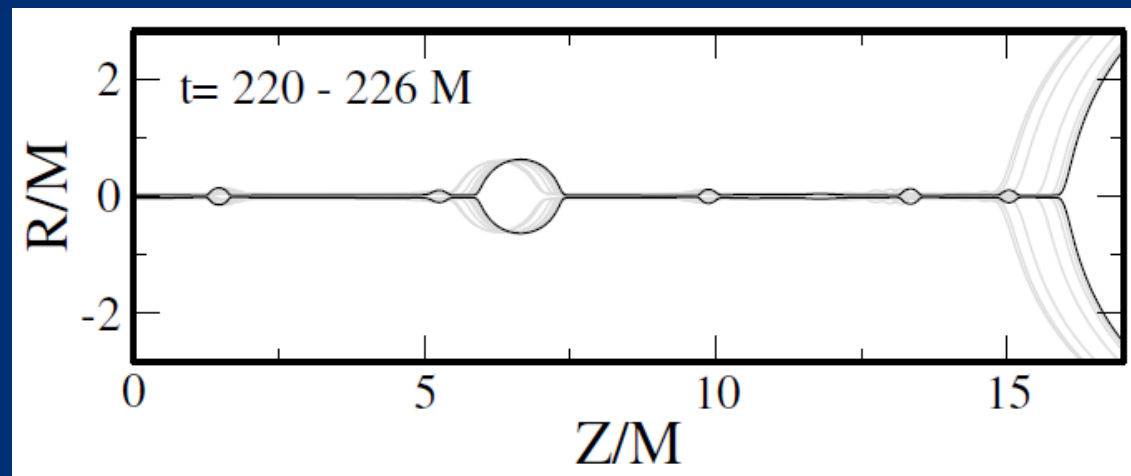
$$K_{BH} = 6; \quad 27(S_{BH} - 1) + 1 = 6$$

$$K_{BS} = 1; \quad 27(S_{BS} - 1) + 1 = 1$$

Apparent Horizon Dynamics



Invariants above evaluated on the apparent horizon at the last time step of the (medium resolution) simulation depicted to right



Properties of satellites and string-segments

- Therefore, the spheres-connected-by-string-segments interpretation seems reasonable. With that interpretation, and that evolution proceeds through a sequence of unstable epochs, we extract the following properties from the horizon:

Gen.	t_i/M	n_s	$R_{s,i}/M$	$R_{AH,f}/M$	$L_{s,i}/R_{s,i}$
1	118.1 ± 0.5	1	2.00	$4.09 \pm 0.5\%$	10.0
2	203.1 ± 0.5	1	$0.148 \pm 1\%$	$0.63 \pm 2\%$	$105 \pm 1\%$
3	223 ± 2	> 1	$0.05 \pm 20\%$	0.1 – 0.2	$\approx 10^2$
4	≈ 227	$> 1(?)$	≈ 0.02	?	$\approx 10^2$

Gen: generation number

t_i : time of initial satellite formation (defined to be time when the areal radius has grown to 1.5 times that of the surrounding string-segment)

n_s : number of satellites that form

$R_{s,i}$: radius of local string segment

$R_{AH,f}$: radius of satellites by the time the simulation was stopped

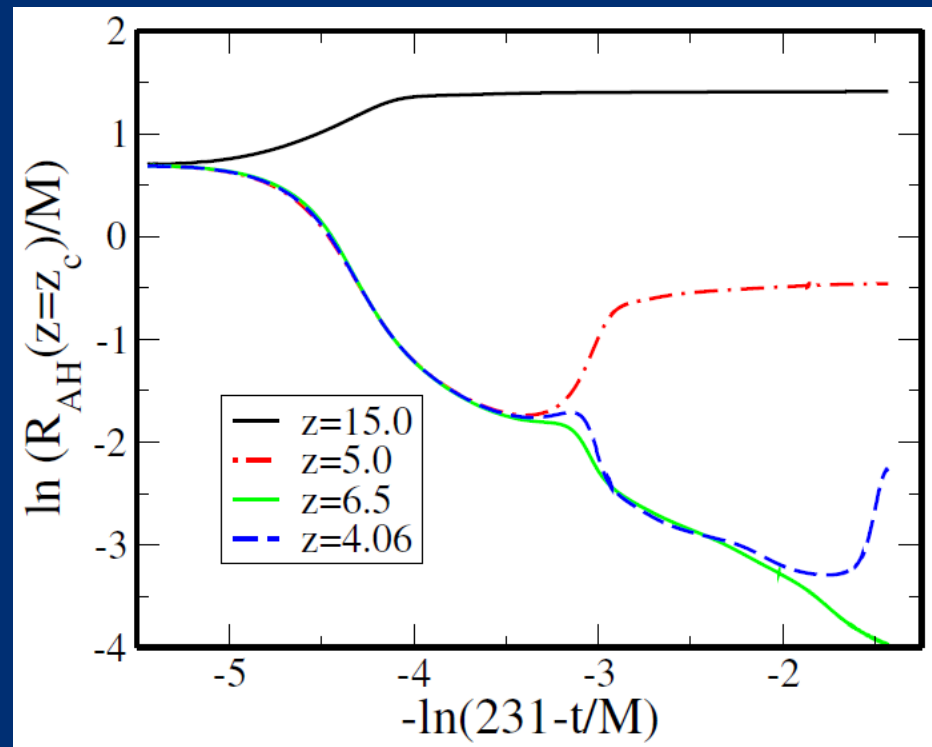
$L_{s,i}/R_{s,i}$: Ratio of length to radius of local string-segment (GL critical ratio ~ 7.2)

Properties of satellites and string-segments

- The dynamics of the apparent horizon also suggests that the instability unfolds in a self-similar manner; if so, transforming to logarithmic coordinates in space and time should reveal this more clearly
- The following shows $R_{AH}(t, w=const.)$ at points (roughly coinciding) with the eventual maxima of satellites, and one representative point that is still string-like near the end of the simulation
- Guess at “pinch-off time” by assuming the time scale for each later generation is a constant fraction X of the preceding one, with the exception of the first generation, whose time scale is controlled by the initial data:

$$\Delta T \sim T_0 + \sum_{i=0}^{\infty} T_1 X^i = T_0 + \frac{T_1}{1 - X}$$

from the data in the table, we get $\Delta T \sim 231M$



Consequences/provocations...

- If Cosmic censorship valid: spacetime accessible to observers does not need QG. Otherwise, QG is required to provide the complete observable solution

GL instability shows CS can be violated 'generically' in higher dims (note: many BHs show this instability. Eg. Myers-Perry BHs)

- For this case (and a number of others that can be mapped to it)
 - Naked singularity has 0 mass. Local spacetime would behave as a Hawking evaporating BH (unless higher curvature corrections kick in)
 - Fluid analogue: nothing drastic takes place at pinch off, 2nd solution (bubbles) proceed smoothly → nothing drastic expected in the spacetime
 - Thus, while formally QG is needed, in practical terms not so much → observer does not need to care about quantum gravity

Properties of satellites and string-segments

- In a fluid with tension, the shrinking neck region exhibits a scaling solution of the form [Eggers, PRL 71 (1993); Miyamoto, JHEP 1010 (2010)]

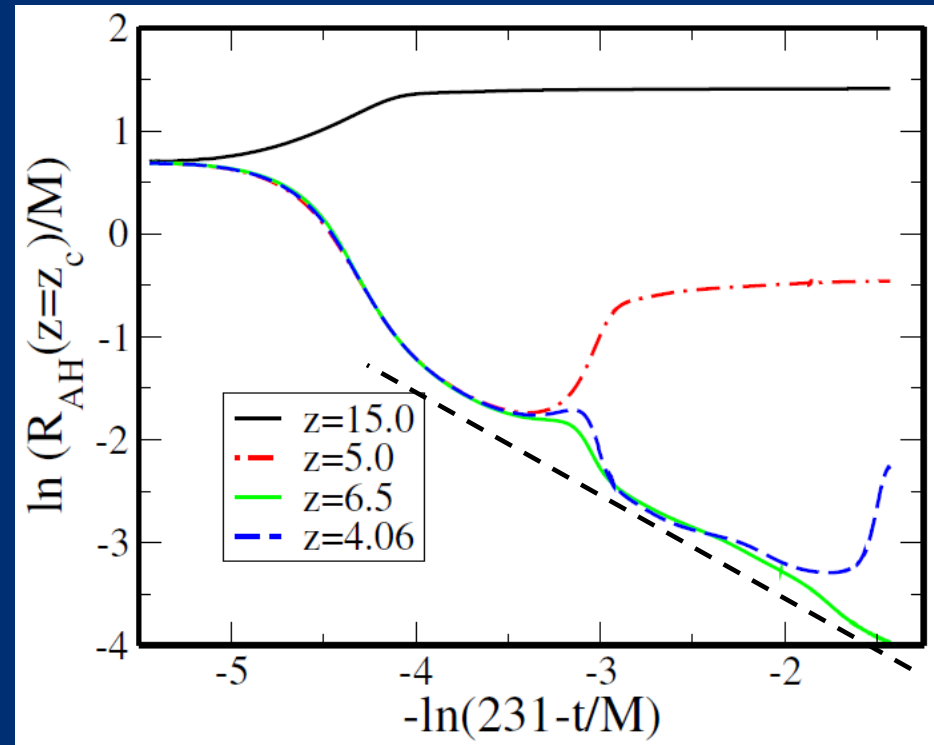
$$r \propto (t_0 - t)$$

or in logarithmic coordinates

$$\frac{d \ln r}{d(-\ln(t_0 - t))} = -1$$

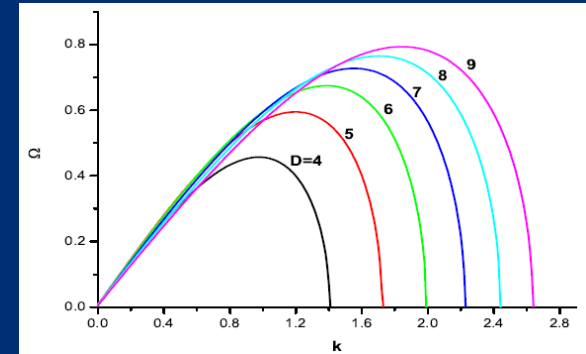
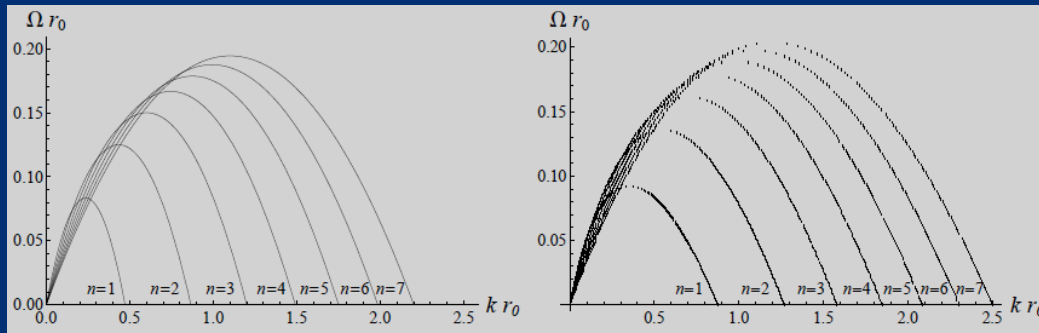
where t_0 is the pinch-off time

- The dashed line overlaid on the figure has slope ~ -1
 - on average seems behavior of thinning string segment seems consistent with a self-similar pinch-off
- *This is the good news...though...*



Not quite the same....

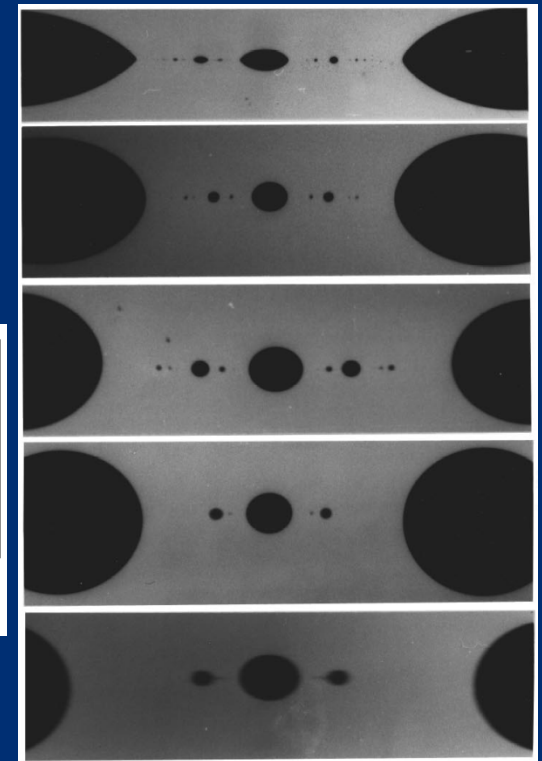
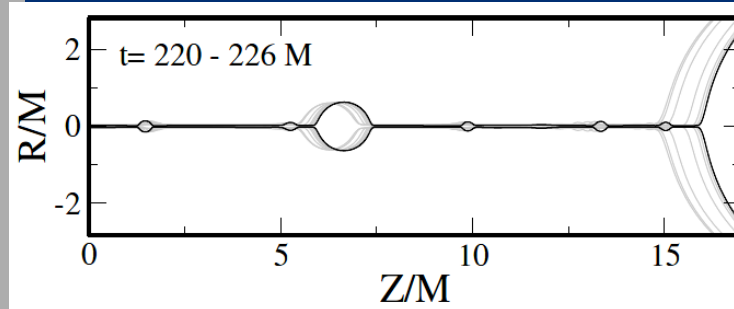
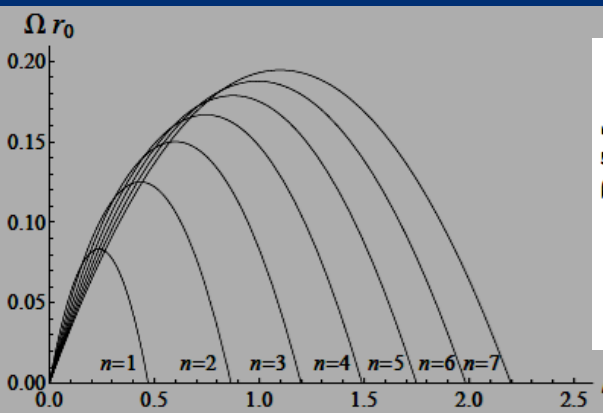
- Rayleigh-Plateau dispersion relation $\Omega \sim \lambda^{-1/2}$
- Gregory-Laflamme dispersion relation $\Omega \sim \lambda^{-1}$



- BUT, ‘cascade’ behavior is ‘mid-wavelength’ driven
- Curiously (i) \rightarrow 2nd order expansion in λ , with ‘just’ intrinsic eqns, turns Ω around [Camps, Emparan]
- Curiously (ii) \rightarrow 1st order expansion in λ , with both intrinsic and extrinsic equations turns around & looks a lot more like RP in derivation.

Final comments

- **As for final state:** extrapolating from the first few generations, pinch-off will be reached in finite asymptotic time, at which time (classically) infinite geometric curvature will be revealed to the exterior universe
- this is then an example in 5D Einstein gravity (and also other dimensions where black holes exhibit similar instabilities) where generic violation of cosmic censorship occurs.
- **As for full solution:** the “true” end-state will thus require some theory of quantum gravity to extend spacetime beyond the pinch-off
- **As for membrane connection:** see how far the hydrodynamic analogy can be extended. Can further connections be uncovered/tested?
- **Next steps:** Understand behavior from:
 - a purely gravity point of view: Thinning/bulging behavior
 - a dynamical p.o.v : NOT a ‘competition of modes’



Fluid / black fold

- Rayleigh-Plateau instability:



- Fluid : Navier-Stokes + cont eqns

$$\delta u^i{}_{,t} = -1/\rho \delta p_{,i}$$

$$\delta u^r{}_{,r} + \delta u^r/r + \delta u^z = 0$$

- Interface: stress balance at interface:

$$p + dp = \sigma \text{Div } n$$

- “Membrane paradigm” (e.g. ‘blackfolds’ [Emparan,Harmark,Niarchos,Obers;Bhattacharyya,Hubeny,Minwalla,Rangami])

- Captures ‘long-wavelength’ dynamics of black branes in far zone $R \gg r_0$

- Einstein Eqns: $D_a T^{ab} = 0$; $T^{ab} K_{ab}{}^A = 0$

$$T_{ab} = (e+P) u_a u_b + P h_{ab}$$

$$e \sim (n+1) r_0^n ; P \sim -e/(n+1)$$

Interestingly, extrinsic eqns:

$$(e X_{,tt} + P X_{,ii}) = 0 \Rightarrow \sigma \sim 1/(n+1) ?$$

What in the world is the end-state of the instability?

Help from outside? Guidance from analogy

- Unstable fluid streams generically break up
 - For the Rayleigh-Plateau instability surface area is also the key explaining why one would expect a long-wavelength instability leading to pinch-off : above a critical length a sequence of spherical droplets has lower energy (due to surface tension) than a cylinder with the same volume/length



➤ Fluid : Navier-Stokes + cont eqns

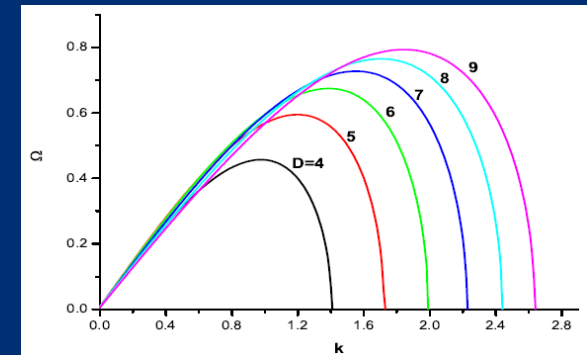
$$\delta u^i_{,t} = -1/\rho \delta p_{,i}$$

$$\delta u^r_{,r} + \delta u^r/r + \delta u^z = 0$$



➤ Interface: stress balance at interface:

$$\rho + dp = \sigma \text{Div } n$$



[Cardoso,Dias 06]