8: Feedback control systems

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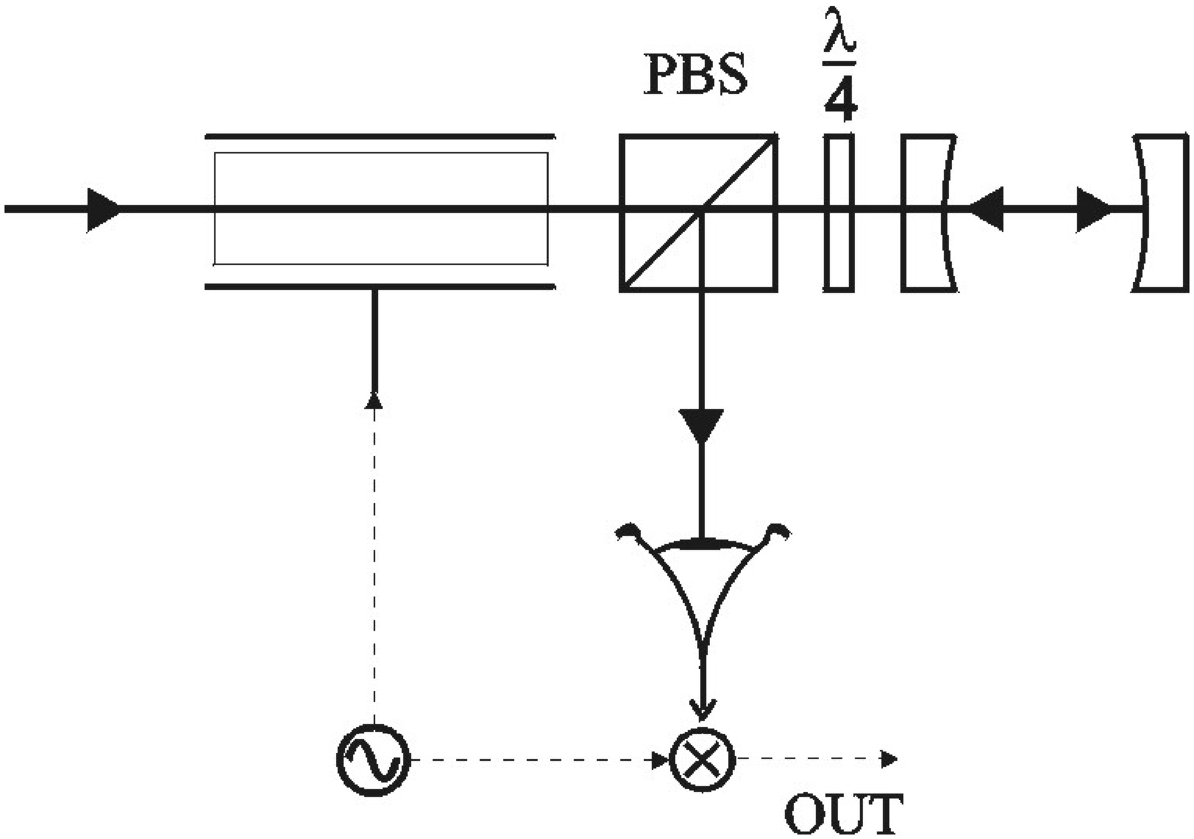
My lectures during this School

1. Overview of gravitational waves and sources
2. Interactions of waves and detectors
3. Shot noise and radiation pressure noise
4. Theory of linear systems
5. Vibration isolation (passive)
6. Optics of Fabry-Perot cavities
7. Thermal noise
8. **Feedback control systems**
9. Description of LIGO and other current detectors
10. Future detectors in space

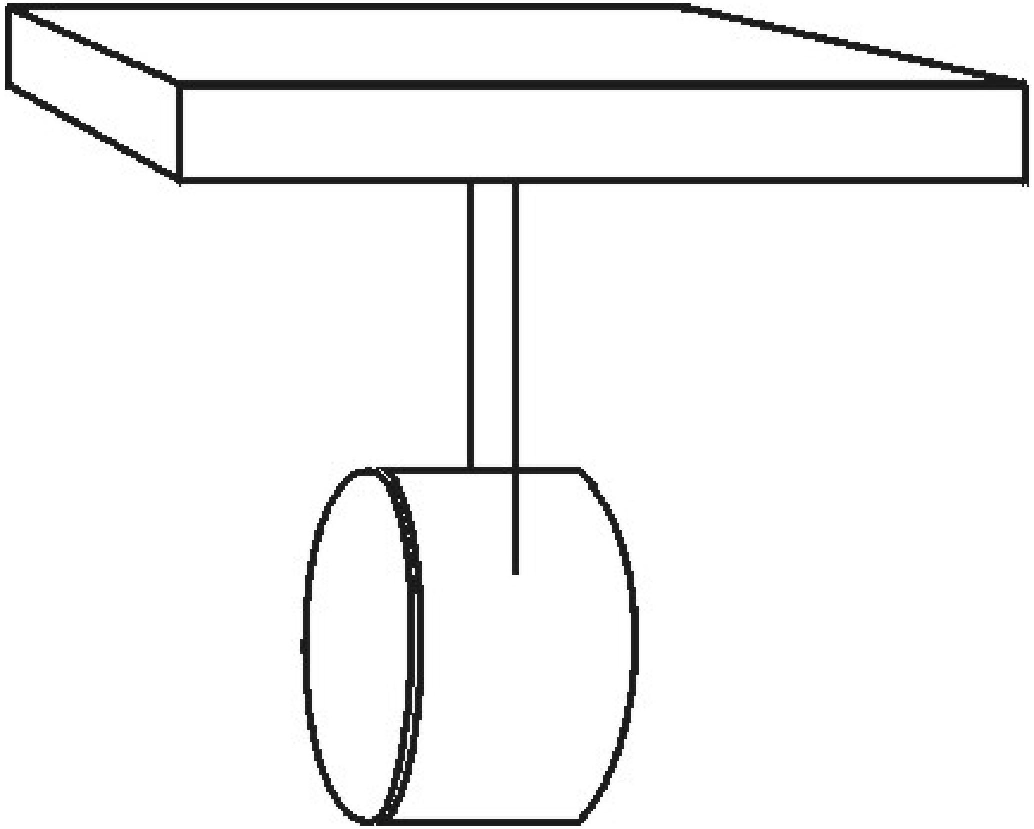
Outline

- Why feedback?
- What is feedback?
- How does feedback work?
- Benefits of feedback
- Feedback example
- Costs of feedback

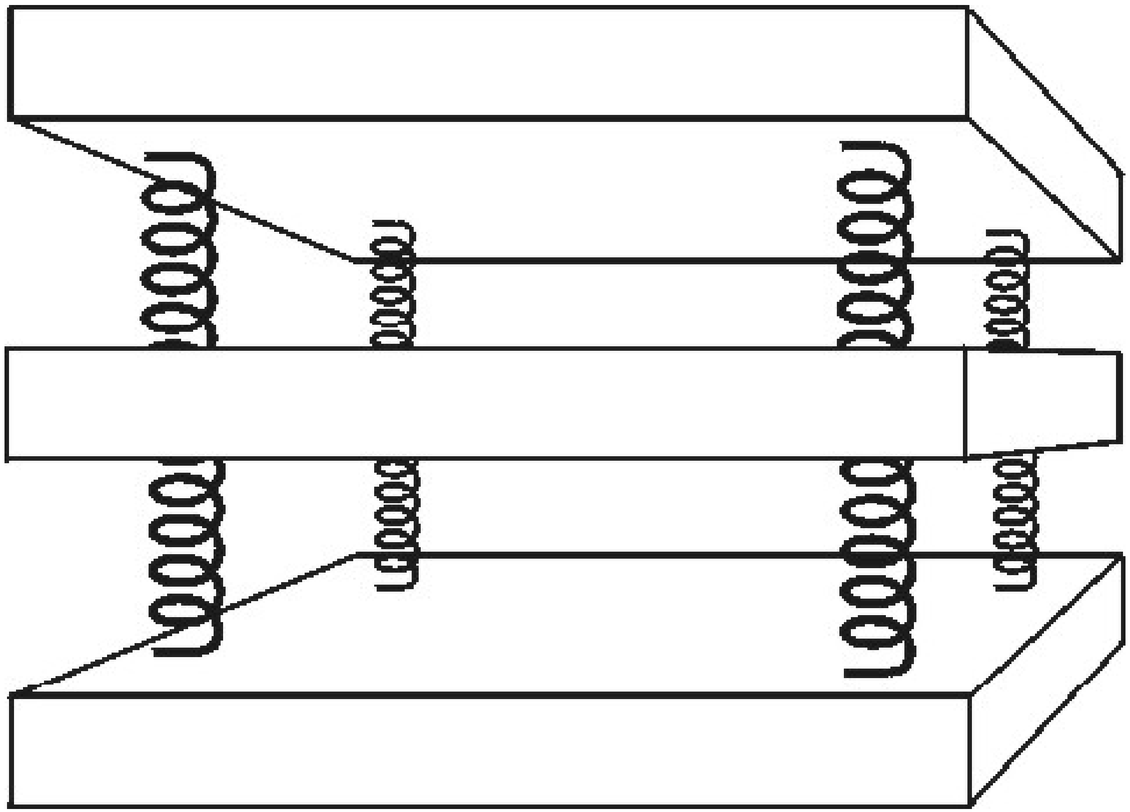
How can we match λ to the length of a resonant cavity?



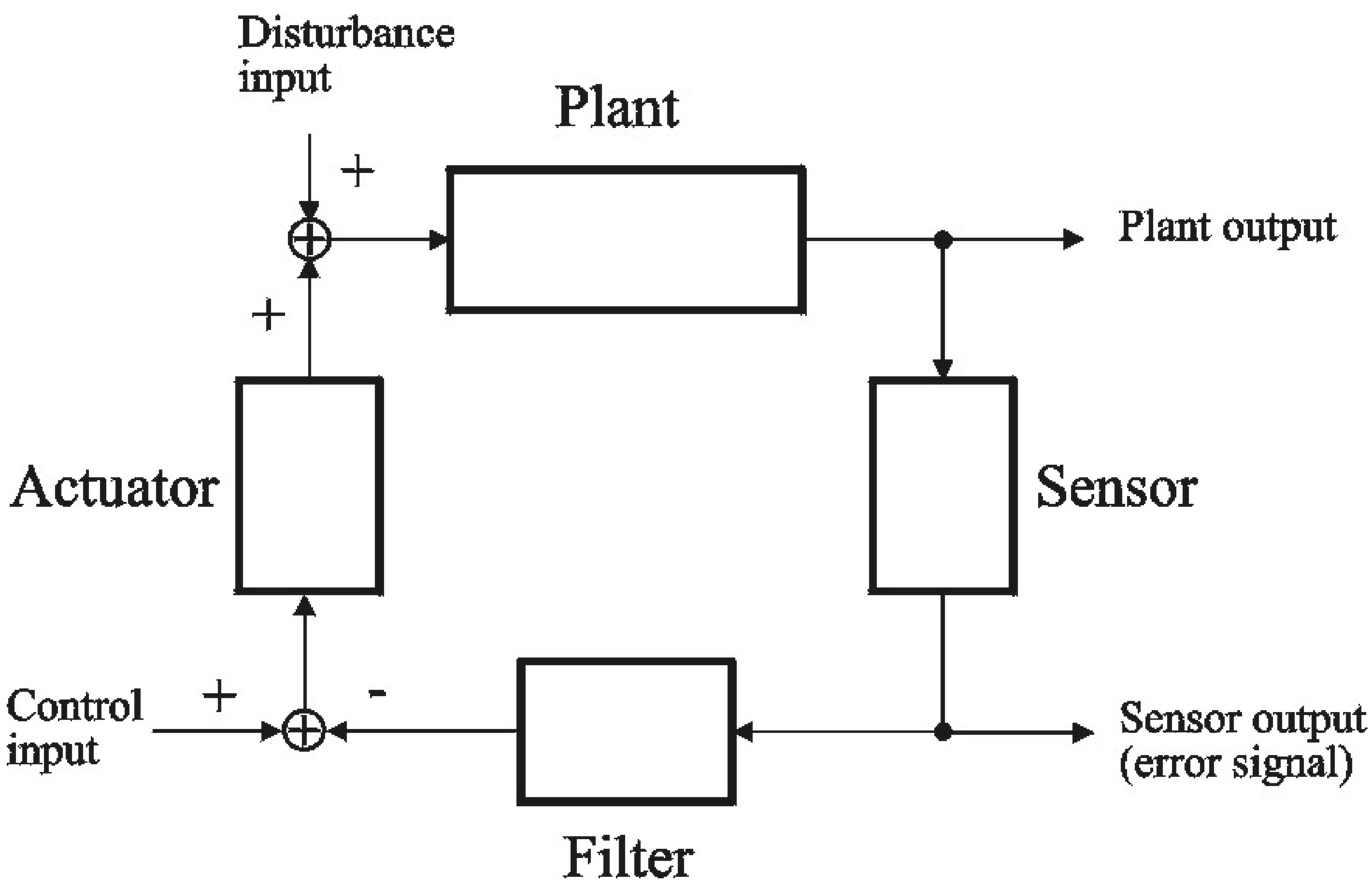
How can we modify the response of a test mass suspension?



How can we improve vibration isolation?



A feedback control system (a.k.a. "servo")



Parts of a servo

- *Plant*: The pre-existing system that you want to control or modify.
- *Sensor*: Generates an (electronic) signal proportional to the plant's output.
- *Actuator*: Acts on the plant to change its output in the desired way.
- *Compensation filter*: (more on this later)

Feedback can hold the plant near a chosen operating point

- Control input is zero. But disturbance inputs cause the plant output to vary.
- Sensor measures the fluctuating plant output.
- Actuator applies a force proportional to the negative of the plant output, thus holding the plant near the chosen operating point.
- *Feedback reduces sensitivity to disturbance inputs.*

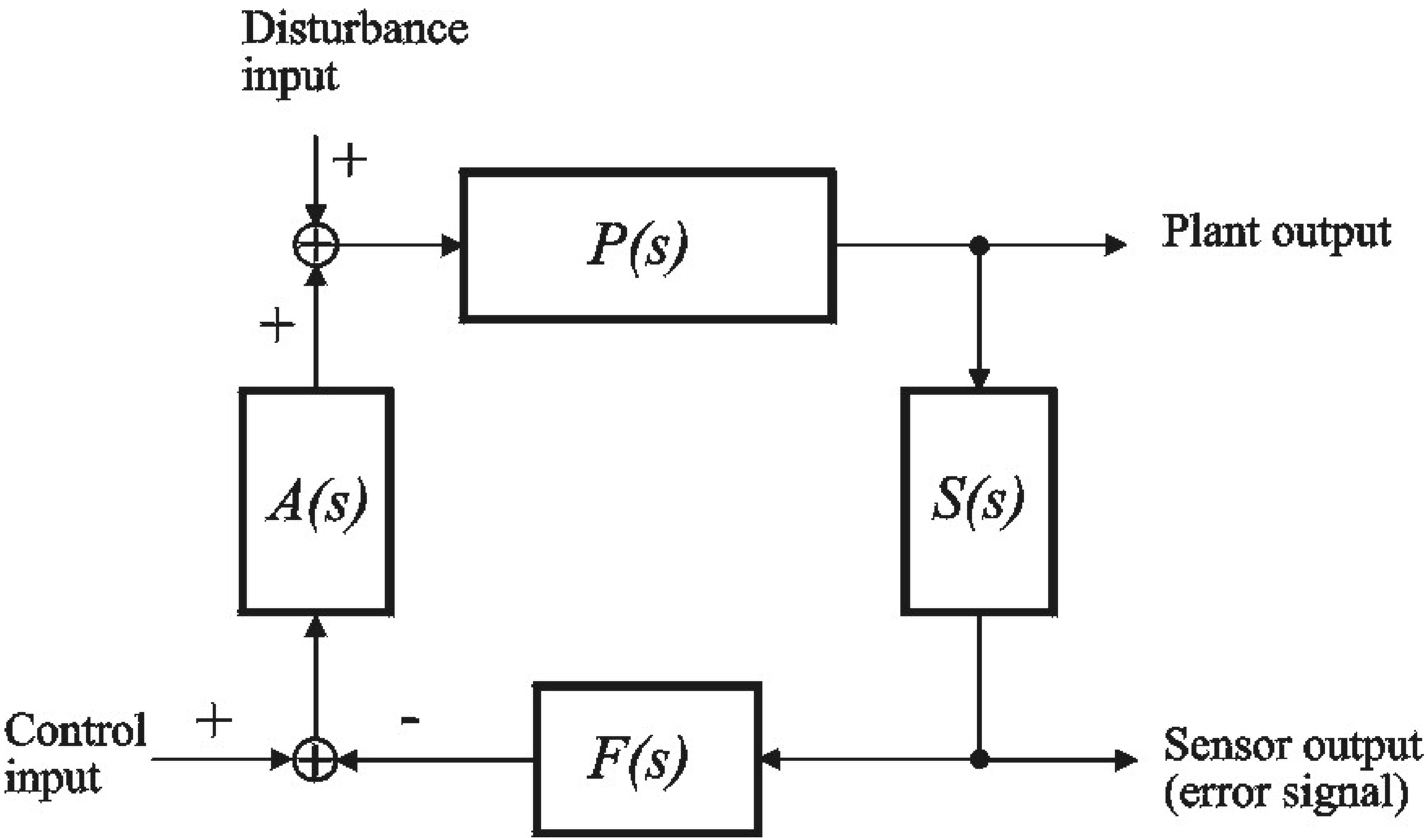
Feedback can modify the dynamics of the plant

Plant responds to control inputs or to disturbance inputs.

Sensor measures the response, which is modified by the compensation filter.

Actuator applies a force that combines with inputs, so that the response of the plant is different than the response to external inputs alone.

Dynamics of a servo



Loop Transfer Function (a.k.a. loop gain)

The *loop transfer function* of a feedback control system is given by

$$G(f) \equiv P(f)S(f)F(f)A(f).$$

G(f) determines the behavior of the servo, how it modifies the performance of the plant, and also the stability of the system.

Benefits of feedback: Closed loop transfer function

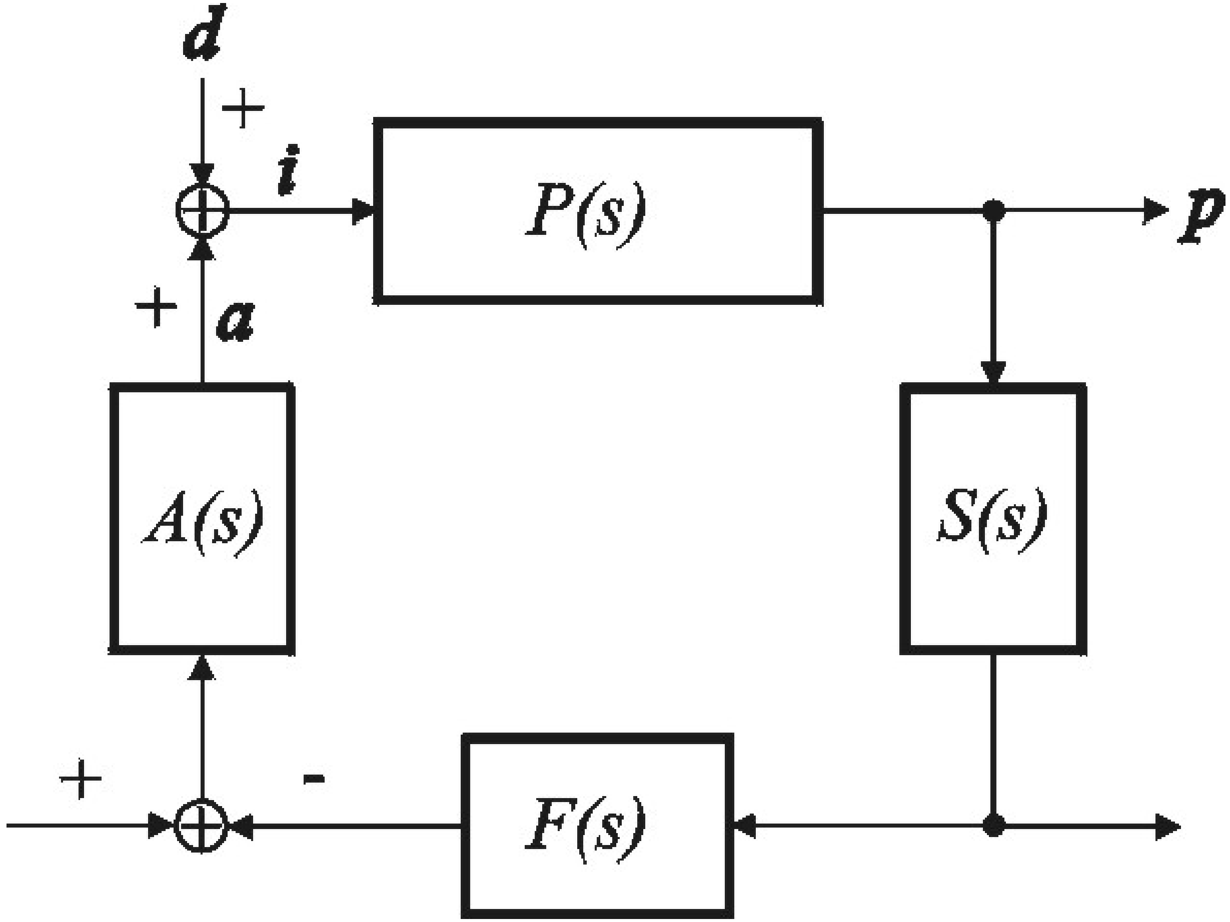
Let $H(f)$ be a transfer function between any two points when the loop *is not* closed.

Let $H_{cl}(f)$ be the same transfer function when the loop *is* closed.

Then

$$H_{cl}(f) = \frac{H(f)}{1+G(f)}.$$

Signals in servo



Derivation of closed loop transfer function

$$i \equiv d + a.$$

$$a = -iPSFA \equiv -iG.$$

$$\text{Thus } i = d - iG, \text{ or } i = \frac{d}{1+G}.$$

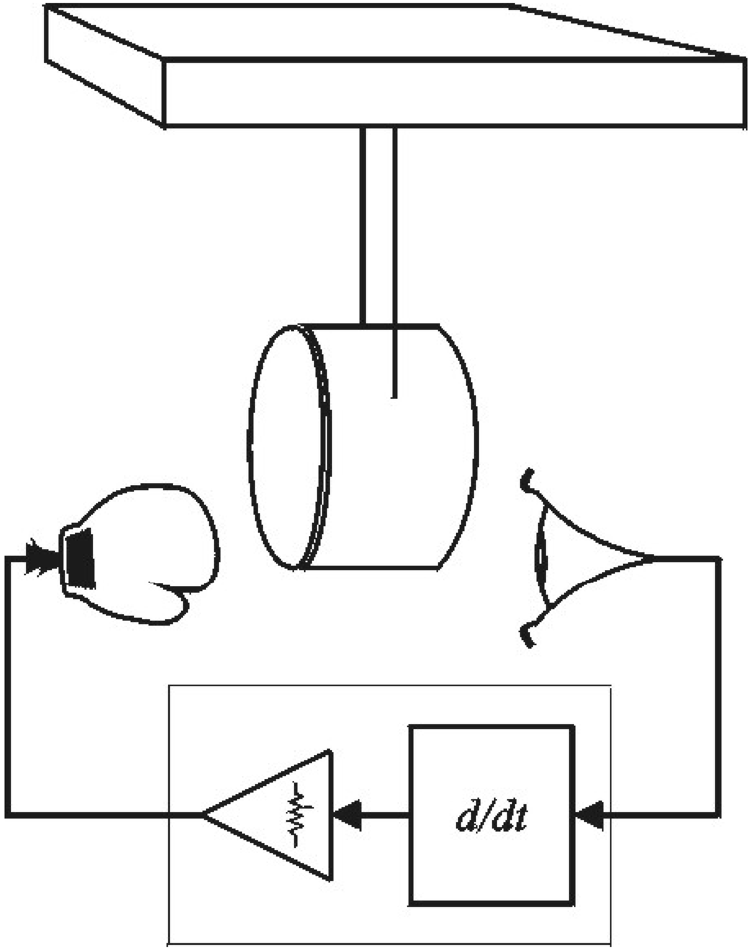
$$\text{So, for ex., } P_{cl} \equiv \frac{p}{d} = \frac{P}{1+G}$$

Benefits of feedback

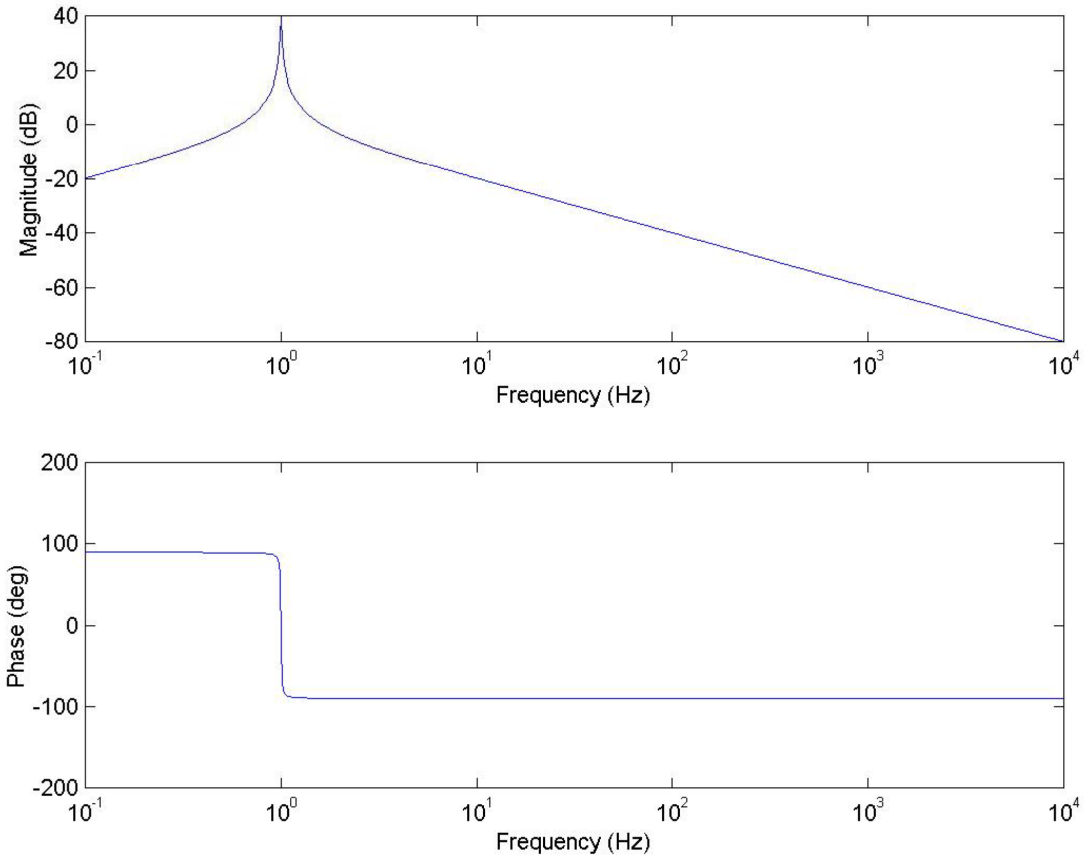
Servo does a lot at frequencies where $1+G$ is large, a little where $1+G$ is of order unity, and almost nothing where $G<<1$.

This is one reason why we say that the benefits of feedback are all encoded in the loop transfer function G.

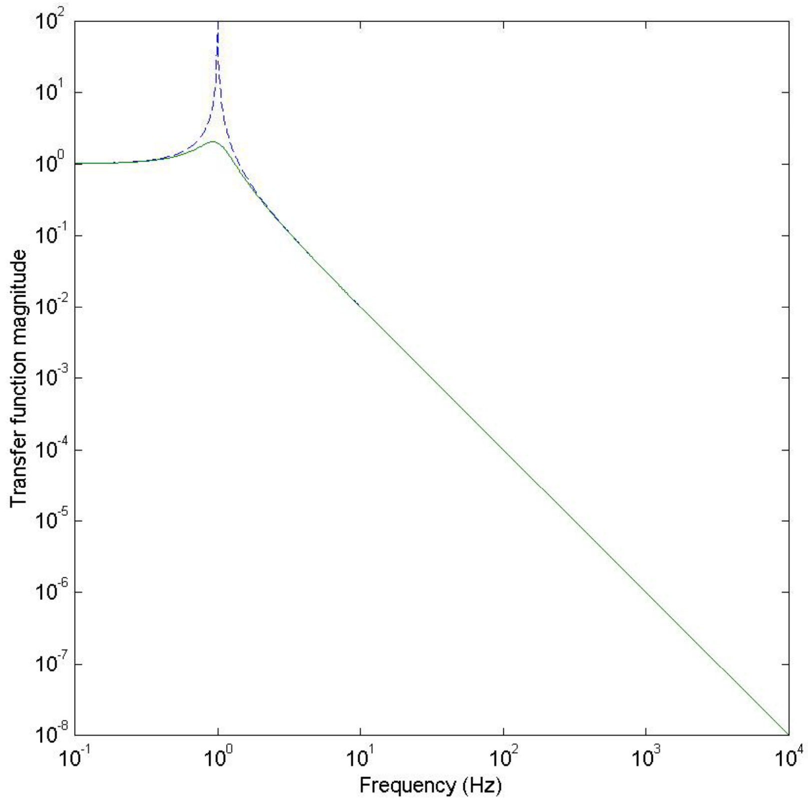
Example: active damping of pendulum



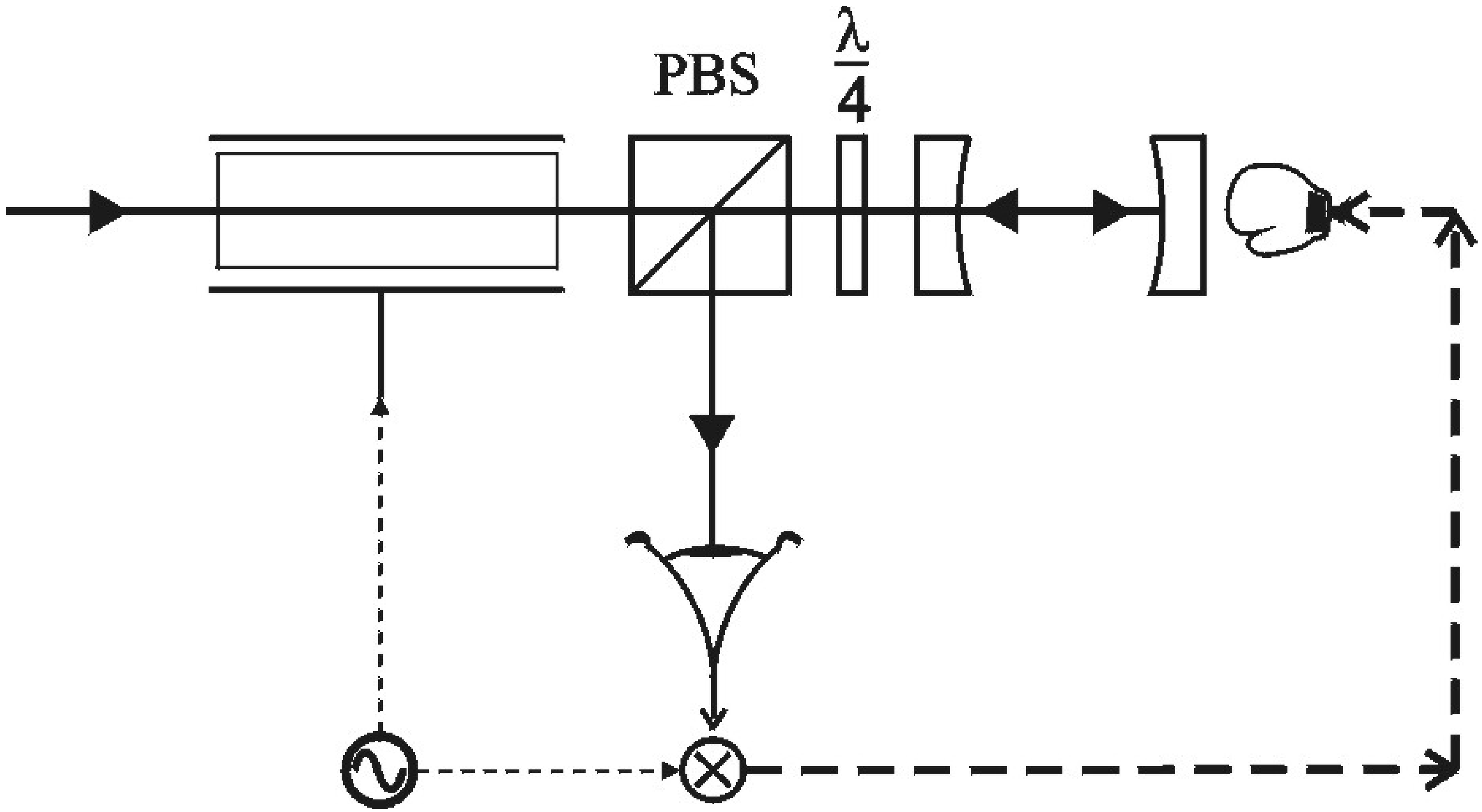
Active damping: loop transfer function



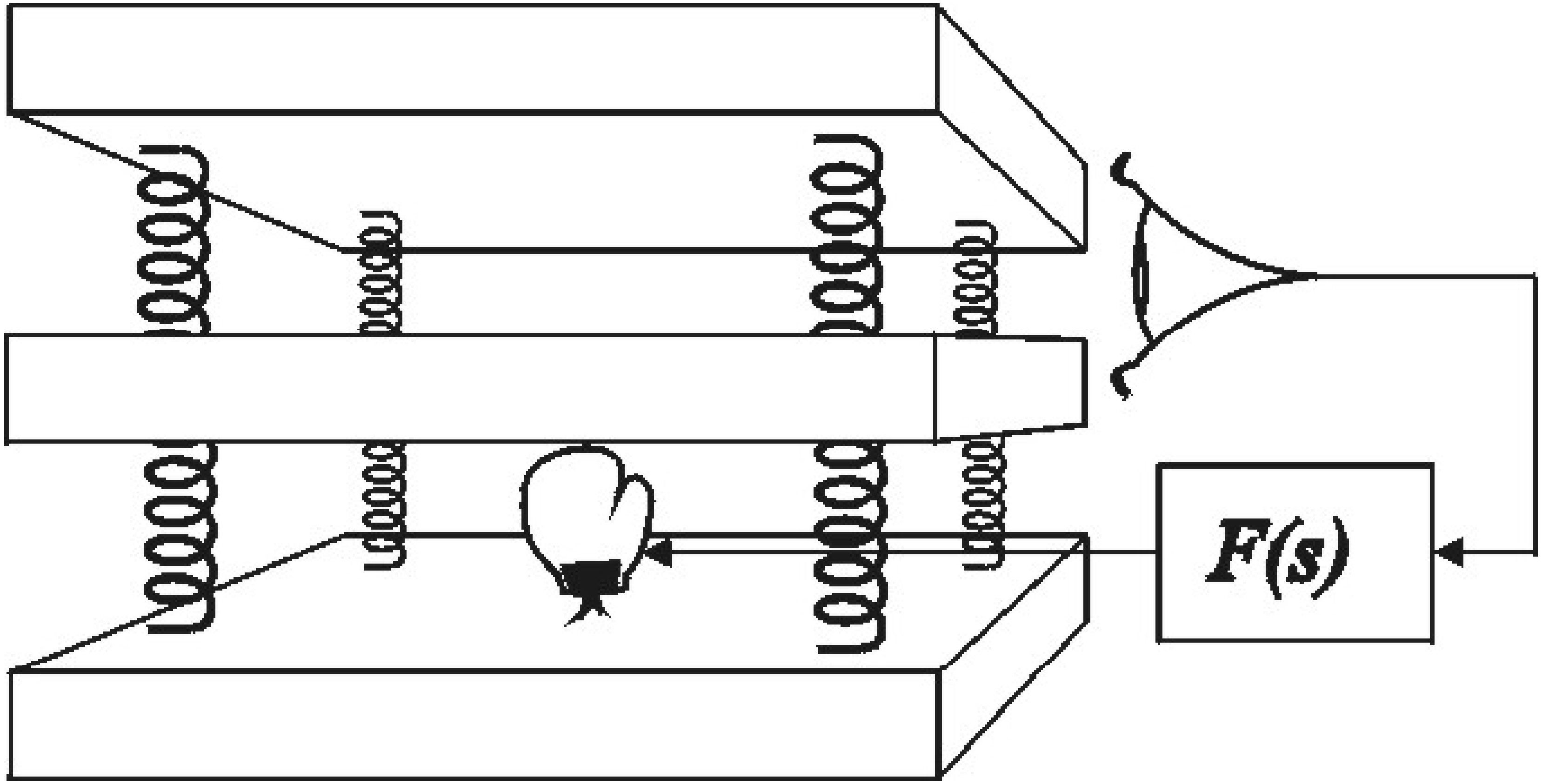
Active damping: Open loop, closed loop transfer functions



Example:
Resonant cavity lock



Example:
Active vibration isolation



Costs of feedback

Some cost: Feedback control systems need extra parts, and those need to be carefully designed.

Biggest "cost": To ensure that the servo will be *stable* when the loop is closed.

Frequency dependence of the loop transfer function G is critical. This is the main role of the compensation filter F.

This is a very rich subject!

Stability and causality

Recall:

$$H_{cl}(f) = \frac{H(f)}{1+G(f)}.$$

What happens if, for some *f*, *G(f)* = -1?

Closed loop response diverges!

Why not just avoid this, e.g. by making *G* large for all *f*?

Causality requires that response of physical systems can't keep fixed magnitude and phase to arbitrarily high *f*.

- Speed of light
- In mechanical systems, the speed of sound is the real limit!

Ensuring stability

Generically, at high frequencies we accumulate large phase lags.

When ϕ = -180 deg, we've picked up an unwanted change of sign.

The standard way to deal with this is by ensuring that the magnitude of the loop transfer function G is small for all f above that point.

Ensuring stability requires careful design of the compensation filter F.

Limits to filter design

Almost always, our first guess at the filter that we'd like is impossible to realize.

For example, I'd love to have a filter that has strongly decreasing gain with frequency, but has constant phase.

A theorem of complex analysis forbids the separate specification of $|G(f)|$ and $\varphi(f)$. Specifying one uniquely specifies the other.

$$|G(f)| \propto f^{-N} \rightarrow \varphi(f) = -N * 90°$$

Stability tests

Feedback designers most often study the Bode plot of *G(f)*, and have developed rules of thumb for estimating when a loop will be stable.

For example, if the phase of *G* only departs from 0 by more than π at frequencies at which the magnitude of *G* is less than 1, the loop is usually stable.

N.B.: <u>There is no rigorous stability test that can always be applied by study of the Bode plot.</u>

Rigorous stability tests

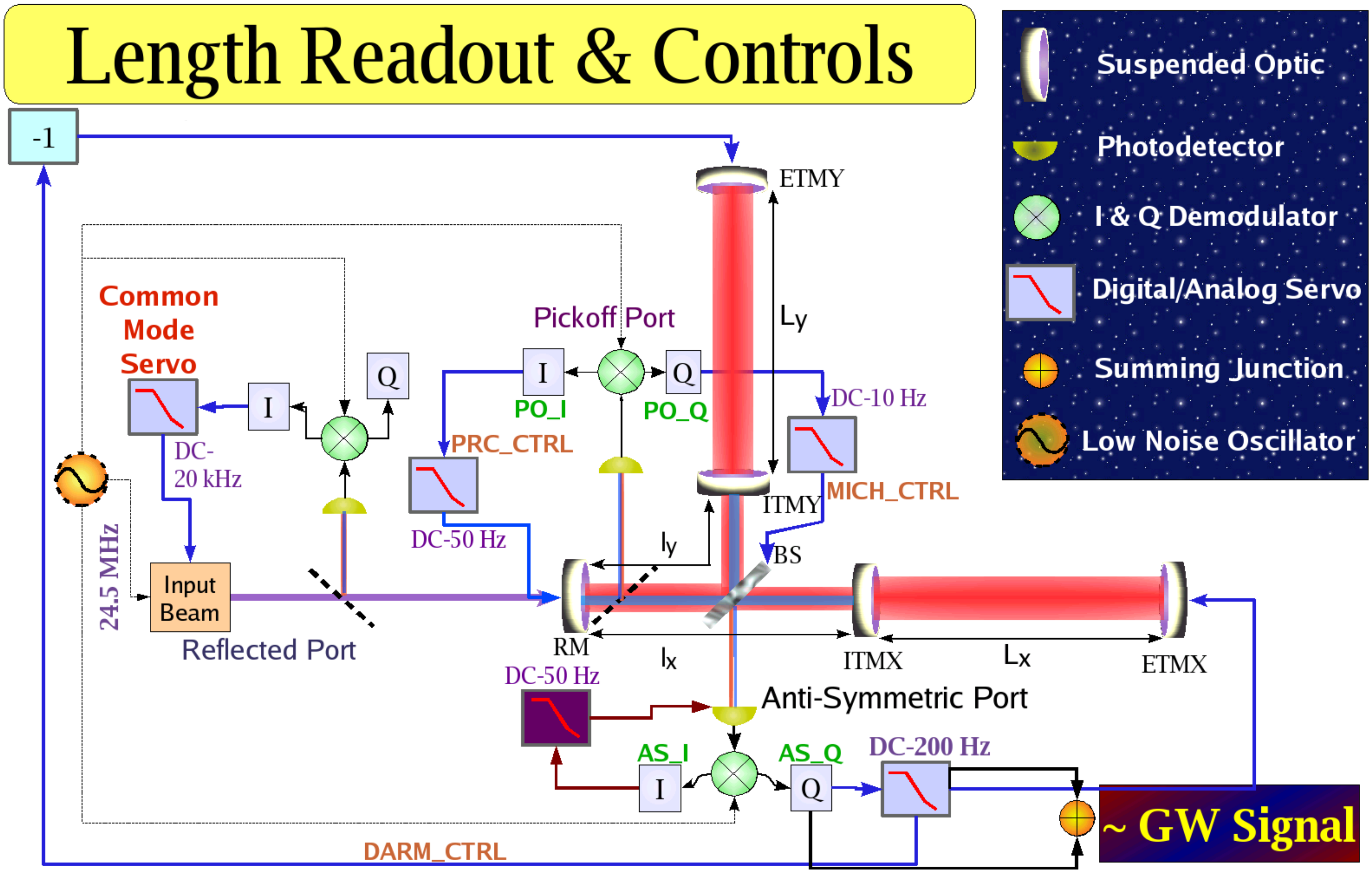
- The Nyquist diagram, in which one plots *G(f)* in the complex plane.
  If there are no encirclements of (-1,0), the loop is stable.
- The root locus method.
  If all of the roots of the closed loop response lie in the left half-plane, the system is stable.

State space methods, a.k.a. "modern control theory"

For complicated systems, it is often best to abandon frequency domain methods, and work directly with a model of the equations of motion of the system.

A well-developed version of this is called *state space control*. There are good books on this subject …

How Does it All Hang Together?



How Does it All Hang Together?