7: Thermal noise

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My lectures during this School

1. Overview of gravitational waves and sources
2. Interactions of waves and detectors
3. Shot noise and radiation pressure noise
4. Theory of linear systems
5. Vibration isolation (passive)
6. Optics of Fabry-Perot cavities
7. **Thermal noise**
8. Feedback control systems
9. Description of LIGO and other current detectors
10. Future detectors in space

Outline

1. Brownian motion and the Fluctuation-Dissipation Theorem
2. Thermal noise in interferometers
3. Internal friction, and how to make it small

Large mechanical noise

How large?

Seismic: $x_{rms} \sim 1\ \mu$m.

Brownian motion:

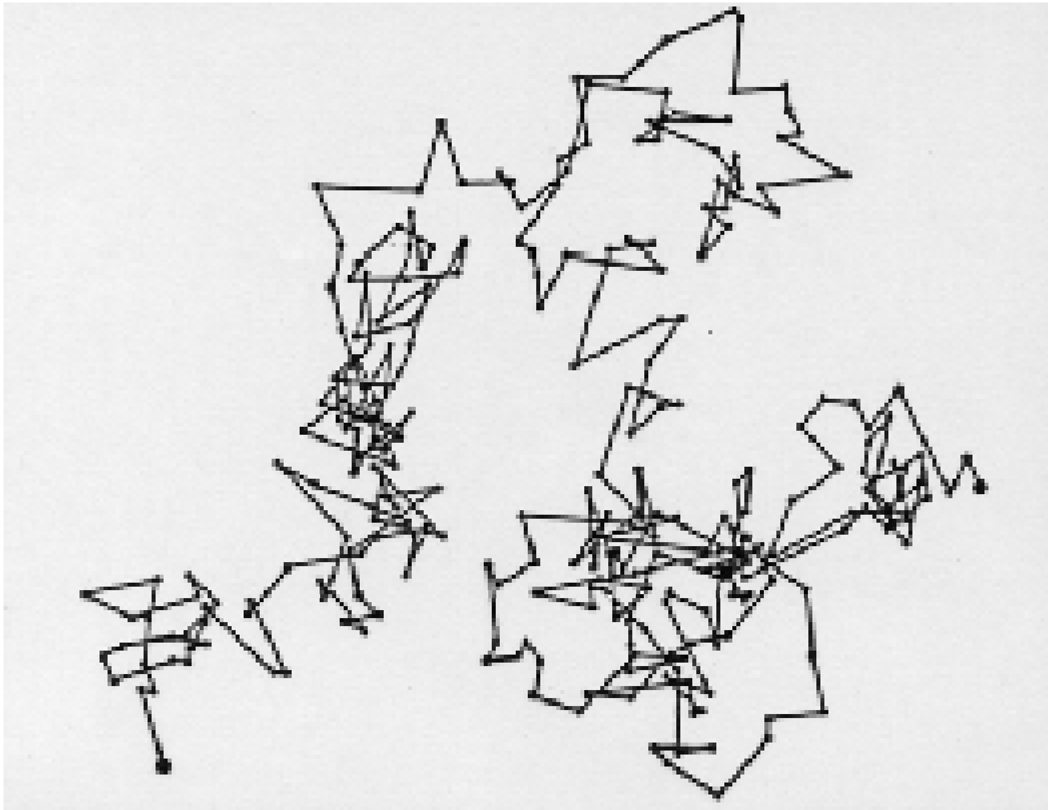
- mirror's CM: ~ 3 x 10^{-12} m.
- mirror's surface: ~ 3 x 10^{-16} m.

If so, can we detect gravitational waves?

Now, we'll focus on Brownian motion.

Brownian motion

In 1827,
Robert Brown
noticed incessant
jiggling of tiny
particles
suspended in
water, as seen
through a
microscope.



Evidence for a ubiquitous "vital force"?

Not just living cells, but small particles of any material.

- He even checked dust ground from a piece of the Great Sphinx!

Did this mean that all material was in some sense animate, i.e. alive?

Or was there a physical explanation?

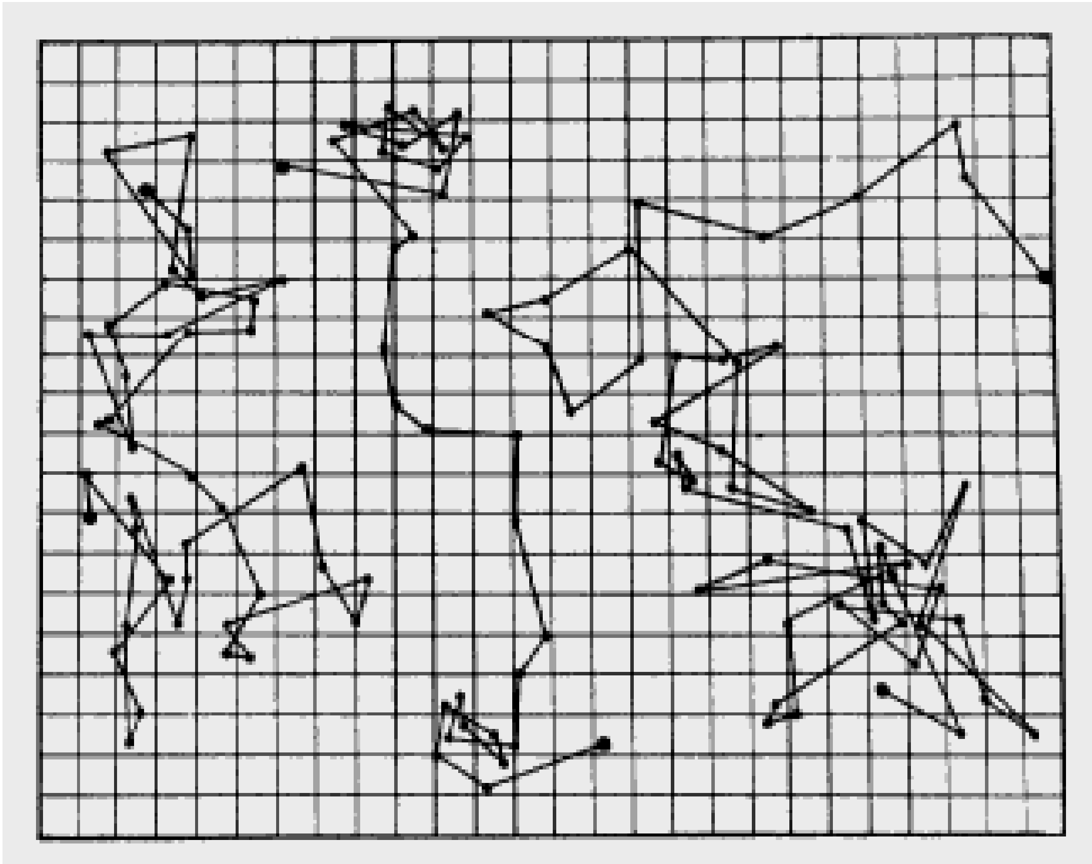
The molecular explanation of Brownian motion

- Water is made of tiny molecules.
- Molecules randomly collide with visible particles.
- Random collisions from all sides yield jiggling motion: a *random walk*.

What experiments showed

Many individual particles were studied.

Keep track of how far a particle drifts in a fixed time interval.



How to find a pattern in all of those random walks?

The graph at right shows <u>endpoints</u> of many random walks.

Scattered all over!

But you can characterize this graph by the typical width of the pattern.

Mathematically, find the average of the square of the distance drifted, or $< x^2 >$.

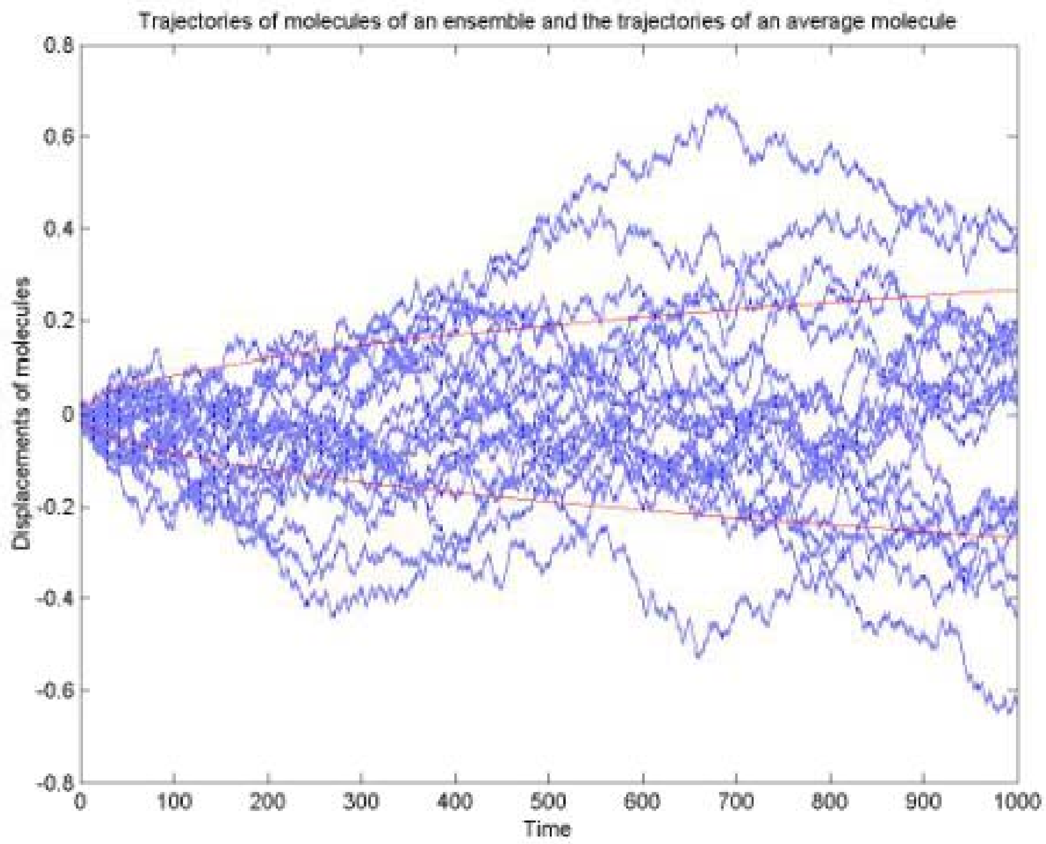
Fig. 10.

How does the typical drift distance grow with time?

Experiments showed that $< x^2 > \propto t$.

Also:

- Drift is greater at higher temperature.
- Drift decreases as drag on particle increases.



Einstein's key contribution

1905: Einstein shows that Brownian particle's random walk obeys

$$\left\langle x^2(t) \right\rangle = 2k_B TBt$$

where B is a coefficient called the *mobility* of the particle.

"Jitter" a result of the existence of atoms.

The first link clear and incontrovertible link between fluctuation and dissipation.

Atoms are real!

Soon, Avogadro's number was determined:

$$N = 6.02 \times 10^{23} \text{ molecules/mole.}$$

Brownian motion is a graphic demonstration that *heat is the microscopic motion of molecules.*

It also suggests an explanation of the relationship between friction and heat.

Thus, Brownian motion gives us a direct window into the microscopic world of atoms.

Other key contributions

- Jean Perrin won the 1926 Nobel Prize for measurements of diffusion, checking Einstein's theory, thereby "proving the existence of atoms".
- Johnson noise, 1928, electrical equivalent of Brownian motion.

Fluctuation-Dissipation Theorem

(Callen *et al.* 1951-2)

For a linear system in thermal equilibrium,

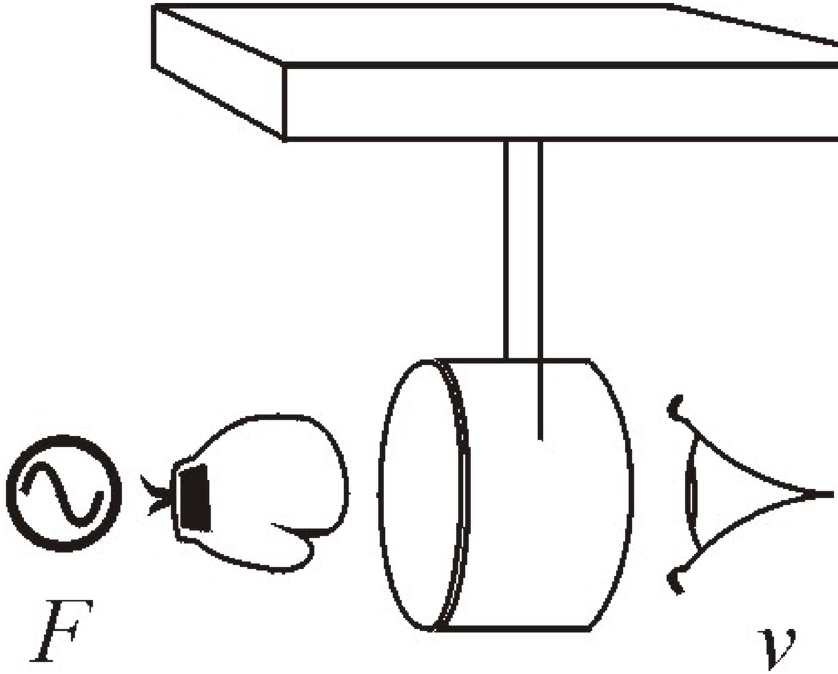
$$S_x(f) = \frac{4k_B T}{(2\pi f)^2}\,\mathrm{Re}[Y(f)].$$

Y(f) is the admittance *v/F;*
Re[*Y(f)*] is proportional to the dissipation in the system.

For lower noise, we can either:

1. lower the dissipation, or
2. lower the temperature.

Real part of admittance Y determines strength of thermal noise



$$Y = v/F$$

Relation to Equipartition

Equipartition Theorem: each d.o.f. has, on average, an energy of $k_B T / 2$.

For a simple harmonic oscillator, this means

$$\frac{1}{2} k x^2 = \frac{1}{2} k_B T,$$

or rms motion of 3×10^{-12} meters, for 10 kg mirror in 1 Hz pendulum at room temp. (!)

Temporal character

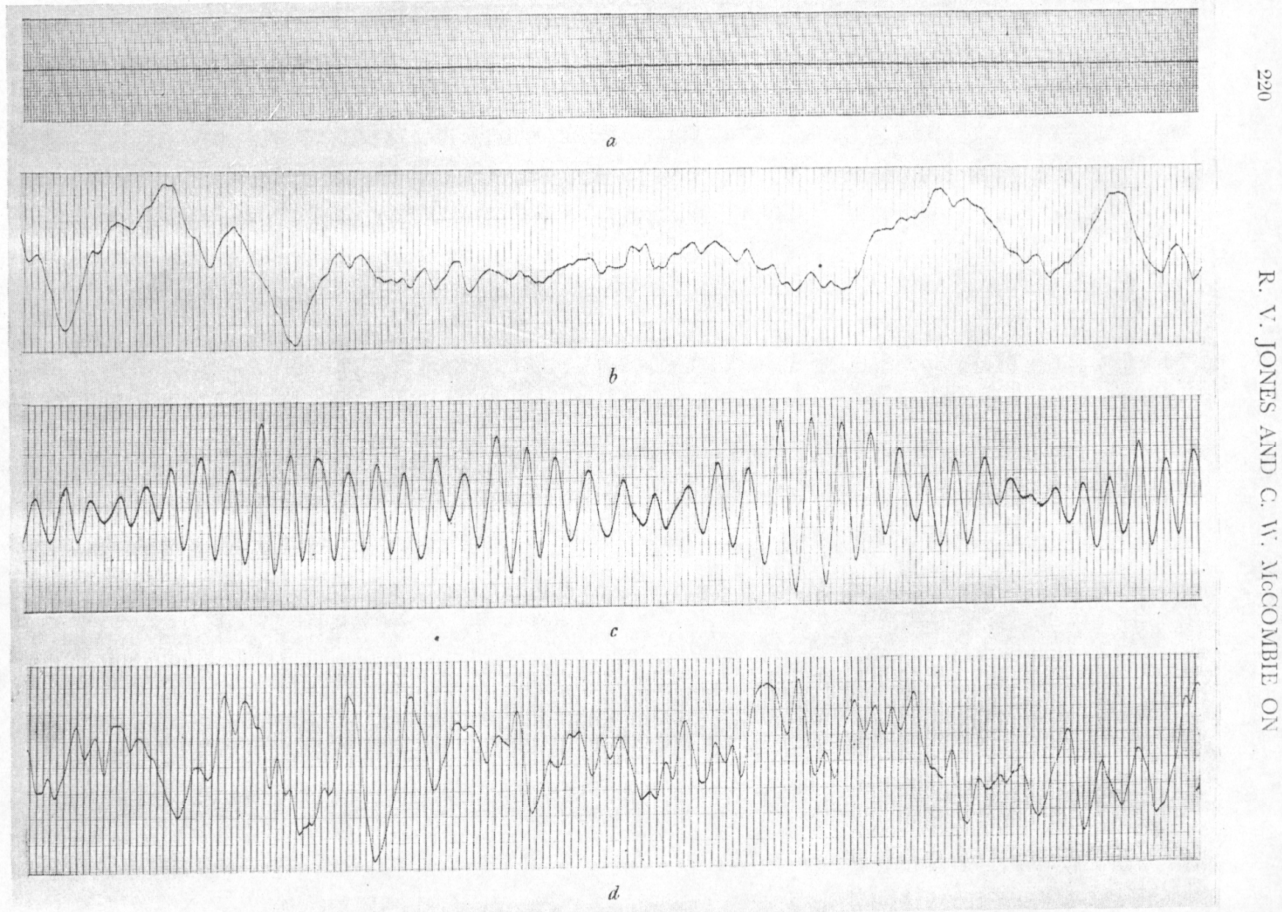
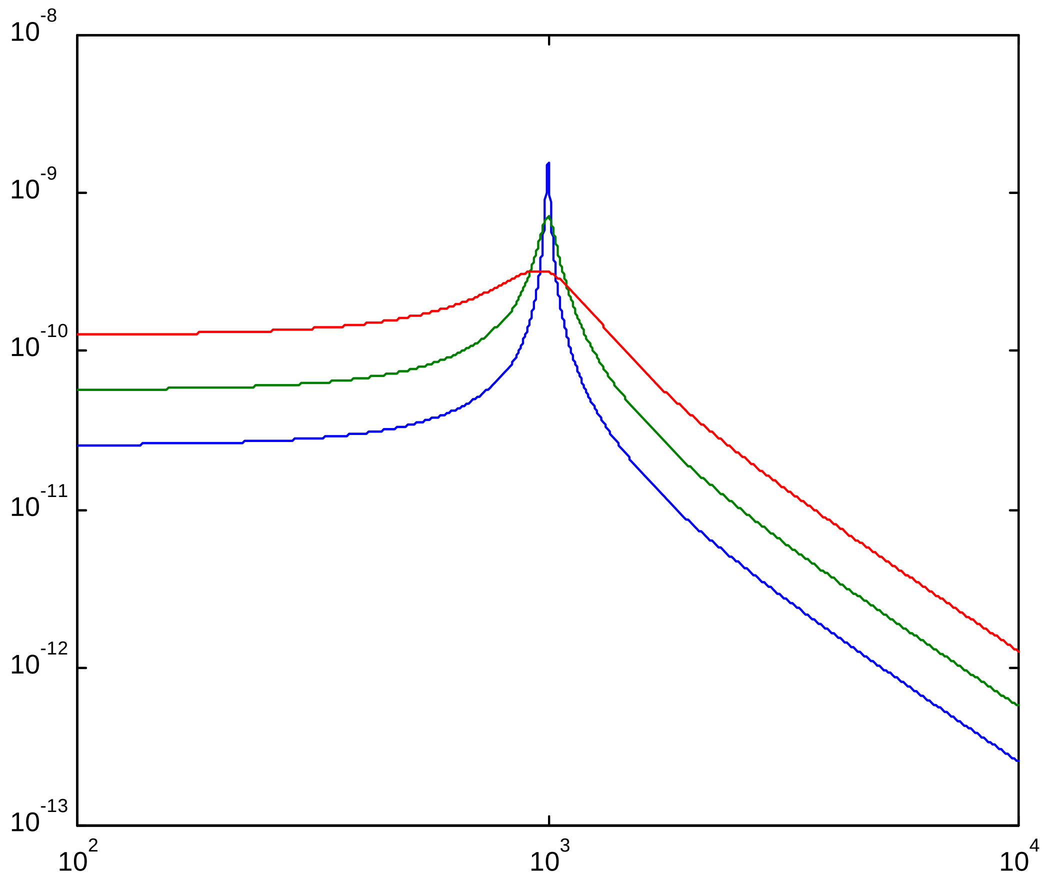


FIGURE 1. *a*. Record of secondary trace, with fixed primary mirror, at approximately twice the sensitivity used in records (*b*), (*c*) and (*d*). Vertical lines at 1 sec. intervals. *b*, *c*, *d*. Records of secondary trace with primary galvanometer (resistance 10·76Ω, free period 2·17 sec.) heat excited (*b*) on open circuit (*c*), and nearly critically damped with external resistance 133·24 ohms (*d*). Secondary galvanometer (free ... Sensitivity about ... per large division. Vertical lines at 0·5 sec. intervals.

Same lesson in frequency domain



Q as a Figure of Merit

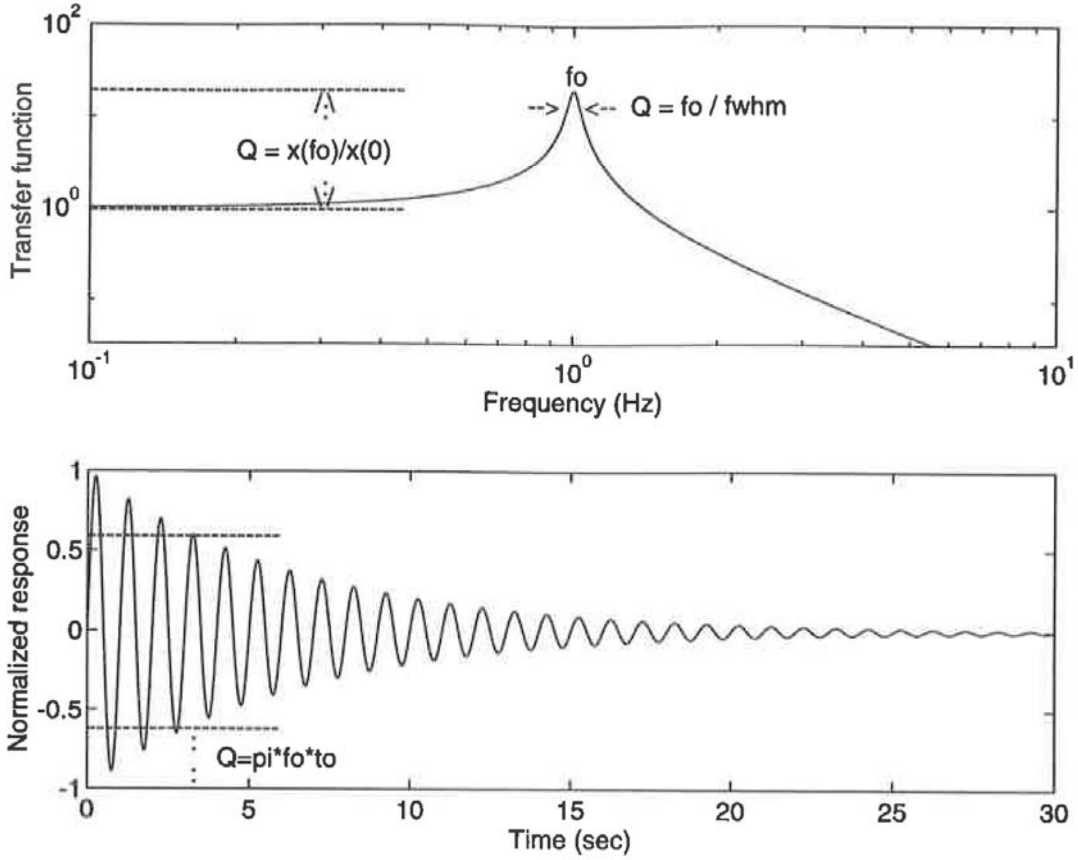
Q: dimensionless measure of the ratio of elastic restoring force to dissipative force.

Measured by width of resonance or by ringdown time of vibration.

Related to "loss angle" ϕ, phase lag between force and displacement, by $\phi = 1/Q$.

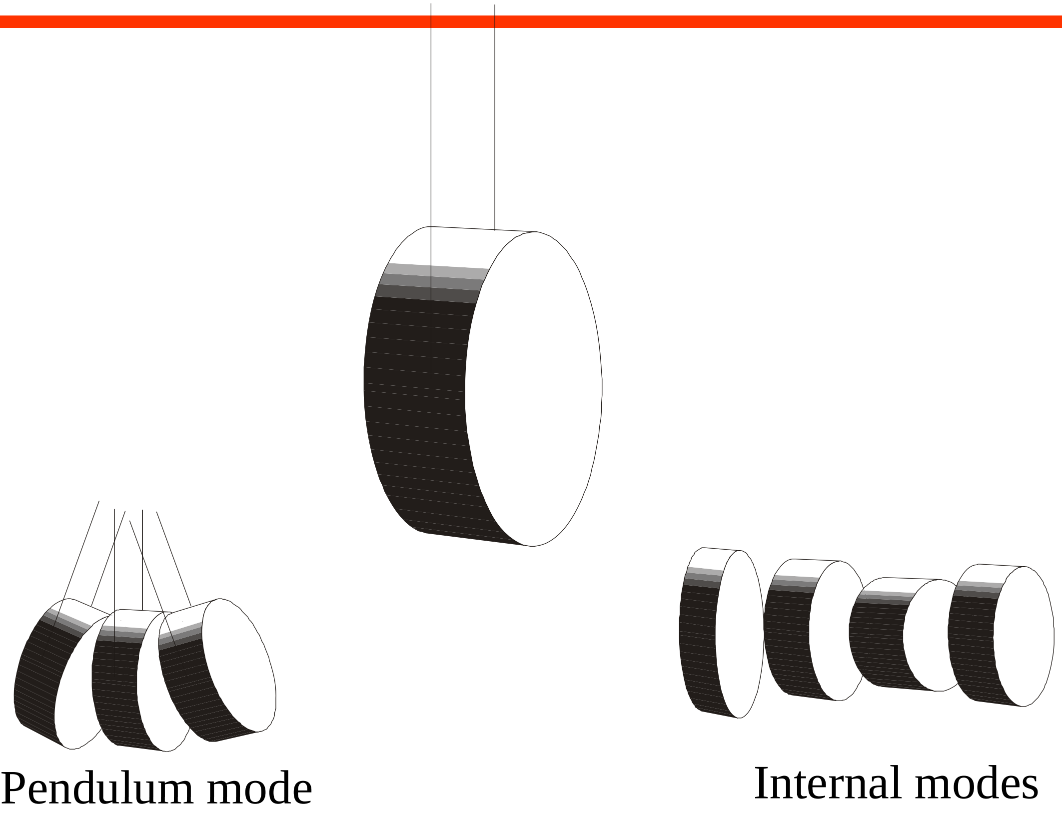
In round numbers: garden variety $Q = 10^3$, good $Q = 10^6$, advanced $Q = 10^8$.

Q



Transfer function
Q = x(fo)/x(0)
fo
--> <-- Q = fo / fwhm
Frequency (Hz)
Normalized response
Q=pi*fo*to
Time (sec)

Interferometer suspensions



$$x^2(f) = \frac{2}{\pi}\frac{k_B T}{k}\frac{1}{f\left[\left(1 - f^2/f_0^2\right)^2 + \phi^2\right]}\phi(f)$$

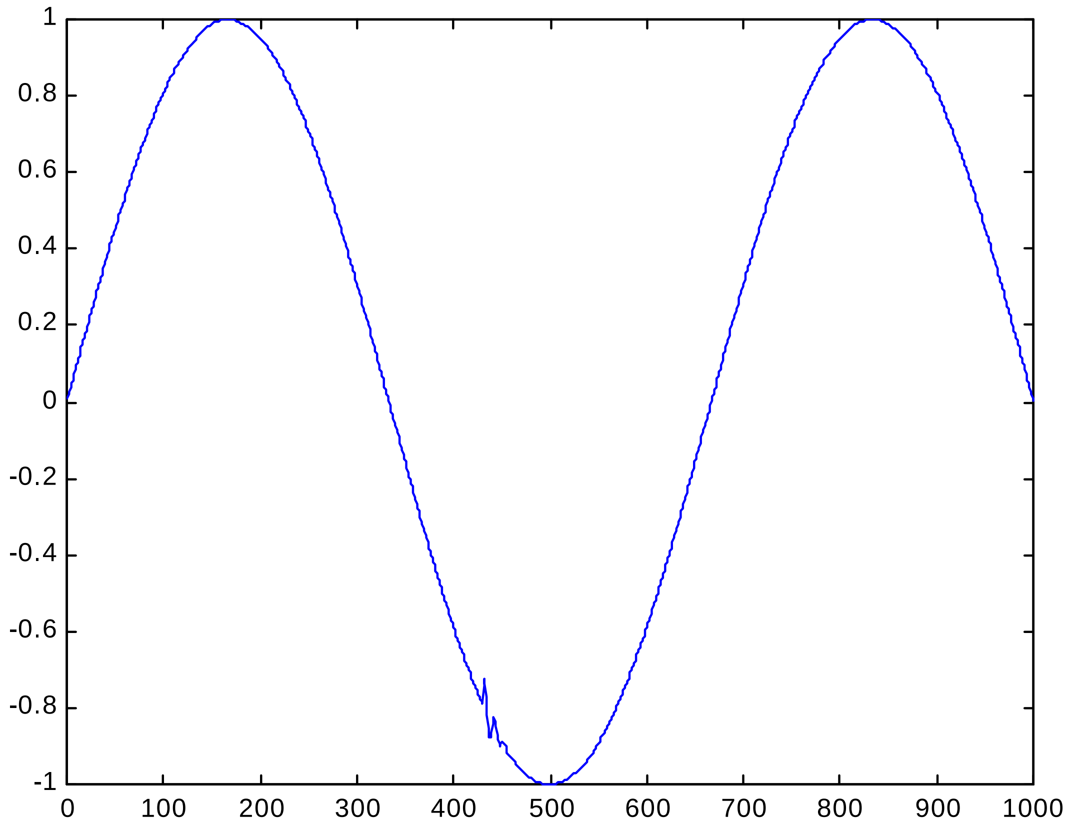
State of the art Q

First LIGO suspensions will have Q around 10^6 for pendulum and internal vibrations.

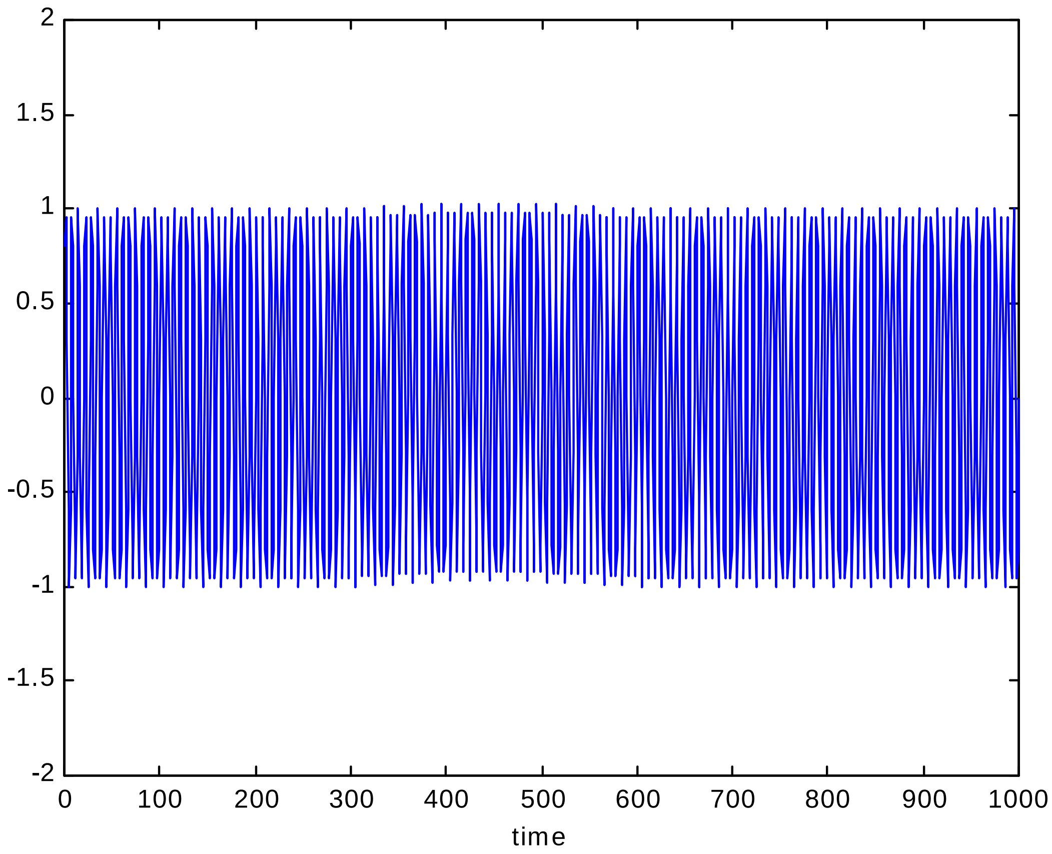
Internal mode Q from use of fused silica.

Pendulum mode Q from steel wires ($Q \sim 10^3$) plus *dissipation dilution effect*: most restoring force in pendulum comes from tension in wires, free of internal friction.

Signal detection in strongly colored noise



Signal detection in strongly colored noise (II)



What determines Q, and thus the level of thermal noise?

- External dissipation

  e.g., air friction, rubbing friction, or eddy current damping

- Internal dissipation processes in materials of test masses and pendulum wires

Craftsmanship: Eliminate external dissipation.

Design goal: Make internal friction small.

How does internal friction work?

Not like a simple dashpot.

No such thing as “internal viscosity”.

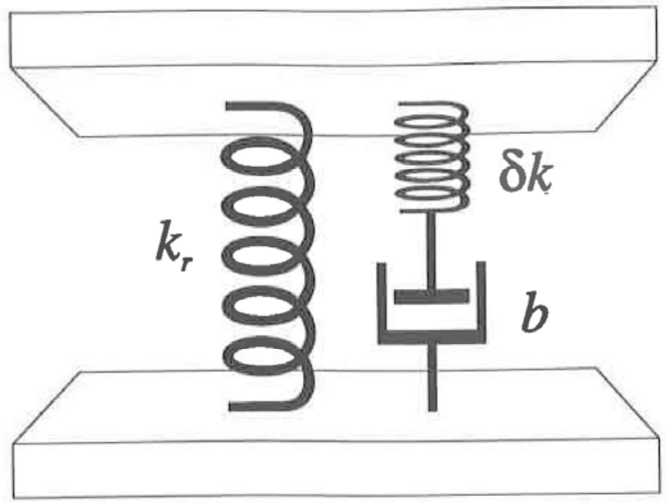
If it did, high frequency modes would be damped more than grave modes, $Q \propto 1/f$.

Xylophones would be impossible.

Dissipation from anelastic relaxation

Many kinds of internal defects (point defects, dislocations, …) can move under stress, given enough time. When they move, material stretches more.

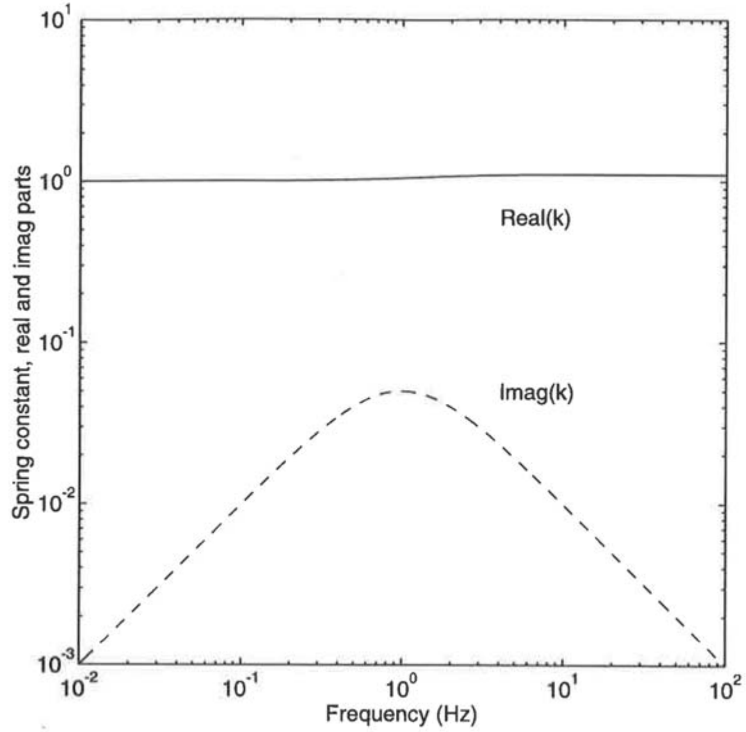
Relaxation with a (single) characteristic time scale can be modeled in this way.



Frequency response of a single relaxation

This kind of process leads not to velocity damping, but to damping concentrated in a range of frequencies about a decade wide.

Sometimes called "Zener damping."



Thermoelastic damping

Flexing fibers exhibit Zener-like damping due to the *thermoelastic effect*.

When flexed, a fiber gets a temperature gradient across it, since one side is stretched and the other side if compressed. (This is the reciprocal effect of thermal expansion.)

Heat flows across the fiber, with a (single) characteristic time scale. As temperature equilibrates, fiber flexes a bit farther.

A relaxation, and thus damping.

Damping in real materials

Aside from thermoelastic damping, one seldom sees Zener-type relaxation peaks in the dissipation.

Why should defects have a single characteristic time scale?

It doesn't take much of a spread in time scales to give dissipation over much wider than a decade in frequency.

Most real materials show (roughly) *frequency-independent* damping. (!)

Q of some useful materials

Rubber: 10 to 30

Aluminum, tungsten, steel: ~ 10^3 to 10^4

Fused silica: up to 2 10^8

Sapphire: 2 to 3 10^8

(all values given at room temperature)

Best practices

- Advanced LIGO:
  - Use fused silica fibers (instead of iLIGO's steel wires).
  - Use "best quality" fused silica for test masses
  - Then, most important thermal noise comes from the high-reflectivity coatings on the mirrors. (!)

- Longer term:
  - Reduce thermal noise by going to low temperatures
    KAGRA is the pathfinder for this
  - For low T, need to replace silica with crystalline material
    all glasses have terrible Q at low temperatures, for deep condensed matter physics reasons