6: Beam folding, especially Fabry-Perot Cavities

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with some additional material by
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My lectures during this School

1. Overview of gravitational waves and sources
2. Interactions of waves and detectors
3. Shot noise and radiation pressure noise
4. Theory of linear systems
5. Vibration isolation (passive)
6. **Optics of Fabry-Perot cavities**
7. Thermal Noise
8. Feedback control systems
9. Description of LIGO and other current detectors
10. Future detectors in space

Outline

1. Why multi-pass? Why not more multi-pass?
2. The operation of Fabry-Perot resonant cavities
3. How can one read out a cavity?

Multipass, phase diff

To make the signal larger, we can arrange for *N* round trips through the arm instead of 1.

$$\Delta\tau = h\frac{2NL}{c} \equiv h\tau_{stor}$$

It is useful to express this as a phase difference, dividing time difference by radian period of light in the ifo:

$$\Delta\phi = h\tau_{stor}\frac{2\pi c}{\lambda}$$

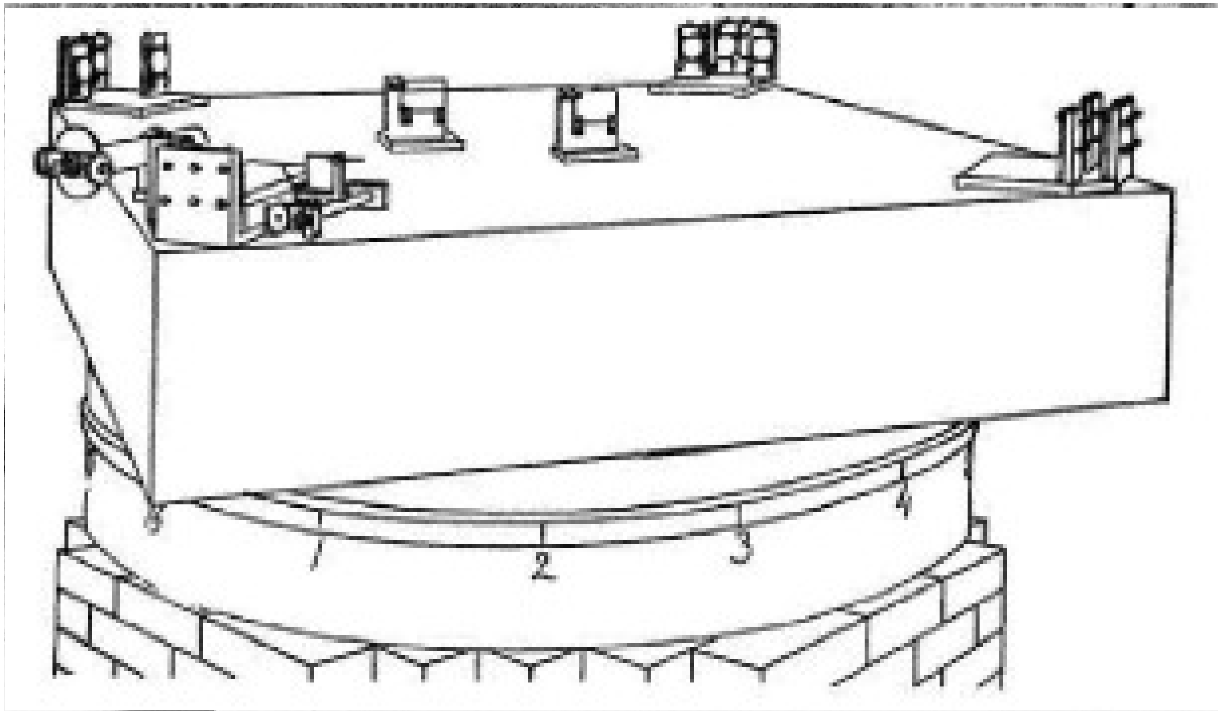
What are the limits of the multi-pass strategy?

- Signal doesn't grow without limit as we increase τ_{stor}. Only grows until $\tau_{stor} \sim 1/f_{grav}$.
- Still, why not make arms much shorter than 4 km, and make <u>many</u> round-trips?
- Displacement noise, expressed in terms of strain, is proportional to number of round trips.
  - More encounters with mirror, larger contribution to (i.e., contamination of) signal.
- Design rule: Build as long an interferometer as you can, then make up the remaining factor to ideal storage time using multi-pass.

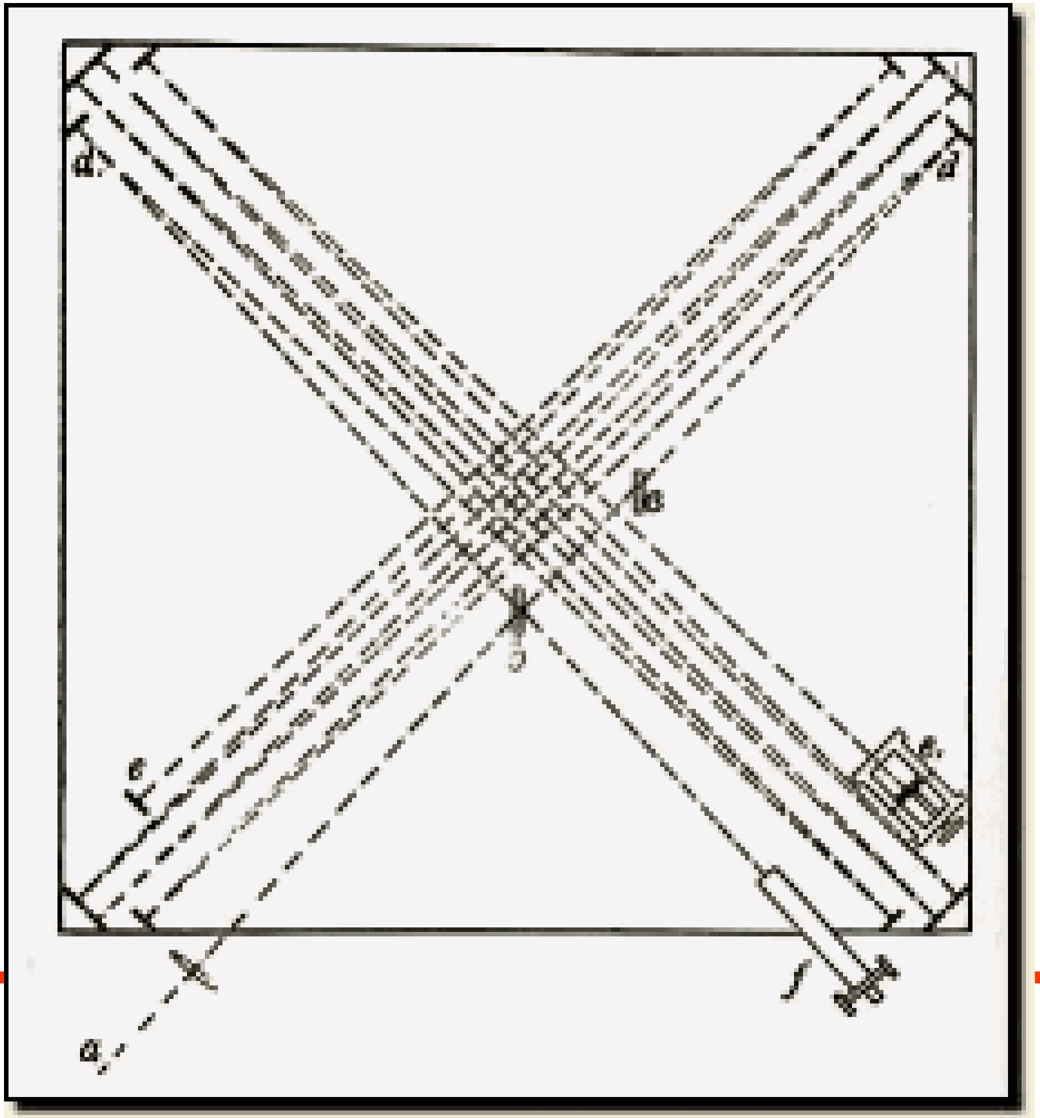
The Michelson-like solution

- Michelson knew the benefits of multi-pass interferometry. He used many mirrors at each end of the interferometer's arms.
- We could do this directly in a gravity wave ifo.
  - Drawback: many more mirrors to point and control.
- We could also make much larger mirrors, which hold all of the reflections.
  - Herriott delay line
  - Drawback: mirror gets very large (> 1 m diam.). This gets very heavy, and also has internal modes at awkwardly low frequencies. (Bad thermal noise.)

Michelson and Morley's apparatus



Michelson-Morley ifo plan view



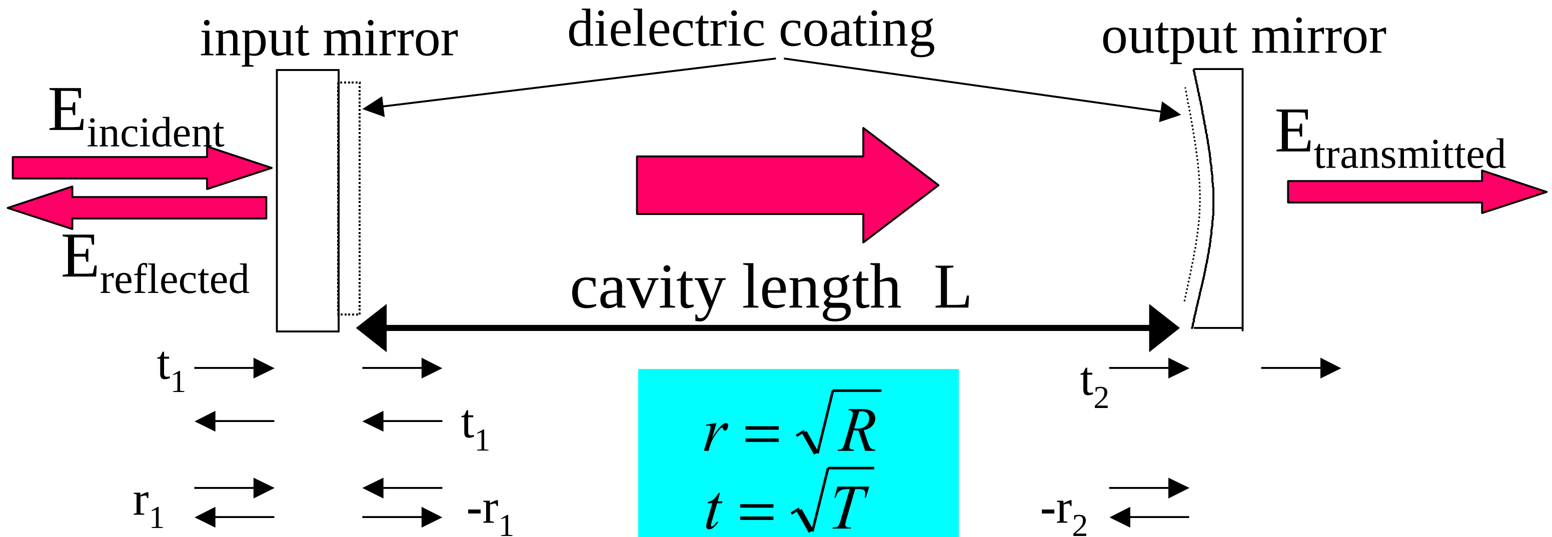
Fabry-Perot cavity to the rescue

- Light can be trapped between two small mirrors, by properly adjusting the distance between them.
  - Interference does the "trapping"
- One (or both) mirrors must have some finite transmission for light to get in and out.

- Minimum mirror size set by diffraction.

Analysis of F-P cavities by Joe Kovalik

Cavity Description



$$r = \sqrt{R}$$
$$t = \sqrt{T}$$

R+T+L=1

$$E_{incident}(t) = E_0 e^{i\omega t}$$

$$E_{transmitted}(t) = E_0 t_1 t_2 e^{i\omega(t-\frac{L}{c})} + E_0 t_1 t_2 r_1 r_2 e^{i\omega(t-\frac{3L}{c})} + E_0 t_1 t_2 r_1^2 r_2^2 e^{i\omega(t-\frac{5L}{c})} + \ldots$$

$$E_{reflected}(t) = E_0 r_1 e^{i\omega t} - E_0 r_2 t_1^2 e^{i\omega(t-\frac{2L}{c})} - E_0 r_1 r_2^2 t_1^2 e^{i\omega(t-\frac{4L}{c})} - E_0 r_1^2 r_2^3 t_1^2 e^{i\omega(t-\frac{6L}{c})} - \ldots$$

Results for Electric field

$$\frac{E_{\text{transmitted}}}{E_{\text{incident}}} = t_1 t_2 e^{-i\omega\frac{L}{c}}\left[1 + r_1 r_2 e^{-i\omega\frac{2L}{c}} + r_1^{\,2} r_2^{\,2} e^{-i\omega\frac{4L}{c}} + \ldots\right]$$

$$= \frac{t_1 t_2 e^{-i\omega\frac{L}{c}}}{1 - r_1 r_2 e^{-i2\omega\frac{L}{c}}}$$

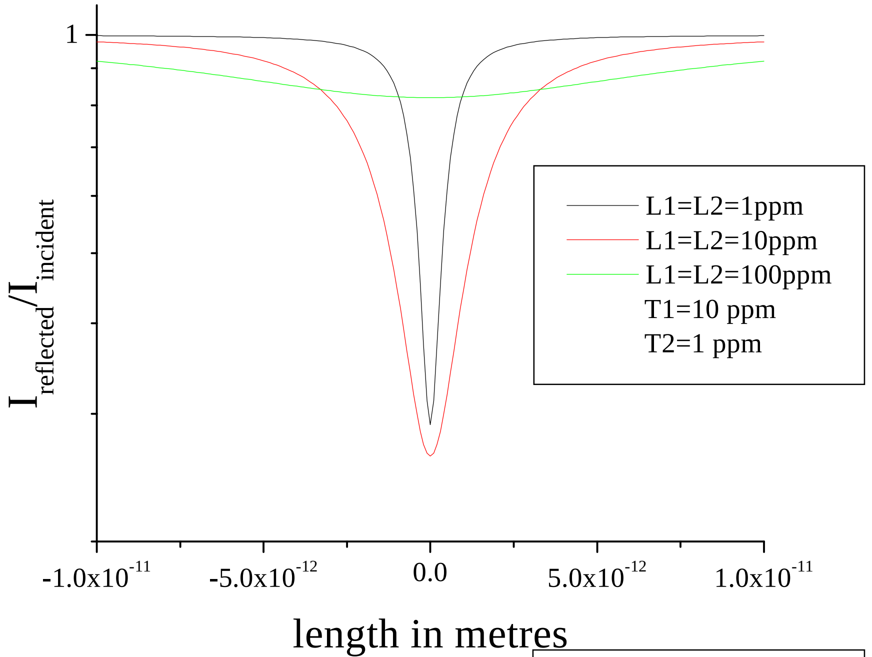
$$\frac{E_{\text{reflected}}}{E_{\text{incident}}} = r_1 - t_1^2 r_2 e^{-i\omega\frac{2L}{c}}\left[1 + r_1 r_2 e^{-i\omega\frac{2L}{c}} + r_1^{\,2} r_2^{\,2} e^{-i\omega\frac{4L}{c}} + \ldots\right]$$

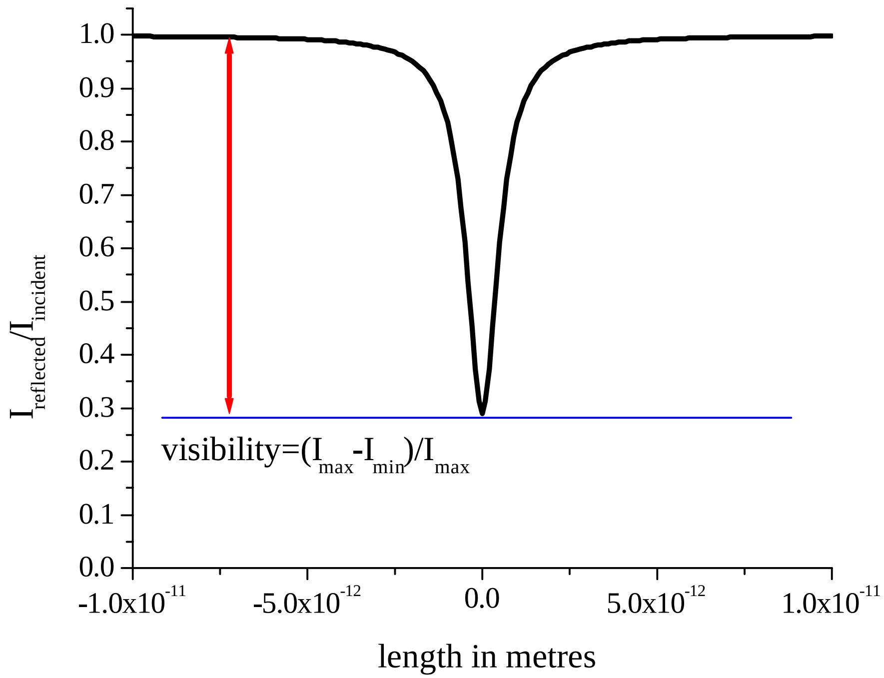
$$= r_1 - \frac{t_1^2 r_2 e^{-i2\omega\frac{L}{c}}}{1 - r_1 r_2 e^{-i2\omega\frac{L}{c}}}$$

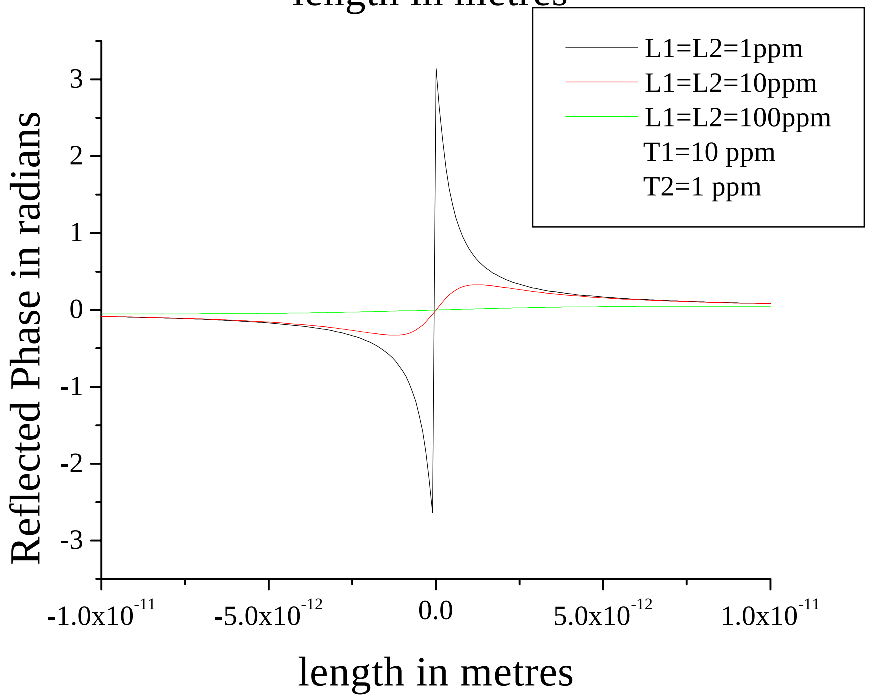
$$= \frac{r_1 - r_2\left(r_1^2 + t_1^2\right) e^{-i2\omega\frac{L}{c}}}{1 - r_1 r_2 e^{-i2\omega\frac{L}{c}}}$$

Reflected Light

- Reflected phase gives signal for locking cavity
  - •Reflection locking
  - •Pound-Drever-Hall Locking







How is a Fabry-Perot cavity like an arm with discrete bounces?

Key benefit of multi-pass arm is increasing the phase change due to grav wave

$$\Delta\phi = h\frac{2NL}{c}\frac{2\pi c}{\lambda}$$

In a Fabry-Perot cavity, high finesse means steep change in phase with respect to length change (incl. due to grav wave!)

See graph on lower right, previous slide.

Frequency response

The calculation above give the response at DC.

With more care, can calculate the response as a function of frequency.

Result: DC response is multiplied by

$$\frac{1}{1 + f/f_{cav}}$$

where

$$f_{cav} = \frac{1 - r_1 r_2}{r_1 r_2} \frac{c}{2L}$$

A smooth one pole roll-off.

Why is shot noise not white in a Fabry-Perot interferometer?

Shot noise in light power has a white (i.e., frequency independent) power spectrum.

No characteristic time scale, no memory.

But, for a given *h*, the arms give less phase shift at $f \gg f_{cav}$ than they do at DC.

Thus, to generate a given amount of output at high frequency, need a larger *h*.

A given amount of noise thus corresponds to a larger *h* at high frequency than it does at DC.