# 3: Shot noise and radiation pressure noise: classical treatment

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# My lectures during this School

1. Overview of gravitational waves and sources
2. Interactions of waves and detectors
3. **Shot noise and radiation pressure noise**
4. Theory of linear systems
5. Vibration isolation (passive)
6. Thermal noise
7. Optics of Fabry-Perot cavities
8. Feedback control systems
9. Description of LIGO and other current detectors
10. Future detectors in space

# Outline

- The challenges of gravitational wave detection
- Measurement noise vs. displacement noise
- Review of interferometer response
- Shot noise
- Shot noise in an interferometer
- Radiation pressure noise and the quantum limit

# Gravity wave detection: challenge and promise

Challenges of gravity wave detection appear so great as to make success seem almost impossible.

The challenges are real, but are being overcome.

# Gravitational wave detection is almost impossible

What is required for LIGO to succeed:

- interferometry with free masses,
- with strain sensitivity of $10^{-21}$ (or better!),
- (which is equivalent to ultra-subnuclear position sensitivity),
- in the presence of much larger noise.

# Interferometry with free masses

What's "impossible": everything!

Mirrors need to be very accurately aligned (so that beams overlap and interfere) and held very close to an operating point (so that output is a linear function of input.)

Otherwise, interferometer is dead or swinging through fringes.

Michelson bolted everything down.

# Strain sensitivity of $10^{-21}$

Why it is "impossible":

Sensitivity $h_{rms}$ can be expressed as

$$h_{rms} \sim \frac{\text{precision to which we can compare arm lengths}}{\text{length of arms}}.$$

Natural "tick mark" on interferometric ruler is one wavelength.

Michelson could read a fringe to $\lambda/20$, yielding $h_{rms}$ of a few times $10^{-9}$.

# Ultra-subnuclear position sensitivity

Why people thought it was impossible:

- Mirrors made of atoms, $10^{-10}$ m.
- Mirror surfaces rough on atomic scale.
- Atoms jitter by large amounts.

# Large mechanical noise

How large?

Seismic: $x_{rms} \sim 1\ \mu$m.

Can you filter it enough?

Thermal:

- mirror's CM: ~ 3 x $10^{-12}$ m.
- mirror's surface: ~ 3 x $10^{-16}$ m.

No filtering is possible. Can lower the temperature, but by enough?

# Gravitational wave detection will succeed very soon

All of these challenges sound impossible.

And yet, all of them can be met.

Detectors of $10^{-21}$ have been built and run.

Detectors 10 or more times better will start operating in a few years, including in India.

With them, we are just about certain to detect gravitational waves.

**This week's goal is to know why we should be confident that this is true.**

# Two classes of noise

Later, we will talk about displacement noise, that is, noise that moves LIGO's mirrors:

seismic noise

thermal noise

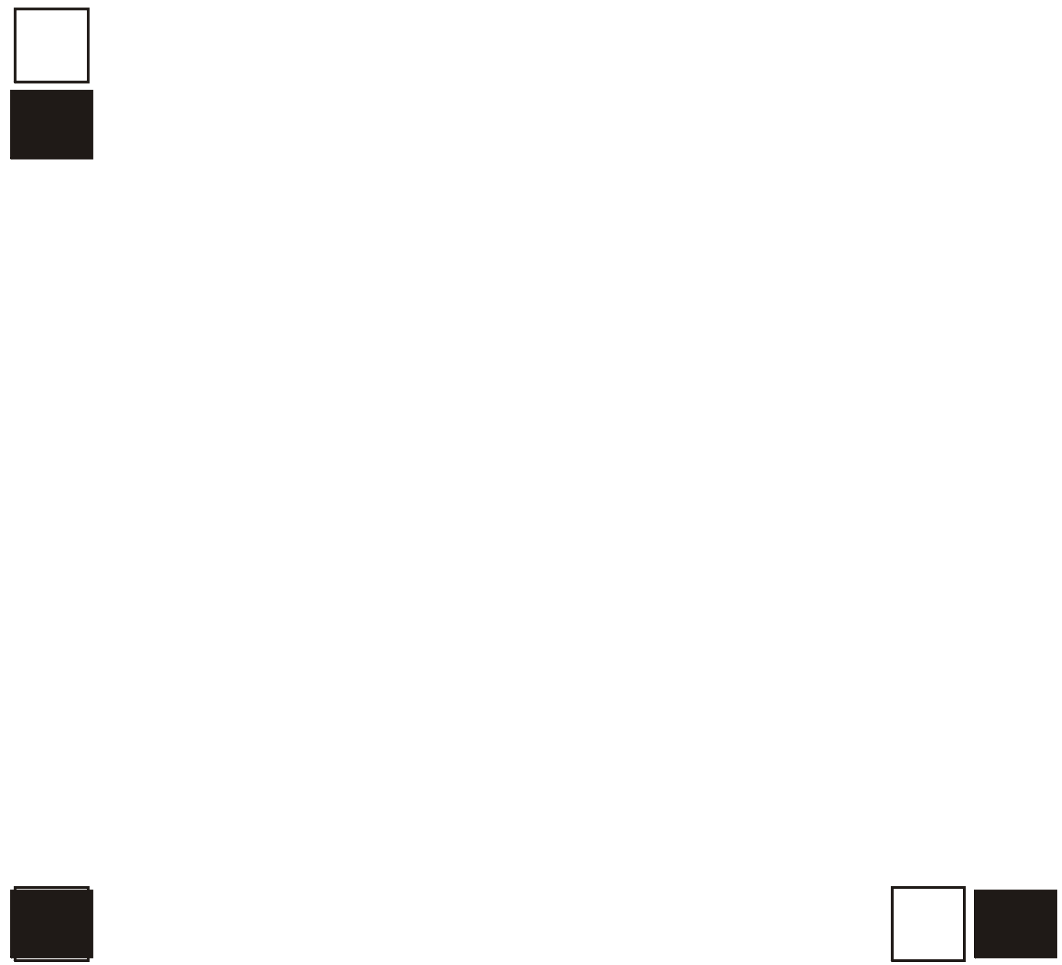
First, let's consider instead the noise that affects our ability to see where the mirrors are.

Generically, one could call this readout noise.

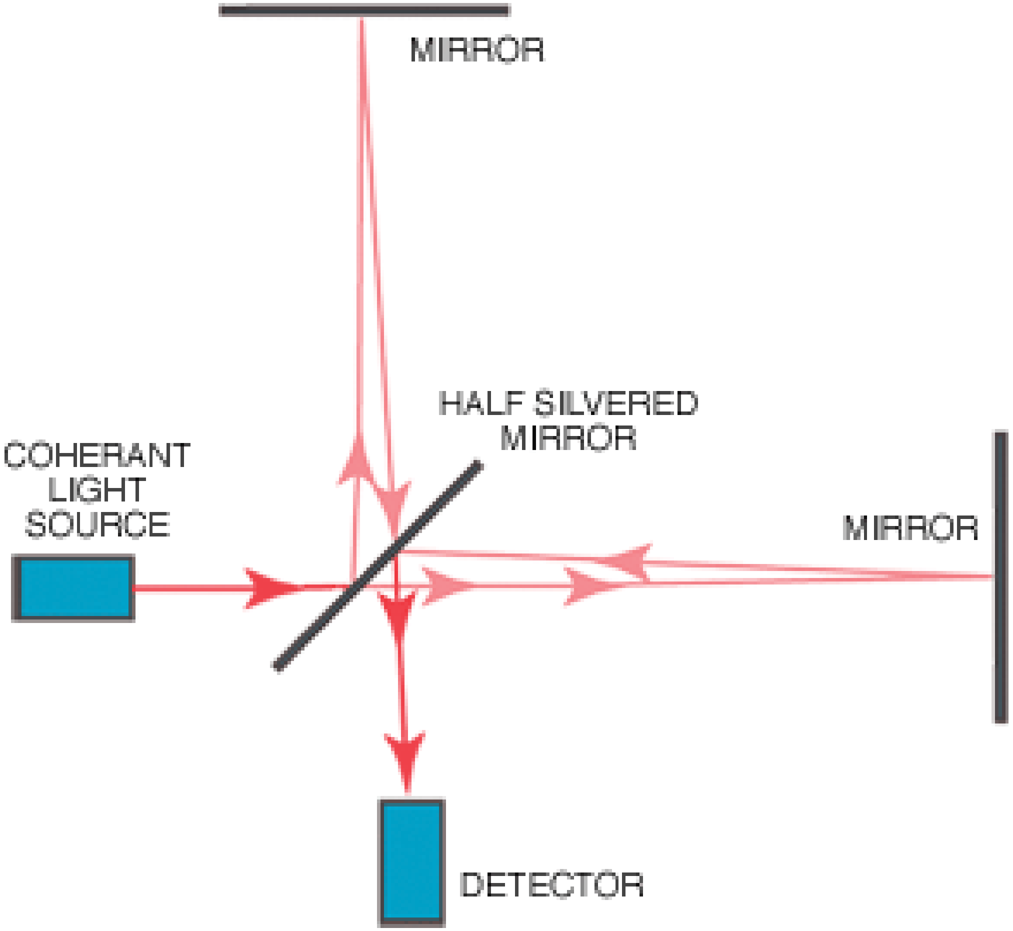
Our main source of readout noise is *shot noise*.

Associated with it is a force that moves the mirrors, *radiation pressure noise*.

# Three test masses



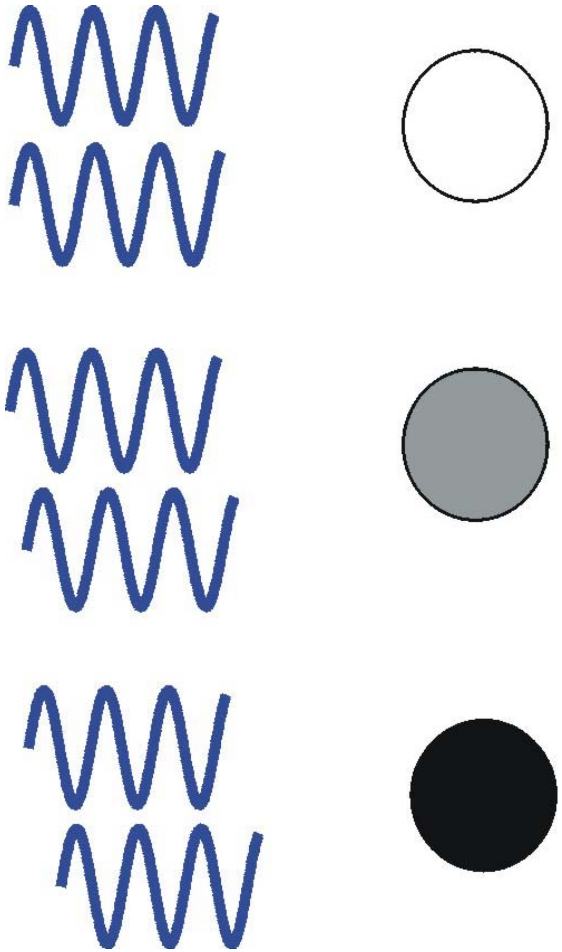
# Interferometer



# A length-difference-to-brightness transducer

Wave from x arm.

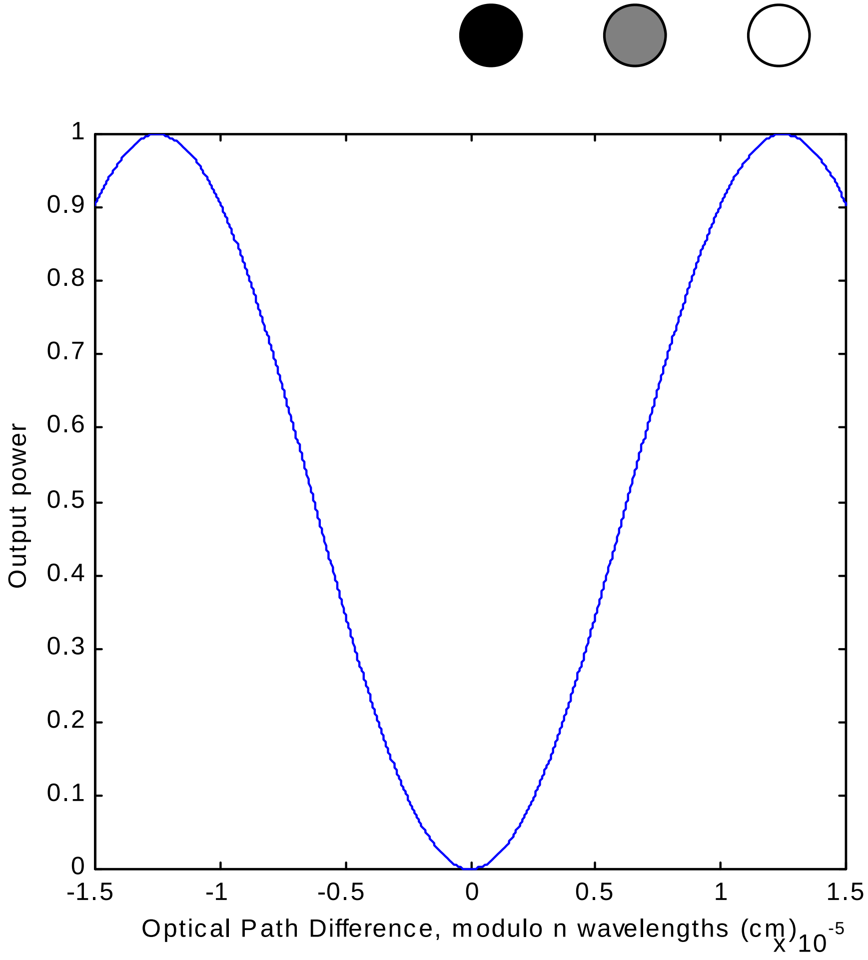
Wave from y arm.



Light exiting from beam splitter.

As relative arm lengths change, interference causes change in brightness at output.

# Interferometer output vs. arm length difference



# Sensitivity of interferometer with "on/off" readout

If we only distinguish between bright and dark output, interferometer wouldn't be very sensitive.

$$h_{rms} \sim \frac{\text{precision to which we can compare arm lengths}}{\text{length of arms}}.$$

$$\text{"on/off" precision } \Delta x \sim \lambda \approx 1\,\mu m$$

$$L = 4\,km \times 25$$

$$\Delta x / L \approx 10^{-11}$$

# The "fringe-splitting" solution

- We require ~10 more orders of magnitude in sensitivity, if we hope to see gravitational waves.
- If so, then we need to know much more than whether we are on the bright or dark point of a fringe.
- We need to know, to 1 part in $10^{10}$, where we are in the fringe.
- Is this possible? Yes.

# Light and photons

A beam of light consists of a stream of photons. A beam with power *P* (in watts) is a stream with a mean flux of

$$\bar{N} = \frac{\lambda}{2\pi\hbar c} P$$

(photons per second.)

# Shot noise

- Nothing guarantees that $\overline{N}$ photons arrive each second. Some seconds, more photons will arrive, while in other seconds fewer arrive.
- The statistics of sets of independent events have been well studied. The behavior is called *Poisson statistics* or *shot noise*.
- Key result: The size of the fluctuation depends simply on the expected value.

# $\sqrt{N}$

When you expect to find $N$ independent events on average, you should also expect that the standard deviation in a set of counts will be $\Delta N = \sqrt{N}$ . The noise grows with $N$, although not as fast as $N$.

What is the *fractional* precision for finding $N$?

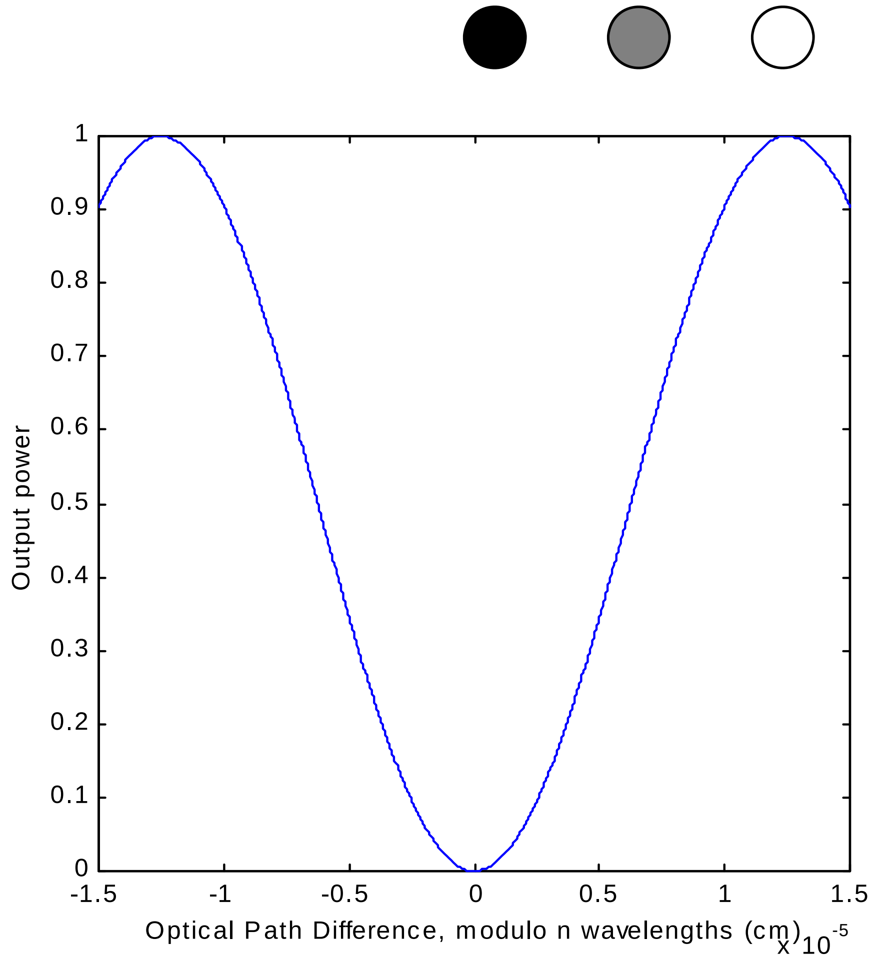
It is $\Delta N/N = \sqrt{N}/N = 1/\sqrt{N}$ . This shows that you can do a more precise job in measuring a flux of photons, if the flux of photons is larger.

# Shot noise in an interferometer

We measure the arm length difference by measuring the power out of the interferometer.

If noise makes the power a bit high, we think the length difference is different from its true value.

# Confusion of noise with signal



# Working out the numbers (for the iLIGO case)

200 W of light @ $\lambda = 1\ \mu$m carries $10^{19}$ photons per second.

$$1/\sqrt{10^{19}} \approx 3\times 10^{-10} \text{ of } (1\,\mu\text{m}/2\pi)$$

We can reach $h = 10^{-21}$ sensitivity by shining 200 W into the LIGO interferometer.

This is what we did in initial LIGO, using a 6 W laser (and the trick called "power recycling".)

# SNR vs. length

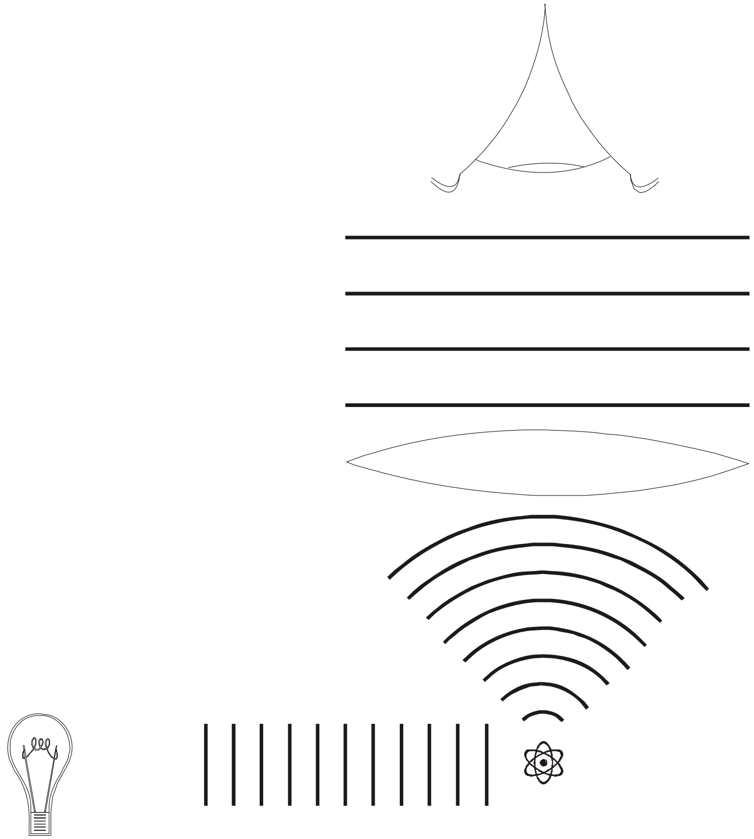
Signal grows with length, shot noise doesn't.

This is why LIGO is long!

Subtlety: Signal grows with the optical path length, which can be even longer if we make the light take many round trips.

Shot noise is independent of the number of bounces off of the mirrors. But displacement noise grows with the number of bounces. So, we can't use this trick too much!

# “The Heisenberg microscope”: a *gedanken* experiment



# The Quantum Limit

A powerful microscope is used to see where an atom is located.

Photons show where the atom is, but they also kick the atom by an unknown amount.

A wide lens gives better position resolution.

But a wide lens admits photons from greater angles.

<u>Position resolution vs. momentum uncertainty.</u>

# Quantum limit for LIGO

Shot noise resolution

$$\Delta x \propto \Delta N / N = 1/\sqrt{N}$$

Shot momentum perturbation

$$\Delta p \propto \Delta F_{rad} \propto \sqrt{N}$$

Position error from momentum perturbation

$$\Delta x_{rad} \propto \sqrt{N}$$

Too much laser power can, under some circumstances, be as bad as too little!

# How big is the fluctuating force?

200 W of light @ $\lambda = 1$ $\mu$m carries $10^{19}$ photons per second. So the typical fluctuation in photon number in a second is $\sqrt{10^{19}} \approx 3 \times 10^{9}$ .

The radiation pressure force associated with the fluctuating photon flux is $N^{1/2}h/\lambda = 2\ 10^{-18}$ newtons.

Multiply this by the number of times the beam encounters the mirror (say 25 for an iLIGO-like ifo.) This causes fluctuating motion of a 10 kg mass of about 2.5 $10^{-19}$ m on a 1 sec time scale.

This was OK in iLIGO. But in aLIGO, radiation pressure noise is important at low frequencies.

# Thinking about this in spectral language…

(… anticipating some results of my lecture from this afternoon.)

A Poisson process has a *white* (i.e., frequency independent) *spectrum*.

Shot noise (as precision in the measurement of arm length difference) is equally bad at all frequencies.

Radiation pressure noise also has a white spectrum, for the same reason. But, it is a noisy force. Pushing on a free mass, it causes noise displacements that have a spectrum $\propto 1/f^2$ .

# Why is there radiation pressure noise in quantum mechanics?

- Understanding of the origin of this noise is subtle. We used semi-classical argument (discreteness of photons, assumption of independent behavior of photons at beamsplitter.)
- Orthodox QM can sound like this argument fails. Dirac wrote, "We must now describe the photon as going partly into each of the two components into which the beam is split."
- If so, no imbalanced force in two arms, thus no noise in a grav wave interferometer. True?

# Radiation pressure noise is real

Some famous physicists thought this was true.

But Dirac also writes about an interferometer where you measure the energy of recoil of a mirror. Then, "wavefunction collapses", and photon will have to "choose" which arm to enter. Get recoil noise.

Subtle.

# The idea of squeezing

Caves (1980) tried to find a way to get this problem right. His analysis of the coupled equations of mirrors/masses and EM field highlighted the role of vacuum fluctuations that enter the “output” port of the interferometer.

Sounds mysterious, but has a practical consequence: If you can modify the properties of the fields entering that port, can modify the trade between shot noise and radiation pressure noise. This modification can be done – it is called squeezing.

Rana will tell you more about this.