2: Interactions of gravitational waves with detectors

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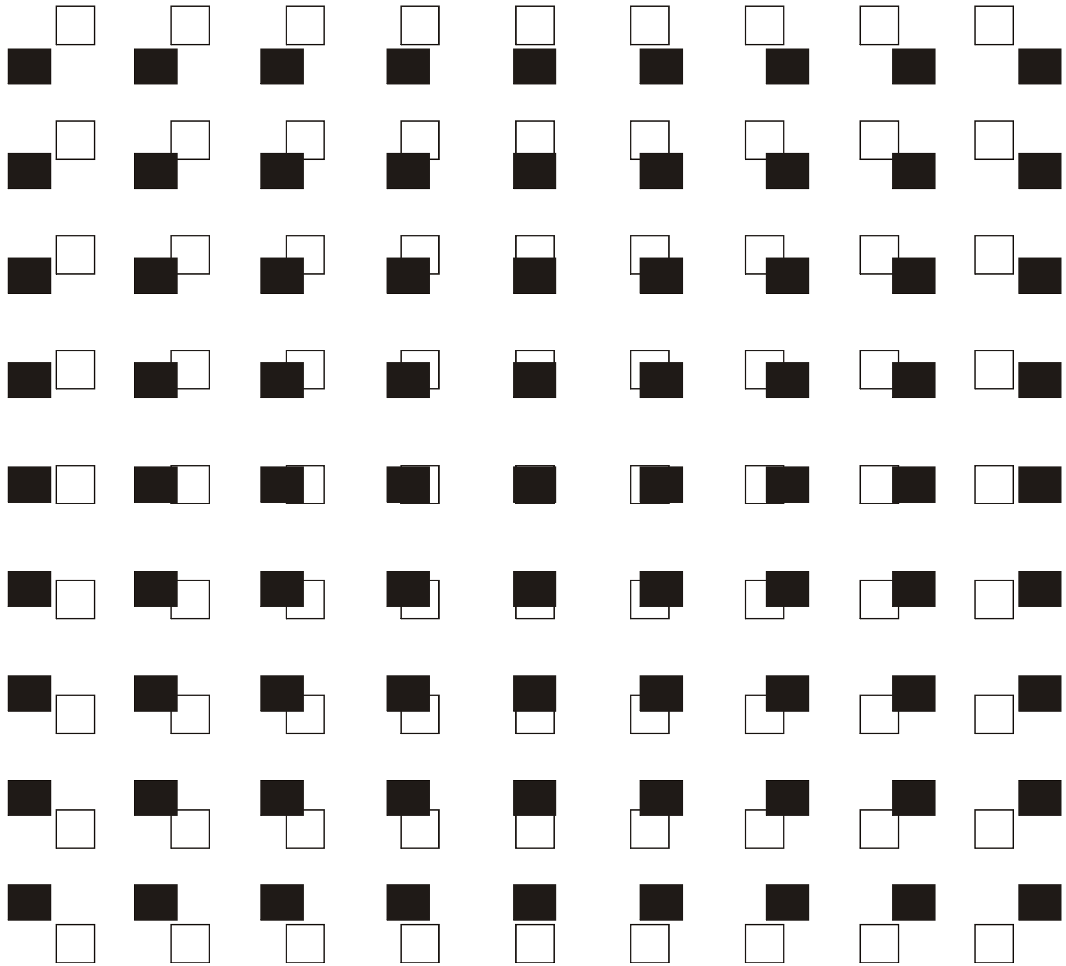
My lectures during this School

1. Overview of gravitational waves and sources
2. **Interactions of waves and detectors**
3. Shot noise and radiation pressure noise
4. Theory of linear systems
5. Vibration isolation (passive)
6. Thermal noise
7. Optics of Fabry-Perot cavities
8. Feedback control systems
9. Description of LIGO and other current detectors
10. Future detectors in space

Outline

1. How does an interferometer respond to gravitational waves?
2. A puzzle: If light waves are stretched by gravitational waves, how can we use light as a ruler to detect gravitational waves?

Freely-falling masses



Distance measurement in relativity…

… is done most straightforwardly by measuring the light travel time along a round-trip path from one point to another. (Felix Pirani, 1956)

- Because the speed of light is the same for all observers.

Examples:

- light clock
- Einstein’s train *gedanken* experiment

The *space-time interval* in special relativity

Special relativity says that the interval

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

between two events is *invariant* (and thus worth paying attention to.)

In shorthand, we write it as $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ with the Minkowski *metric* given as

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Generalize a little

General relativity says almost the same thing, except the metric can be different.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The trick is to find a metric $g_{\mu\nu}$ that describes a particular physical situation.

The metric carries the information on the space-time curvature that, in GR, embodies gravitational effects.

Gravitational waves

Gravitational waves propagating through flat space are described by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

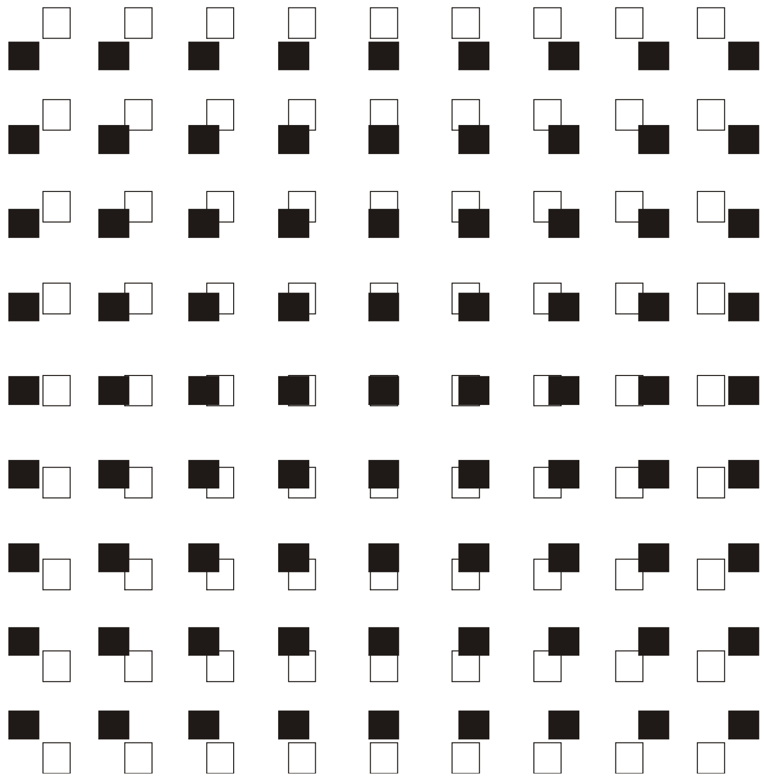
A wave propagating in the *z*-direction is described by

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Two free parameters implies two polarizations

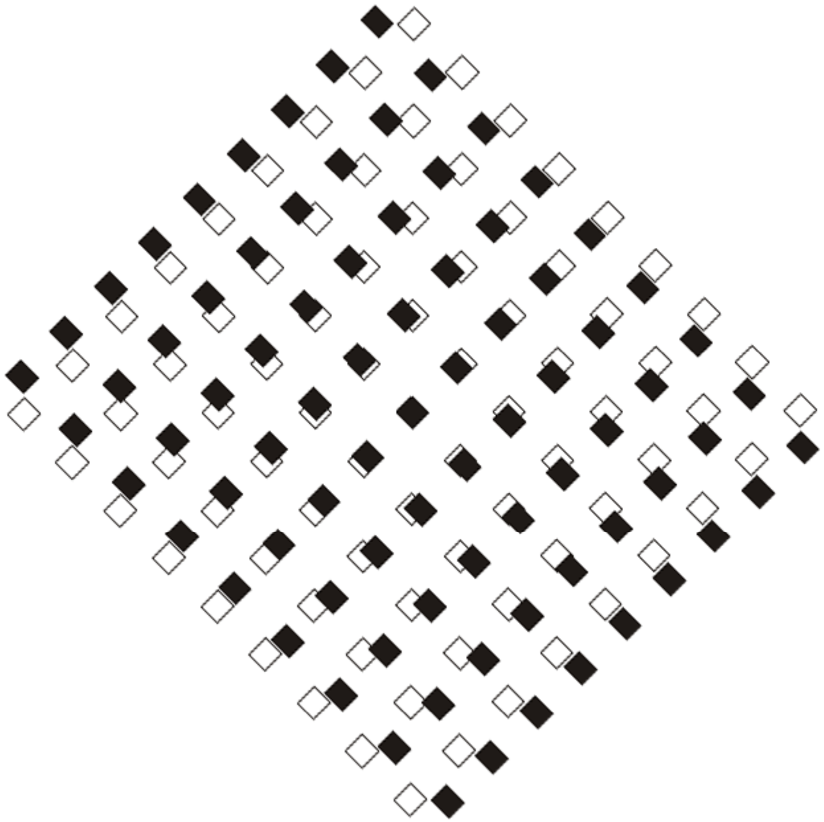
Plus polarization

$$\hat{h}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

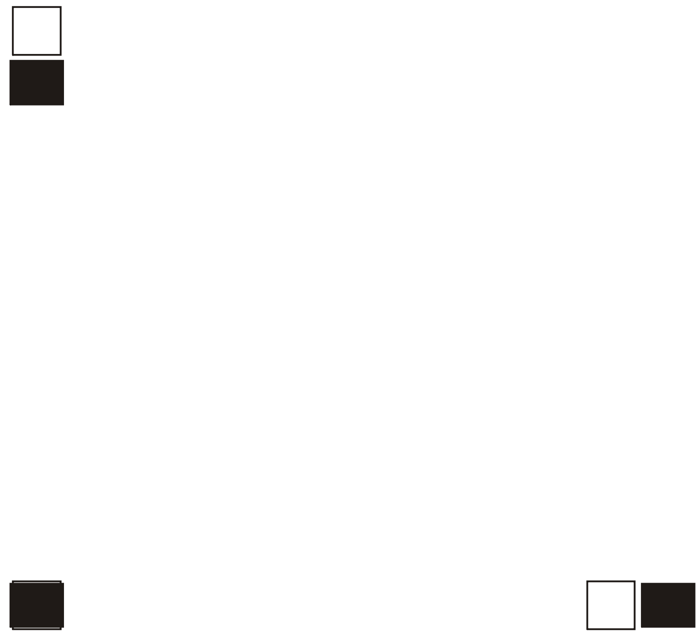


Cross polarization

$$\hat{h}_\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Three test masses



Solving for variation in light travel time

For light moving along the x axis, we are interested in the interval between points with non-zero dx and dt, but with $dy = dz = 0$:

$$ds^2 = 0 = -c^2 dt^2 + (1 + h_{11}) dx^2$$

Solving for variation in light travel time: start with *x* arm

$$ds^2 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2 = 0$$

- *h(t)* can have any time dependence, but for now assume that *h(t)* is constant during light's travel through ifo.
- Rearrange, take square root, and replace square root with 1st two terms of binomial expansion

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{11}\right) dx$$

- then integrate from $x = 0$ to $x = L$:

$$\Delta t = h_{11} L / 2c$$

Solving for variation in light travel time (II)

In doing this calculation, we choose coordinates that are marked by free masses.

"Transverse-traceless (TT) gauge"

Thus, end mirror is always at $x = L$.

Round trip back to beam-splitter:

$$\Delta t = h_{11} L / c$$

y-arm (h_{22} = - h_{11} = $-h$):

$$\Delta t_y = -hL / c$$

Difference between x and y round-trip times:

$$\Delta \tau = 2hL / c$$

Multipass, phase diff

To make the signal larger, we can arrange for *N* round trips through the arm instead of 1.

More on this in a later lecture.

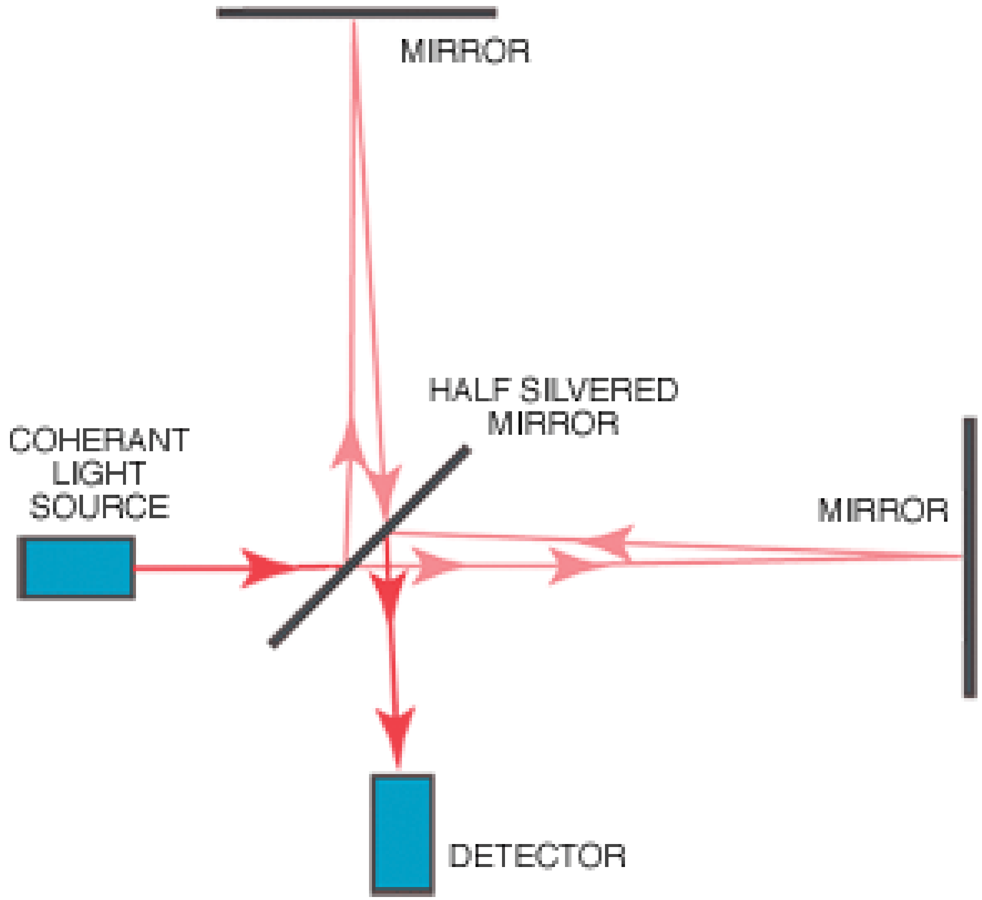
$$\Delta\tau = h\frac{2NL}{c} \equiv h\tau_{stor}$$

It is useful to express this as a phase difference, dividing time difference by radian period of light in the ifo:

$$\Delta\phi = h\tau_{stor}\frac{2\pi c}{\lambda}$$

Sensing relative motions of distant free masses

Michelson interferometer



How do we make the travel-time difference visible?

In an ifo, we get a change in output power as a function of phase difference.

At beamsplitter, light beams from the two arms are superposed. Thus, at the port away from laser

$$\left|E_{out}\right| = E_0 \cos\Delta\phi$$

and at the port through which light enters

$$\left|E_{refl}\right| = E_0 \sin\Delta\phi$$

Output power

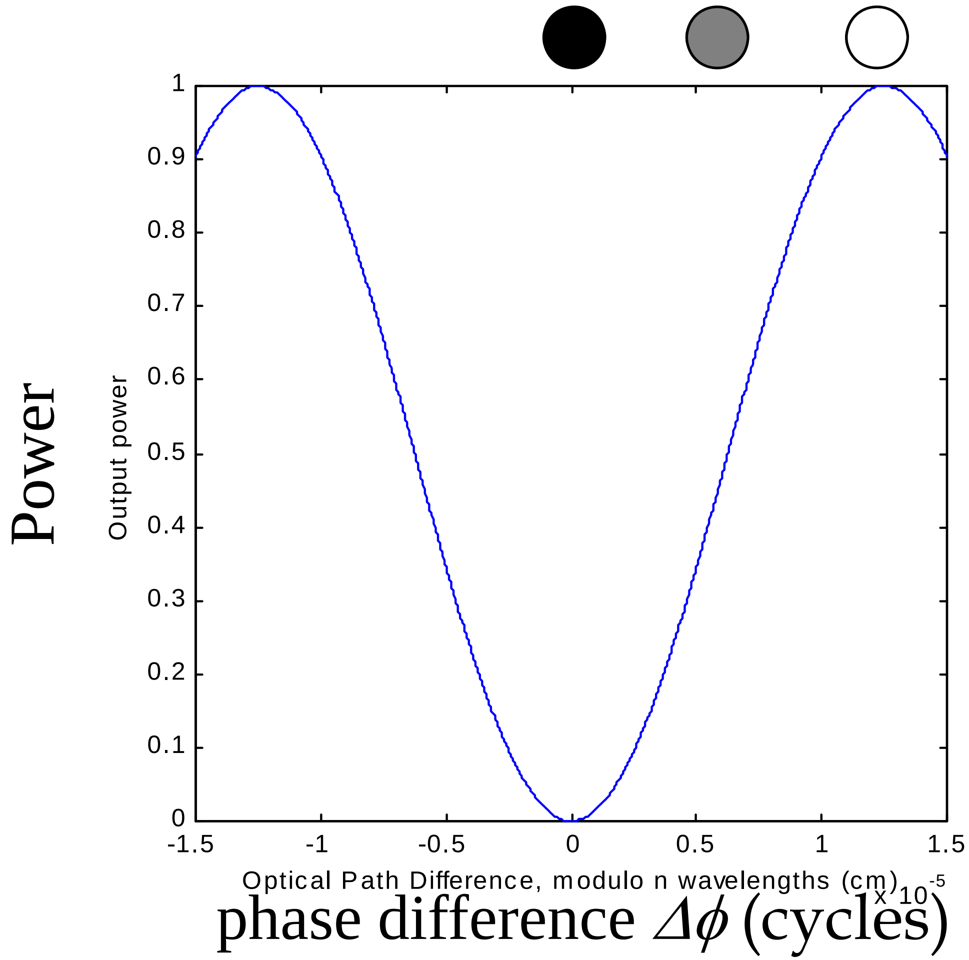
We actually measure the optical power (not the electric field) at the output port (recall $P \propto E^2$)

$$P_{out} = P_{in} \cos^2 \Delta\phi$$
$$= \frac{P_{in}}{2}\left(1 + \cos 2\Delta\phi\right)$$

Note that energy is conserved:

$$P_{out} + P_{refl} = P_{in}\left(\cos^2 \Delta\phi + \sin^2 \Delta\phi\right) = P_{in}$$

Interferometer output vs. arm length difference



Ifo response to *h(t)*

- Free masses are free to track time-varying *h*.
- As long as τ_{stor} is short compared to time scale of *h(t)*, then output tracks *h(t)* faithfully.
  - If not, then put time-dependent *h* into integral of slide 13 before carrying out the integral.
  - Response "rolls off" for fast signals.
- This is what is meant by interferometers being *broad-band* detectors.
- But, noise is stronger at some frequencies than others. (More on this later.) This means some frequency bands have good sensitivity, others not.

Interpretation

A gravitational wave's effect on one-way travel time:

$$\Delta t = \frac{h}{2}\frac{L}{c}$$

Just as if the arm length is changed by a fraction

$$\frac{\Delta L}{L} = \frac{h}{2}$$

In the TT gauge, we say that the masses didn't move (they mark coordinates), but that the separation between them changed.

Metric of the space between them changed.

Comparison with rigid ruler, force picture

We can also interpret the same physics in a different picture, using different coordinates.

Here, define coordinates with rigid rods, not free masses.

With respect to a rigid rod, masses <u>do</u> move apart.

In this picture, it is as if the gravitational wave exerts equal and opposite forces on the two masses.

How are tests masses realized?

Interferometer: test masses are 40 kg cylinders of fused silica, suspended from fine wires as pendulums

Effectively free for horizontal motions at frequencies above the resonant frequency of the pendulum (~1 Hz)

This justifies modeling them as free masses.

The "rubber ruler puzzle"

If a gravitational wave stretches space, doesn't it also stretch the light traveling in that space?

If so, the "ruler" is being stretched by the same amount as the system being measured.

And if so, how can a gravitational wave be observed using light?

How can interferometers possibly work?

Handy coordinate systems

In GR, freely-falling masses play a special role, whose motion is especially simple.

It makes sense to use freely-falling masses to mark out coordinate systems to describe simple problems in gravitation.

For gravitational waves, the *transverse-traceless gauge* used on previous slides is marked by freely-falling masses as well.

A gravitational wave stretches light

Imagine many freely-falling masses along arms of interferometer.

Test case: imagine that a *step function* gravitational wave, with amplitude h and + polarization, encounters interferometer.

Along x arm, test masses suddenly farther apart by $(1+h/2)$.

Wavefronts near each test mass stay near the mass. (No preferred frames in GR!)

The wavelength of the light in an interferometer is stretched by a gravitational wave.

Are length changes real?

Yes.

Interferometer arms really do change length.

We could (in principle) compare arm lengths to rigid rods.

It is important not to confuse our coordinate choices with facts about dynamics.

"Eppur si muove"

OK, so how can interferometers work?

- Argument above proves that there is no *instantaneous* response to a gravitational wave.
- But, we don't just care about the instantaneous response. We can wait.
- Clearly, new light produced by the laser (after gravitational wave has passed by) isn't affected by the gravitational wave, so if we wait to measure using all "new light", it <u>must</u> reveal the changed arm lengths.

The time-dependent response

The x arm was lengthened by the gravitational wave.

Light travels at *c*. So, light will start to arrive late, as it has to traverse longer distance than it did before the wave arrived.

Delay builds up until all light present at wave's arrival is flushed out. Then delay stays constant at $\Delta\tau = h(2NL/c)$.

New light isn't stretched, so it serves as the good ruler

In the end, there is no puzzle.

The time it takes for light to travel through stretched interferometer arms is still the key physical concept.

Interferometers *can* work.