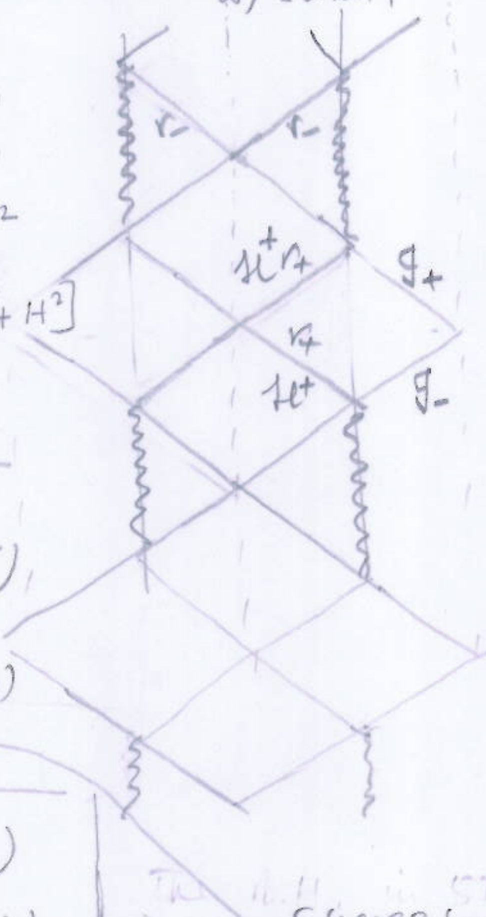
+ Green's functⁿ

D1-D5 : Hawking radiation

ref: Phys. Rep. w/ Sumit



The classical solution

$$S_{IIB} = \frac{1}{G_N^{(10)}}\int d^{10}x\sqrt{g}\left[e^{-2\Phi}\left(R+(\partial\Phi)^2+H^2\right) + e^{-\Phi}(F^{(3)})^2\right]$$

$\downarrow$

$$= \frac{V}{G_N^{(6)}}\int d^6x\sqrt{g}\left\{e^{\sigma-2\Phi}\left[R+(\partial\Phi)^2+\overbrace{(\partial h_{67})^2}^{\phi^2}+H^2\right] + e^{\sigma-\Phi}(F^{(3)})^2\right\}$$

2-charge BPS

$$ds^2 = f_1^{-1/2} f_5^{-1/2}\, dx_\mu dx^\mu + f_1^{1/2} f_5^{1/2}(dr^2 + r^2 d\Omega_3^2) + f_1^{1/2} f_5^{-1/2}\, \underbrace{dy_i dy_i} \leftarrow T^4 \text{ (or K3)}$$

6D Black string (zero-size)

Entropy $S = S_{Bek} = 0$ (classically)

'α'-correction' $S_{\text{Wald-Sen}} = \begin{cases} 4\pi\sqrt{Q_1 Q_5} & (K3) \\ 2\pi\sqrt{2Q_1Q_5} & (T^4) \end{cases}$

Atish 0409148 Eq.(23)

with Sumit Eq.(1.3)

Absorption cross-section

$$\boxed{\sigma_{cl}(\omega) = \pi^3 \ell^4 \omega}$$

$$\ell^4 \equiv r_1^2 r_5^2$$

[Temp T = 0]

How classical? $h_{00} = \frac{r_1^2+r_5^2}{r^2}$

$c_1 = \frac{g_s\alpha'}{\tilde v}$, $c_5 = g_s\alpha'$

$G_N^{(10)} = g_s^2\alpha'^4 = g_s^2 l_s^8$

$G_N^{(6)} = \frac{g_s^2\alpha'^4}{V} = \frac{g_s^2\alpha'^4}{\tilde v\alpha'^2} = \frac{g_s^2}{\tilde v}\alpha'^2$

$[L]^4$

$G_N^{(6)}\frac{M}{4R_5} = (r_1^2 + r_5^2) \sim c_5 Q \sim g_s l_s^2 Q$ | $\frac{Q_1}{\tilde v} \sim Q_5 \sim Q$

$\frac{M}{R_5} \sim \frac{g_s l_s^2 Q}{g_s^2 l_s^4} \sim \frac{Q}{g_s l_s^2}$ matches D1 brane

Charges

$F^{(3)} \sim \frac{c_1 Q_1}{r^3}\, dr\wedge dt\wedge dx_5$

$+ c_5 Q_5\, d\chi\wedge d\theta\wedge d\phi$

$*F^{(3)}_{mag} \sim \frac{c_5 Q_5}{r^3}\, dr\wedge dt\wedge dx_5 \wedge d^4y$

$h_{00} \sim \frac{c_1 Q_1 + c_5 Q_5}{r^2}$

[2.29 Phys rep.]

KK charges

$$\left.\begin{aligned} h_{05} + B_{05} &\sim \frac{P}{r^2} \\ h_{05} - B_{05} &\sim \frac{\bar P}{r^2} \end{aligned}\right\} (?)$$

$(P, \bar P) = (0,0)$ 2-charge system

$P \neq 0$ $\bar P = 0 \Rightarrow$ BPS 3-charge

$P \neq 0$ $\bar P \neq 0$ $\Rightarrow$ non-BPS 3-charge

$$\frac{1}{R_5} \sim \frac{Q}{g_s} \sim \frac{Q^2}{(g_s Q)}$$ in string units (recall in Maldacena's scaling N' fixed)

$$g_s^2 l_s^4 = l_{P,6}^4 \Rightarrow l_P = \sqrt{g_s}\, l_s$$

$$\frac{M}{R_5} \sim \frac{Q^2}{g_s Q}\cdot\frac{1}{l_s^2} \sim \frac{Q^2}{g_s l_s^2}\cdot\frac{1}{Q} \sim \frac{Q}{l_{P,6}^2}$$

$\therefore$ tension of black string $= Q$ in Planck units !

In the limit $Q \to \infty$, the solution is classical.

What does this say about $G_N^{(6)}$?

$$G_N^{(6)} \simeq \frac{g_s^2 l_s^4}{\tilde{V}} \simeq \frac{l_{P,6}^4}{\tilde{V}}$$ held fixed in

$$S = \frac{1}{G_N^{(6)}} \int d^6x \sqrt{g}\, e^{-2\Phi}\Big[\underset{\substack{\uparrow \\ \text{classical}}}{R} + (\partial h_{67})^2\Big]$$

$$H = H_{class} + H(h_{67}) \qquad \omega\, \underbrace{\alpha^\dagger_{67}\alpha_{67}}_{N_\alpha} \sim N_\alpha \omega \sim N_\alpha \cdot \frac{1}{R}$$

$$= \left(\frac{Q}{l_P^2}\right) R_5 + \frac{N_\alpha}{R_5}$$ (Planck units)

$$= Q R_5 + \frac{N_\alpha}{R_5}$$

Need $Q \gg \frac{N_\alpha}{R_5^2}$ which is trivially satisfied if $R_5 \gtrsim l_s$ which anyway is necessary for a classical gravity description.

[i.e. $Q \gg \frac{l_s^2}{R_5^2} N_\alpha$]

$$\boxed{\begin{array}{l}\text{Hence as long as} \\ \qquad Q \gg N_\alpha \\ (\text{e.g. } Q \gg 1,\ N_\alpha \simeq O(1)) \\ \text{we can ignore the back-reaction}\end{array}}$$

Microscopic descripti

$$\frac{M}{R_5} = Q_1 \cdot \frac{1}{g_s l_s^2} + Q_5 \frac{V}{g_s l_s^6} = \text{tension of } Q_1 \text{ D1-branes and } Q_5\text{- D5-branes}$$

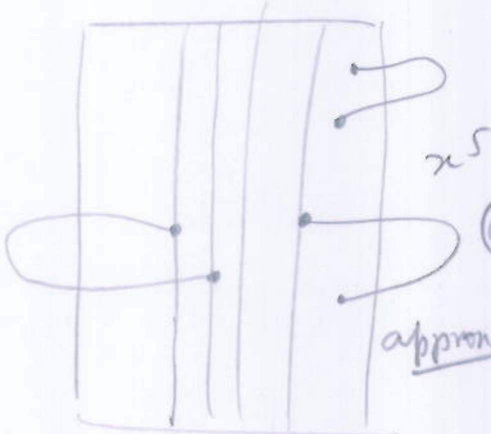
masses add up since the grav. attracti and RR repulsin cancel bet. D1 and D5 (BPS)

[why is there an RR force bet. D1 and D5?

Q_1 sources $F^{(3)}_{el}$, Q_5 sources $F^{(3)}_{mag}$

in presence of B fields

$$S_{D5} \sim Q_5 \int \underbrace{C^{(2)}}_{\propto Q_1} \wedge B^{(2)} \wedge B^{(2)} \quad]$$



(Q_1, Q_5) bifundamental

approx. $4Q_1Q_5$ bose oscillators $\alpha^i_A(\omega)$ | $i = 1,2,3,4$

$4Q_1Q_5$ ferm " $\gamma^a_A(\omega)$ | $A = 1, \dots, Q_1Q_5$

| $a = 1,2,3,4$

$x^{6,7,8,9}$

modes $q_\omega \sim e^{-i\omega(t - x_5)}$

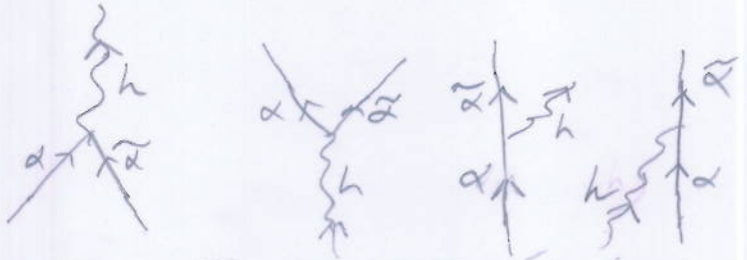
$$q^i_{A,\omega}(x_5 + 2\pi R_5) = q^i_{A+1,\omega}(x_5)$$

$$\omega = 2\pi \frac{n}{R_5} \qquad e^{-i\frac{m}{R_5}(t \pm x_5)}$$

$$\{\alpha^i_A\} \longrightarrow \underline{\underline{\alpha^i}}$$

Coupling to gravity

$$\boxed{\int d^6x (\partial h)^2 + \int \sqrt{G_N^{(6)}}\, \alpha^6 \tilde{\alpha}^7 h_{67} d^2\xi}$$

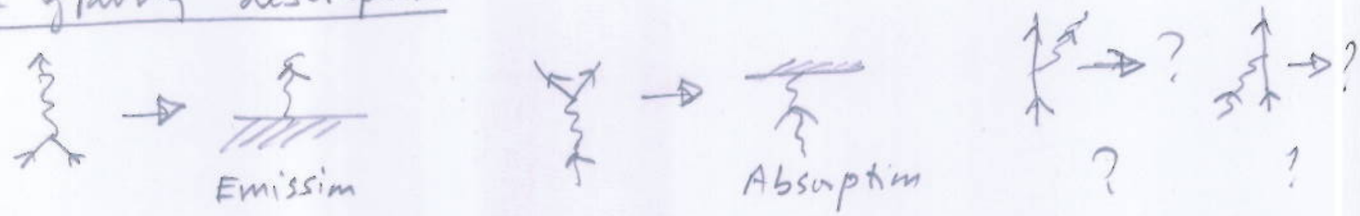


~~Big state~~

$(\alpha^+_n)^{N_n} (\tilde{\alpha}^+_m)^{N_m} |0\rangle$ $\sim \sum$ $\sqrt{G_N}$ $\sqrt{G_N}$ = $G_N P$

Brane   vertex op.   geometry

The gravity description



Emission: $|N_{\omega/2}, \tilde N_{\omega/2}\rangle \xrightarrow{S_{int}} |N_{\omega/2}-1, \tilde N_{\omega/2}-1\rangle \otimes h^+_\omega|0\rangle$

$$|BH, M = \frac{P+\bar P}{R_5}\rangle \longrightarrow |BH, M = \frac{P+\bar P}{R_5} - \omega\rangle \otimes h^+_\omega |0\rangle$$

$$\langle f|S_{int}|i\rangle \sim = \langle\Omega| h_\omega \frac{(\alpha_{\frac{\omega}{2}})^{N'_{\frac{\omega}{2}}} (\tilde\alpha_{\frac{\omega}{2}})^{\tilde N'_{\frac{\omega}{2}}}}{\sqrt{N'_{\frac{\omega}{2}}!}\sqrt{\tilde N'_{\frac{\omega}{2}}!}} \left(h^+_\omega \alpha_{\frac{\omega}{2}} \tilde\alpha_{\frac{\omega}{2}}\right) \frac{(\alpha^+_{\omega/2})^{N_{\omega/2}} (\tilde\alpha^+_{\omega/2})^{\tilde N_{\frac{\omega}{2}}}}{\sqrt{N_{\omega/2}!}\sqrt{\tilde N_{\frac{\omega}{2}}!}} |\Omega\rangle$$

$$\delta_{N_{\omega/2}, N'_{\frac{\omega}{2}}+1}\, \delta_{\tilde N_{\frac{\omega}{2}}, \tilde N'_{\frac{\omega}{2}}+1}$$

Exercise.

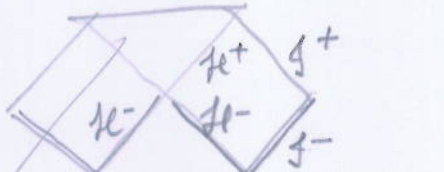
$$\langle 0| \frac{(\alpha)^{N-1}}{\sqrt{(N-1)!}} \alpha \frac{(\alpha^+)^N}{\sqrt{N!}} |0\rangle = \sqrt{N} \langle 0| \frac{\alpha^N}{\sqrt{N!}} \frac{\alpha^{+N}}{\sqrt{N!}} |0\rangle = \sqrt{N}$$

$$\therefore \mathcal{M}_{fi} = \sqrt{N_{\omega/2}}\sqrt{\tilde N_{\omega/2}}$$

$$P \sim \frac{1}{\Omega_N} \sum_{\{N', N\}} |\mathcal{M}_{fi}|^2 \sim \frac{1}{\Omega_N} \sum_{\{N_\omega\}} (N_{\omega/2})(\tilde N_{\omega/2}) \sim \langle N_{\omega/2}\rangle\langle \tilde N_{\omega/2}\rangle$$

Compare with Hawking's calculati

~~$|in\rangle = |0, U\rangle$ Unruh $a_\omega|0,U\rangle = 0$~~



$$e^{-i\omega u} = e^{-i\omega\left(-\frac{1}{\kappa}\ln(-\kappa U)\right)} = (-\kappa U)^{i\omega/\kappa}$$

$$\phi = \sum_\omega \tilde a_\omega e^{-i\omega U} + a_\omega e^{-i\omega V}$$

$$\int e^{-i\omega u} e^{i\omega' U} dU = \alpha_{\omega\omega'} = \Gamma($$

$(-\kappa U)^{i\omega/\kappa}$

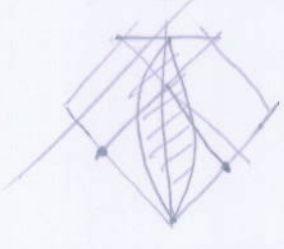
$a_\omega|0,U\rangle = 0 = \tilde a_\omega|0,U\rangle$

$$\beta_{\omega\omega'} = \alpha_{\omega(-\omega')}$$

$$\phi = \sum b_\omega e^{-i\omega u} + \tilde b_\omega e^{-i\omega V}$$

$+\left(\frac{x\kappa}{i\omega'}\right)^{i\omega/\kappa}$

$$\beta_{\omega\omega'} = e^{-\pi\omega/\kappa}\alpha_{\omega\omega'}$$



$$b_\omega|out\rangle = 0 = \tilde b_\omega|out\rangle$$

Hawking's calculation

$$a_\omega |0\rangle_U = 0 = \tilde a_\omega |0\rangle_U \qquad \phi = \sum_\omega \underbrace{a_\omega e^{-i\omega v}}_{\mathcal{J}^-} + \underbrace{\tilde a_\omega e^{-i\omega U}}_{\mathcal{H}^-}$$

$$e^{-i\omega U} = \sum_{\omega'>0} \bar\alpha_{\omega\omega'} e^{-i\omega' u} + \beta_{\omega\omega'} e^{i\omega' u}$$

$$e^{-i\omega u} = \sum_{\omega'} \alpha_{\omega\omega'} e^{-i\omega U} + \beta_{\omega\omega'} e^{i\omega' U}$$

$$\phi = \sum_\omega \Big[a_\omega \left(\gamma_{\omega\omega'} e^{-i\omega V} + \delta_{\omega\omega'} e^{+i\omega V} \right) + a^+_\omega \left(\qquad \right)$$
$$+ \tilde a_\omega \left(\alpha_{\omega\omega'} e^{-i\omega u} + \beta_{\omega\omega'} e^{i\omega u} \right) \Big]$$
$$+ \tilde a^+_\omega \left(\bar\alpha_{\omega\omega'} e^{i\omega u} + \bar\beta_{\omega\omega'} e^{-i\omega u} \right)$$

$$b_\omega = \tilde a_\omega \alpha_{\omega\omega'} + \tilde a^+_\omega \bar\beta_{\omega\omega'}$$

$${}_U\langle 0 | b^+_\omega b_\omega | 0 \rangle_U = \sum_{\omega'} |\beta_{\omega\omega'}|^2 = (e^{\beta\omega} - 1)^{-1}$$

$$\mathrm{Tr}_{\tilde b} |0\rangle_U {}_U\langle 0| = \rho_\beta$$

$$|0\rangle_U = \sum e^{-\frac{\beta\omega}{2} b^+_\omega b^{+\prime}_\omega} |0\rangle_b \otimes e^{-\beta\omega/2\, c^+_\omega c^{+\prime}_\omega} |0\rangle_c$$

b production by $|0\rangle_U$

$${}_b\langle 0 | b_\omega |0\rangle_U \longrightarrow |0\rangle_b |0\rangle_{b'} + e^{-\beta\omega/2}\, b^+ |0\rangle_b\, b^{+\prime} |0\rangle_{b'}$$
$$+ \cdots$$

$${}_U\langle 0 | \underbrace{b^+ b^{+\prime}}\, | 0 \rangle_{b,b'} \sim e^{-\beta\omega/2}$$

$\boxed{b \text{ is like } h\,!}$ $\qquad \langle N_\omega \rangle = (e^{\beta\omega} - 1)^{-1}$

Detectors and Green's fns.

'scalar charge' cf. $J_0 = e\dot{x}_0$

$$ig\langle E,\psi|\int_{-\infty}^{+\infty} d\tau\, m(\tau)\,\phi[x(\tau)]\,|0_M, E_0\rangle$$

$|\psi\rangle = \sum \psi_k |1_{\vec{k}}\rangle$

$\cancel{\int_{-\infty}^{+\infty} d\tau\, \langle E|m(\tau)|E_0\rangle\, \langle\psi|\,\phi\,|0_M\rangle}$

~~$|\psi\rangle$ is of the form~~ $\cancel{A_{\vec{k}}\,|1_{\vec{k}}\rangle \qquad |1_{\vec{k}}\rangle = a^\dagger_{\vec{k}}|0_M\rangle}$

$\cancel{\langle 1_{\vec{k}}|\,\phi[x(\tau)]\,|0_M\rangle = \langle 1_{\vec{k}}|e^{+i\vec{k}\cdot\vec{x}(\tau) - i\omega t(\tau)}\, a^\dagger_{\vec{k}}|0_M\rangle = e^{i\vec{k}}}$

$$\int d\tau\, d^4x\, \underbrace{\langle E|\,m(\tau)\,\delta^4(x - x(\tau))\,|E_0\rangle}\, \underbrace{\langle 1_{\vec{k}}|\,\phi(x)\,|0_M\rangle}$$

$$\iint_{d\tau d^4x} e^{i(E-E_0)\tau}\langle E|m(0)|E_0\rangle\,\delta^4(x - x_{cl}(\tau))\int d^3k\,\frac{1}{\sqrt{2\omega}}\,e^{i\vec{k}\cdot\vec{x} - i\omega t}\,\psi^*_{\vec{k}}$$

Imagine: detector is infinitely heavy and does not jiggle about q.m. mechanically

$$x(\tau) \simeq x_{cl}(\tau) + \cancel{x_{qu}(\tau)}$$

Example: ~~non~~inertial detector

$$x^\mu(\tau) = (\gamma\tau, \gamma\vec{v}\tau) = u^\mu\tau \qquad \gamma = (1-v^2)^{-1/2}$$

$$\int\frac{d^3k}{\sqrt{2\omega_k}}\int d\tau\, e^{i(E-E_0)\tau}\,\psi^*_{\vec{k}}\,\langle E|m(0)|E_0\rangle\, e^{-ik_\mu u^\mu\tau + ik_\mu x_0^\mu}$$

$$\delta(E - E_0 + k_\mu u^\mu)$$

$$-k_\mu u^\mu = (-\gamma\omega + \gamma\vec{k}\cdot\vec{v}) = \gamma(\omega - \vec{k}\cdot\vec{v}) > 0 \quad \text{for } \underline{m>0}$$

$\omega = \sqrt{\vec{k}^2 + m^2}$

rest frame ($=0$ only if $m = 0$)

$E \geq E_0 \qquad -k_\mu u^\mu > 0$

hence $\delta(\ldots)^{+ve} = 0$

$\therefore\ \mathcal{A}[\psi_{\vec{k}}] = 0$ for any $\psi_{\vec{k}}$ $\Rightarrow$ no particle detected

Detector does not get excited.

More generally

$|E,\psi\rangle$

$\uparrow$

$|E_0, 0_M\rangle$

$$g^2\sum_E |\langle E|m(0)|E_0\rangle|^2\,\mathcal{F}(E - E_0)$$

$$\mathcal{F}(E) = \int d\tau\int d\tau'\, e^{-iE(\tau-\tau')}\,G^+(x(\tau), x(\tau'))$$

Singleton's paper

$$G^+_{(x,t,x',t')} = -\frac{1}{4\pi} \ln\left[(\Delta u_i - i\varepsilon)(\Delta v_i - i\varepsilon)\right]$$

$$\Delta u_i = u_i(x^0, t^0) - u_i(x', t')$$

Mink. vacuum $\phi(x,t) = \sum a_\omega e^{-i\omega V} + \tilde{a}_\omega e^{-i\omega U} + h.c.$

$a_\omega |0_M\rangle = 0$

Mink. observer

$$\cancel{x = v^1 \tau \quad t = v^0 \tau} \quad \cancel{-(v^0)^2 + (v^1)^2 = -1}$$

$$\cancel{t = \sqrt{1+(v^1)^2}\,\tau} \quad \cancel{v^0 = \sqrt{1+(v^1)^2}}$$

$$\cancel{v = t + x =} \qquad dx^\mu dx_\mu = -d\tau^2$$

$$t = \gamma\tau, \quad x = \gamma v_1 \tau \qquad -(v^0)^2 + (v^1)^2 = -1$$

$$V = t + x = \gamma(1+v_1)\tau \qquad \gamma \cancel{\tfrac{1}{c}}, \ \tfrac{\vec{v}}{c}\gamma \quad \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$U = \gamma(1-v_1)\tau$$

$$\Delta U \Delta V = \gamma^2(1-v_1^2)\Delta\tau^2 = \Delta\tau^2 \qquad -\tfrac{1}{c^2}\gamma^2 + \tfrac{v^2}{c^2}\gamma^2 = \frac{-(1-v^2)}{c^2}\gamma$$

$$(\Delta u - i\varepsilon)(\Delta v - i\varepsilon) = \Delta\tau^2 - i\varepsilon\,\gamma\Delta\tau\left[1 + \cancel{v_1} + 1 - \cancel{v_1}\right] \qquad = -\frac{1-v^2}{1-v^2} = -1$$

$|\vec{v}| < 1$

$$= \Delta\tau^2 - i\varepsilon\, 2\gamma\Delta\tau$$

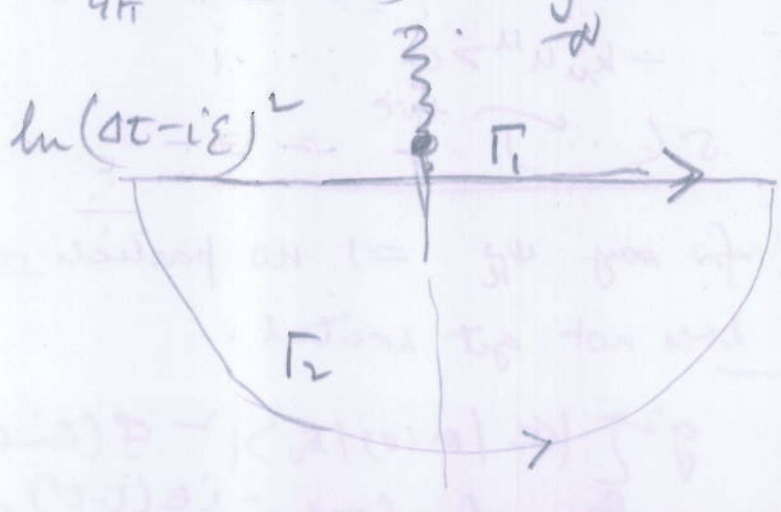
$$= (\Delta\tau - i\varepsilon\gamma)^2 \qquad \underline{\Delta\tau > 0}$$

$$= (\Delta\tau - i\tilde{\varepsilon})^2$$

$$G^+ = -\frac{1}{4\pi}\ln(\Delta\tau - i\tilde{\varepsilon})^2 \qquad \int_{-\infty}^{+\infty} G^+ e^{-i(E-E_0)\tau} d\tau$$

$E - E_0 > 0$

$\tau = -i\infty$ OK



$$\int_{\Gamma_1} = \int_{\Gamma_2} = 0$$

$$G^+_{(x,x')} = \text{+ve freq. Wightmann fn} = \langle 0|\phi(x)\phi(x')|0\rangle$$

$$\langle 0|\phi(x')\phi(x)|0\rangle = G^-(x,x')$$

$$iG_F(x,x') \equiv \langle 0|T(\phi(x)\phi(x')|0\rangle$$
$$= \theta(t-t')G^+(x,x')$$
$$+\theta(t'-t)G^-(x,x')$$

$G^+(x(\tau), x'(\tau))$ depends on the particle trajectory.

~~FFD detector~~ Schwarzschild detector

$$G_S = -\frac{1}{2\pi}\ln(\Delta u \Delta v)$$

Kruskal $G_K = -\frac{1}{2\pi}\ln(\Delta U \Delta V)$

$$(U_1 - U_2)(V_1 - V_2) \qquad U = -\frac{1}{\kappa}e^{-\kappa u}$$

$$= -\frac{1}{\kappa^2}\left(e^{-\kappa(t_1 - r_1^*)} - e^{-\kappa(t_2 - r_2^*)}\right)\left(e^{\kappa(t_1 + r_1^*)} - e^{\kappa(t_2 + r_2^*)}\right)$$

$$= -\frac{1}{\kappa^2}\Big[e^{2\kappa r_1^*} + e^{2\kappa r_2^*} - e^{-\kappa(t_2 - t_1)}e^{\kappa(r_1^* + r_2^*)}$$
$$- e^{\kappa(t_2 - t_1)}e^{\kappa(r_1^* + r_2^*)}\Big]$$

$$e^{\kappa(r_1^* + r_2^*)}\underline{\cosh\kappa(t_2 - t_1)}$$

only time dependence is in the form of $\cosh\kappa(t_2 - t_1)$ which is periodic in imag. time

$$t \equiv t + i\frac{2\pi}{\kappa}$$
$$\underline{\beta}$$

Better done in Singleton's paper

Rindler observer

$$x = \frac{1}{a}\,\mathrm{ch}\, a\tau \qquad u = -\frac{1}{a} e^{-a\tau}$$
$$t = \frac{1}{a}\,\mathrm{sh}\, a\tau \qquad v = +\frac{1}{a} e^{a\tau} \qquad \underline{\xi = \mathrm{const}}$$

$$\Delta u = -\frac{1}{a} e^{-a\tau} + \frac{1}{a} e^{-a(\tau+\Delta\tau)}$$
$$= -\frac{1}{a} e^{-a(\tau+\frac{\Delta\tau}{2})}\Big[\underbrace{e^{+a\Delta\tau/2} - e^{-a\Delta\tau/2}}_{2\,\mathrm{sh}\frac{a\Delta\tau}{2}}\Big]$$

$$\Delta v = +\frac{1}{a} e^{a\tau} - \frac{1}{a} e^{a(\tau+\Delta\tau)}$$
$$= -\frac{1}{a} e^{a(\tau+\frac{\Delta\tau}{2})}\, 2\,\mathrm{sh}\frac{a\Delta\tau}{2}$$

$$\frac{1}{4\pi}\ln(\Delta u - i\varepsilon)(\Delta v - i\varepsilon) = -\frac{4}{a^2}\ln\left(\mathrm{sh}^2 \frac{a\Delta\tau - i\varepsilon}{2}\right) \qquad \int (\,)\, e^{-i\omega\Delta\tau}\, d\tau \to \frac{1}{E(e^{\beta E}-1)}$$

$$\int_{-\infty}^{+\infty} e^{\left(\frac{a}{2}\right)(\Delta\tau - i\varepsilon)} \qquad \left(e^{\frac{a}{2}\Delta\tau} - e^{-\frac{a}{2}\Delta\tau}\right)^2$$
$$e^{-i\omega\Delta\tau} \qquad e^{a(\Delta\tau - i\varepsilon)} + e^{-a(\Delta\tau - i\varepsilon)} - 2$$
$$\frac{d}{d\tau}\; e^{(a-i\omega)} - e^{-a-i\omega} \qquad \left(e^{\beta\omega} - 1\right)^{-1}$$

$\cancel{\Delta\tau = \Delta\tau +}$

Singularity: $\boxed{\Delta\tau = i\frac{2n\pi}{a}} + i\varepsilon$ $\quad n = -1, -2, \dots$

$$\left(e^{\beta\omega} - 1\right)^{-1} \to e^{-n\beta\omega} \qquad e^{-i\omega\Delta\tau} \to e^{-i\omega\left(i\frac{2n\pi}{a}\right)}$$

$$\boxed{\beta = \frac{2\pi}{a}} \qquad e^{-\frac{2\pi}{a}\beta\omega|n|} \qquad n = -1, -2, \dots$$

Rindler vacuum $\qquad \phi = \sum b_\omega e^{-i\omega v} + \tilde{b}_\omega e^{-i\omega u} + \text{c.c.}$

$$b, \tilde{b}\, |0_R\rangle = 0$$

$$G^+ = -\frac{1}{4\pi}\ln[(\Delta u - i\varepsilon)(\Delta v - i\varepsilon)]$$

Rindler observer $\quad u = \eta - \xi_0, \; v = \eta + \xi_1$

$$\Delta u = \Delta v = \Delta\eta \qquad \int G^+ e^{-i\omega\Delta\eta}\, d\Delta\eta$$

$$\int_{-\infty}^{+\infty} \ln(\Delta\eta)\, e^{-i\omega\Delta\eta}\, d\Delta\eta = 0$$

Schwarzschild geom. (Eternal ?)

both are solutions in 2D

$$\phi \sim a_\omega e^{-i\omega u} + \tilde{a}_\omega e^{-i\omega v} + c.c.$$

$a, \tilde{a} |0_B\rangle = 0$ also called (0_S)

FIDO | $\boxed{\text{Boulware vacuum}}$

Schwarzschild observer ($r =$ fixed)

(1) $\rightarrow ds^2 = -\left(1 - \frac{2M}{r}\right) du\, dv$ $u = t + r_*$, $v = t - r_*$

$\Delta u = \Delta t = \Delta v$

$\Delta\tau^2 = -ds^2 = \left(1 - \frac{2M}{r}\right) \Delta t^2$

Boulware $= -\left(1 - \frac{2M}{r}\right)(-dt^2)$ for $r =$ constant.

$\Delta\tau = \sqrt{1 - \frac{2M}{r}}\, \Delta t$

$$G^+ = -\frac{1}{4\pi} \ln\left[\left(\sqrt{1-\frac{2M}{r}}\right)^{-1} \Delta\tau - i\varepsilon\right]^2$$

same as for Minkowski observer in Mink vacuum.

$\Rightarrow \underbrace{f(E)}_{SB} = 0$

Schwarzschild observer   Boulware vacuum

Boulware vacuum is not physical near horizon since coord. system (1) is not regular. $(u \to \infty)$

[Eddington-Finkelstein Freely falling observer ?

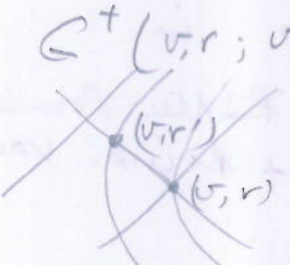
$v =$ const, ~~$r = r(t)$~~

$ds^2 = 0$

must use affine parameter $\lambda = r$

$r = r(\lambda) = \lambda$

$G^+(v, r; v', r') =$ not defined ?!



$$\int d^2p \frac{e^{ip_\mu x^\mu}}{p^2} \sim \int d\omega\, dp \frac{e^{i(\omega - p)x}}{\omega^2 - p^2} \sim \int dp_+ dp_- \frac{e^{ip_+ x}}{p^+ p^-}$$

$$\sim \int \frac{dp_+}{p_+} e^{ip_+ x} \int \frac{dp_-}{p_-}$$

NOT CONCLUSIVE]

Hartle - Hawking vacuum

$$ds^2 = -\frac{2M}{r} e^{-\frac{r}{2m}} dU dV \qquad \text{Kruskal coordinates}$$

$$\phi \sim a_\omega e^{-i\omega U} + \tilde{a}_\omega e^{-i\omega V}$$

$$U = -\frac{1}{\kappa} e^{-\kappa u} \qquad V = +\frac{1}{\kappa} e^{\kappa v}$$

Schwarzschild observer (FIDO)

$$\cancel{dt} \qquad v = t + R_* \quad U = -\frac{1}{\kappa} e^{-\kappa(t - R_*)}$$
$$u = t - R_* \quad V = \frac{1}{\kappa} e^{\kappa t (t + R_*)}$$

$$(-dU\,dV) = \frac{dt^2}{\kappa^2} e^{2\kappa R_*} \qquad r = R \text{ fixed}$$

$$+d\tau^2 = -ds^2 = +\frac{2M}{R} e^{-\frac{R}{2m}} e^{2\kappa R_*} \frac{1}{\kappa^2} dt^2 \qquad \kappa = -\frac{1}{4M}$$

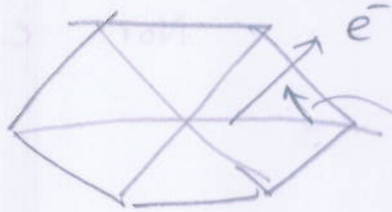
$$G^+ = -\frac{1}{4\pi} \ln((\Delta U - i\varepsilon)(\Delta V - i\varepsilon))$$
$$= -\frac{1}{4\pi} \ln\left(\sinh^2\left(\frac{1}{2\kappa}\Delta t - i\varepsilon\right)\right)$$

$$\int G^+ e^{-iE\Delta\tau} = \int_{-\infty}^{+\infty} G^+(\Delta t)\, e^{-iE\left(\frac{2M}{r} e^{-\frac{r}{2m}}\right)^{1/2} \frac{e^{\kappa R_*}}{\kappa} \Delta t} d(\Delta t)$$

$$= \frac{1}{E(e^{\beta E} - 1)} \qquad \text{as in Rindler observer + Mink. vacuum.}$$

FIDO sees a thermal bath in $|0_K\rangle$

$\boxed{\text{Unruh vacuum}}$



$$ds^2 = (\ )\, dU\, dv$$

FIDO $\phi = a_\omega e^{-i\omega v} + \tilde{a}_\omega e^{-i\omega U}$

~~Δt~~

$$\ln(\Delta v\, \Delta U) \qquad \Delta v = \Delta t$$
$$\Delta U = -\frac{1}{\kappa} e^{+\kappa R_* - \kappa t - \kappa \frac{\Delta t}{2}} \sinh \kappa \frac{\Delta t}{2}$$

$$\Delta v = \Delta t$$

The Planckian spectrum again

$$T_{SU} = \frac{1}{8\pi M\sqrt{1-\frac{2M}{R}}} = \frac{\kappa}{2\pi\sqrt{1-\frac{1}{2\kappa R}}} \qquad \kappa = \frac{1}{4M}$$

$$T_{RM} = \frac{(2M/R)^2}{8\pi M\sqrt{1-\frac{2M}{R}}} = \frac{a}{2\pi\sqrt{1-\frac{1}{2aR}}} \times \left(\frac{1}{2aR}\right)^2$$

[illegible] - Mink space is

$$ds^2 = (1+2aR)\,dt^2 - \frac{dR^2}{1+2aR}$$

$$\therefore \quad T_{RM} < T_{SU}$$

If temp is higher, one is in a grav-field. !!