Hawking Radiation

Plan :

1. Semiclassical quantization, probe approximation and QFT in curved spacetime
2. Time-dependent background metric and particle creation
3. Non-inertial observers in flat space, Rindler spacetime :
   - → particle density for different choice of vacua ($|0_M\rangle$, $|0_R\rangle$) 1
   - → detector response and Green's function
   - → $|0_M\rangle$ as a thermofield double in terms of Rindler oscillators
   - → Entanglement entropy and AdS/CFT
4. Schwarzschild black hole
   - → Coordinate systems and Penrose diagram for Eternal B.H. and collapse
   - → 2D schwarzschild : 2

     Eternal BH vs Rindler

     $|0_K\rangle \longleftrightarrow |0_M\rangle$

     $|0_S\rangle \longleftrightarrow |0_R\rangle$

     $|0_U\rangle$
   - → Green's functions
   - → (Stress tensors)
5. Back reaction, black hole decay, issues of unitarity and equivalence principle
6. D1-D5 system : a microscopic description of Hawking radiation and black hole decay 3
   - → information loss vs thermalization
   - → AdS_2/CFT_2 examples ($|B\rangle$)

§1. Semiclassical q.g., probe approx. & QFT in curved spacetime

Consider a functional integral involving the spacetime metric $g_{\mu\nu}$ and other fields ϕ_i

$$Z = \int \mathcal{D}g_{\mu\nu}\, \mathcal{D}\phi \exp\left(iS[g_{\mu\nu}, \phi_i]\right)$$

$\hookrightarrow S_2 = \frac{1}{2}\int d^4x \sqrt{g}\,(\partial^\mu\phi_2\partial_\mu\phi_2 + V_2(\phi_2))$

e.g. $S = S_1 + S_2$, $S_1 = \frac{1}{16\pi G_N}\int d^4x \sqrt{g}\left(R + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 + V(\phi_1)\right)$

In general, there may be (multiple) saddle points of the functional integrals in the limit $G_N \to 0$. Around any saddle point $\bar{g}, \bar{\Phi}_1$

$$Z = e^{i\frac{S_1^{(0)}[\bar{g},\bar{\Phi}_1]}{G_N}} \int \mathcal{D}h_{\mu\nu}\,\mathcal{D}\delta\phi_1\,\mathcal{D}\phi_2\; e^{i\left(S_1^{(2)}[h_{\mu\nu}, \delta\phi_1] + S_2[\bar{g}, \phi_2] + \sqrt{G_N}\, S_1^{(3)}[\bar{g}, \bar{\Phi}_1, h, \delta\phi] + O(G_N)\right)}$$

$\bar{g}, \bar{\Phi}_1,$

where $\bar{g}, \bar{\Phi}_1$ satisfy $\frac{\delta S_1}{\delta\phi_1} = 0 = \frac{\delta S_1}{\delta g_{\mu\nu}}$ $\quad$ $\bar{\Phi}_1$ can describe collapsing shell

$$\frac{S_1^{(0)}}{G_N}[\bar{g},\bar{\Phi}_1] \equiv S_1[\bar{g},\bar{\Phi}_1]$$

$\longrightarrow$ String theory: ϕ_1 = dilaton, moduli $\quad$ cf. D1-D5 system later
(no ϕ_2 to begin with)

$\longrightarrow$ AdS/CFT:

$$Z_{CFT} = \int \mathcal{D}A_\mu \mathcal{D}\Phi_i\, e^{iS}, \quad S = \underbrace{\frac{N}{\lambda}\int d^4x\, \mathrm{Tr}\, F_{\mu\nu}^2}_{O(N^2)} + \underbrace{\frac{1}{N}\int d^4x\, \mathcal{O}\,\delta\phi_{1,B}}_{O(1)}$$

$\mathcal{O}$: e.g. $\mathrm{Tr}\,\Phi_1^2$

$$S_1^{(2)}[\bar{g}, \bar{\Phi}_1, h_{\mu\nu}, \delta\phi_1] + S_2[\bar{g}, \phi_2] \Rightarrow \text{QFT in curved spacetime}$$

<u>Probe approximation</u>: back reaction of q.m. fluctuations on $\bar{g}_{\mu\nu}$ (and $\bar{\Phi}_1$) can be neglected.

e.g. $\frac{S_1^{(0)}}{G_N}[\bar{g},\bar{\Phi}_1] \gg S_1^{(2)}[\bar{g},\bar{\Phi}_1, h_{\mu\nu}, \delta\phi_1]$ $\quad$ $\to$ e.g. a decaying B.H.

OK if $G_N \to 0$ unless $S_1^{(0)}[\bar{g},\bar{\Phi}_1] \to 0$ ~~at the same time~~ also

QFT in curved spacetime (Birrell & Davies 1982, Preskill Caltech lectures, Hawking – CMP 1974, ...)

Let $S = -\frac{1}{2}\int d^dx \sqrt{\bar{g}}\left[\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right]$

We wish to quantize this theory. E.g. to compute

$$Z[J] = \int \mathcal{D}\phi\, e^{iS + i\int d^dx\sqrt{g}\,\phi J}, \quad \langle T_{\mu\nu}\rangle = \sqrt{g}\frac{\delta}{\delta g^{\mu\nu}}\ln Z[0]$$

These clearly depends on $\bar{g}_{\mu\nu}$, but $Z[\bar{g}]$, $\langle T_{\mu\nu}^{ren}\rangle$, ... also on the choice of the 'vacuum'.

- Why is the vacuum not unique?

① In Minkowski space there is a unique Poincaré-invariant vacuum $|0_M\rangle$

$$P_\mu, J_{\mu\nu}|0_M\rangle = 0$$

this satisfies $a_{\vec{k}}|0_M\rangle = 0$. What's $a_{\vec{k}}$?

$\rightarrow$ Solutions of KG equation (in an inertial frame $\vec{x}, t$)

$$A_{\vec{k}}(x) = e^{i\vec{k}\cdot\vec{x} - i\omega t} \qquad \overline{A_{\vec{k}}}(x) = e^{-i\vec{k}\cdot\vec{x} + i\omega t}$$

$$\sum_k = \int \frac{d^3\vec{k}}{(2\pi)^3\sqrt{2\omega}} \qquad \omega = \sqrt{\vec{k}^2 + m^2}$$

$$\phi(x) = \sum_k a_{\vec{k}} \underbrace{A_{\vec{k}}(x)}_{e^{-i\omega t}} + a^\dagger_{\vec{k}}\overline{A_{\vec{k}}}(x)$$

More generally $\{A_i(x), \overline{A_i}(x)\}$

$$\Rightarrow H = i\partial_t = \omega > 0, \text{ +ve frequency}$$

With this choice

$$\underbrace{\phi(x)|0_M\rangle}_{} = \sum_k e^{-i\vec{k}\cdot\vec{x} + i\omega t}|\vec{k}\rangle$$

$$e^{iHt}\phi(\vec{x})\cancel{e^{-iHt}}|0\rangle = e^{iHt}\sum_k e^{-i\vec{k}\cdot\vec{x}}|\vec{k}\rangle = e^{iHt}|\vec{x}\rangle$$

$$\therefore \phi(\vec{x})|0\rangle = |\vec{x}\rangle, \text{ OK.}$$

$\rightarrow$ In another inertial frame $x^\mu = \Lambda^\mu{}_\nu x'^\nu$

$$a_{\vec{k}'} = U^\dagger_\Lambda a_{\vec{k}} U_\Lambda \quad \therefore \quad a_{\vec{k}'}|0_M\rangle = U^\dagger_\Lambda a_{\vec{k}}|0_M\rangle = 0$$

$\therefore |0_M\rangle$ does not depend on the choice of the inertial frame.

$\rightarrow$ non-inertial frame: e.g. $|0_R\rangle \neq |0_M\rangle$ $\quad \{b_k e^{ik\xi - i\omega\eta} + cc\}$, $b_k|0_R\rangle = 0$

② In a spacetime $\bar{g}_{\mu\nu}$, there could be 2 asymp. Mink. in the past and future

$$ds^2 = -dt^2 + \alpha^2(t)\,dx^2 \qquad (2\text{-dim})$$

$$= \alpha^2(\eta)\,[-d\eta^2 + dx^2]$$

$|0_{out}\rangle$ $\quad \alpha = \alpha_{out}$

time-dependent geometry

$|0_{in}\rangle$ $\quad \alpha = \alpha_{in}$

shift this part to p.4

One mode expansion

$$\left(-\partial_\eta^2 - k^2 - m^2\alpha^2(\eta)\right)\phi = 0$$

$$\phi = \sum_k a_k A_k(\eta)\, e^{ikx} + a_k^\dagger \overline{A_k(\eta)}\, e^{-ikx}$$

where
$$A_k(\eta) \begin{cases} \xrightarrow{\eta \to -\infty} & e^{-i\omega_{in}\eta} \\ \xrightarrow{\eta \to +\infty} & \boxed{T_k e^{-i\omega_{out}\eta} + R_k e^{i\omega_{out}\eta}} \end{cases} \qquad \omega_{\substack{in \\ out}} = \sqrt{k^2 + m^2\alpha^2_{\substack{in \\ out}}}$$

i.e. $A_k(\eta)$ has only +ve freq. modes in the infinite past: $\phi \xrightarrow[\eta\to-\infty]{} \sum_k a_k e^{ikx - i\omega_{in}\eta} + a_k^\dagger e^{-ikx + i\omega_{in}\eta}$

This matches Minkowski modes (with $m^2 = m_{in}^2 = m^2\alpha_{in}^2$)

Hence define $|0_{in}\rangle$ by

$$a_k |0_{in}\rangle = 0$$

A second mode expansion

$$\phi = \sum b_k B_k(\eta)\, e^{ikx} + b_k^\dagger \overline{B_k(\eta)}\, e^{-ikx}$$

$$B_k(\eta) \xrightarrow[\eta\to+\infty]{} e^{-i\omega_{out}\eta}$$

$$\Rightarrow \phi(\eta, x) \xrightarrow[\eta\to\infty]{} \sum_k b_k e^{ikx - i\omega_{out}\eta} + b_k^\dagger e^{-ikx + i\omega_{out}\eta}$$

as in Minkowski

Define $|0_{out}\rangle$ by $b_k |0_{out}\rangle = 0$

$$|0_{out}\rangle = |0_{in}\rangle \ ?$$

(4) More generally, $\exists$ no canonical choices at all.

③ In a black hole space time (eg. a collapse) it's Minkowski in the infinite past but the future consists of BH interior $\cup$ asympt. flat

(4) To page (4a)

There is no canonical choice of modes in the B.H. interior. For eternal BH there is no canonical choice either in the past or the future.

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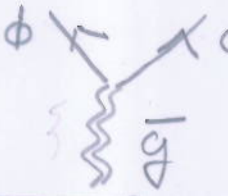
§2. Time-dependent background and particle creation

$$ds^2 = \alpha^2(\eta)\left(-d\eta^2 + dx^2\right) \equiv C(\eta)\left(-d\eta^2 + dx^2\right)$$

$$S = -\frac{1}{2}\int dx\, d\eta \left(-(\partial_\eta \phi)^2 + (\partial_x \phi)^2 + \underbrace{m^2 \alpha^2(\eta)\, \phi^2}_{\text{time-dependent potential}}\right)$$

$\left(\text{cf. } \psi^\dagger \psi \overline{A_0(t)}\right)$

$\bar{g}_{00}(\eta)\, \phi\phi$



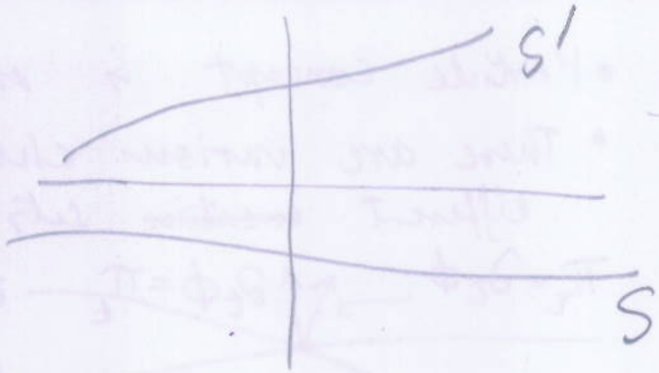
$\Rightarrow$ particle creation (as in Schwinger pair creation)

Put insert here from p. L3

From the above qualitative argument we expect that $|0_{in}\rangle$, viewed in the Heisenberg picture will contain particles of the out-modes.

Before proving this, we need a bit of technology:

(5)



Let $A_i(x), \overline{A_i}(x)$ be a complete, orthonormal set of solutions of

$$D^\mu \partial_\mu \phi - m^2 \phi = 0$$

The completeness is meant in the sense that on a certain surface S (spacelike or null) any function $f(x)$ can be expanded as a lin. comb. of $A_i(x)$ and $\overline{A_i}(x)$ on S. Together with the time-evolution, this means that any solution $\phi(x)$ can be expanded as

$$\phi(x) = \sum_i a_i A_i(x) + a_i^\dagger \overline{A_i}(x)$$

The split between $A_i(x)$ and $\overline{A_i}(x)$ now is arbitrary since no choice of 'time' is used implied.

Orthonormality $\quad (f, g) = -i \int_S f \overleftrightarrow{\partial_\mu} \bar{g} \sqrt{\gamma}\, n^\mu d^3x$

[The scalar product is independent of S for on-shell f, g]

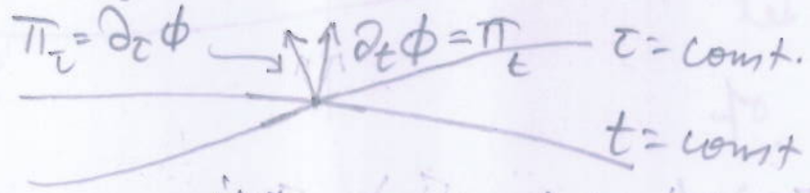
We have chosen $\quad (A_i, A_j) = -(\overline{A_i}, \overline{A_j}) = \delta_{ij}$
$\quad (A_i, \overline{A_j}) = 0$

The vacuum $|0, a\rangle$ is defined by $a_i|0, a\rangle = 0$

① → • Particle concept is ambiguous.

• There are various choices of 'times' leading to different ~~creation~~ sets of creation / ann. ops.

e.g.

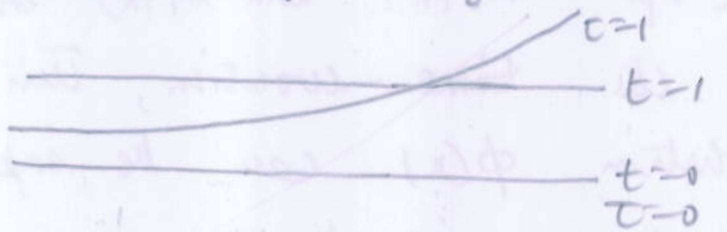


$$\phi\big|_{t=0} = \sum_k a_k e^{ikx} + a_k^+ e^{-ikx} = \sum_k (a_k + a_{-k}^+)\, e^{ikx}$$

$$i\dot\phi = -(\ \ ) + (\ \ ) = (a_k - a_{-k}^+)$$

$\longrightarrow$ $\phi, \dot\phi$ together separately determine $\{a_k\}, \{a_k^+\}$

• Particle concept is global, history-dependent



} t-modes and τ-modes are different

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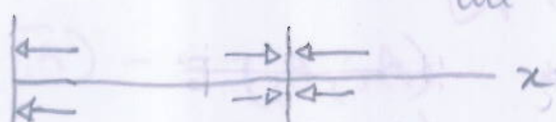
② $\longrightarrow$ $\langle 0_{in} | T_\mu^{\nu\,ren}(b) | 0_{in} \rangle = \begin{pmatrix} E & 0 \\ 0 & P \end{pmatrix}$ $\quad T_\mu^{\mu\,ren} \neq 0$

$(E, P) = \cancel{\frac{1}{C}\partial_\eta^2(\ln C) \pm \sqrt{C}\,\partial_\eta^2 \frac{1}{\sqrt{C}}}$ (~~η-independent~~)!

$\boxed{T_\eta^x = 0}$ no particle flux $\longrightarrow$ or $\longleftarrow$
(would be in conflict with homogeneity and $x \to -x$ symmetry)

T_0^0 = energy density

T_1^1 = pressure (flux of 1-momentum in the 1-direction)



boundary wall (net outward force)

fictitious wall (no net force)

$|0_{in}\rangle$ = bath of b-particles with non-trivial energy-momentum (in equilibrium since

Suppose $\exists$ a second ortho-complete mode expansion

$$\phi = \sum b_k B_i(x) + b_k^\dagger \overline{B_i(x)}$$

with $B_i = \alpha_{ij} A_j + \beta_{ij} \overline{A}_j$ ← α, β Bogoliubov coeffs.

Exercise: (i) $\alpha_{ij} = (B_i, A_j)$, $\beta_{ij} = -(B_i, \overline{A}_j)$

(ii) $a_i = \alpha_{ji} b_j + \beta_{ji} b_j^\dagger$

(iii) $A_i = \overline{\alpha}_{ji} B_j - \beta_{ji} \overline{B}_j$

(iv) $b_i = \overline{\alpha}_{ij} a_j - \overline{\beta}_{ij} a_j^\dagger$

(v) $\sum_j \overline{\alpha}_{ij} \alpha_{kj} - \overline{\beta}_{ij} \beta_{kj} = \delta_{ik}$

Def. $b_i |0, b\rangle = 0$

Ex. Show that

$$\langle 0, a | b_i^\dagger b_i | 0, a\rangle = \langle 0, a | \sum_k^{(-)} a_k \beta_{kj} \left(-\overline{\beta}_{ij} a_j^\dagger\right) | 0, a\rangle$$

$$= \sum_j |\beta_{ij}|^2$$

This shows that

$\boxed{\text{The vacuum of the } a\text{-modes contains } b\text{-particles}}$

① to pg (6a)

Ex. Show that the β coefficients are non-zero in the 2D cosmology example

Proof: $A_k(\eta) \xrightarrow{\eta \to +\infty}$ lin. comb. of $\left(e^{-i\omega_{out}\eta}, e^{i\omega_{out}\eta}\right)$

$= $ lin. comb. of $(B_k(\eta), B_k^\dagger(\eta))$

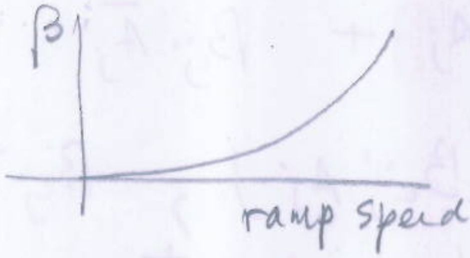
Q.E.D

Hence $\langle 0_{in} | b_k^\dagger b_k | 0_{in}\rangle \neq 0$.

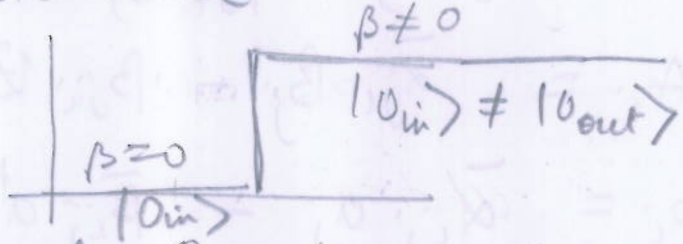
③ to pg. (6b) ②

(3) $\rightarrow$ Adiabatic limit $|0_b\rangle = |0_a\rangle$
since the a-modes smoothly go over to the b-modes

(6b)



cf. Kibble-Zurek



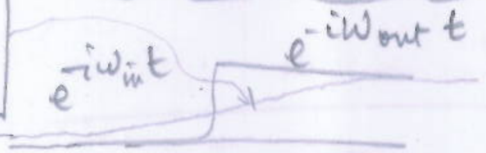
Quantum Quench

Relation to time-dependent perturbation theory

$$|0_{in}\rangle \xrightarrow{U_t} |0_{in}(t)\rangle \neq |0_{out}\rangle$$

$$\underbrace{1 - |\langle 0_{out}|0_{in}\rangle|^2}_{} = \text{prob. to jump to excited state} \propto |\beta|^2$$

Adiab. limit



no reflection in the adiabatic limit

§ 3. Non-inertial observers in flat space: Rindler spacetime

Uniformly accelerated observers in Minkowski space $X^\mu(\tau)$

$$\ddot{x}^\mu \ddot{x}_\mu = \bar{a}^2$$

$$\therefore \ddot{x}^\mu = \bar{a}(\operatorname{sh}\alpha\tau, \operatorname{ch}\alpha\tau)$$

$$\Rightarrow x^\mu = \cancel{\alpha\,\bar{a}}\,\frac{\bar{a}}{\alpha^2}(\operatorname{sh}\alpha\tau, \operatorname{ch}\alpha\tau)$$

$$\dot{x}^\mu = (\bar{a}/\alpha)(\operatorname{ch}\alpha\tau, \operatorname{sh}\alpha\tau)$$

$$-1 = \dot{x}^\mu \dot{x}_\mu = \bar{a}^2/\alpha^2 \qquad \bar{a} = \alpha$$

$$x^\mu = \frac{1}{\alpha}(\operatorname{sh}\alpha\tau, \operatorname{ch}\alpha\tau) \to \text{hyperbolas}$$

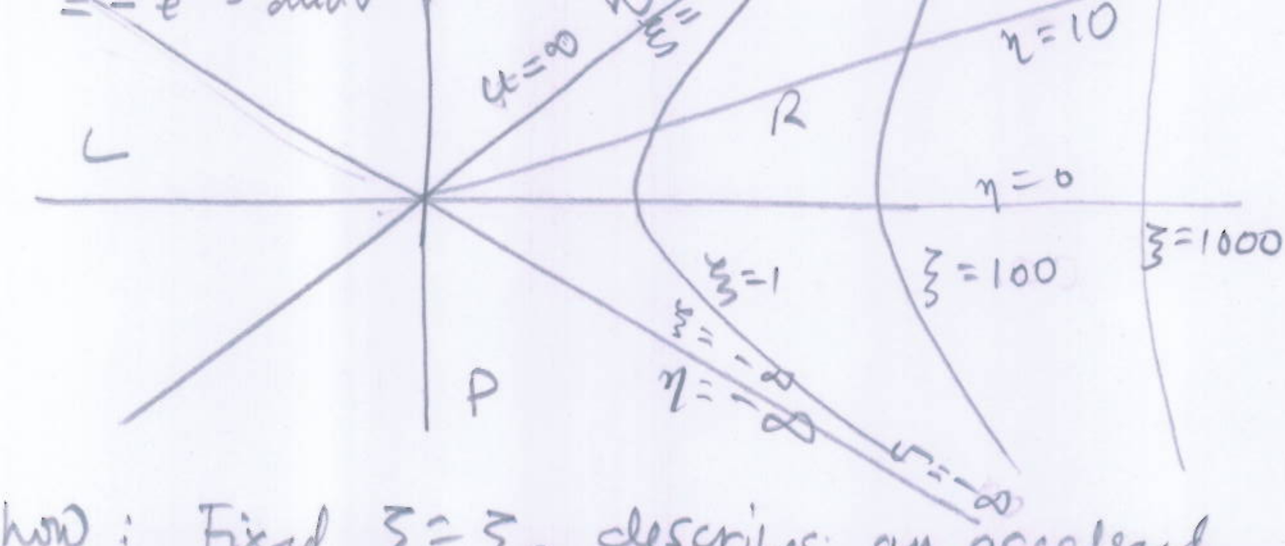
Ex: Show $\ddot{x}^\mu \ddot{x}_\mu = \alpha^2$

A Family of accelerated observers:

$$x = \frac{1}{a}e^{a\xi}\operatorname{ch} a\eta \qquad U = t-x = -\frac{1}{a}e^{-au} \qquad u = \eta - \xi$$

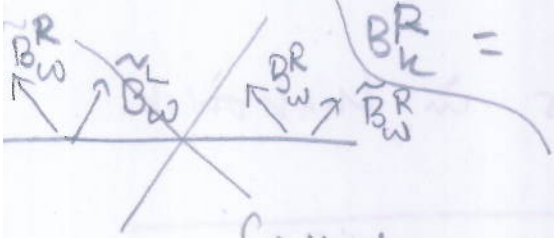
$$t = \frac{1}{a}e^{a\xi}\operatorname{sh} a\eta \qquad V = t+x = \frac{1}{a}e^{av} \qquad v = \eta + \xi$$

$$ds^2 = -dU\,dV$$
$$= -e^{2a\xi}\,du\,dv$$



Ex: Show: Fixed $\xi = \xi_0$ describes an accelerated observer with $\sqrt{\ddot{x}^\mu \ddot{x}_\mu} = a e^{-a\xi_0}$

(2) $\longrightarrow$ $\phi = \sum_k b^L_k B^L_k + b^R_k B^R_k + c.c.$



$$B^R_k = e^{ik\xi - i\omega\eta} \xrightarrow[\text{rt. moving}]{k>0} e^{-i\omega(\eta-\xi)} = e^{-i\omega u} = \tilde{B}^R_\omega(u)$$

$$\xrightarrow[\text{left-moving}]{k<0} e^{-i\omega(\eta+\xi)} = e^{-i\omega v} = B^R_\omega(v)$$

Similarly

$$B^L_k \xrightarrow[\text{left!}]{k>0} e^{i\omega\eta + i\omega\xi} = e^{i\omega v} = B^L_\omega(v)$$

$$\xrightarrow[\substack{k<0\\ \text{right!}}]{} e^{i\omega(\eta-\xi)} = e^{i\omega u} = \tilde{B}^L_\omega(u)$$

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(1) $\longrightarrow$ On the left, we define modes $\sim e^{i\omega\eta}$ $\left(-i\frac{\partial}{\partial\eta} > 0\right)$
since η increases towards the past
($-\eta$ increases towards the future)

⑧

L: $U = \frac{1}{a} e^{-au}$ Again
$V = -\frac{1}{a} e^{av}$ $\quad ds^2 = -e^{2a\xi} du\, dv$

η

F: $U = \frac{1}{a} e^{-au}$
$V = \frac{1}{a} e^{av}$ $\quad ds^2 = e^{2a\xi} du\, dv = e^{2a\xi}(d\eta^2 - d\xi^2)$

Space-like! time-like!

P: $U = -\frac{1}{a} e^{-au}$
$V = -\frac{1}{a} e^{av}$

Rindler modes $(B_k^{R,L})$

$$S = -\frac{1}{2}\int d\eta\, d\xi \left(-(\partial_\eta \phi)^2 + (\partial_\xi \phi)^2\right)$$
$$= -\frac{1}{2}\int dt\, dx \left(-(\partial_t \phi)^2 + (\partial_x \phi)^2\right)$$

$m^2 = 0$ conformal 2D

$(-\partial_\eta^2 + \partial_\xi^2)\phi = 0$
$\partial_u \partial_v \phi = 0$ in R, L

② to pg (8a)

$$\phi = \sum \tilde{b}_\omega^R \tilde{B}_\omega^R + b_\omega^R B_\omega^R + \sum \tilde{b}_\omega^L \tilde{B}_\omega^L + b_\omega^L B_\omega^L + \text{c.c.}$$

$\partial_U \partial_V \phi = 0$

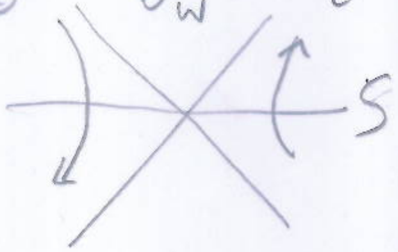
$$\phi = \sum_\omega \tilde{a}_\omega \underbrace{e^{-i\omega U}}_{\tilde{A}_\omega} + \sum a_\omega \underbrace{e^{-i\omega V}}_{A_\omega} + \text{c.c.}$$

$a_\omega, \tilde{a}_\omega |0_M\rangle = 0$

① to pg (8a)

$B_\omega^L = e^{+i\omega v}\,\Theta(-V)$
$\tilde{B}_\omega^L = e^{+i\omega u}\,\Theta(U)$

$B_\omega^R = e^{-i\omega v}\,\Theta(V)$
$\tilde{B}_\omega^R = e^{-i\omega u}\,\Theta(-U)$



$B_\omega^L, \tilde{B}_\omega^L, B_\omega^R, \tilde{B}_\omega^R$ / and c.c. provide a complete basis on S

$|0_R\rangle$: $b_\omega^{L,R}, \tilde{b}_\omega^{L,R} |0_R\rangle = 0$

Claim: $|0_M\rangle = \mathcal{N}\, e^{+\sum_\omega e^{-\frac{\beta\omega}{2}}\left(b_\omega^{L\dagger} b_\omega^{R\dagger} + \tilde{b}_\omega^{L\dagger} \tilde{b}_\omega^{R\dagger}\right)} |0_R\rangle$

$\beta = \frac{2\pi}{a}$

Bogoliubov

Fourier transform

$$B^R_\omega(V) = (aV)^{i\omega/a}\,\theta(V) = \int_0^\infty \frac{d\omega'}{\pi}\left(\alpha^R_{\omega\omega'} e^{-i\omega' V} + \beta^R_{\omega\omega'} e^{+i\omega' V}\right)$$

etc.
$$\beta^R_{\omega\omega'} = \alpha^R_{\omega(-\omega')}$$

This gives

$$a_{\omega'} = \sum_{\omega'}\left(\alpha^R_{\omega\omega'} b^R_\omega + \bar{\beta}^R_{\omega\omega'} b^{R\,\dagger}_\omega + \alpha^L_{\omega\omega'} b^L_\omega + \bar{\beta}^L_{\omega\omega'} b^{L\,\dagger}_\omega\right)$$

$$\Rightarrow \qquad \text{,,} \qquad |0_M\rangle = 0$$

Ex: can't solve $|0_M\rangle$ from just this equation

Hint: $(\partial_x + x + \partial_y + y) f(x,y) = 0$ cannot be solved (inadequate equations) $\left(b^R_\omega, b^{R\,\dagger}_\omega, b^L_\omega, b^{L\,\dagger}_\omega\right)$

Show that

$$(aV)^{i\omega/a}\Big|_{\text{analytic in LHP of } V} = B^R_\omega + e^{-\frac{\beta\omega}{2}}\,\bar{B}^L_\omega \equiv F^R_\omega$$

$$= \text{lin. comb. of } \{e^{-i\omega' V}\ \ \omega' > 0\}$$

Pf. For $-V$, use $e^{-\pi i} V$

$$\therefore\ \bar{B}^L_\omega = (-aV)^{i\omega/a} \qquad V<0$$
$$= \left(aVe^{-\pi i}\right)^{i\omega/a} \qquad V<0$$
$$= (aV)^{i\omega/a} e^{\pi\omega/a} \qquad V<0$$

$$\therefore\ e^{-\frac{\pi\omega}{a}}\,\bar{B}^L_\omega = (aV)^{i\omega/a} \quad V<0$$

(10a)

① $\longrightarrow \langle 0_M | b^{R\dagger}_\omega b^R_\omega | 0_M \rangle = \dfrac{e^{-\beta\omega/2}}{2\,sh\,\frac{\beta\omega}{2}} = \dfrac{e^{-\beta\omega/2}}{e^{\beta\omega/2} - e^{-\beta\omega/2}}$

$= \dfrac{1}{e^{\beta\omega} - 1}$ = Bose-Einstein distribution

$\therefore$ the Minkowski-vacuum appears to be a thermal bath of Rindler particles at temp $\beta = \frac{2\pi}{a}$ $\quad T = \frac{a}{2\pi}$

(For the black hole $a \longrightarrow \kappa = \frac{1}{4M}$, $T = \frac{1}{8\pi M}$)

For fermions $|\alpha|^2 + |\beta|^2 = 1$ $\Rightarrow \dfrac{e^{-\beta\omega/2}}{2\,ch\,\beta\omega/2} = (e^{\beta\omega} + 1)^{-1}$ = Fermi Dirac

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③ Incorporate $\tilde{b}_{R,L}$

$(\tilde{b}^R_\omega - e^{-\beta\omega/2}\, \tilde{b}^{L\dagger}_\omega)|0_M\rangle = 0$ — (ii)

(i) & (ii) $\Rightarrow (b^R_k - e^{-\beta\omega_k/2} b^{L\dagger}_k)|0_M\rangle = 0 \quad \forall k$

Also $(b^L_k - e^{-\frac{\beta}{2}\omega_k} b^{R\dagger}_k)|0_M\rangle = 0$

$\therefore |0_M\rangle = \exp\left[\sum_k e^{-\beta\omega_k/2} b^{R\dagger}_k b^{L\dagger}_k\right]|0_R\rangle$

$= \prod_k \sum_{n_k=0}^{\infty} (e^{-\frac{\beta}{2}\omega_k n_k}) \dfrac{(b^{R\dagger}_k)^{n_k} (b^{L\dagger}_k)^{n_k}}{n_k!} |0_R\rangle$

$= \prod_k \sum_{n_k} e^{-\frac{\beta}{2} n_k \omega_k} |n_k, n_k\rangle$

$= \sum_{\{n_k\}} e^{-\frac{\beta}{2}\sum_k \omega_k n_k} |\{n_k\}, \{n_k\}\rangle \qquad n^L_k = n^R_k \;\forall k$

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② $\quad (b_1 - \beta b_2^\dagger)|\psi\rangle = 0 \quad (b_2 - \beta b_1^\dagger)|\psi\rangle = 0$

Go to the representation $b_i = \frac{\partial}{\partial x_i}$, $b_i^\dagger = x_i$

$b_i|0\rangle = 0 \Rightarrow \frac{\partial}{\partial x_i}\psi_0(x) = 0 \Rightarrow \psi_0(x) = 1$ up to normalization

$\left.\begin{array}{l}(b_1 - \beta b_2^\dagger)|\psi\rangle = 0 \Rightarrow (\frac{\partial}{\partial x_1} - \beta x_2)\psi = 0 \\ (b_2 - \beta b_1^\dagger)|\psi\rangle = 0 \quad\;\; (\frac{\partial}{\partial x_2} - \beta x_1)\psi = 0\end{array}\right\} \Rightarrow \psi = C e^{\beta x_1 x_2}$

Hint: $\psi = F(x_1 x_2, \frac{x_1}{x_2})$

$$\phi = b'^R_\omega F^R_\omega + b'^L_\omega F^L_\omega + (\text{tildes}) + c.c.$$ (10)

Hence $|0_M\rangle$ can equivalently be defined by

$$b'^{R,L}_\omega,\ \tilde{b}'^{R,L}_\omega\ |0_M\rangle = 0$$

Ex: Show:

$$b^R_\omega = N\left(b'^R_\omega + e^{-\beta\omega/2}\, b'^{L\dagger}_{-\omega}\right)$$

$$= \frac{1}{\sqrt{2\sinh \beta\omega/2}}\left(e^{\beta\omega/4}\, b'^R_\omega + e^{-\frac{\beta\omega}{4}}\, b'^{L\dagger}_{-\omega}\right)$$

b^R etc. Check $|\alpha|^2 - |\beta|^2 = 1$

Inverses:

$$b'^R_\omega = \frac{1}{\sqrt{2\sinh\frac{\beta\omega}{2}}}\left(e^{\beta\omega/4}\, b^R_\omega - e^{-\beta\omega/4}\, b^{L\dagger}_\omega\right)$$

1 $a'^R_\omega |0_M\rangle = 0$

to pg(10a) $\Rightarrow \left(b^R_\omega - e^{-\beta\omega/2}\, b^{L\dagger}_\omega\right)|0_M\rangle = 0$ — (i)

(2) "

$$|0_M\rangle = e^{\sum_\omega e^{-\beta\omega/2}\left(b^{R\dagger}_\omega b^{L\dagger}_\omega + (\text{tildes})\right)}|0_R\rangle$$

(3) "

$$= \prod_\omega \sum_{n_\omega=0}^{\infty} \frac{\left(e^{-\beta\omega/2}\right)^{n_\omega}}{n_\omega!}\left(b^{R\dagger}_\omega\right)^{n_\omega}\left(b^{L\dagger}_\omega\right)^{n_\omega}|0_R\rangle$$

$$= \prod_\omega \sum_{n_\omega}\left(e^{-\frac{\beta}{2}\omega}\right)^{n_\omega}\frac{\left(b^{R\dagger}_\omega\right)^{n_\omega}}{\sqrt{n!}}\frac{\left(b^{L\dagger}_\omega\right)^{n_\omega}}{\sqrt{n!}}|0_R\rangle$$

$$= \sum_{\{n_\omega\}} e^{-\sum_\omega (\beta/2)\,\omega\, n_\omega}\,|n^R_\omega, n_\omega\rangle$$

$\downarrow R$ $\downarrow L$

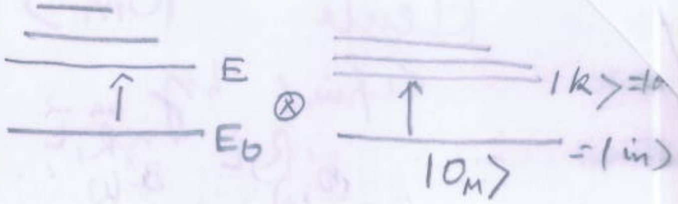
$$H_L = \sum_\omega \omega\, b^{L\dagger}_\omega b^L_\omega$$

$$H_R = \sum_\omega \omega\, b^{R\dagger}_\omega b^R_\omega$$

$\Rightarrow n^L_\omega = n^R_\omega\ \forall \omega$

Normalization const. $= \frac{1}{\sqrt{Z_1}}$

Detector response



$$S_{int} = g\int \mu(\tau)\,\phi(x(\tau))\,d\tau$$

$$\mu(\tau) = e^{iH\tau}\mu(0)e^{-iH\tau}$$

$$\mathcal{A} = |\langle out|\int H_{int}(\tau)\,|in\rangle|^2\,d\tau$$

$$= g\int e^{i(E-E_0)\tau}\langle E|\mu(0)|E_0\rangle\,\phi(x(\tau))\,d\tau$$

$$\sum_{out}|\;|^2 = g^2\sum_{E}|\langle E|\mu(0)|E_0\rangle|^2\Big[\int_{-\infty}^{+\infty}\underbrace{\langle 0_M|\phi(x(\tau))\,\phi(x(\tau'))|0_M\rangle}_{G^+_M(x,x')} e^{i(E-E_0)(\tau'-\tau)}\,d(\tau'-\tau)\Big]$$

$x = x(\tau)$, $x' = x(\tau')$ $\qquad [\cdots] = \mathcal{F}(E-E_0)$

$$G^+_M(x-x') = -\frac{1}{4\pi}\ln\left[(\Delta U - i\varepsilon)(\Delta V - i\varepsilon)\right]$$

$U = -\frac{1}{a}e^{-au}$ $\qquad u = \eta - \xi_0$ $\qquad d\tau^2 = \frac{1}{a^2}e^{-2a\xi_0}d\eta^2$

$V = \frac{1}{a}e^{av}$ $\qquad v = \eta + \xi_0$ $\qquad \eta = e^{-a\xi_0}\tau$

$$\Longrightarrow \mathcal{F}_{RM}(E-E_0) \propto \frac{1}{(E-E_0)}\left[\frac{1}{e^{\beta(E-E_0)}-1}\right] \rightarrow \text{Bose Einstein}$$

$$\frac{\langle E|\mu(0)|E_0\rangle}{E-E_0}$$ detector characteristic