Generation of strong Electro-Magnetic Fields in HIC and their impact on the properties of hot nuclear matter

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Motivation of the heavy ion collision

Onset of QGP - journey towards characterizing the hadronic substructure

1. The contradiction between the limiting temperature concept of hadronic matter from **Hegedorn model** and monotonically increase of energy density with collision energy concept from **Fermi-Landau-Pomeranchuk model**, indicates existence of a deconfined quark phase.

2. The mass difference between the hadrons and their constituent current quark masses explained by the **spontaneous chiral symmetry breaking of QCD**, with the non-vanishing chiral condensates $\bar{\psi}\psi$, indicates the signature of a phase transition from quark ground state to a state of confined Goldstone bosons (pion,kaon and eta).

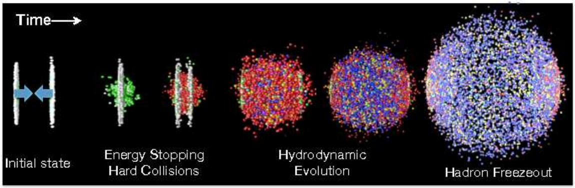
3. The **quark confinement** at low energies forming color neutral hadronic bound states and the **asymptotic freedom** at high energies with QCD coupling decreasing with increasing momentum scale indicate the existence of a phase transition, beyond which a deconfined QGP phase can be predicted.

4. A direct motivation to understand the deconfined QGP comes from cosmology which suggests that **a few microseconds after the Big Bang, the universe was made up of quark-gluon plasma at a temperature of about 200 MeV ($\sim 2 \times 10^{12}$K).** To understand the evolution of the early universe it is necessary to study the properties of QGP at very high temperatures.

5. Another motivation comes from **neutron stars** whose core density is so high that quarks and gluons may not remain confined, leading to a QGP phase at high baryon density.

Dynamics of the heavy ion collision

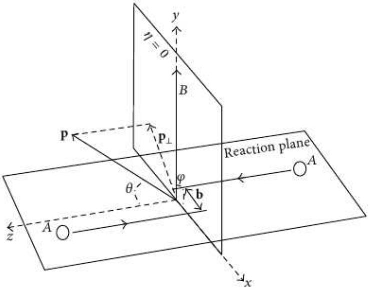
Different stages of heavy ion collision



1. Experimental results indicate that matter produced in heavy ion collision at RHIC and LHC might be in a deconfined state of quarks and gluons called QGP.
2. Due to considerable interaction strength this system of liberated quarks and gluons reaches its thermalized state early.
3. This thermalized quark matter expands and gradually cools down as the system evolves with time and beyond a critical temperature it converts into hadronic matter.
4. The hadron gas further expands to reduce energy density and finally reaches the freeze-out.
5. Electromagnetic responses and dissipative processes occur during the evolution of the system which are quantified by different transport coefficients.

Origin of Magnetic Field

In the center of mass frame Biot-Savart law gives



- Electric charge on the ion
- Lorenz factor$=\sqrt{s_{NN}}/2m_N$

$$B \sim \gamma \, Ze \, \frac{b}{R^3}$$

- Radius of colliding ions
- Impact parameter

Origin and Properties of Electromagnetic Field

An estimation of the magnetic field at heavy ion collision

RHIC parameters for $Au + Au$ collision

$$\sqrt{s_{NN}} = 200\text{GeV/A}$$
$$\gamma = 100$$
$$Z = 79$$
$$b = R_{Au} \approx 7fm$$

$$\boxed{eB \approx m_\pi^2}$$

LHC parameters for $Pb + Pb$ collision

$$\sqrt{s_{NN}} = 2.76\text{TeV/A}$$
$$\gamma = 1.38 \times 10^3$$
$$Z = 82$$
$$b = R_{Pb} \approx 7fm$$

$$\boxed{eB \approx 10m_\pi^2}$$

Origin and Properties of Electromagnetic Field

Strongest magnetic field ever existed in nature

$$eB \approx m_\pi^2 \sim 10^{18}\, G$$

1. Strongest magnetic field created on earth in a form of EM shock wave $\sim \mathbf{10^7}$ Gauss.
2. Magnetic field of a neutron star $\sim \mathbf{10^{10}} - \mathbf{10^{13}}$ Gauss.
3. Magnetic field of a magnetar $\sim \mathbf{10^{15}}$ Gauss.

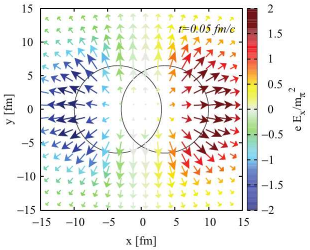
Motivation to study

1. Created field is well above Schwinger critical value $F = m_e^2/e$.
2. Classical electrodynamics breaks down.
3. Pair creation is operative by vacuum excitations.

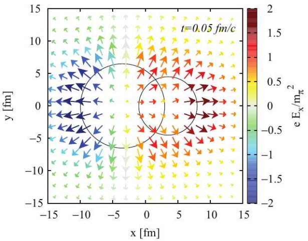
Electric field in the heavy ion collision

Origin :

1. Fluctuation in the proton position of same species collision nuclei.
2. Difference in proton numbers of colliding nuclei - asymmetric collision.



E.field in Au-Au collision



E.field in Cu-Au collision

Theoretical framework to observe the evolution of fields

Maxwell Equations

$$\nabla \cdot \vec{E} = \rho, \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$$

electrical conductivity

$$\vec{J} = \boxed{\sigma} \vec{E}$$

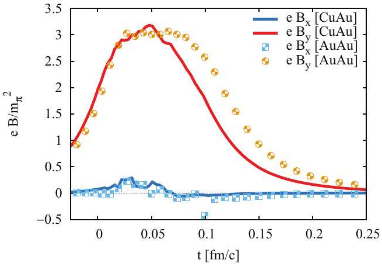
$\rho \equiv$ Woods-saxon nuclear distribution

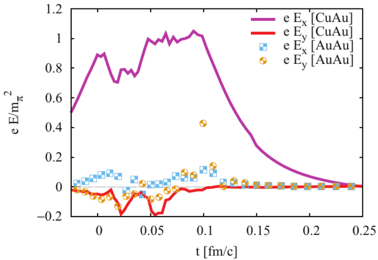
$$eB = \alpha_{em} \frac{b\gamma}{\left\{b^2 + \gamma^2 (t-z)^2\right\}^{3/2}}$$

$$\frac{B_{t=0}}{B_{t=b}} = \frac{1}{\gamma^3} << 1 (\sim 10^{-6} \text{ for RHIC})$$

Conclusion : EM fields falls rapidly in the later stages of heavy ion collisions

The EM fields in heavy ion collision are observed to decay down quite early leaving their imprints on early stage processes only

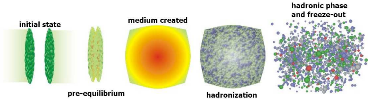


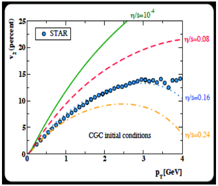


Ref : V. Voronyuk, V. D. Toneev, S. A. Voloshin and W. Cassing, Phys. Rev. C **90** (2014) no.6, 064903

Effect of EM fields on QGP observables

Hydrodynamics - theoretical modeling of the space-time evolution of QGP





1. Due to high temperature and density, the direct access of QGP properties are not very feasible. Only information is available from the final state particle spectra.
2. An analytical or numerical method, describing the evolution of the system from first principle is necessary.
3. Hydrodynamics offer an sensibly accurate description the collective dynamics of the system.
4. Hydrodynamical predictions exhibit considerable agreement with the experimental data proving its validity in heavy ion collisions.

Effect of EM fields on QGP observables

Hydrodynamics - Theoretical framework and guiding equations

$$\partial_\mu \boxed{T^{\mu\nu}} = 0, \qquad \partial_\mu \boxed{J^\mu} = 0$$

- Stress energy tensor
- Any conserved current

Hydrodynamics 4 velocity - as an explicit function of space time

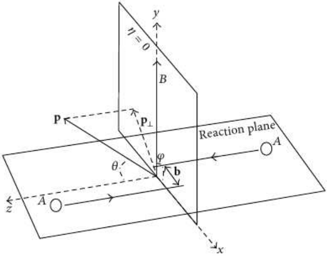
$$\boxed{u^\mu = u^\mu(x) = \gamma(1, \vec{v})}$$

Equation of state for 1+1 longitudinal expansion under dissipative hydrodynamics

$$\boxed{\frac{d\epsilon}{d\tau} + \frac{\epsilon + P}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2}}$$

Effect of EM fields on QGP observables

Magnetohydrodynamics - QGP collective properties under the influence of strong magnetic field



- Magnetic field breaks the spherical symmetry of plasma by exerting Lorenz force on the charged particle along the reaction plane.
- The expanding plasma exhibits azimuthally anisotropic flow in the reaction zone.

- The viscous pressure tensor becomes azimuthally anisotropic in the transverse plane due to the suppression of momentum transfer in the direction perpendicular to the magnetic field.
- The hydrodynamic evolution equations are greatly modified by the magnetic field.

Effect of EM fields on QGP observables

Electromagnetic probes - an authentic signature of QGP

- The thermal photons and dileptons coming out of the plasma interact with its surroundings only through e.m. interaction with smaller interaction strength as compared to strong interaction($\alpha_{em} << \alpha_s$).
- Hence have large mean free path compared to the system size to pass through the collision region and reach the detector without losing much information.
- They are produced from all stages of the evolving system and hence probe the entire space-time history.
- Their production rate and the momentum distribution depend on the momentum distribution of the initial quarks and gluons governed by the thermodynamic conditions of plasma.
- They are regarded as deep probes of QGP.

Sensitivity of the electromagnetic probes on EM responses

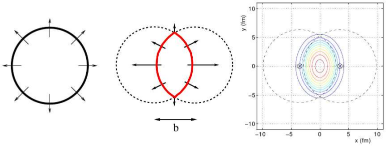
Emission rate of soft photons

$$\omega \frac{d\Gamma_\gamma}{d^3\vec{p}}\Big|_{\omega=|\vec{p}|} = \frac{\alpha_{EM}}{\pi^2 e^2} \frac{\underbrace{\sigma}_{\text{Electrical conductivity}}\ \omega}{e^{\omega/T}-1}$$

1. The soft photon and dilepton emission rates directly depend upon the electrical conductivity.
2. As a result the hydrodynamic description of the p_T spectra and elliptic flow of thermal photon and dileptons are also sensitive to EM responses and their temperature behavior.

Effect of EM fields on QGP observables

Collective flows - The ultimate signature of QGP

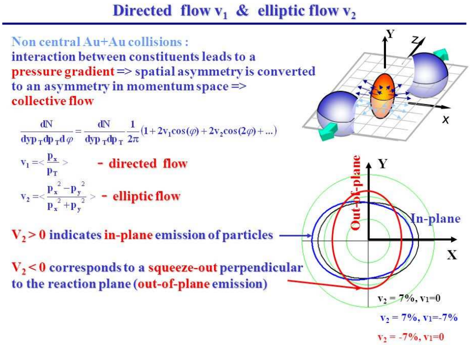


1. In non central heavy ion collisions or if the colliding nuclei are deformed, the azimuthal symmetry of nuclear overlap zone breaks and anisotropic transverse flow patterns develops.
2. The pressure gradient is steeper along the reaction plane than its transverse direction.
3. Interactions being sensitive to the anisotropic density gradient, redirect the momentum flow in the direction of stronger density gradient.
4. The net result is a momentum space anisotropy with more momentum flowing into the reaction plane than out of it.
5. The momentum anisotropy manifests itself as an azimuthal anisotropy of the observed hadron spectra.

Effect of EM fields on QGP observables

Flow coefficient - a signature of QGP

1. The angle-momentum distribution of the outgoing particle is expanded in Fourier series.
2. The polar plot of the first harmonics shifts the fireball without changing its shape - **directed flow**
3. The polar plot of the second harmonics makes the fireball elliptic - **elliptic flow**



Effect of EM fields on QGP observables

Sensitivity of directed flow on EM responses

1. The early stage EM field creates a dipole deformation in the charge distribution of the medium.
2. The early stage charge asymmetry is frozen in the systems collective motion.
3. The charge dependent directed flow of observed hadrons becomes sensitive to the charged dipole formed at the early stage, that is reflected by the system's electrical conductivity.

The distribution of the final state charged particle

$$\frac{dN_\pm}{d\phi} = \boxed{\bar{N}_\pm}\,[1 + 2\{\boxed{v_1} \pm \boxed{A}\,(\boxed{d_e} - v_1)\}\,cos\,\boxed{\phi}\,) + O\{(Ad_e)^2\}]$$

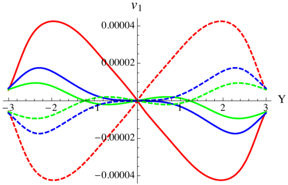
- Angle average of particle number distribution
- Azimuthal distribution of total number of particles without the effect of EM field
- Charge anisotropy parameter
- Measure of dipole deformation in the medium
- Azimuthal angle of the outgoing particle with reaction plane

$$Ad'_e \propto \sigma$$

Direct consequence of the EM responses on the collective flow

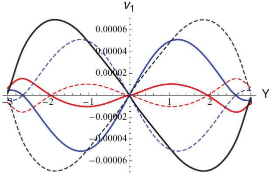
Effect of EM fields on QGP observables

Results as the evidence of electromagnetic responses in HIC



v_1 of positively and negatively charged pions with different p_T

Ref : U. Gursoy, D. Kharzeev and K. Rajagopal, Phys. Rev. C 89 (2014) no.5, 054905



v_1 of protons and antiprotons with different p_T

The EM fields present in the early stages of HIC act as an electric accelerator on the charges and affect the measured distribution of final state particles separately according to their charges

The extracted results are very useful indicator that whether the created matter is in perturbative on non-perturbative regime

An estimation of the signatures of EM responses in HIC

The induced current in the medium is related to the electric field by the electrical conductivity which characterizes the the charge transport property of the matter.

$$\vec{J} = \sigma_{el}\vec{E}$$

1. σ_{el} is directly related to the low energy photon and dilepton emission rates. Due to having longer wavelengths the electromagnetic spectra are very relevant observables in heavy ion collisions.
2. The charge-dependent directed flow of the observed hadrons is sensitive to the charge deformation of the system created by the induced current, which provides information about electric conductivity of the qgp.
3. The production and later diffusion of the primordial magnetic fields of early universe and in spiral galaxies depends crucially on σ_{el}.

Solving the relativistic transport equation in kinetic theory approach

Relativistic Transport Equation

- Electromagnetic field tensor

$$p_k^\mu \partial_\mu f_k + q_k \boxed{F^{\alpha\beta}} p_\beta \frac{\partial f_k}{\partial p_k^\alpha} = \boxed{-\,C[f_k]} = -\omega_k \frac{\delta f_k}{\boxed{\tau_k}} = -\omega_k \frac{f_k^0(1 \pm f_k^0)\,\boxed{\phi_k}}{\tau_k}$$

- Collision term
- Thermal relaxation time of partons
- Deviation of particle distribution function from equilibrium

Thermodynamic quantities in a multicomponent system

1. Particle diffusion flow $\quad I_k^\mu = N_k^\mu - x_k N^\mu$
2. Particle 4-flow for the k^{th} species $\quad N_k^\mu(x) = \int \frac{d^3\vec{p_k}}{(2\pi)^3 p_k^0} p_k^\mu f_k(x, p_k)$
3. Particle fraction $\quad x_k = \frac{n_k}{n}$
4. Total particle 4-flow $\quad N^\mu(x) = \sum_{k=1}^{N} N_k^\mu(x)$

Expression for electrical conductivity

Microscopic definition of electric current density

$$J^\mu = \sum_{k=1}^{N-1} (q_k - q_N) I_k^\mu$$

$$I_a^{(0)\mu} = 0$$

$$I_a^\mu = \sum_{k=1}^{N} (q_{ak} - x_a) \int \frac{d^3 \vec{p_k}}{(2\pi)^3 p_k^0} p_k^\mu \underbrace{f_k^0 (1 \pm f_k^0) \phi_k}_{\delta f_k}$$

1. The zeroth order contribution to the diffusion flow resulting from equilibrium distribution function vanishes.
2. The first finite contribution results from next to leading order in particle momentum distribution.

Macroscopic definition of electric current density

$$J^\mu = \sigma_{el} E^\mu$$

Expression of electrical conductivity

$$\sigma_{el} \sim q_Q^2 \;\; \tau$$

Determination of thermal relaxation time of quarks and gluons

- Degeneracy factor

Inverse of thermal relaxation time

$$\tau_k^{-1} \equiv \Gamma_k = \frac{\nu_l}{2}\frac{1}{2\omega_k}\int d\Gamma_{p_l}\, d\Gamma_{p'_k}\, d\Gamma_{p'_l}(2\pi)^4\delta^4(p_k+p_l-p'_k-p'_l)$$
$$\times \langle|M_{k+l\to k+l}|^2\rangle \frac{f_l'^0(1\pm f_k'^0)(1\pm f_l^0)}{(1\pm f_k^0)}$$

- Interaction rate
- Scattering amplitudes

The behavior of the thermal relaxation time

$$\tau^{-1} \sim T\alpha_s^2 ln\{\frac{1}{\alpha_s}\}$$

Quasi-particle description of hot QCD medium

Effective fugacity quasi-particle model(EQPM)

Ref : V. Chandra and V. Ravishankar,

Phys. Rev. D 84 (2011) 074013

EQPM maps the hot QCD medium effects in terms of the effective equilibrium distribution function of quasi-partons which describes the strong interaction effects in terms of effective fugacities $z_{g,q}$.

Quasi-parton equilibrium distribution function

Effective fugacity

$$f_{g/q} = \frac{\boxed{z_{g/q}} \exp[-\beta E_p]}{\left(1 \mp z_{g/q} \exp[-\beta E_p]\right)}$$

Energy dispersion relation

Contribution from hot QCD medium

$$\omega_{g/q} = E_p + \boxed{T^2 \partial_T ln(z_{g/q})}$$

The EOSs incorporating the QCD interactions modeled from -

1. Improved perturbative QCD computations at $O(g^5)$ and $O(g^6 \ln(1/g) + \delta)$.
2. (2+1)-flavor lattice QCD simulations.

Charge renormalization and effective coupling

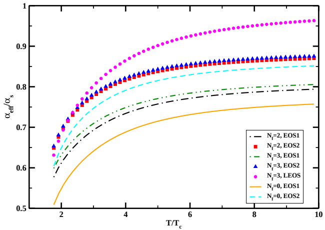
Debye mass in semi-classical transport theory

$$
\begin{aligned}
m_D^2 &= 4\pi\alpha_s(T)\Big(-2N_c\int\frac{d^3p}{(2\pi)^3}\partial_p f_g(\vec{p}) \\
&+ 2N_f\int\frac{d^3p}{(2\pi)^3}\partial_p f_q(\vec{p})\Big)
\end{aligned}
$$

$\alpha_s(T)\ T^2(\frac{N_c}{3}+\frac{N_f}{6})$

$$
m_D^2 = \boxed{m_D^2(\text{Ideal})} \times \frac{\frac{2N_c}{\pi^2}\text{PolyLog}[2,z_g] - \frac{2N_f}{\pi^2}\text{PolyLog}[2,-z_q]}{\frac{N_c}{3}+\frac{N_f}{6}}
$$

Effective coupling constant using various EOS as a function of T/T_c



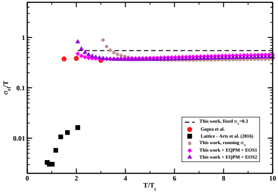
Effective Coupling

EQPM contribution

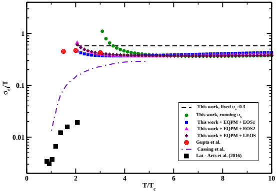
$$
\alpha_{eff} \equiv \alpha_s(T)\ \boxed{\frac{\frac{2N_c}{\pi^2}PolyLog[2,z_g] - \frac{2N_f}{\pi^2}PolyLog[2,-z_q]}{\frac{N_c}{3}+\frac{N_f}{6}}}
$$

Results of Electrical conductivity the leading log cross section

(σ_{el}/T) vs. (T/T_c) for N_f=2



(σ_{el}/T) vs. (T/T_c) for N_f=3

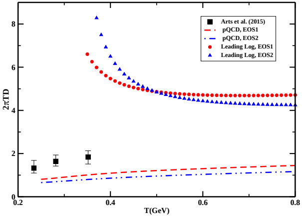


Ref : S. Mitra and V. Chandra,
Phys. Rev. D 94 (2016) no.3, 034025

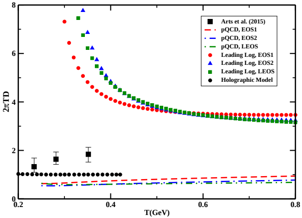
1. Due to the leading log effect the magnitude of σ_{el}/T is quite larger than pQCD case.
2. The quenched lattice measurement of electrical conductivity from Gupta *et al.* upto $T/T_c \sim 3$ shows good agreement with the current estimation of σ_{el}.
3. For 3-flavor case beyond $T/T_c \sim 3$, the estimations of σ_{el} is observed to match with the trend from Cassing *et al.* and agrees with their statement that above $T \sim 5T_c$, σ_{el}/T becomes approximately constant (≈ 0.3).

Results of Diffusion coefficient

$(2\pi DT)$ vs. (T/T_c) for N_f=2



$(2\pi DT)$ vs. (T/T_c) for N_f=3

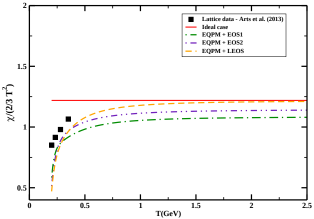


Ref : S. Mitra and V. Chandra,
Phys. Rev. D 94 (2016) no.3, 034025

1. The leading log results turn out to be much higher due to the logarithmic term as compare to the pQCD situation.
2. The pQCD results are comparable to the lattice and holographic results in the range of temperature $0.2 - 0.4$ GeV.

Results of charge susceptibility

Charge susceptibility as a function of T/T_c



Ref : S. Mitra and V. Chandra, Phys. Rev. D 94 (2016) no.3, 034025

Einstein's relation

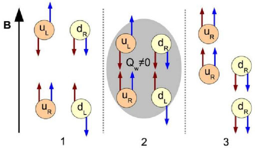
$$\sigma_{el} = \overset{\text{susceptibility}}{\boxed{\chi}} \; D$$

1. In lower temperature region, ranging from $0.2 - 0.35$ GeV, our results show good agreement with the lattice data.

2. Different EOSs within EQPM show discrete effects on χ, where the ones with LEOS are closer to the lattice data the most.

Chiral magnetic effect in HIC

A chiraly symmetric QGP at high temperature is a fundamental prediction of QCD



1. The magnetic field makes the quark spin aligned with $\vec{B}$ by spin polarization $\langle\vec{s}\rangle \propto (Qe)\vec{B}$.
2. Quarks with specific chirality have their momentum direction correlated with spin orientation, $\vec{p} \uparrow\uparrow \vec{s}$ for RH quarks and $\vec{p} \uparrow\downarrow \vec{s}$ for LH quarks.
3. In the presence of chirality imbalance ($\mu_5 = N_R - N_L \neq 0$), there is a net correlation between average spin and momentum $\langle\vec{p}\rangle \propto \mu_5\langle\vec{s}\rangle$ converting a LH up/down quark to RH one by reversing its momentum.
4. So the momentum of the quarks will also follow the magnetic field according to their charges $\langle\vec{p}\rangle \propto (Qe)\mu_5\vec{B}$.
5. The resulting quark currents $\vec{J} \propto (Qe)\mu_5\vec{B}$ will make the up and down quarks move in different direction creating a charge separation within the medium.

Identifying CME in the charged particle spectra extracted from QGP

The charge separation in the QGP fireball due to CME across the reaction plane can be observed in the azimuthal distribution of the outgoing charged hadrons

$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_1 cos\phi + 2v_2 cos(2\phi) + \cdots + 2a_\pm sin\phi$$

Coefficients of the CME induced charge separation signal

$$\boxed{a_+ = -a_- \propto \mu_5 |\vec{B}|}$$

Experimental Challenges

1. Chirality imbalance μ_5 arises from fluctuations and charges sign from event to event.
2. Upon averaging over many events $\langle sin\phi \rangle \sim \langle \mu_5 |\vec{B}| \rangle$ term vanishes.

Possible solution

1. Measurement of event by event fluctuations $\sim \langle (\mu_5 |\vec{B}|)^2 \rangle$ of such terms by observing the azimuthal correlations for same and opposite pair of charges is the next best attempt.

Conclusion and outlook

Concluding remarks···

1. In the early stages of the relativistic heavy ion collisions an EM field is generated that is sufficiently strong for particle production by vacuum fluctuation.

2. Although these fields decay very fast, due to strongly interacting nature of QGP are imprinted in the collective motion of the system which is being reflected in the signals extracted from HIC experiments.

3. The current state of the art research dealing with the main challenge of identifying the proper dynamics that can explain the system under both strong QCD color forces and EM fields, that ofter produce quantitatively similar effects.

Possible outlooks···

1. Since the fields are sharply time dependent ones, these time varying fields are needed to properly implemented in evolution hydrodynamics, for which theoretical formalisms beyond Schwinger mechanism is needed to be explored.

2. The inclusion of a temperature dependent electrical conductivity is needed to describe the charge transport around transition temperature (T_c) instead of a constant one, could be a matter of immediate future investigations.