# Wave-kinetic theory and simulations for filamentation and modulational instabilities

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# Contents



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# Particles are also waves

Individual particles ⟶ Distribution function ⟶ Wigner function ⟶ Wave function

J.J. Thompson: “Cathode rays are not waves, but a stream of particles (electrons).” Nobel Prize, 1906.

G.P. Thompson: “Sorry Dad, but electrons are waves after all.” Nobel Prize, 1937.

# Waves are also particles

Individual particles ⟵ Distribution function ⟵ Wigner function ⟵ Wave function

A. Einstein: “Photons are particles.” Nobel Prize, 1921.

This presentation: “Photons are particles, and can be accelerated. The same applies to other types of waves.”

# The math behind it

For an EM wave in unmagnetised plasma

$$\omega^2 = \omega_p^2 + c^2 k^2$$
$$\hbar^2 \omega^2 = \hbar^2 \omega_p^2 + c^2 \hbar^2 k^2$$
$$E^2 = (m_0 c^2)^2 + c^2 p^2$$

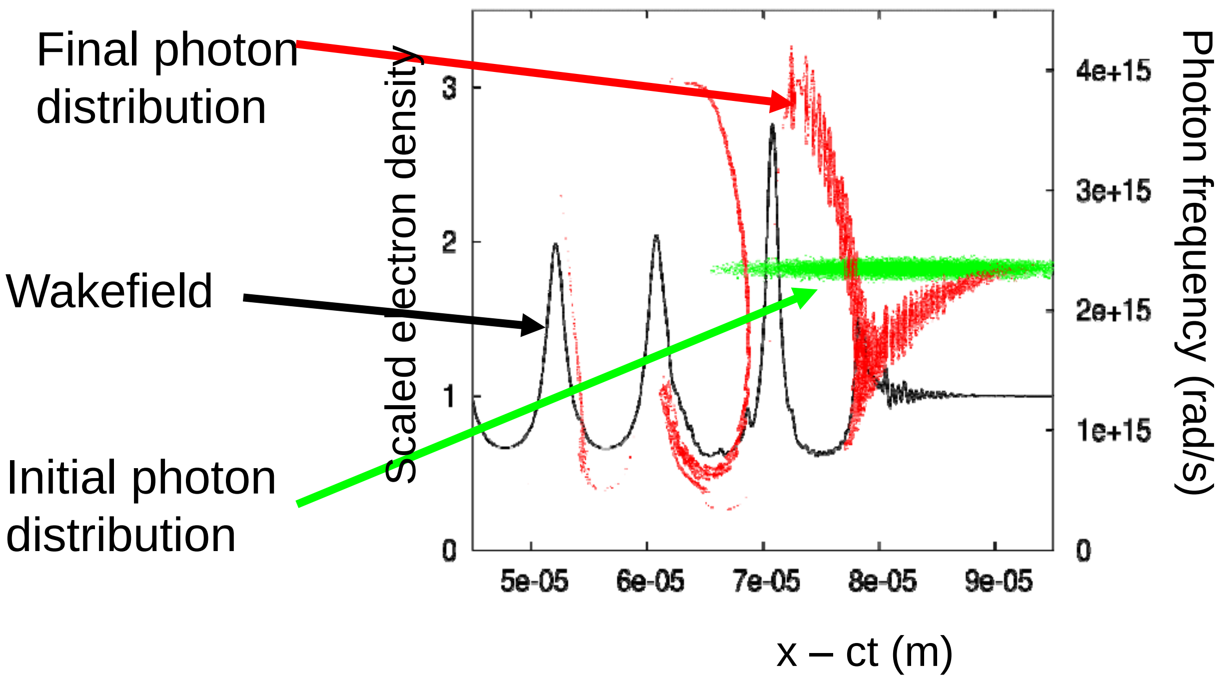
$$\frac{dx}{dt} = \frac{\partial H}{\partial p} = \frac{\partial \omega}{\partial k} \qquad \frac{dk}{dt} = -\frac{1}{\hbar}\frac{\partial H}{\partial x} = -\frac{\partial \omega}{\partial x}$$

We can use a particle model to simulate this!

# Wigner function

$$W(t,x,k)=\int \vec{E}(x+s/2)\cdot\vec{E}^*(x-s/2)\exp(iks)ds \sim N(t,x,k)$$

*Example: photon dynamics in a wakefield*




Experimental spectrum

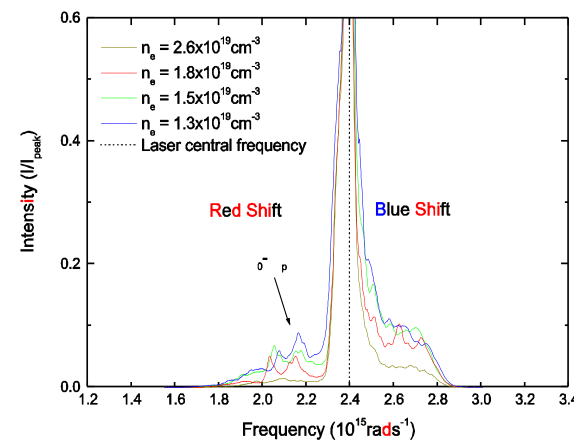
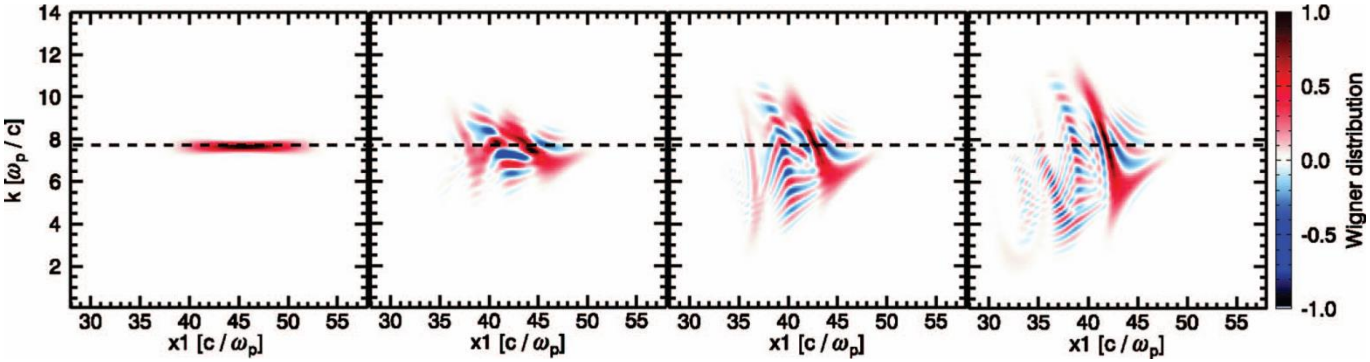


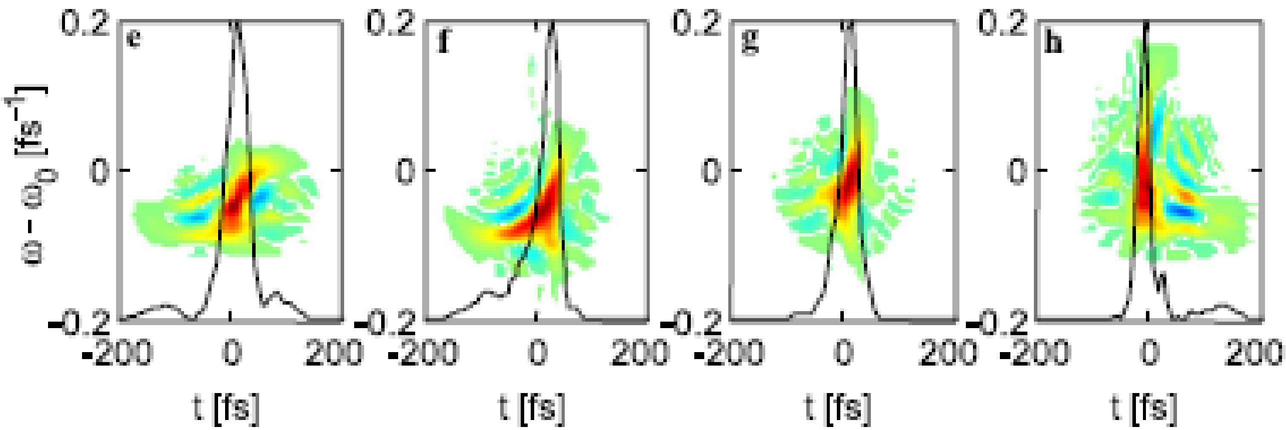

Image taken from simulations using a dedicated wave-kinetic code

# Photon acceleration in laser-wakefield experiments

PIC simulations by J. Vieira (2006).




Experimental data by J. Schreiber (2010): novel expt. development

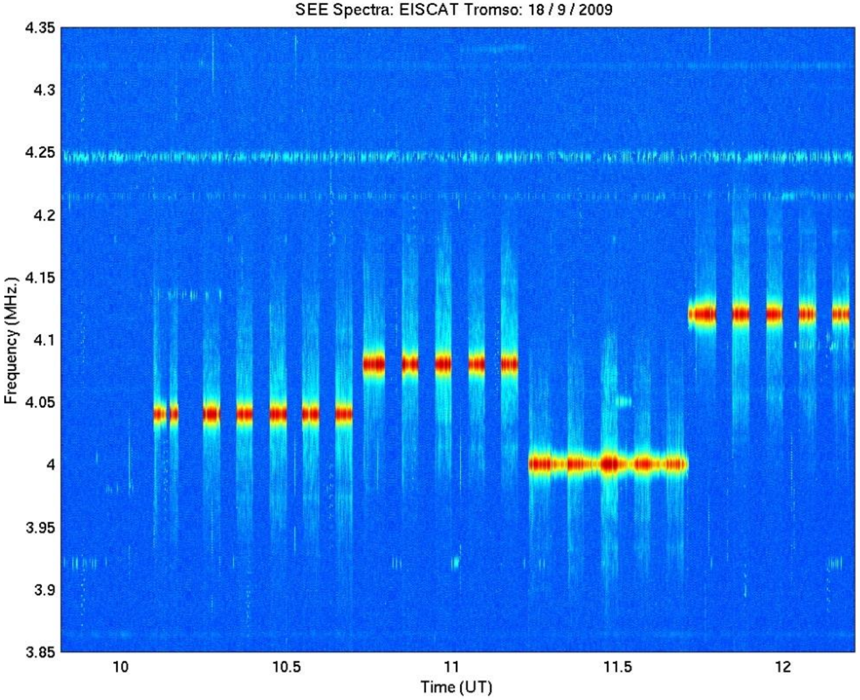



Murphy, Trines et al., PoP **13**, 033108 (2006).
J. Schreiber et al., PRL **105**, 235003 (2010).

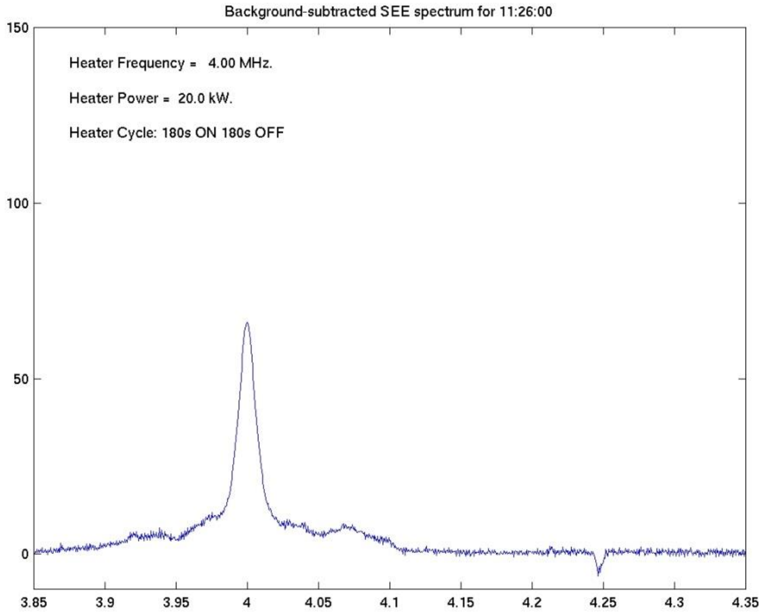
# Eiscat ionosphere experiment

Photon acceleration can also be seen in radar wave experiments

Backscattered radar signal




Detail showing up-/downshift

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# Modulational instability

Modulational instability in long pulse-plasma interaction will lead to:

— Bunching of photons in both real and momentum space

— Redshift of some parts of the pulse, blueshift of other parts

— Non-Raman spectral peak splitting:

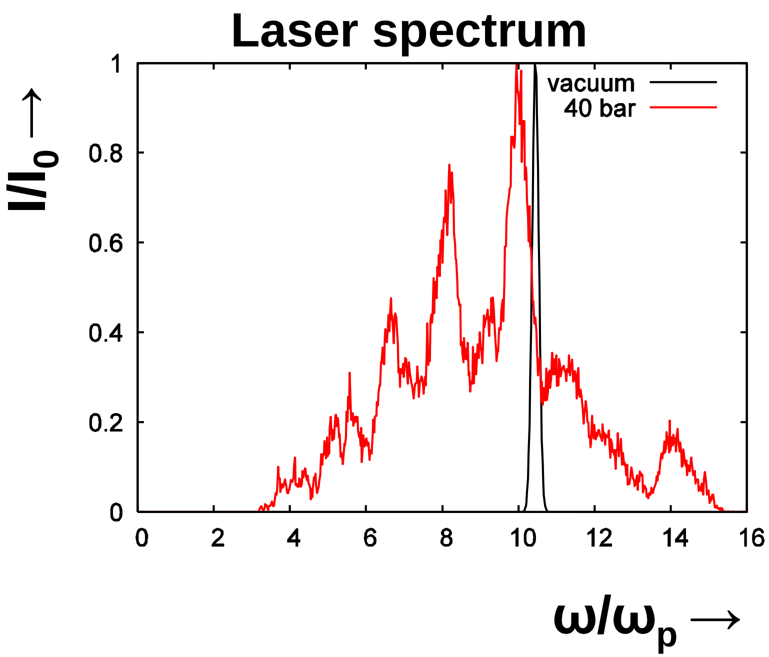
$$\omega = \omega_0 \pm n\omega_p \pm \Delta\omega$$

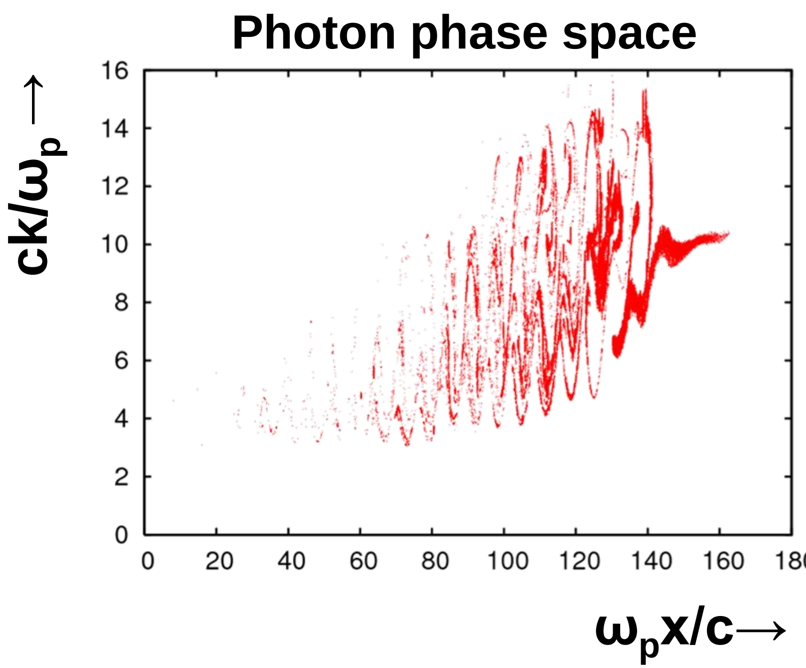
This should be visible in both experiments and simulations

Modulations to a probe pulse can be used to diagnose wakefield

# Wave-kinetic simulations

Photon kinetic simulations nicely reproduce spectral structure



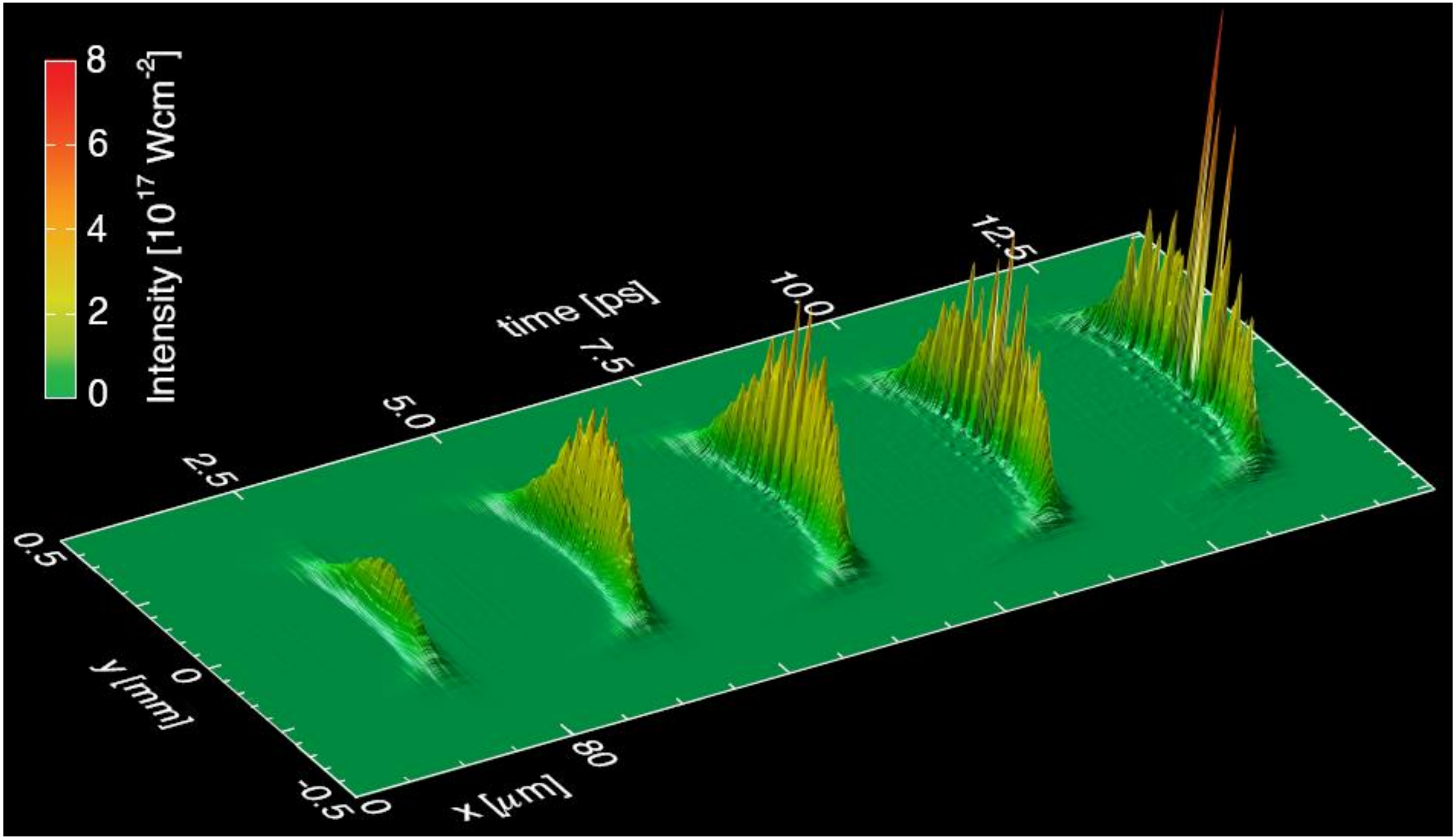




Simulations don't explain everything, e.g. blueshift of entire spectrum (ionisation effect?)

# Filamentation instability

- The ‘transverse’ equivalent of the modulational instability
- A light beam or particle beam entering a plasma breaks up in the transverse direction
- This spoils the envelope of this beam
- It also spoils the effectiveness of this beam
- We need to study filamentation to prevent or at least control it

# Example: Raman amplification




For a $2*10^{15}$ W/cm$^2$ pump and $\omega_0/\omega_p = 10$, the probe is amplified, but destroyed by filamentation

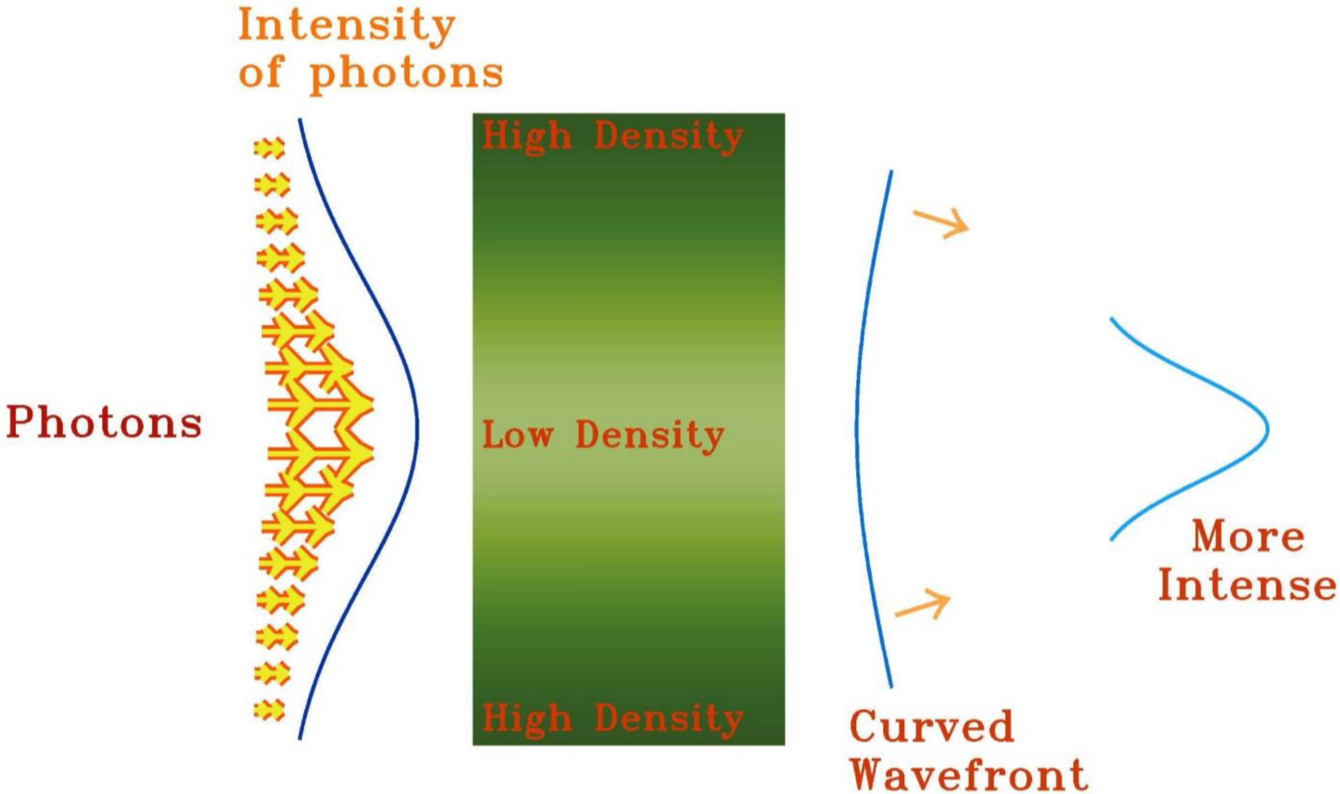
Trines et al Nature Physics 2010

# Theory

Three types of filamentation:

- Relativistic: For intense lasers the electrons experience a relativistic mass increase. This changes the local phase and group speed of the laser. Transverse modulations of laser intensity  are enhanced.
- Ponderomotive: 3 dimensional force, which repels electrons from areas of high intensity to low intensity. Moderately quick.
- Thermal: The laser loses energy to the electrons in the plasma due to electron-ion collisions. The electrons are heated and expand creating low density regions for the laser beam to filament. Dominant in high Z targets.

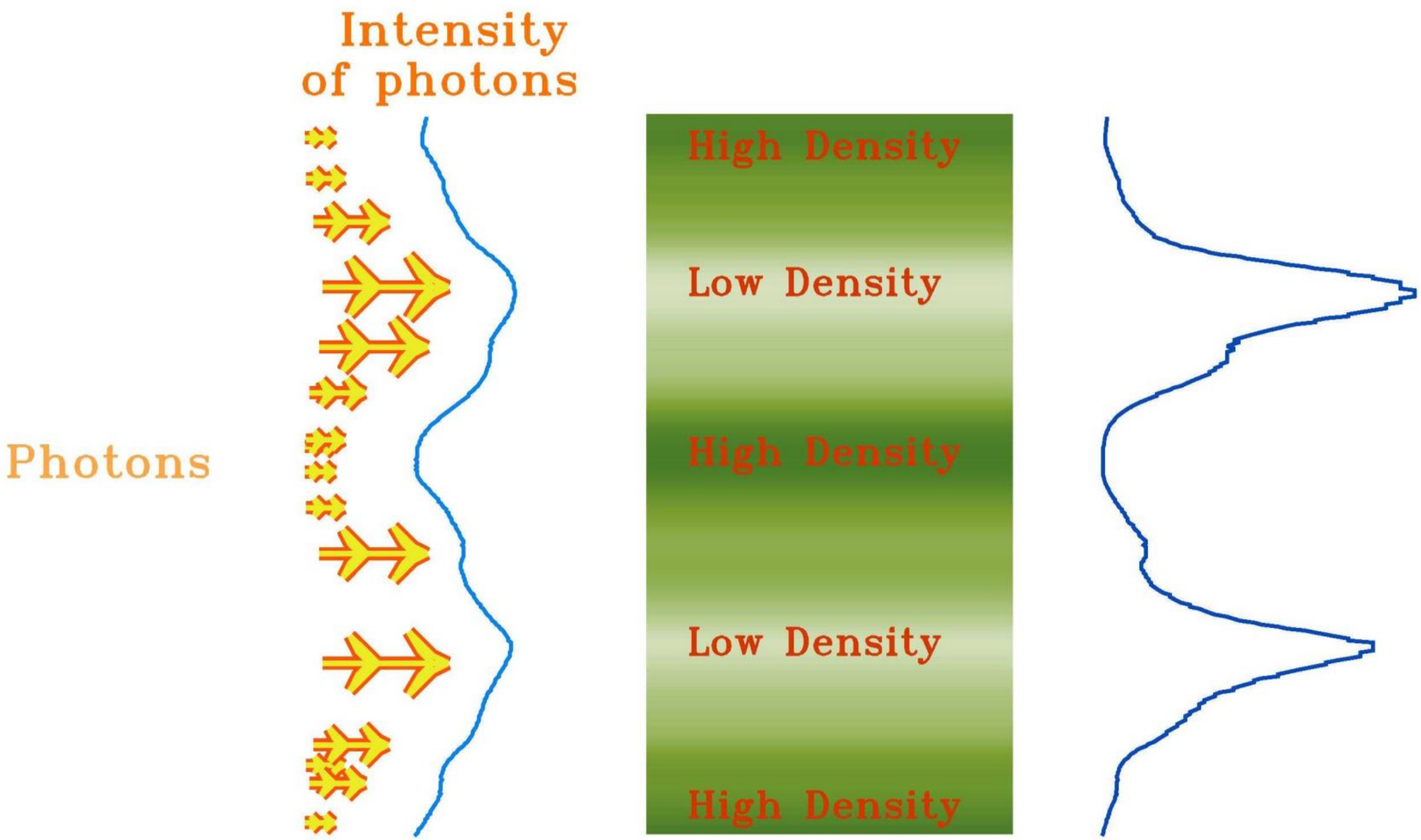
# Self-Focusing




Physical mechanism for self-focusing driven by the ponderomotive force, relativistic mass increase or thermal effects.

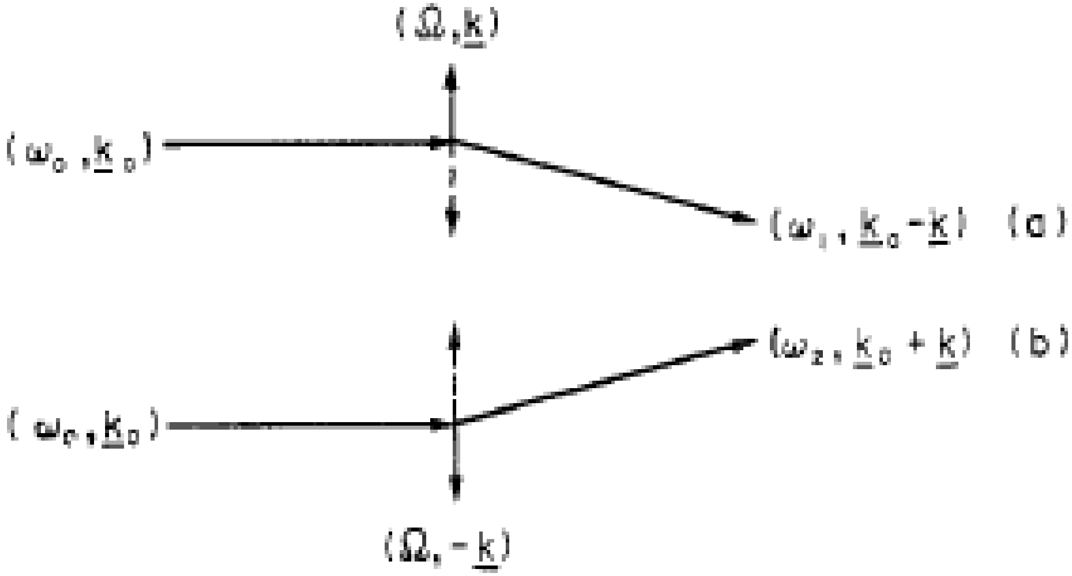
$\underline{F}_{pond} \propto -\underline{\nabla} I$

# Filamentation



# Filamentation as a four-wave process

An initial plane wave (wave 1) scatters from a density perturbation (wave 2) into a Stokes and anti-Stokes wave (waves 3 and 4)




$$c^2\nabla^2\mathbf{E} - \partial^2\mathbf{E}/\partial t^2 - \omega_{pe}^2\mathbf{E} = i4\pi e\omega\delta n\mathbf{v} + 4\pi n_0 e v_e \mathbf{v}$$

$$\delta n = i n_0 e\langle(\mathbf{v}\times\mathbf{B})_y\rangle/k_y k_B T_e - n_0\delta T_e/T_e$$

# Ponderomotive vs Thermal Filamentation

Ponderomotive filamentation $\propto I\lambda^2$

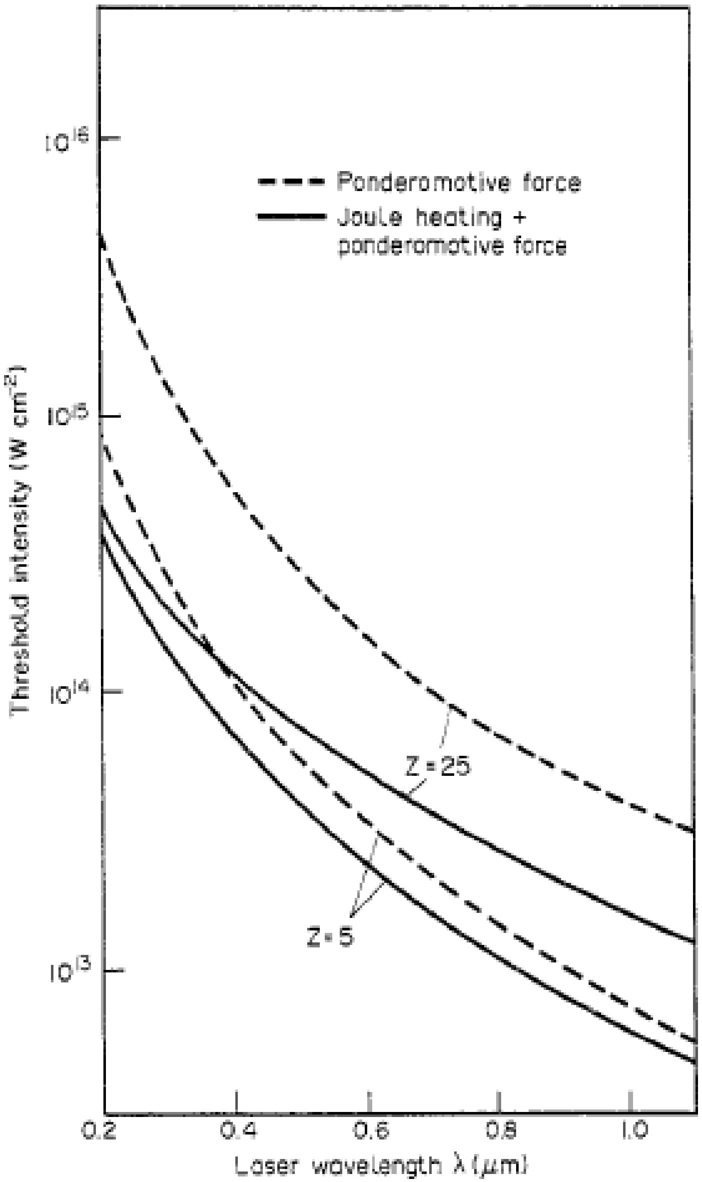
Thermal filamentation $\lambda_{mfp} < L$ filament width

Threshold:

$$\left(\frac{v_0}{v_{Te}}\right)^2_{\text{Threshold}} = \frac{8\omega_0^2}{\omega_{pe}^2}\left(\left[(0.065/k_0^2\lambda_{\text{mfp}}^2)^2 + (\gamma_T/\omega_0)^2\right]^{1/2} - 0.065/k_0^2\lambda_{\text{mfp}}^2\right).$$

Where $v_0$ is the electron quiver velocity in the laser field.

# Threshold Intensity for thermal and ponderomotive filamentation

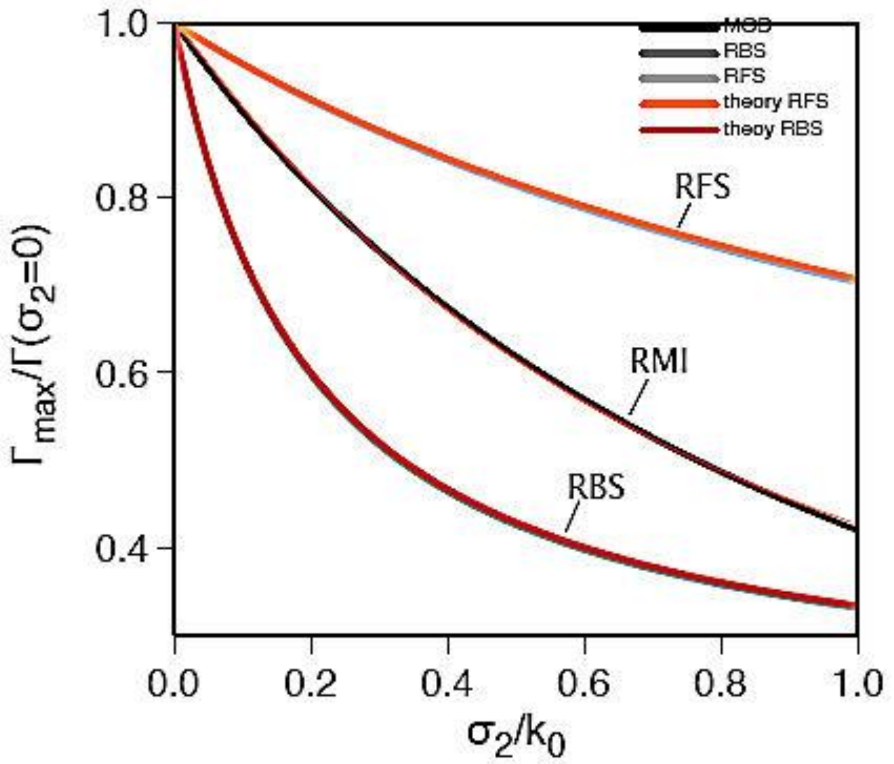



Both thermally driven and ponderomotive filamentation need to be investigated

The threshold intensity is higher for short wavelengths.

At x-ray wavelengths threshold intensity may not be reached.

# Bandwidth Results For Raman forward and backward scattering




**Maximum growth rate as a function of the photon distribution width.**

Raman Backscatter growth rate much more controlled by bandwidth.

Raman forward scatter a four wave process like filamentation is less affected by bandwidth.

# Advanced wave kinetics

**The fast waves need not be photons**

**The slow waves need not be wake fields**

**Examples**

- Drift wave/zonal flow
- Rossby wave/zonal flow
- Langmuir/ion-acoustic wave
- Photon/gravitational wave

**Modulational instability of particular interest**

**Applications range from planetary atmospheres to tokamaks and solar flares, and many more**

# Drift waves

**Drift waves:**

- Transverse ES waves in magnetized plasma
- Wave vector *k* perp. to magnetic field *B*
- Electric field *E* parallel to *k*, perp. to *B*
- Plasma oscillations perp. to *E* and *B*, driven by the *drift velocity*

**Lower hybrid drift modes**…

- are important for the physics of magnetized plasma edges,
- control the particle and energy transport in tokamaks, in the magnetopause boundary layer [1], in stellar flares,…
- have many applications in astrophysics, tokamak physics,…

**We will explore the interaction between drift waves and zonal flows, and compare to real-life configurations, using the first ever wave-kinetic code for drift waves**

[1] R. Bingham *et al.*, Physica Scripta **T113**, 144 (2004).

# "Kinetic" drift wave theory

We use the kinetic model for 2-D drift waves by Smolyakov *et al.*, Lashmore-Davies *et al.*

A.I. Smolyakov, P.H. Diamond, and V.I. Shevchenko, *Phys. Plasmas* **7**, 1349 (2000).

C.N. Lashmore-Davies, D.R. McCarthy, and A. Thyagaraja, *Phys. Plasmas* **8**, 5121 (2001).

Fluid model for the plasma (el. static potential Φ(r)):

$$\frac{\partial \Phi}{\partial t} = \int \frac{k_x k_y}{\left(1 + k_x^2 + k_y^2\right)^2} N_k d^2 k$$

Particle model for the "driftons" (number density $N_k$):

- Drifton number conservation;
- Hamiltonian: $\omega_i = k_x \frac{\partial \Phi}{\partial x} + \frac{k_y V_*}{\left(1 + k_x^2 + k_y^2\right)}; \qquad V_* = -\frac{1}{n_0}\frac{\partial n_0}{\partial x}$
- Equations of motion: from the Hamiltonian

# Simulations

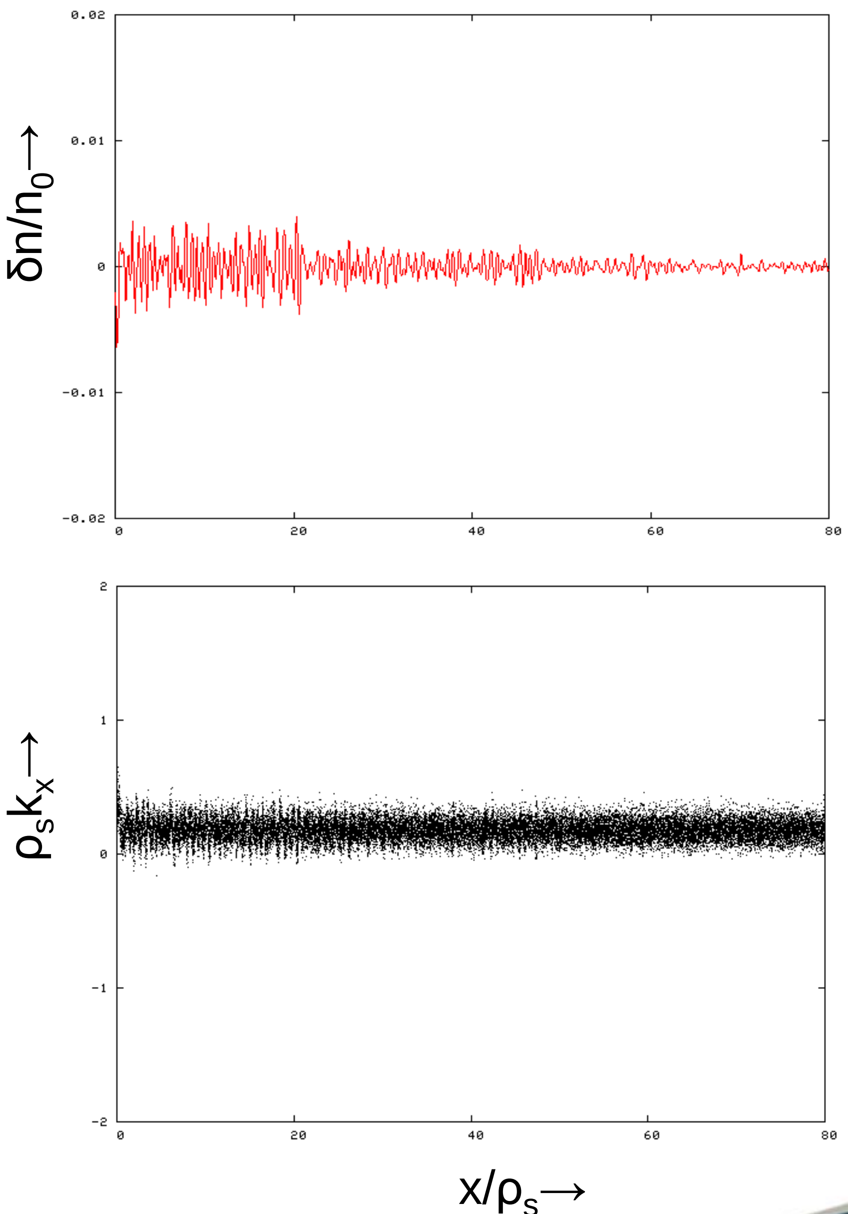
**We simulated drift waves using the quasi-particle method:**

- Two spatial dimensions, slab geometry,
- Homogeneous, broadband drifton distribution,
- Plasma density profile: 2-D Gaussian (tokamak-like)

**We have obtained the following results:**

- Modulational instability of drift modes,
- Excitation of  a zonal flow,
- Solitary wave structures drifting outwards,
- Zonal flow growth controlled by density gradient.

# Simulation results



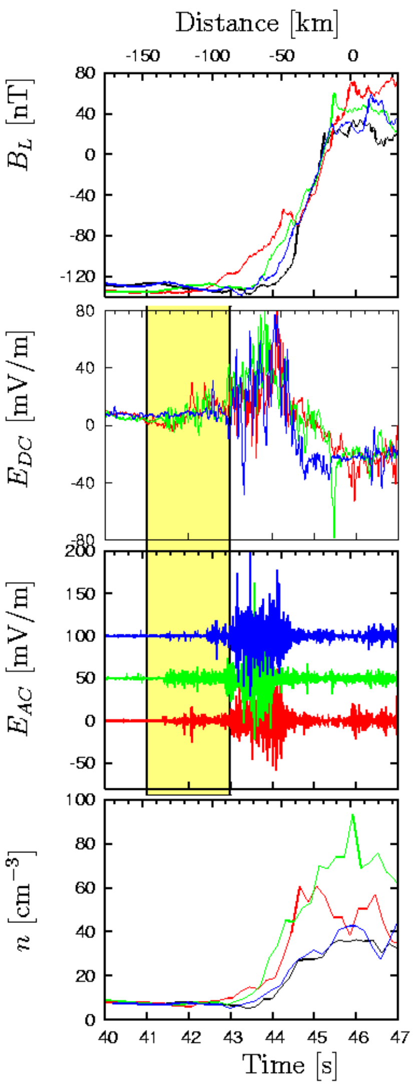

Background plasma profile: (∇n)/n increases with radius

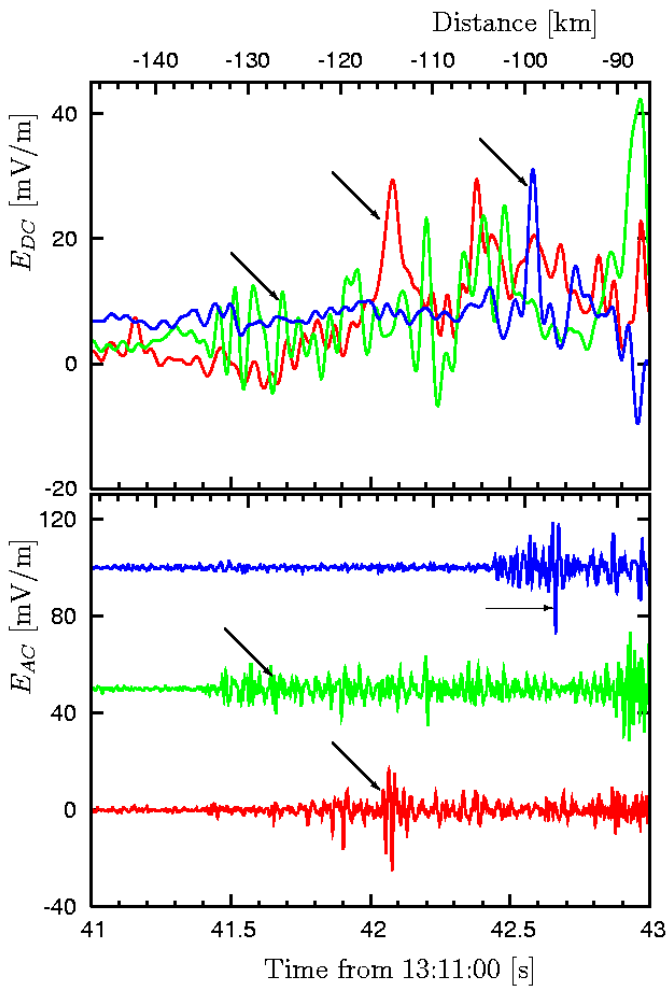
Radial ES field and plasma density fluctuations versus radius r :

- Excitation of a zonal flow at small background density gradients,
- Propagation of solitary structures towards regions with higher gradients.
- Bunching and drift of drift modes under influence of zonal flow

R. Trines *et al.*, Phys. Rev. Lett. **94**, 165002 (2005).

# Cluster satellite data: magnetopause



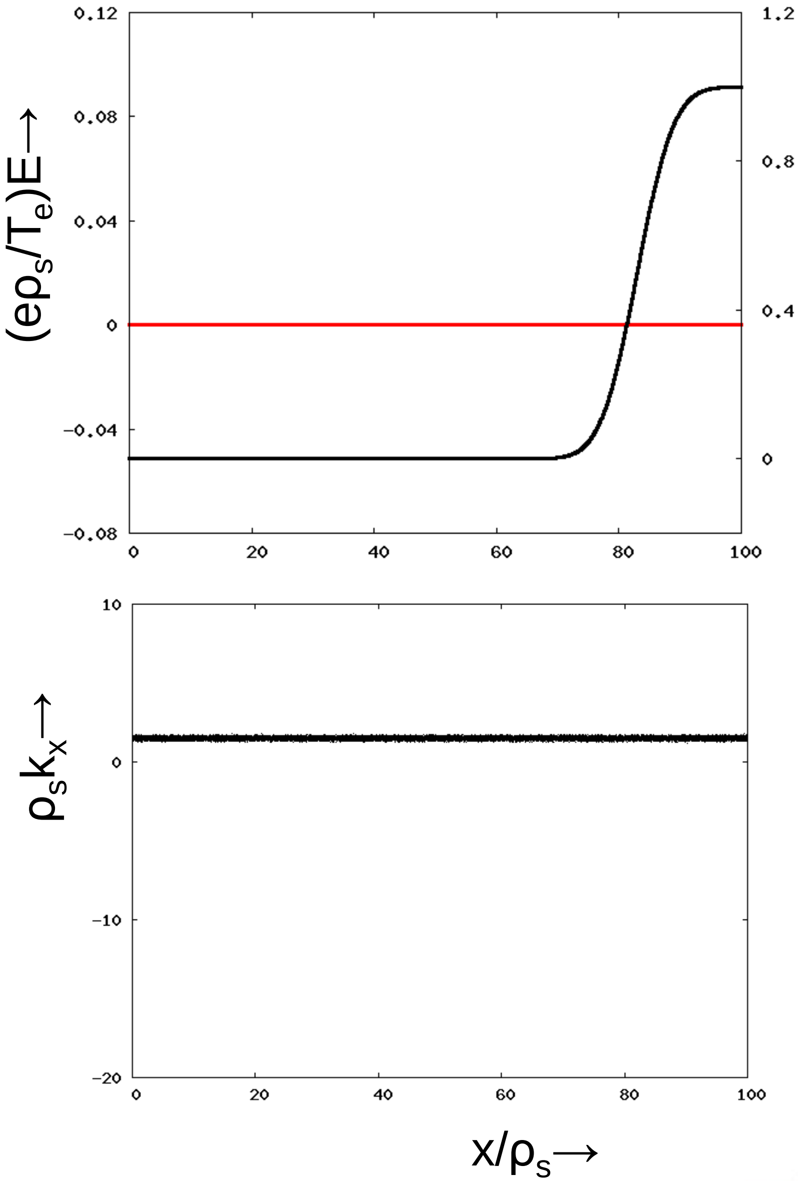




See the solitary structures moving down the density gradient in the DC field…

Observe the accompanying bursts of turbulence in the AC field…

Ordered structures arising from turbulence!

# Cluster versus simulations




Simulations exhibit formation of solitary structures, just like the Cluster observations;

Zonal flow wavelength: (0.2-0.25)*$\rho_s$ for obser-vations, (0.5-0.7)*$\rho_s$ for simulations;

Structure size: (0.8-0.9)*$\rho_s$ for observations, (0.7-1.0)*$\rho_s$ for simulations;

Structure speed: about (0.08-0.1)*$c_s$ for obser-vations, (0.02-0.05)*$c_s$ for simulations.

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# Summary and conclusions

## Wave kinetics: a powerful new approach

- Simple wave description, simple implementation
- Very versatile
- Provides powerful new diagnostics

## Photon acceleration

- Explains spectral modulations for short and long pulses
- Is being used to develop a real-time wakefield diagnostic
- Close ties to the modulational instability

## Filamentation instability

- Potentially damaging to laser-plasma interactions
- Needs to be understood to be controlled
- Relativistic and ponderomotive filamentation easily simulated via PIC; thermal filamentation under investigation

# Summary and conclusions

## Drift wave turbulence: spontaneous soliton formation

- A new result, first discovered in simulations, then fully explained from existing analytic theory
- Already identified in Cluster observations of the magnetopause
- Probable extension to tokamaks and other laboratory plasma devices

## Behaviour of drift modes dictated by zonal flow

- Explains formation of solitary structures at plasma edge
- Good agreement with observations by Cluster at the magnetopause

## Synergy between wave-kinetic and full-PIC simulations will lead to better understanding of these instabilities, and thus to better control