Theory and simulations of turbulence

Mahendra K. Verma
Indian Institute of Technology Kanpur

mkv@iitk.ac.in
http://turbulence.phy.iitk.ac.in

Ref: M. K. Verma, Phys. Rep. 2004

MHD Equations

velocity field

Pressure

Ext. Force

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla\frac{p}{\rho} + \frac{\mathbf{J}\times\mathbf{b}}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{F}$$

$$\partial_t \mathbf{b} + (\mathbf{u}\cdot\nabla)\mathbf{b} = (\mathbf{b}\cdot\nabla)\mathbf{u} + \eta\nabla^2\mathbf{b}$$

magnetic field

magnetic diffusivity

kinematic viscosity

Incompressibility

$$\nabla\cdot\mathbf{u} = 0$$

$$\nabla\cdot\mathbf{b} = 0$$

Prandtl no = $\frac{\nu}{\eta}$

Reynolds no = $\frac{UL}{\nu}$

Mag. Reynolds no= $\frac{UL}{\eta}$

Kraichan-Iroshniknov model (1964,1965)

Time scale = $1/(kB_0)$

$$E^b(k) = A(\Pi B_0)^{1/2} k^{-3/2}$$

Generalisation by Dobrowonly et al. (1980)

Random scattering of Alfven waves

$$\Pi^{+} \approx \Pi^{-} \approx \frac{1}{B_0} E^{+}(k) E^{-}(k) k^3 = \Pi$$

Kolmogorov-Like spectrum

Time scale $\tau^{\pm} \sim 1/(kz^{\mp})$

$$\Pi^{\pm} = E^{\pm}(k)[E^{\mp}]^{1/2} k^{5/2}$$

$$\left(\frac{\Pi^{+}}{\Pi^{-}}\right)^{2} = \frac{E^{+}(k)}{E^{-}(k)}$$

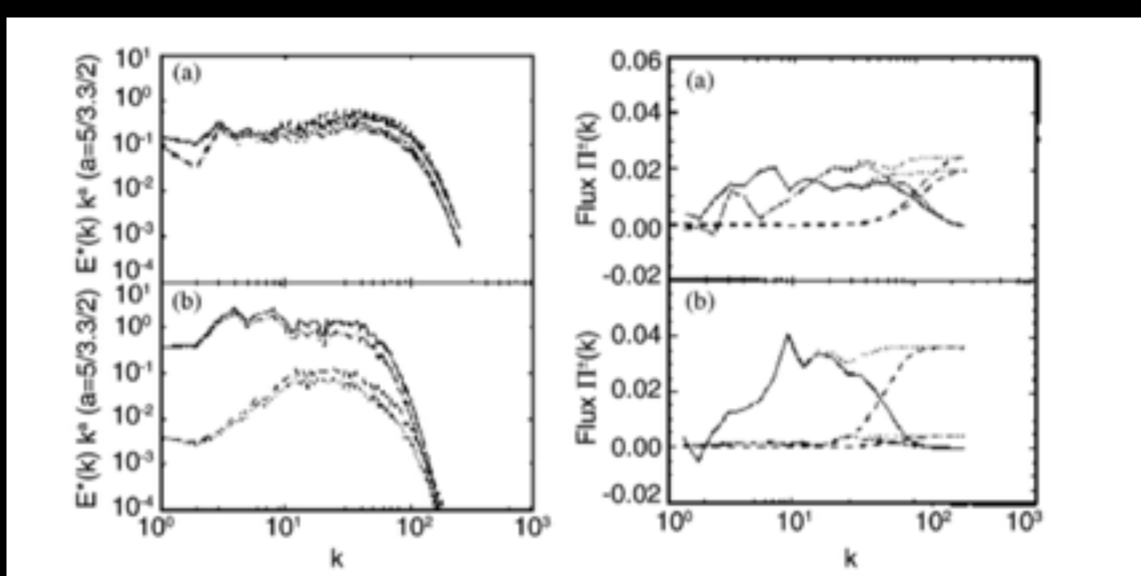
Marsch, 1990;
Verma et al….
Goodrich and Sridhar, 1995, 2003
(critical balance)

Verma et al. 1994,1996

2D DNS: Testing flux

$$\Pi^+ \approx \Pi^- \approx \frac{1}{B_0} E^+(k) E^-(k) k^3 = \Pi .$$

$$\left(\frac{\Pi^+}{\Pi^-} \right)^2 = \frac{E^+(k)}{E^-(k)}$$



Kolmogorov wins over KID

Why Kolmogorov's spectrum in MHD

Verma, 1999

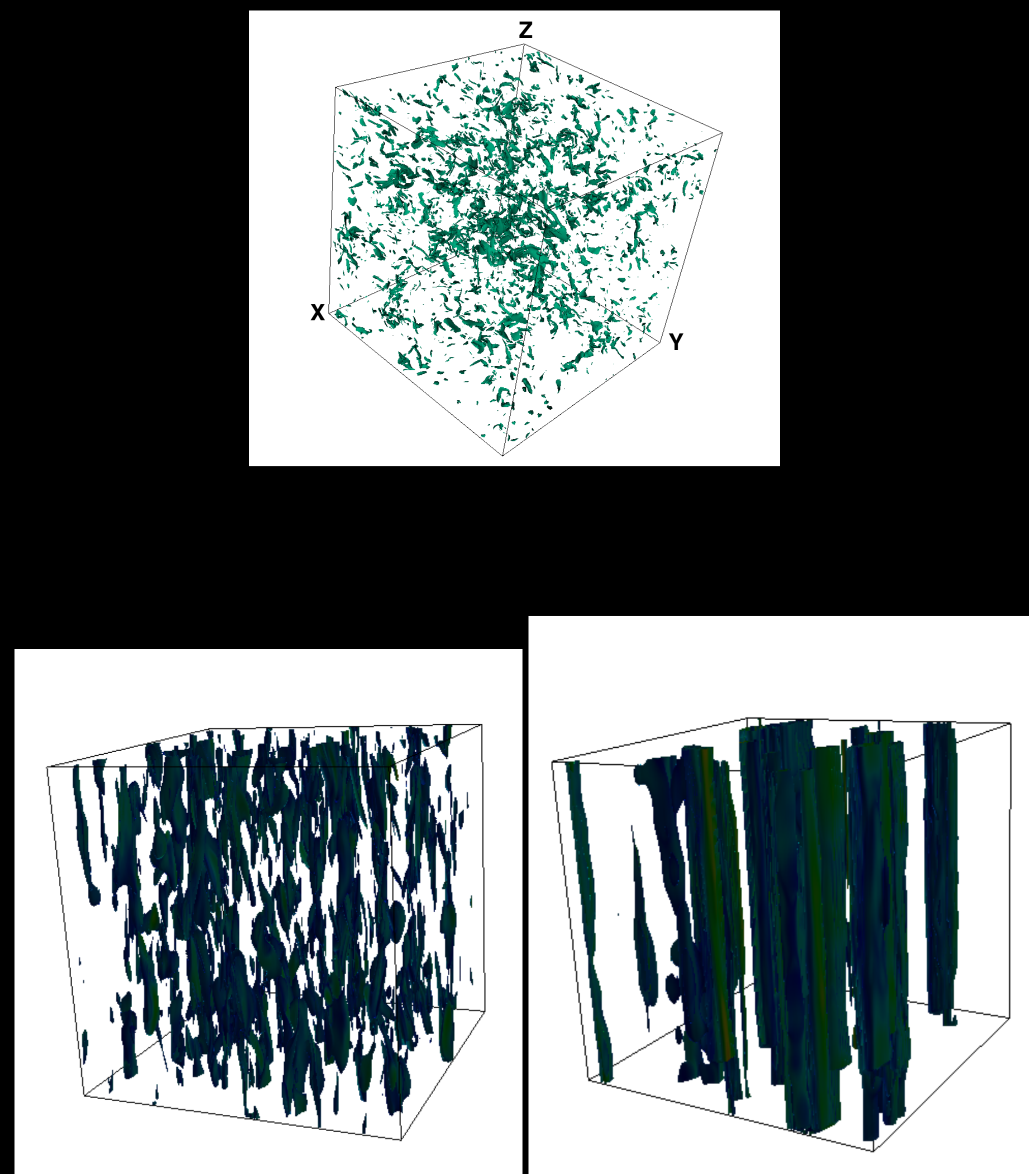
Waves at a scale gets scattered by B_0 of large scale.

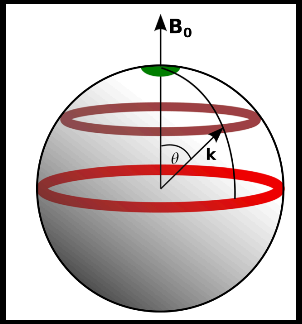
Renormalisation group analysis

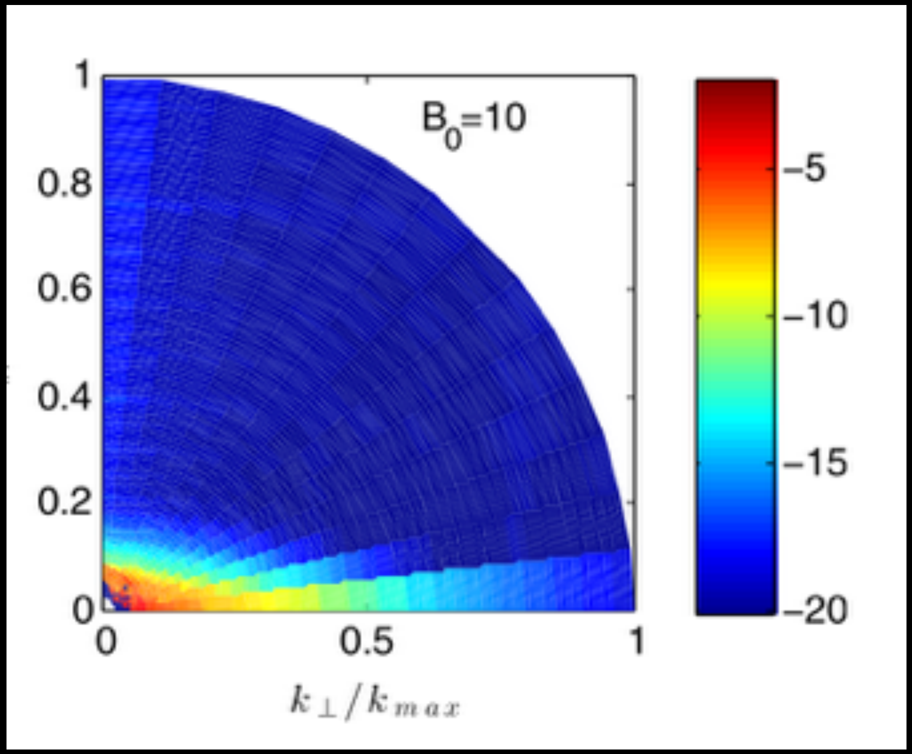
$B_0(k) \approx \Pi^{1/3} k^{-1/3}$

$$E^b(k) = A(\Pi B_0)^{1/2} k^{-3/2}$$ → $E(k) \approx \Pi^{2/3} k^{-5/3}$

What happens when $B_0 >> \delta B$


Z
X
Y





Quasi2D flow

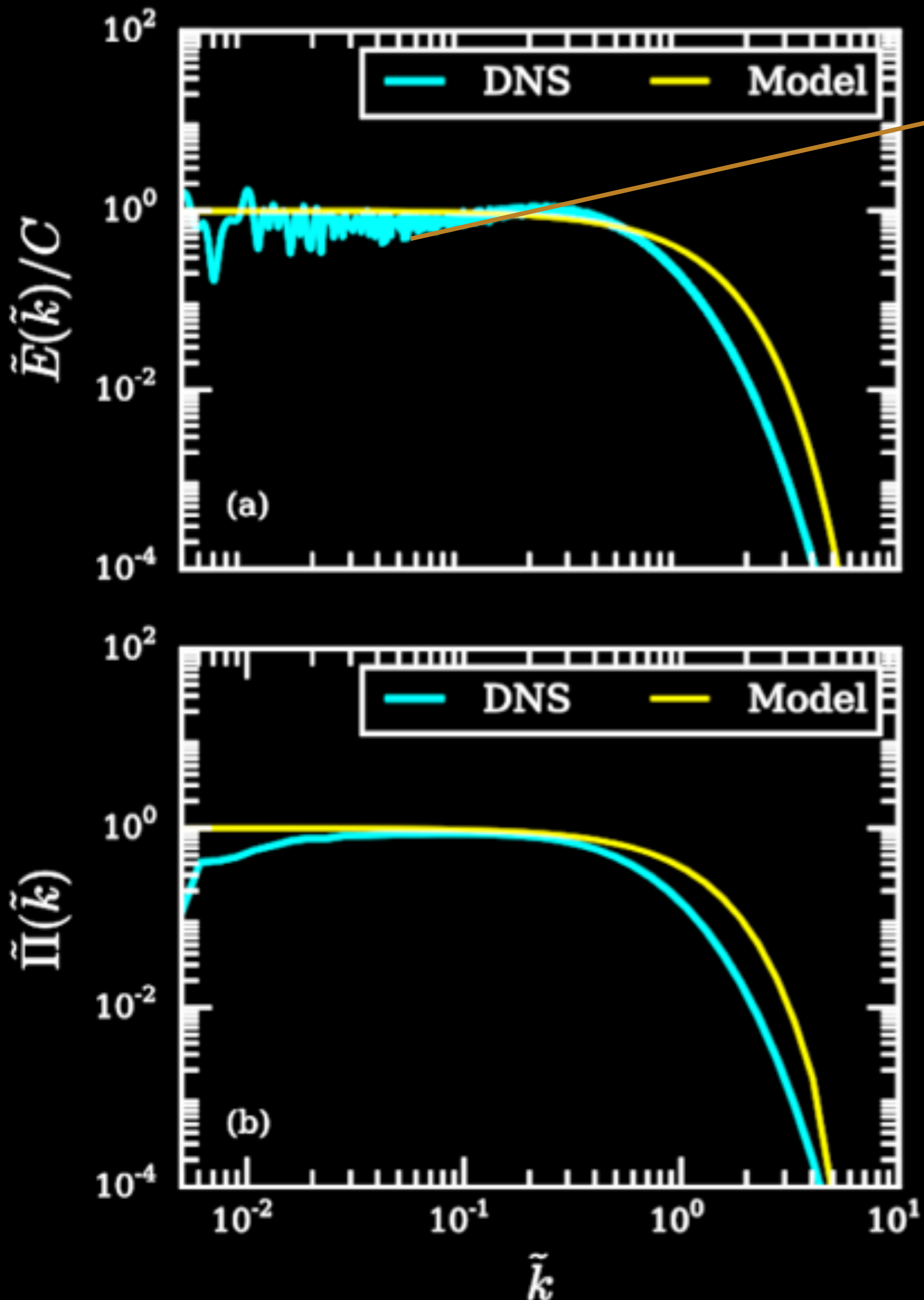
Flux is suppressed.

Quantitive theory not present

Using the code Tarang

Sunder et al., PoP, 2017

Spectrum for Rayleigh Benard Convection



$E_u(k)k^{5/3}$

Pao's model, 1965

$$E(k) = K_{\text{Ko}} \epsilon^{2/3} k^{-5/3} \exp(-\tilde{k}^{4/3})$$

$$\Pi(k) = \epsilon \exp(-\tilde{k}^{4/3})$$

$$\tilde{k} = k / k_d$$

Verma et al., 2014,2017

4096^3 grid simulation on Shaheen (KAUST)

$\varepsilon = 1.6 \times 10^{-3}$

Dynamo process in plasmas

Mahendra K. Verma
Indian Institute of Technology Kanpur

Collaborators: Rohit Kumar (iitk)
Ravi Samtaney (KAUST)

mkv@iitk.ac.in
http://turbulence.phy.iitk.ac.in

Mag energy grows with time at large length scales

Conditions for dynamo

Mag Reynolds no: $Rm = UL/\eta > 1$

Schekochihin, Isakov, Proctor, Cowley, 2004-2007

Ponty et al., 2005, 2007

How does large-scale B field grow?

Inverse cascade of B?

Stapanov & Plunian, 2006, 2007, 2012

Dynamo in protostar

We test if Rm >>1.

Verma et al., Pramana 2013

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nature physics

Turbulent amplification of magnetic fields in laboratory laser-produced shock waves

J. Meinecke[1*], H. W. Doyle[1], F. Miniati[2], A. R. Bell[1], R. Bingham[3,4], R. Crowston[5], R. P. Drake[6], M. Fatenejad[1,7], M. Koenig[8], Y. Kuramitsu[9], C. C. Kuranz[6], D. Q. Lamb[7], D. Lee[7], M. J. MacDonald[6], C. D. Murphy[10], H-S. Park[11], A. Pelka[8], A. Ravasio[8], Y. Sakawa[9], A. A. Schekochihin[1], A. Scopatz[7], P. Tzeferacos[1,7], W. C. Wan[6], N. C. Woolsey[5], R. Yurchak[8], B. Reville[1,12] and G. Gregori[1,7*]

Can the above ideas be applied here?
Or has it been applied already?

The flow is compressible, relativistic…

Still, we could get some ideas about the
B field in laser-induced plasmas using MHD.

Spitzer formula

$$\nu \sim 2.21 \times 10^{-15} T^{5/2}/(4\rho),$$

$$\eta \sim 4\pi^{3/2} m_e^{1/2} e^2 c^2 / (2 \times (2k_B T)^{3/2} \times 0.6),$$

$T=10^4$K, $\rho=3\times10^{-6}$ gm/cc

$\nu=2$ cm^2/s

$\eta=3\times10^7$ cm^2/s

mag Prandtl no: $Pm=\nu/\eta\approx10^{-7}$

$$L = R/10 \approx 100\ R_s/10$$

$$Rm \approx 3\times10^9$$

Hence dynamo is active in the protostar.

$$B = ?$$

Assume equipartition after 1 eddy turnover time.

$$\rho\ U^2 = B^2$$

$$\Rightarrow B \approx 2\times10^4\ G$$

1 Eddy turnover time = L/U = 200 earth days

Conclusions

Porting the ideas of MHD dynamo may be very useful.