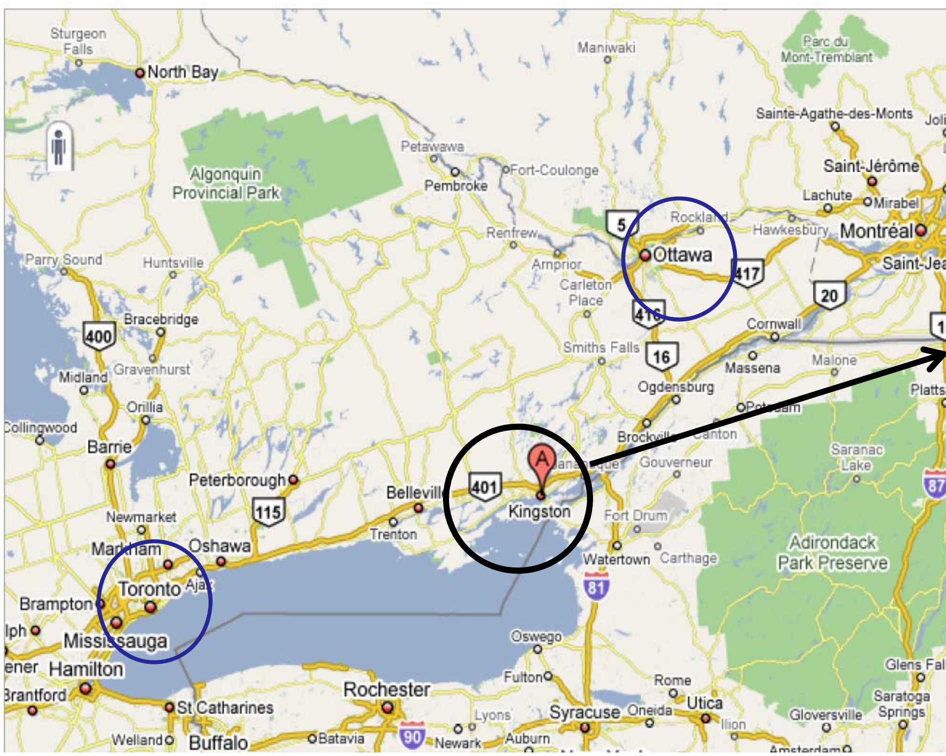
S. Hughes, Queen's University, Canada

Basic topics of the 4(5?) lectures:

- L1) Slow-light photonic crystal waveguides and disorder-induced scattering
- L2) Quantum light sources in photonic crystal waveguides
- L3) Quantum dot cavity-QED – polaron master equation approach
- L4) Purcell effect and mode volumes for leaky optical cavities and plasmonic resonators

Queen’s University (est. 1841)

- Older than “Canada” (founded: 1867) by 26 years!
- Reputation for scholarship and strong spirit
- ~20,000 students
- Excellent engineering physics program



Outline – L1

- Introduction to photon crystals (PCs) and slow-light PC waveguides
- Theory of disorder-induced scattering – the "intrinsic disorder problem"

History of photonic crystals
"semiconductors for light"

VOLUME 58, NUMBER 20 PHYSICAL REVIEW LETTERS 18 MAY 1987

Inhibited Spontaneous Emission in Solid-State Physics and Electronics

Eli Yablonovitch

Bell Communications Research, Navesink Research Center, Red Bank, New Jersey 07701

(Received 23 December 1986)

It has been recognized for some time that the spontaneous emission by atoms is not necessarily a fixed and immutable property of the coupling between matter and space, but that it can be controlled by modification of the properties of the radiation field. This is equally true in the solid state, where spontaneous emission plays a fundamental role in limiting the performance of semiconductor lasers, heterojunction bipolar transistors, and solar cells. If a three-dimensionally periodic dielectric structure has an electromagnetic *band gap* which overlaps the electronic *band edge*, then spontaneous emission can be rigorously forbidden.

Citations:
9,306 (March 12, 2012)
12,904 (March 17, 2014)

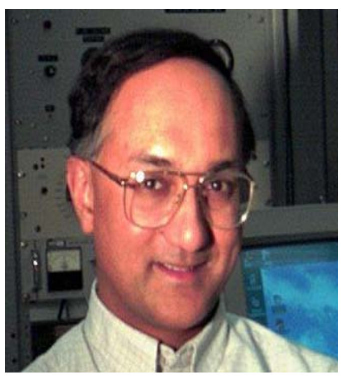
History of photonic crystals

VOLUME 58, NUMBER 23 PHYSICAL REVIEW LETTERS 8 JUNE 1987

Strong Localization of Photons in Certain Disordered Dielectric Superlattices

Sajeev John
Department of Physics, Princeton University, Princeton, New Jersey 08544
(Received 5 March 1987)

A new mechanism for strong Anderson localization of photons in carefully prepared disordered dielectric superlattices with an everywhere real positive dielectric constant is described. In three dimensions, two photon mobility edges separate high- and low-frequency extended states from an intermediate-frequency pseudogap of localized states arising from remnant geometric Bragg resonances. Experimentally observable consequences are discussed.



Citations:
6,199 (March 12, 2012)
8,977 (March 17, 2014)

P. W. Anderson, Phys Rev 109, 1492 (1958)

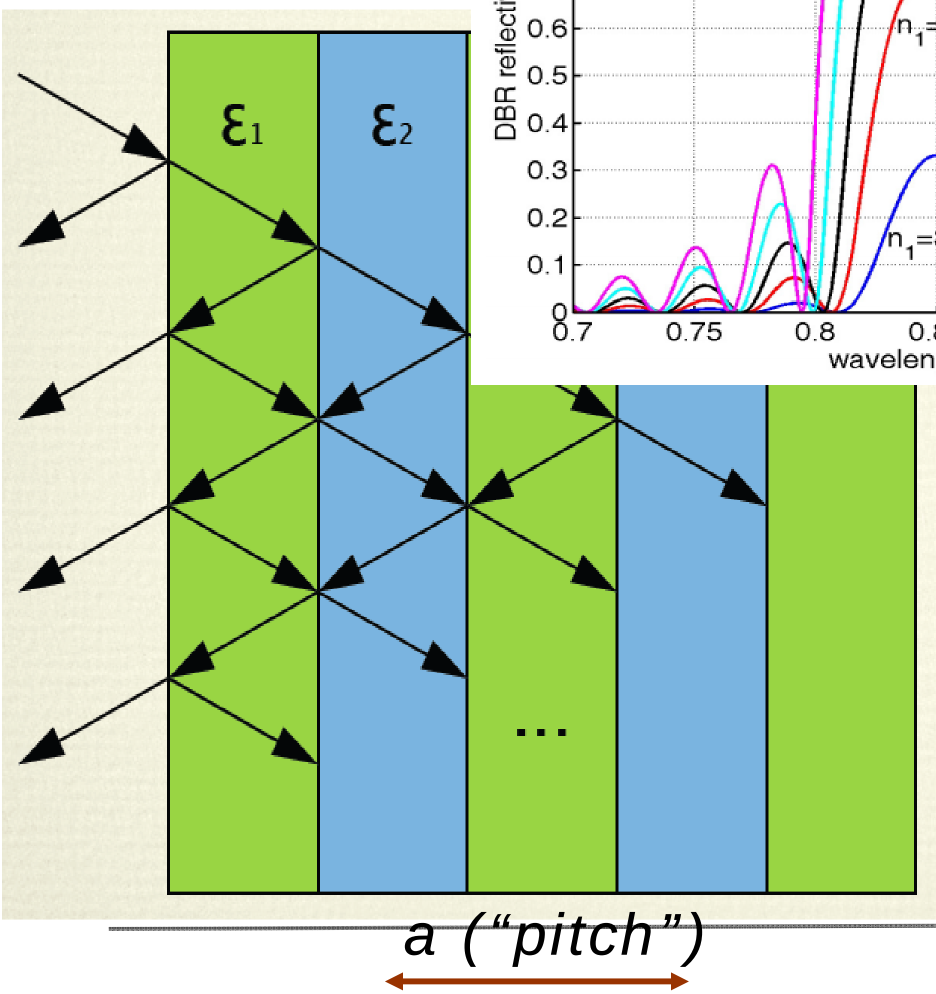
Extended states can be localized by the presence of system disorder.

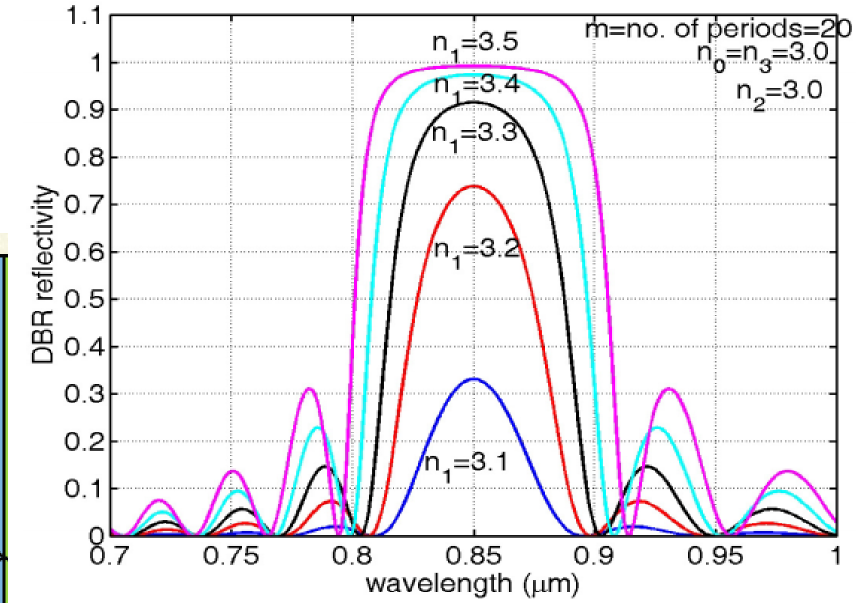
Anderson localization, also known as strong localization, is the absence of diffusion of waves in a random medium.

This phenomenon is different to weak localization, which is the precursor for Anderson localization.

Nobel Prize in Physics (1977)

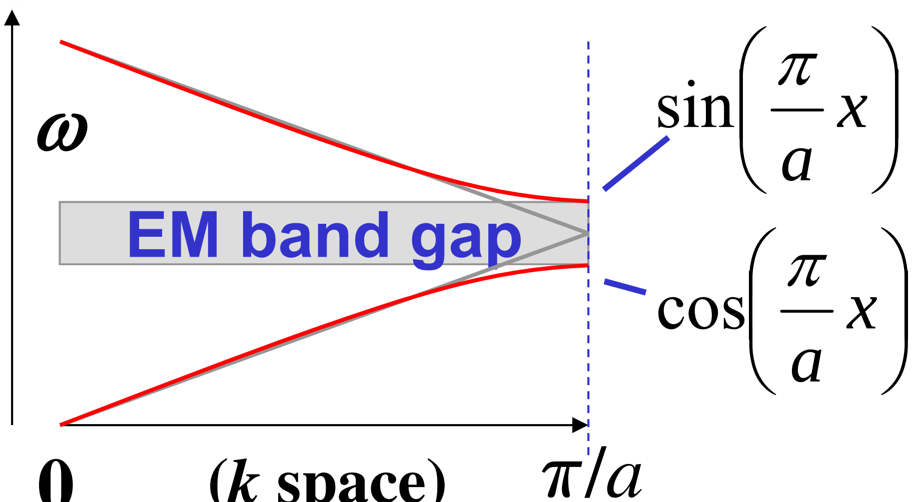
DBR (Distributed Bragg Reflector)



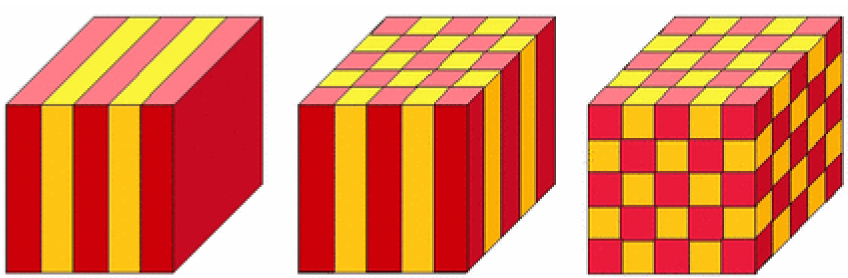


$$r_{12} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)$$

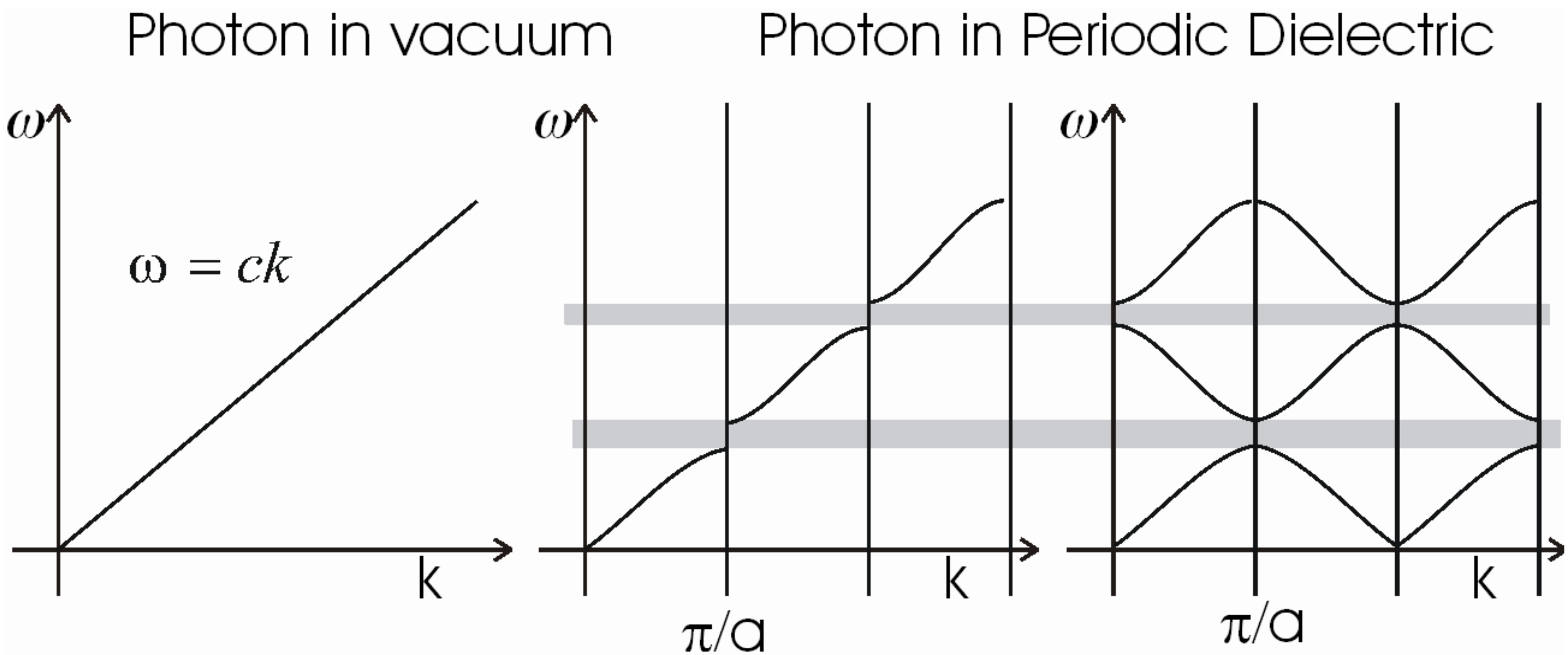
$$t_{12} = \left(\frac{2n_1}{n_1 + n_2}\right)$$



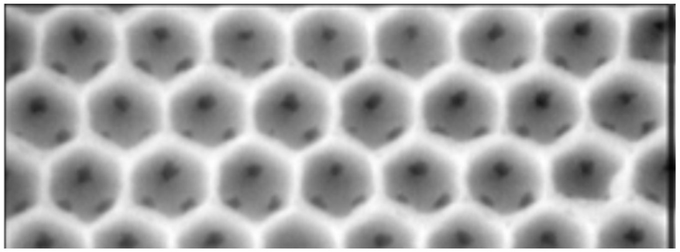
Effect of dielectric periodicity on photons

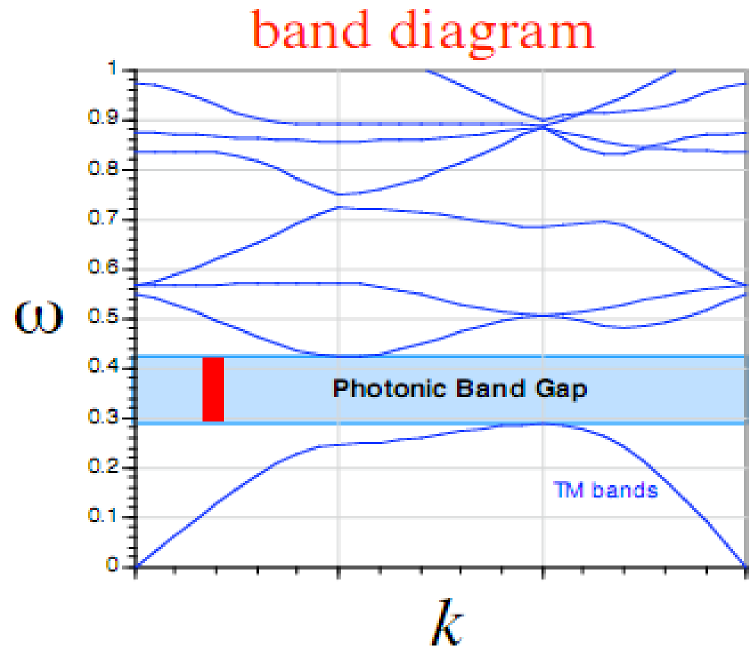


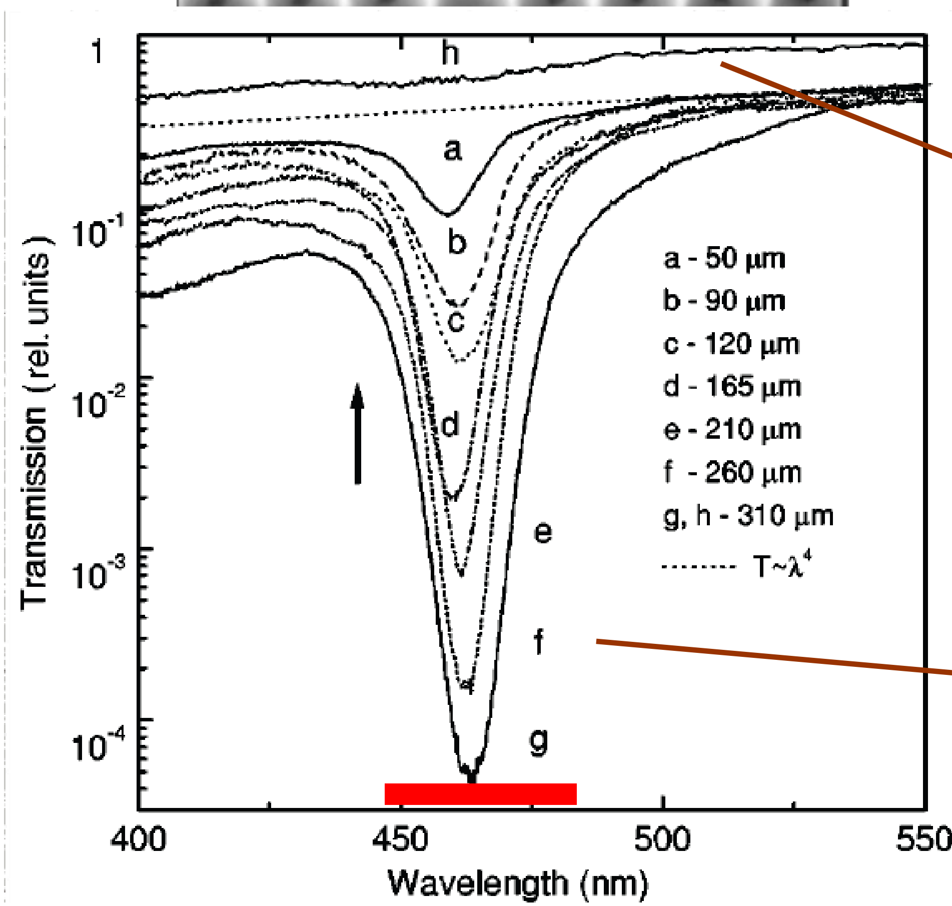
Source: http://ab-initio.mit.edu/photons/tutorial/



Inverse Opals (FCC Lattice)







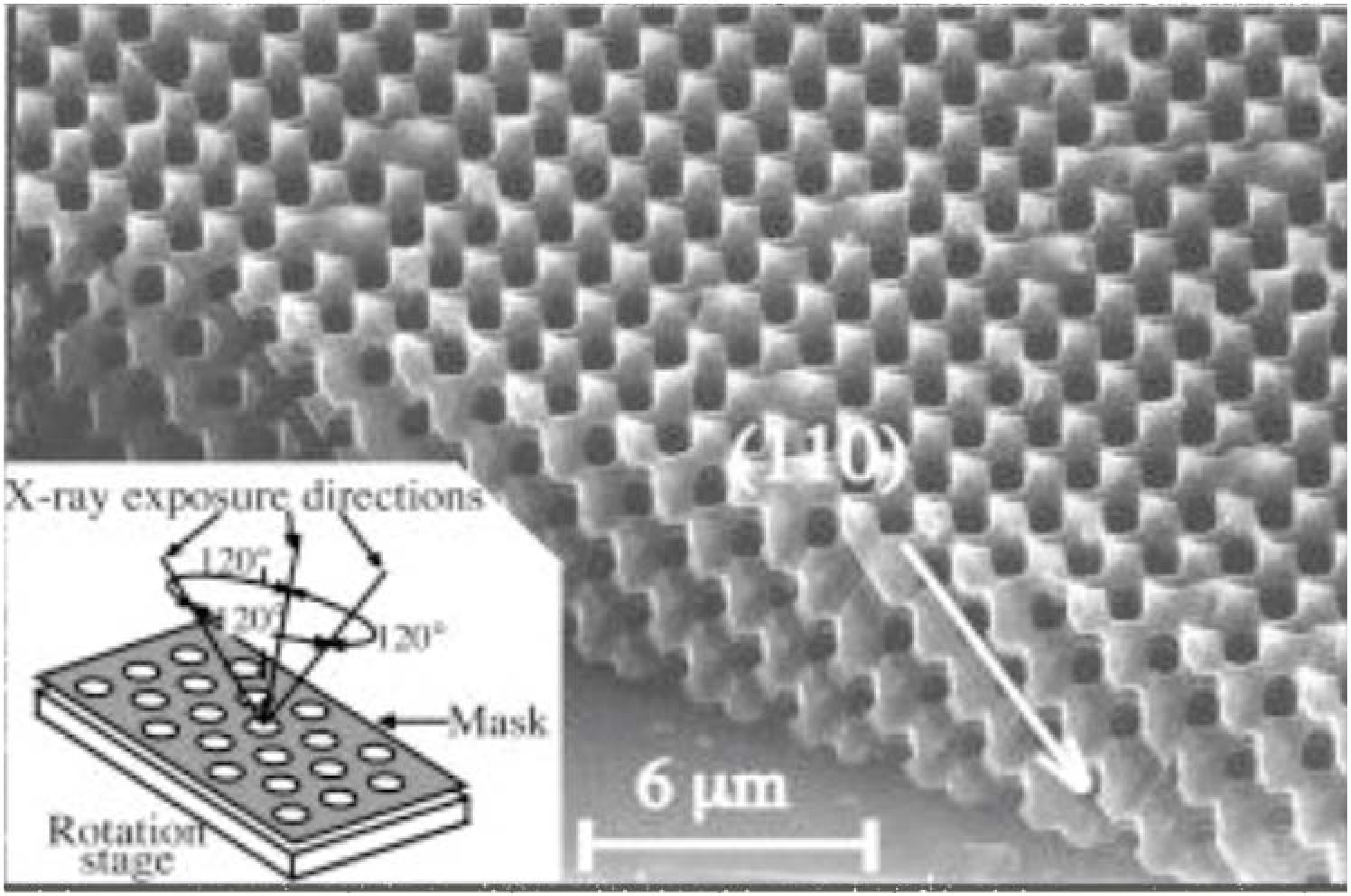
Index matched solution

Vlasov et al. PRB 60, 1555 (1999).

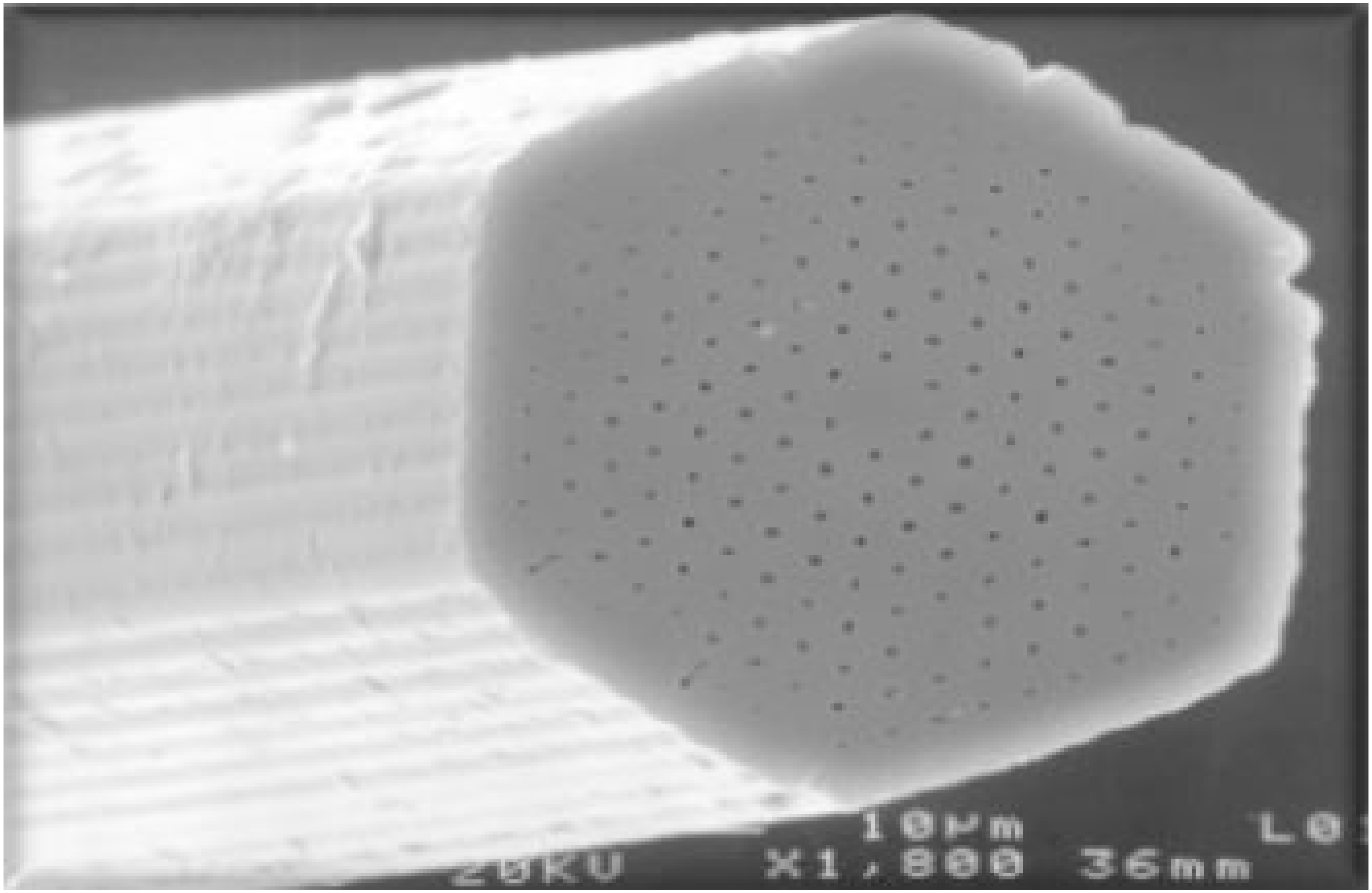
Observing a PBG (photonic bandgap)

“Yablonovite”

Concept: Yablonovitch et al. Phys. Rev. Lett. 67, 2295 (1991).



Photonic Crystal (Holey) Fibers



Origin of *photonic* bandgaps

Classical EM Wave Equation from Maxwell equations:

$$\frac{1}{\varepsilon(\mathbf{r})}\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) = \frac{w^2}{c^2}\mathbf{E}(\mathbf{r})$$

$$\mathcal{L}\,\mathbf{E}(\mathbf{r}) = \frac{w^2}{c^2}\mathbf{E}(\mathbf{r})$$

If eigen-operator is periodic, e.g., via $\varepsilon(x)$:

$$\mathbf{E}(x) = \mathbf{E}_k(x)e^{ikx}$$

$$\mathbf{E}_k(x+a) = \mathbf{E}_k(x)$$

➡ Exploit Bloch's Theorem

Photon Green Function & local density of states (LDOS)

Photon Green function (dipole response)

$$\left[\nabla \times \nabla \times - \varepsilon(\mathbf{r})\frac{\omega^2}{c^2}\right] \mathbf{G}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{w^2}{c^2}\delta(\mathbf{r} - \mathbf{r}')$$

Projected LDOS:

$$\rho_i(\mathbf{r}) \propto \mathrm{Im}[\mathbf{G}_{ii}(\mathbf{r}, \mathbf{r}, \omega)]$$

Scattering problem:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\mathrm{h}}(\mathbf{r}) + \int d\mathbf{r}'\, \mathbf{G}^{\mathrm{h}}(\mathbf{r}, \mathbf{r}'; \omega)\Delta\varepsilon(\mathbf{r}')\mathbf{E}(\mathbf{r}')$$

Photon G and mode expansions

- Analytically from the completeness of PC modes

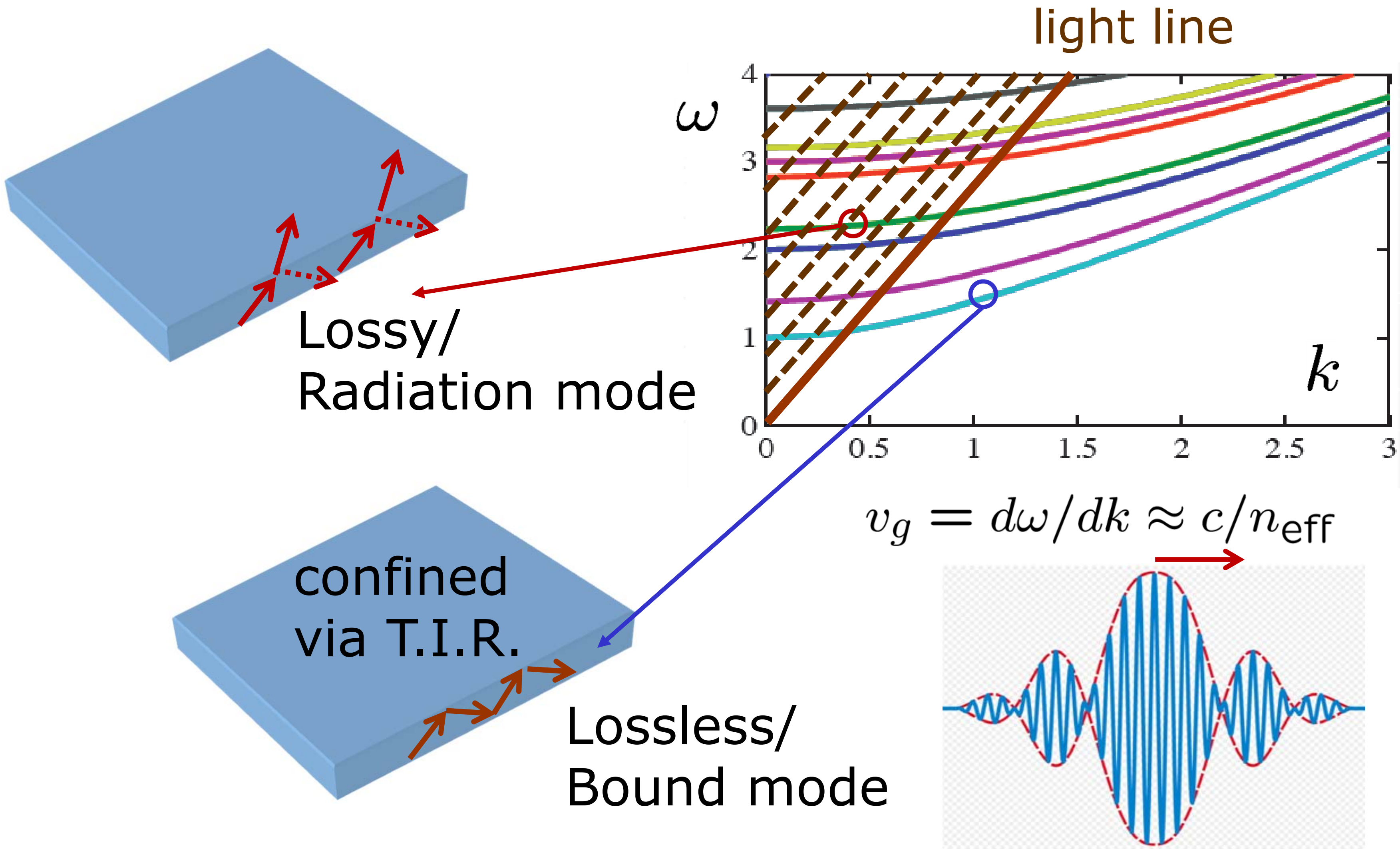
$$\mathbf{G}(\mathbf{r},\mathbf{r}';\omega) = \sum_k \left[\frac{w^2\mathbf{E}_k^T(\mathbf{r})[\mathbf{E}_k^T(\mathbf{r}')]^*}{\omega^2-\omega_k^2} - \mathbf{E}_k^L(\mathbf{r})[\mathbf{E}_k^L(\mathbf{r}')]^*\right]$$

$$\delta(\mathbf{r}-\mathbf{r}') = \sum_k \left(\varepsilon(\mathbf{r})\mathbf{E}_k^T(\mathbf{r})[\mathbf{E}_k^T(\mathbf{r}')]^* + \varepsilon(\mathbf{r})\mathbf{E}_k^L(\mathbf{r})[\mathbf{E}_k^L(\mathbf{r}')]^*\right)$$

- Or direct solution – dipole in numerically-exact Maxwell solver (e.g. FDTD)

$$\left[\nabla\times\nabla\times - \varepsilon(\mathbf{r})\frac{\omega^2}{c^2}\right]\mathbf{G}(\mathbf{r},\mathbf{r}';\omega) = \frac{w^2}{c^2}\delta(\mathbf{r}-\mathbf{r}')$$

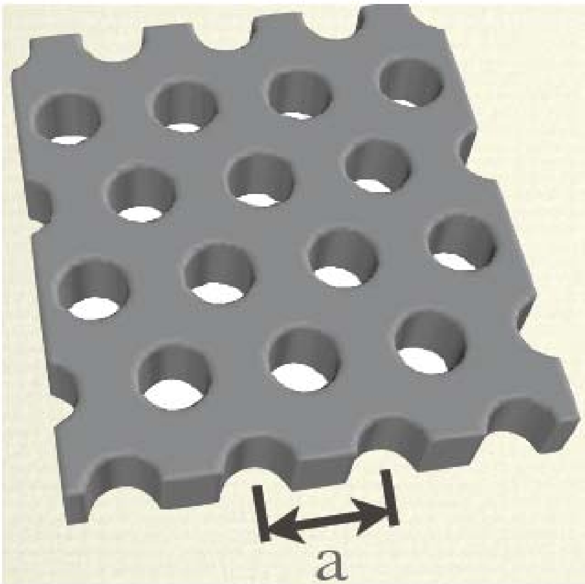
Band structure for semiconductor slab

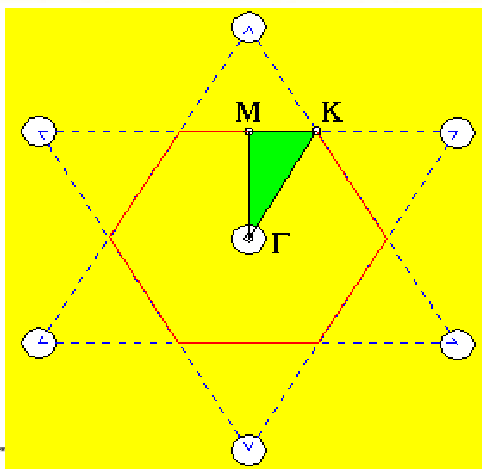


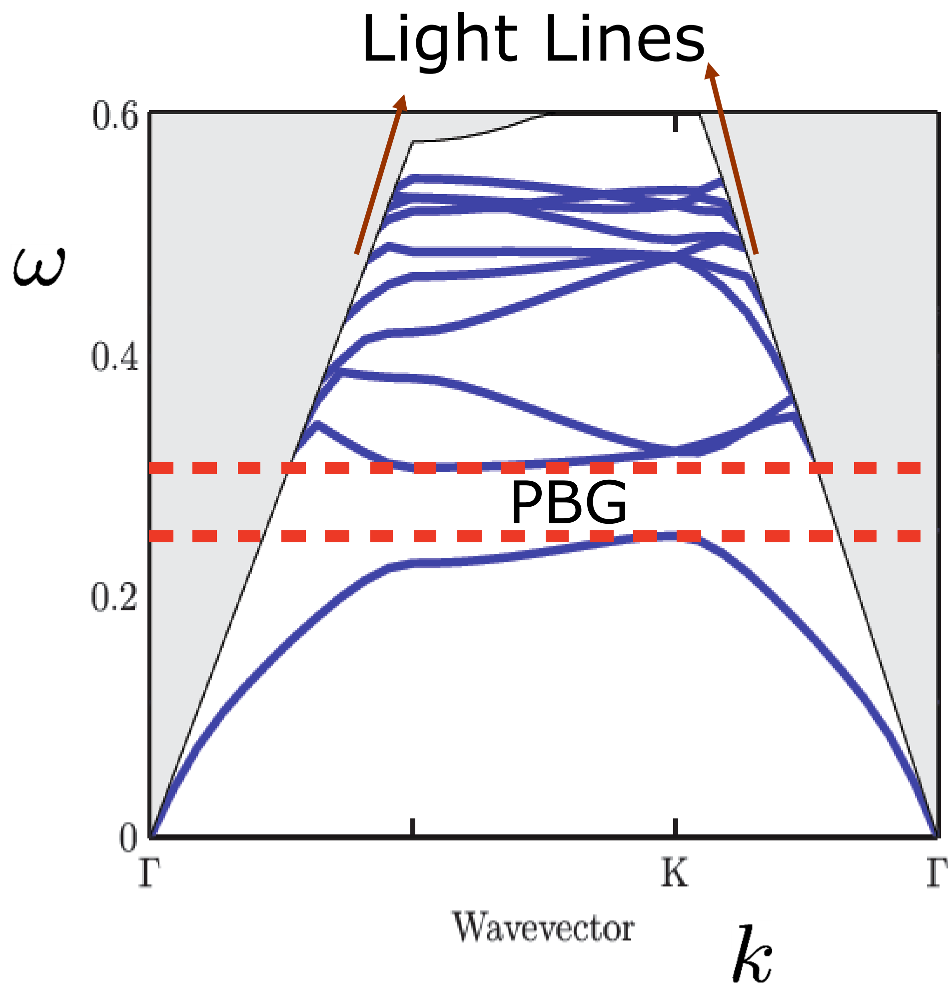
$$v_g = d\omega/dk \approx c/n_{\text{eff}}$$

Planar (slab) PC band structure

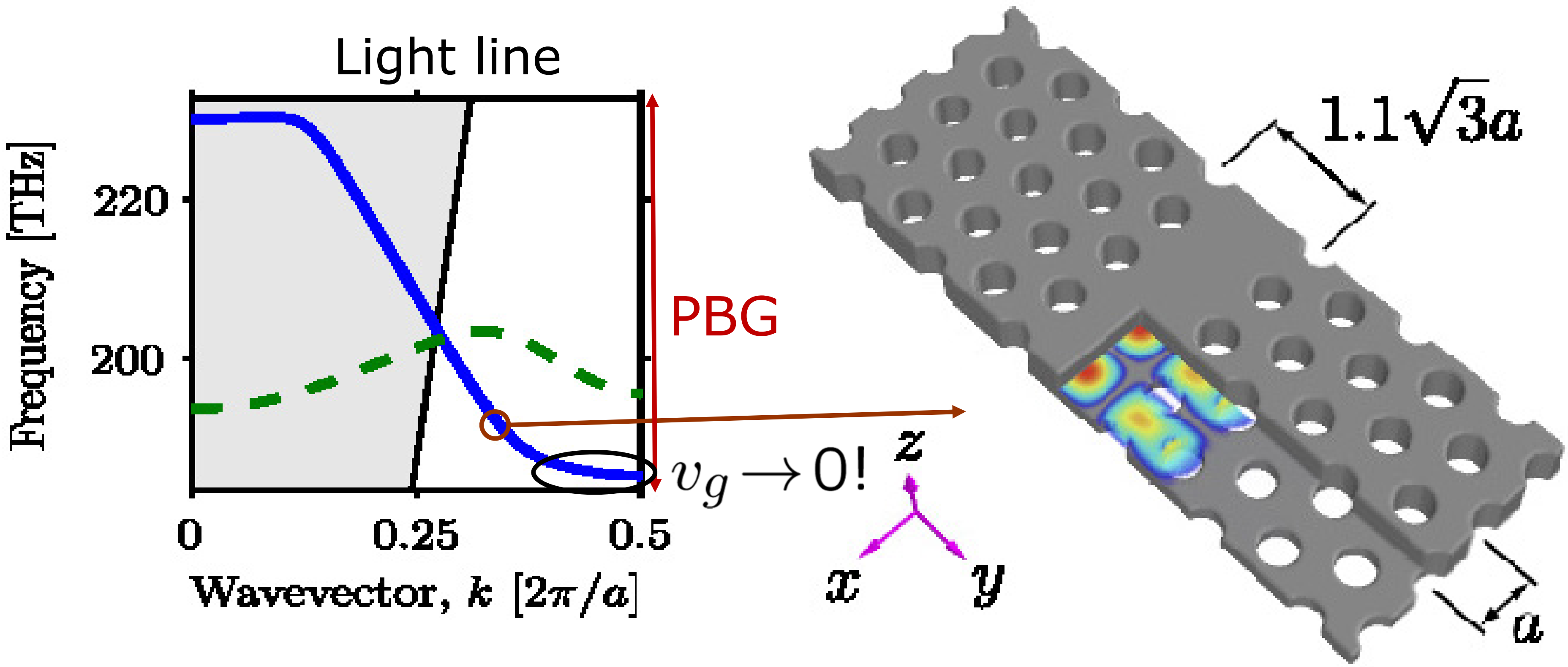
PC slabs:







Planar PC waveguide - Bloch Modes

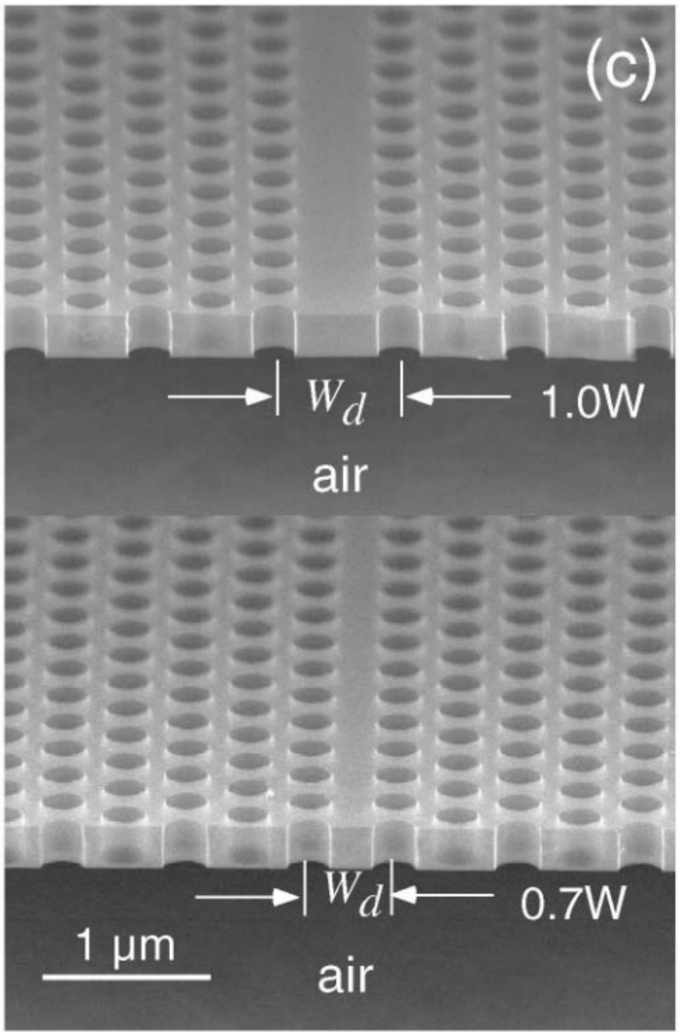


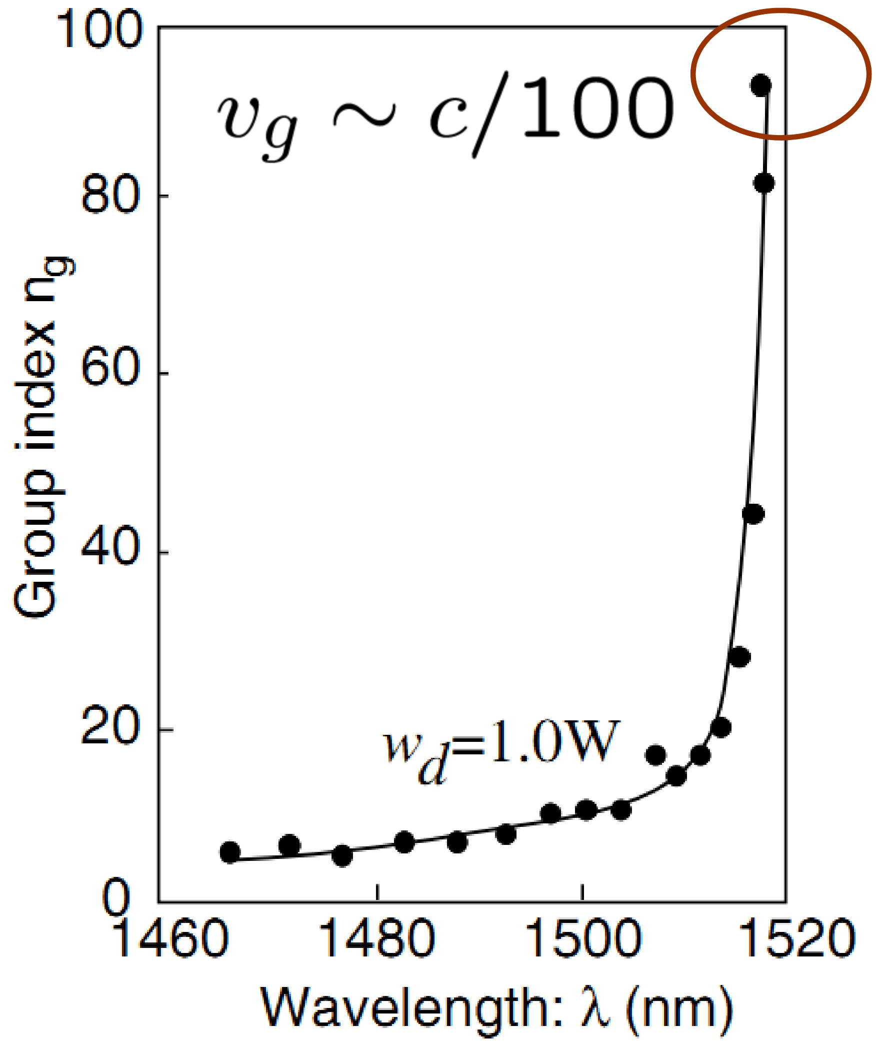
Bloch's Theorem

$$\mathbf{E}(x) = \mathbf{E}_k(x)e^{ikx}$$
$$\mathbf{E}_k(x+a) = \mathbf{E}_k(x)$$

Early experiments (NTT Group, Japan)

"W1" and "W0.7" guides





Notomi et al, PRL 87, 253902 (2001)

Why slow light?

Figure 1 Faster is not always better, as the fable of the hare and the tortoise demonstrates. The same is true in photonics, where slow light can outperform fast light in many aspects.

- Fundamentally interesting
- Local field enhancements: better optical sensing, better nonlinear optics, better quantum optics, …

$$E_{\text{loc}} \sim 1/v_g$$

- Optical buffering

Source: T. Krauss, Nature Photonics 2, 448 (2008)

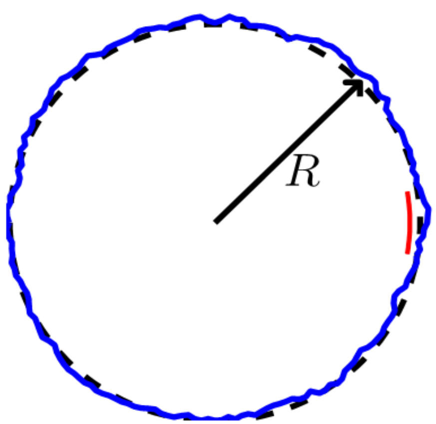
Outline

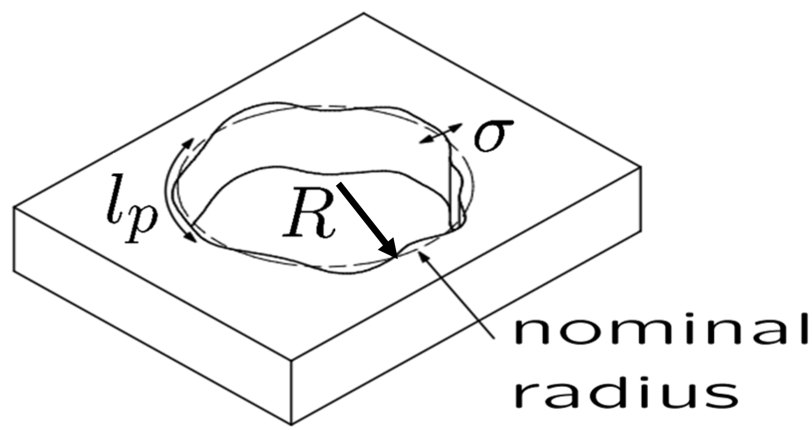
- Introduction to photon crystals (PCs) and slow-light PC waveguides

- **Theory of disorder-induced scattering – the intrinsic "disorder problem"**

The "Disorder Problem"

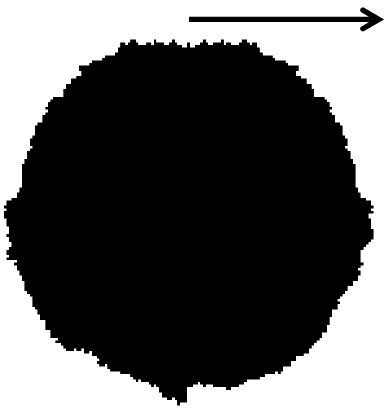
- What is the effect of naturally disordered holes (i.e., *unavoidable* fabrication imperfections) ?
- Theory must describe scattering from very small dielectric (nm-size) bumps.





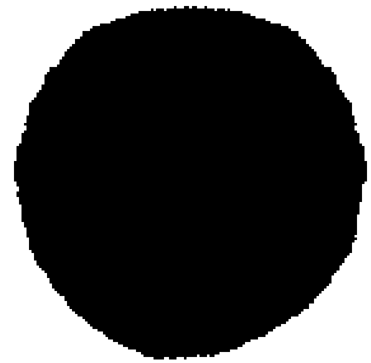
Numerically Generated Disordered Holes

$R \sim 100\,\mathrm{nm}$



$\sigma = 2\,\mathrm{nm}$

$l_{\mathrm{corr}} = 40\,\mathrm{nm}$



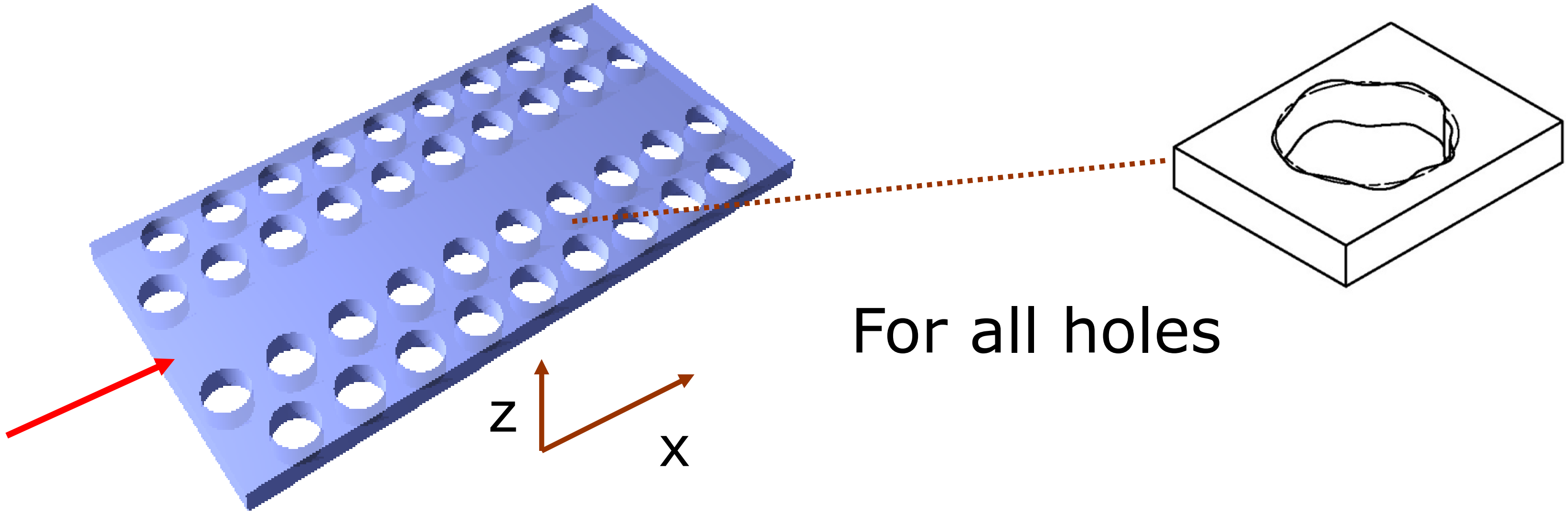
$\sigma = 2\,\mathrm{nm}$

$l_{\mathrm{corr}} = 100\,\mathrm{nm}$

$$\langle \delta R(\mathbf{r}) \delta R(\mathbf{r}') \rangle = \sigma^2 e^{-R|\phi - \phi'|/l_{\mathrm{corr}}} \to \Delta\varepsilon(\mathbf{r}) \Delta\varepsilon(\mathbf{r}')$$

$\Delta\varepsilon(\mathbf{r}) \longrightarrow$ Disordered dielectric constant from *fabrication imperfections*

Disorder-induced scattering model



incident — scattered

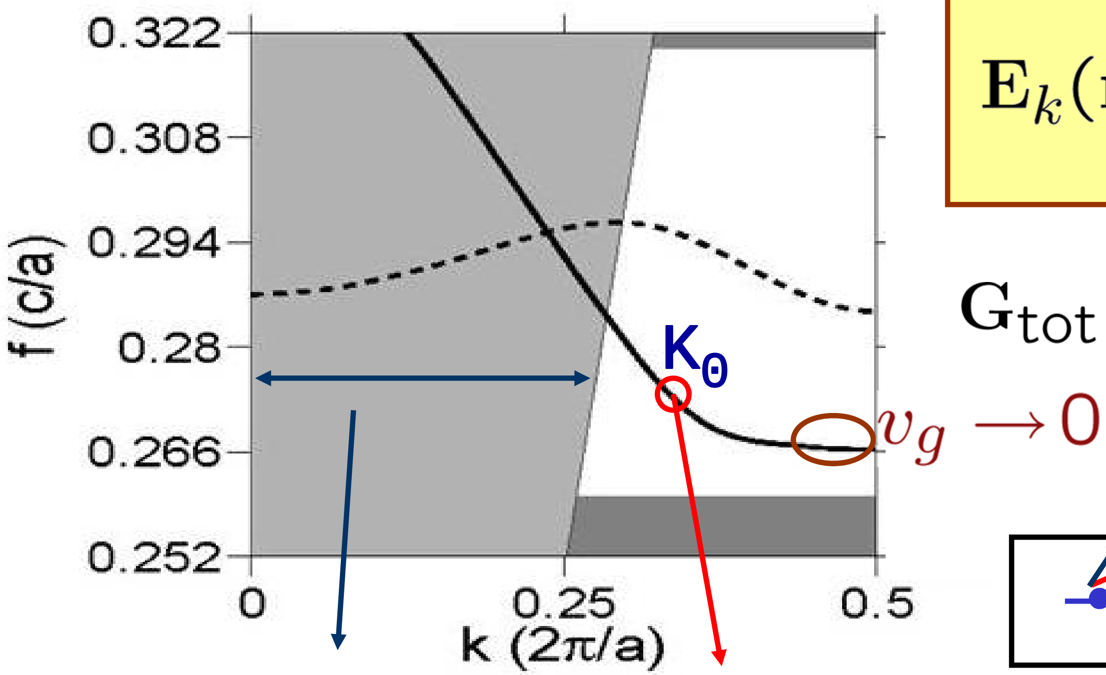
$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^0(\mathbf{r}) + \int d\mathbf{r}'\, \mathbf{G}^0(\mathbf{r},\mathbf{r}';\omega)\varepsilon_{\text{dis}}(\mathbf{r}')\mathbf{E}(\mathbf{r}')$$

$\mathbf{E}^0$ → lossless waveguide mode

$\mathbf{G}^0$ → Green function of ideal PC

$\varepsilon_{\text{dis}} = \Delta\varepsilon$ → Disorder

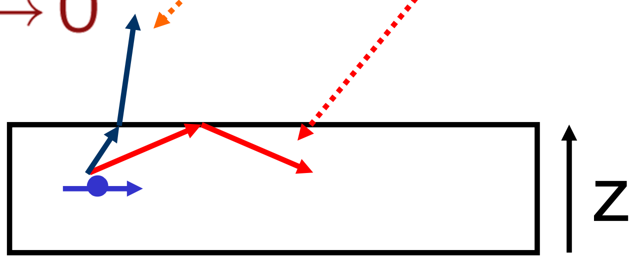
Green-function mode decomposition



$$\mathbf{E}_k(\mathbf{r}) = \sqrt{\frac{a}{L}}\,\mathbf{f}_k(\mathbf{r})e^{ikx}$$

$$v_g \rightarrow 0$$

$$\mathbf{G}_{\text{tot}} = \mathbf{G}_{\text{rad}} + \mathbf{G}_{\text{bound}}$$



$$\mathbf{G}_{\text{bound}}(\mathbf{r},\mathbf{r}';\omega) = \frac{ia\omega}{2v_g}\Big[H(x-x')\,\mathbf{f}_{k_0}(\mathbf{r})\mathbf{f}^*_{k_0}(\mathbf{r}')e^{ik_0(x-x')} + H(x'-x)\,\mathbf{f}_{-k_0}(\mathbf{r})\mathbf{f}^*_{-k_0}(\mathbf{r}')e^{-ik_0(x-x')}\Big]$$

Obtaining the waveguide Green function

With $\int_{\text{cell}} d\mathbf{r}\varepsilon(\mathbf{r})|\mathbf{f}_k(\mathbf{r})|^2 = 1$

$$\mathbf{G}_{\text{bound}}(\mathbf{r},\mathbf{r}';\omega) = \frac{a}{L}\sum_k \left[\frac{w^2\mathbf{f}^*_{k_\omega}(\mathbf{r})\mathbf{f}^*_{k_\omega}(\mathbf{r}')e^{ik_\omega(x-x')}}{\omega^2-\omega_k^2}\right]$$

$$= \frac{a}{L}\frac{L}{2\pi}\int dk \left[\frac{w^2\mathbf{f}^*_{k_\omega}(\mathbf{r})\mathbf{f}^*_{k_\omega}(\mathbf{r}')e^{ik_\omega(x-x')}}{\omega^2-(\omega_k+i\delta)^2}\right]$$

$$= \frac{ia\omega}{2v_g}\left[H(x-x')\,\mathbf{f}_{k_\omega}(\mathbf{r})\mathbf{f}^*_{k_\omega}(\mathbf{r}')e^{ik_\omega(x-x')}\right.$$

$$\left.+\,H(x'-x)\,\mathbf{f}^*_{k_\omega}(\mathbf{r})\mathbf{f}_{k_\omega}(\mathbf{r}')e^{-ik_\omega(x-x')}\right]$$

Using use complex pole integration with $d\omega/dk = v_g$

Perturbation theory shows that backloss dominates disorder-induced scattering

SH, Ramunno, Young, Sipe, PRL 94, 33903 (2005)

Incoherent backscatter loss (thin-sample approximation):

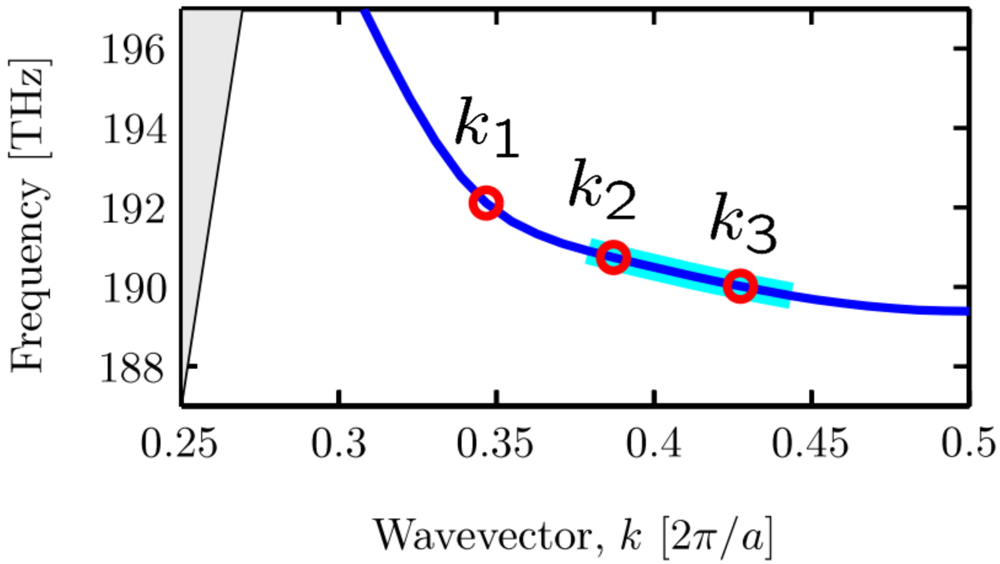
$$\langle \alpha_{\text{back}}(\omega) \rangle = \frac{a^2 \omega^2}{4 v_g^2} \int \int \langle \Delta\varepsilon(\mathbf{r}) \Delta\varepsilon(\mathbf{r}') \rangle [\mathbf{e}_k(\mathbf{r}) \cdot \mathbf{e}_k(\mathbf{r})] \times [\mathbf{e}_k^*(\mathbf{r}') \cdot \mathbf{e}_k^*(\mathbf{r}')] e^{2ik(x-x')} d\mathbf{r} d\mathbf{r}',$$

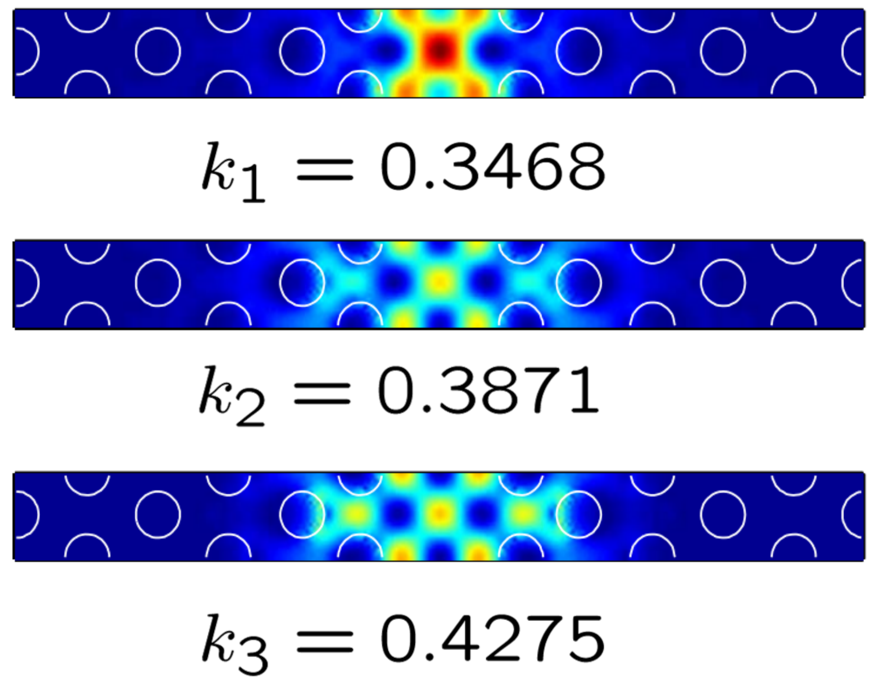
$$\propto \frac{1}{v_g^2} Fun(v_g)$$

See also, Povinelli et al, APL (2005);Gerace and Andreani, OL (2005); Song et al PRL (2010)

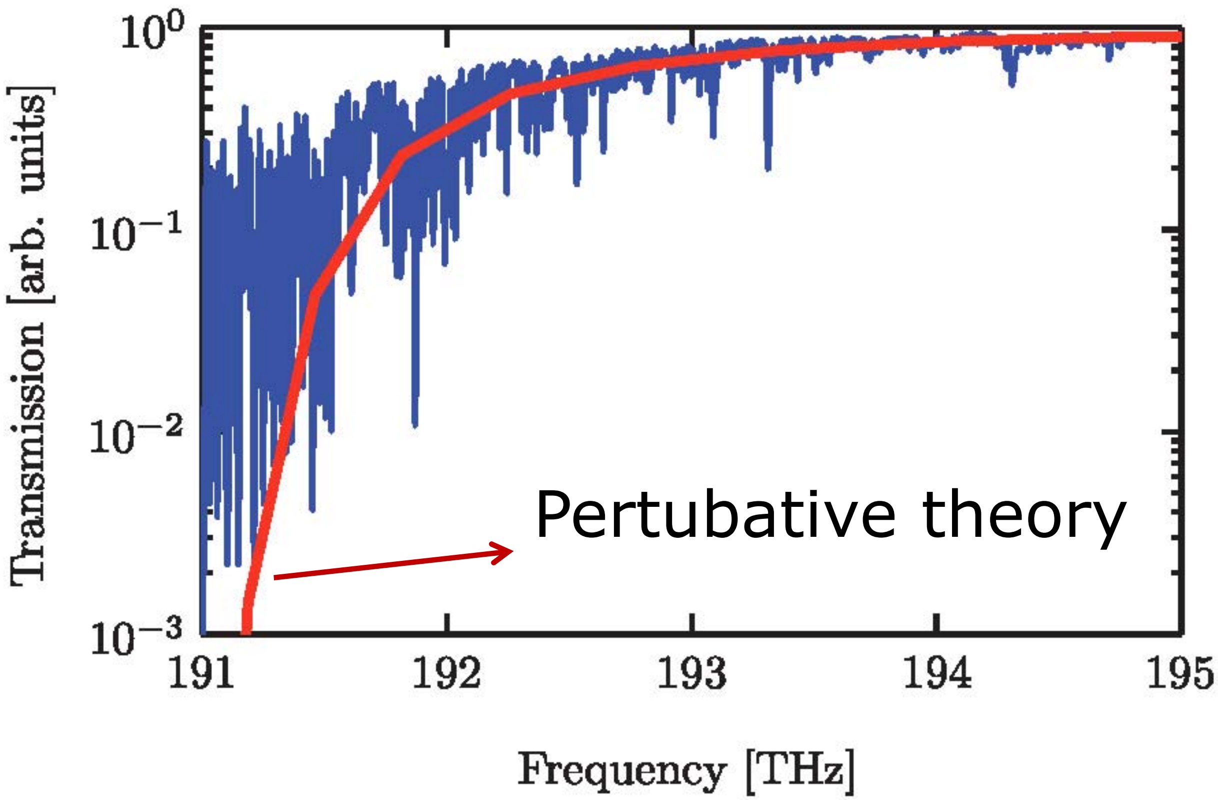
Bloch modes also change with frequency

M. Patterson, SH, S. Schulz, D. M. Beggs, T. P. White, L. O'Faolain, and T. F. Krauss, PRB (2009)



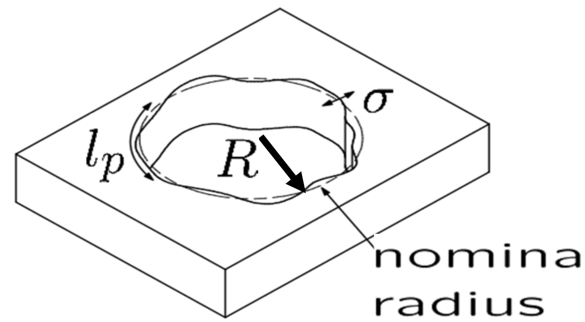


Experiment vs Perturbation Theory



$l_{\mathrm{corr}} = 40\,\mathrm{nm}$

$\sigma = 2\,\mathrm{nm}$

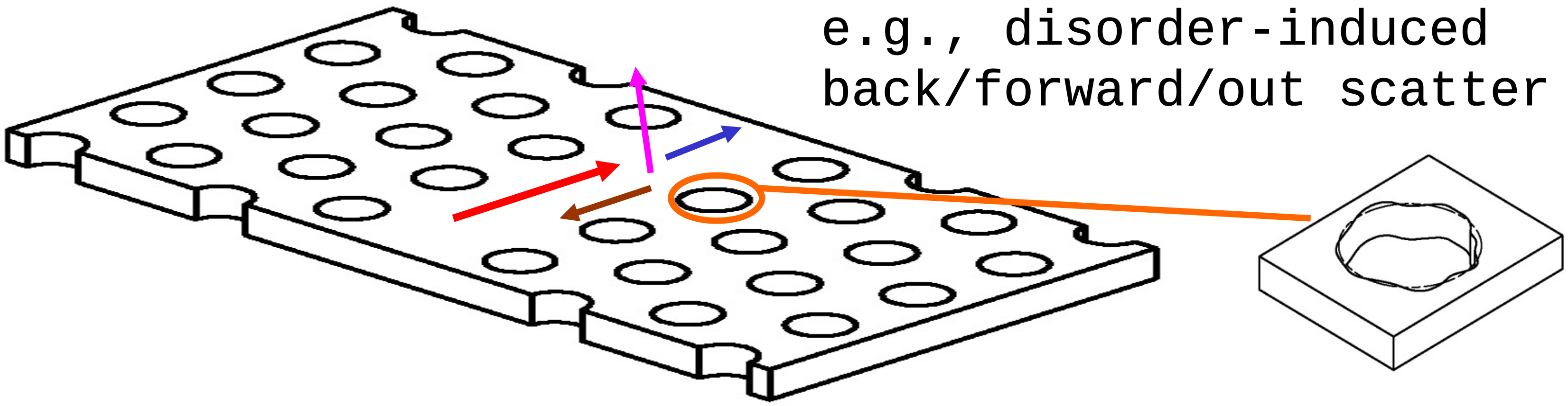


1mm "W1" GaAs, data from Thales Group, France

Multiple Scattering Theory

forward backward

$$\mathbf{E}(\mathbf{r},\omega) = E_0 \left[\mathbf{e}_k(\mathbf{r})e^{ikx}\psi_\mathsf{f}(x) + \mathbf{e}_{-k}(\mathbf{r})e^{-ikx}\psi_\mathsf{b}(x)\right] + \cdots$$



e.g., disorder-induced back/forward/out scatter

Forward (f) and backward (b) Bloch mode amplitudes:

forward-forward, forward-back, forward-out

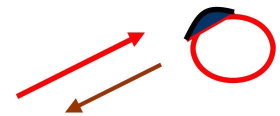
$$v_g \frac{d\psi_\mathsf{f}(x)}{dx} = ic_\mathsf{ff}(x)\psi_\mathsf{f}(x) + ic_\mathsf{fb}(x)e^{-i2kx}\psi_\mathsf{b}(x) + ic_\mathsf{fr}(x)\psi_\mathsf{f}(x)$$

$$-v_g \frac{d\psi_\mathsf{b}(x)}{dx} = ic_\mathsf{bb}(x)\psi_\mathsf{b}(x) + ic_\mathsf{bf}(x)e^{i2kx}\psi_\mathsf{f}(x) + ic_\mathsf{br}(x)\psi_\mathsf{b}(x)$$

Coupling coefficients/disorder statistics

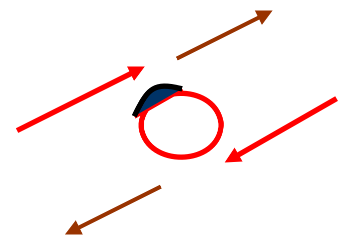
Forward-to-backward coupling:

$$c_{\text{fb}}(x) \;=\; \frac{\omega a}{2}\int \mathbf{e}_k^*(\mathbf{r})\cdot\mathbf{e}_k^*(\mathbf{r})\Delta\varepsilon(\mathbf{r})\,dydz$$



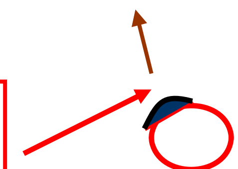
Forward-forward or back-back :

$$c_{\text{ff}}(x) \;=\; c_{\text{bb}}(x) = \frac{\omega a}{2}\int \mathbf{e}_k^*(\mathbf{r})\cdot\mathbf{e}_k(\mathbf{r})\Delta\varepsilon(\mathbf{r})\,dydz$$



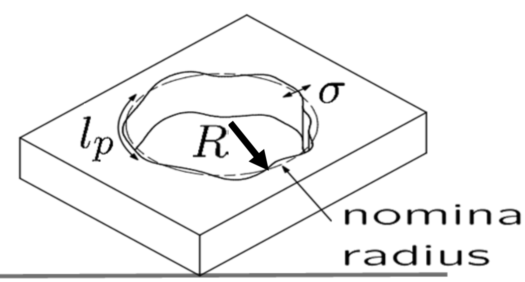
Radiation/out scatter (above light line):

$$c_{\text{fr}}(x) = \frac{\omega a}{2}\int \Delta\varepsilon(\mathbf{r})\mathbf{e}_k^*(\mathbf{r})\cdot\mathbf{G}_{\text{rad}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{e}_k(\mathbf{r}')\Delta\varepsilon(\mathbf{r}')\,dydzd\mathbf{r}'$$



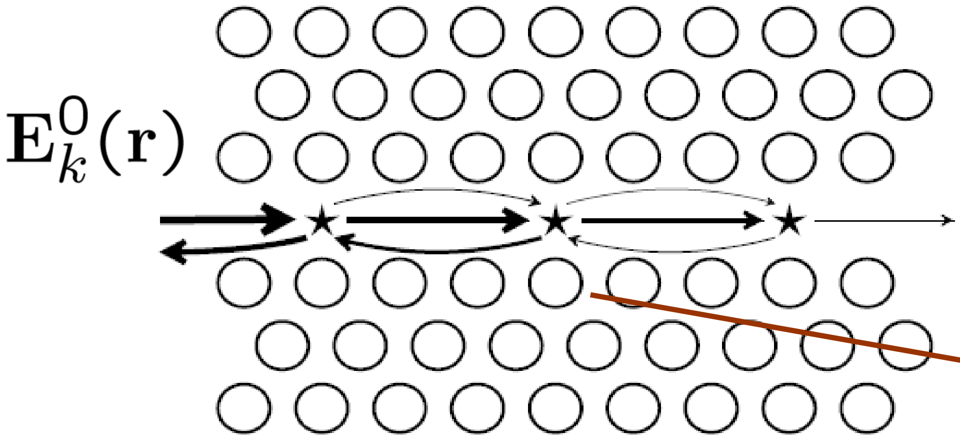
Disorder Function/Statistics:

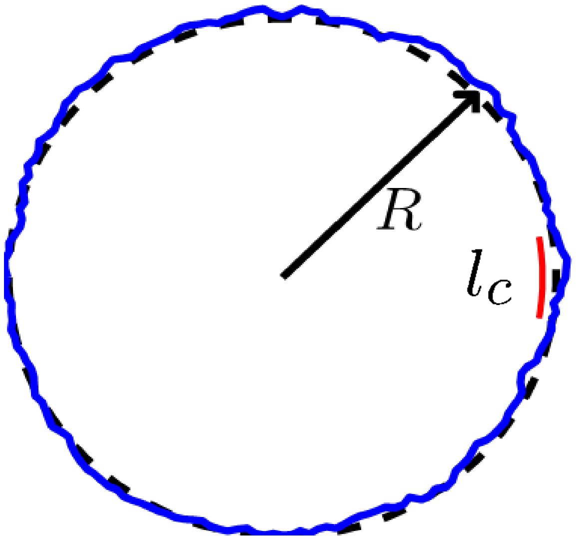
$$\langle\delta R(\mathbf{r})\delta R(\mathbf{r}')\rangle = \sigma^2 e^{-R|\phi-\phi'|/l_p} \rightarrow \Delta\varepsilon(\mathbf{r})\Delta\varepsilon(\mathbf{r}')$$



Numerical Complexities -- work done by Mark Patterson

Self-consistently solve coupled-mode Eqs. over **thousands of disordered unit cells**



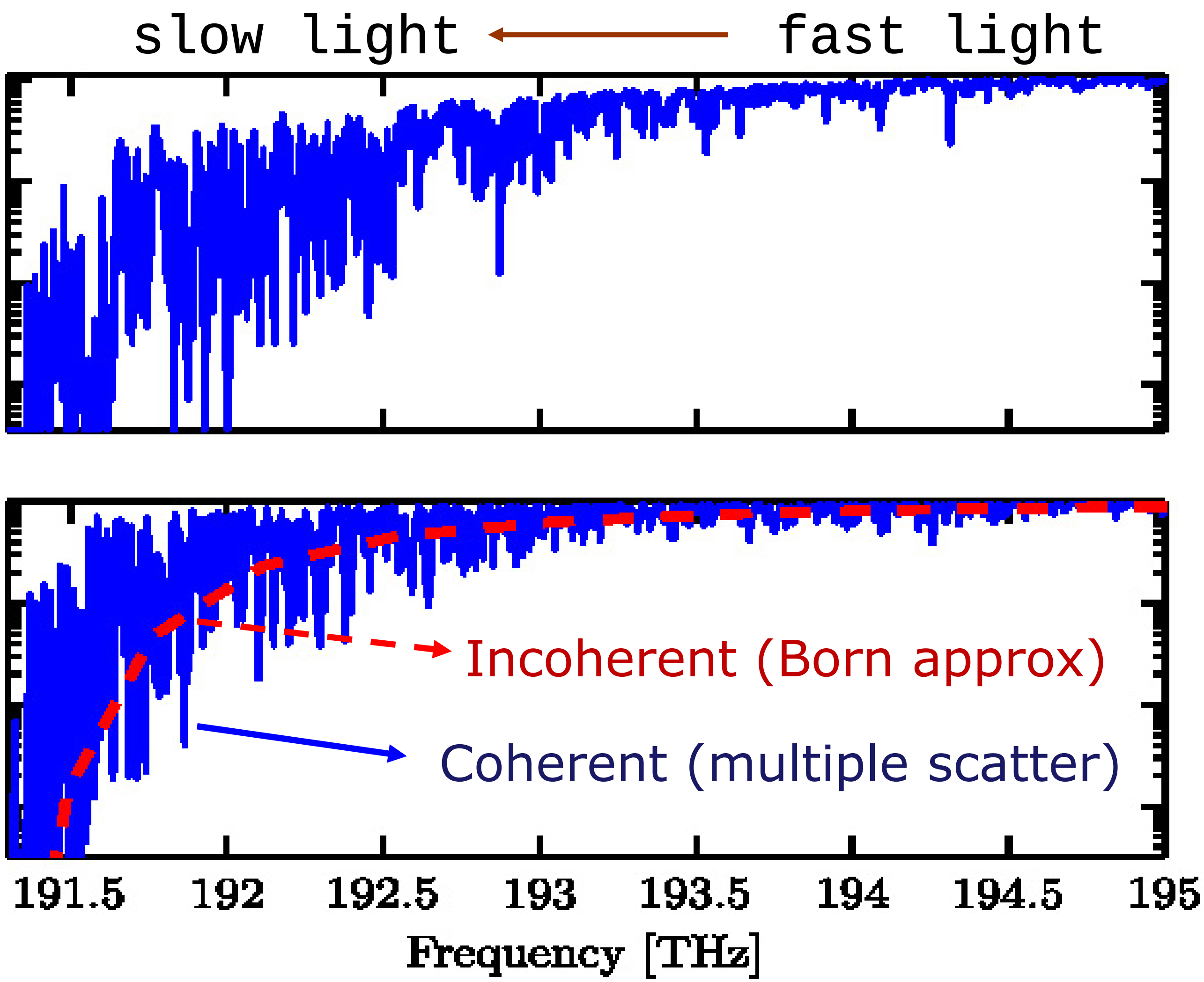


Requires **at least 10 points per unit cell., fully 3D**

See Patterson and Hughes, J. Opt. 12, 104013 (2010)
Patterson et al. PRL 102, 253903 (2009)

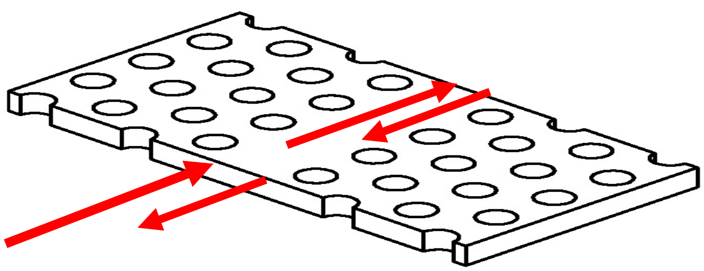
Transmission through a 1mm GaAs PC (L=2400a)

☺ Q: Which one is theory (no fitting parameters)?

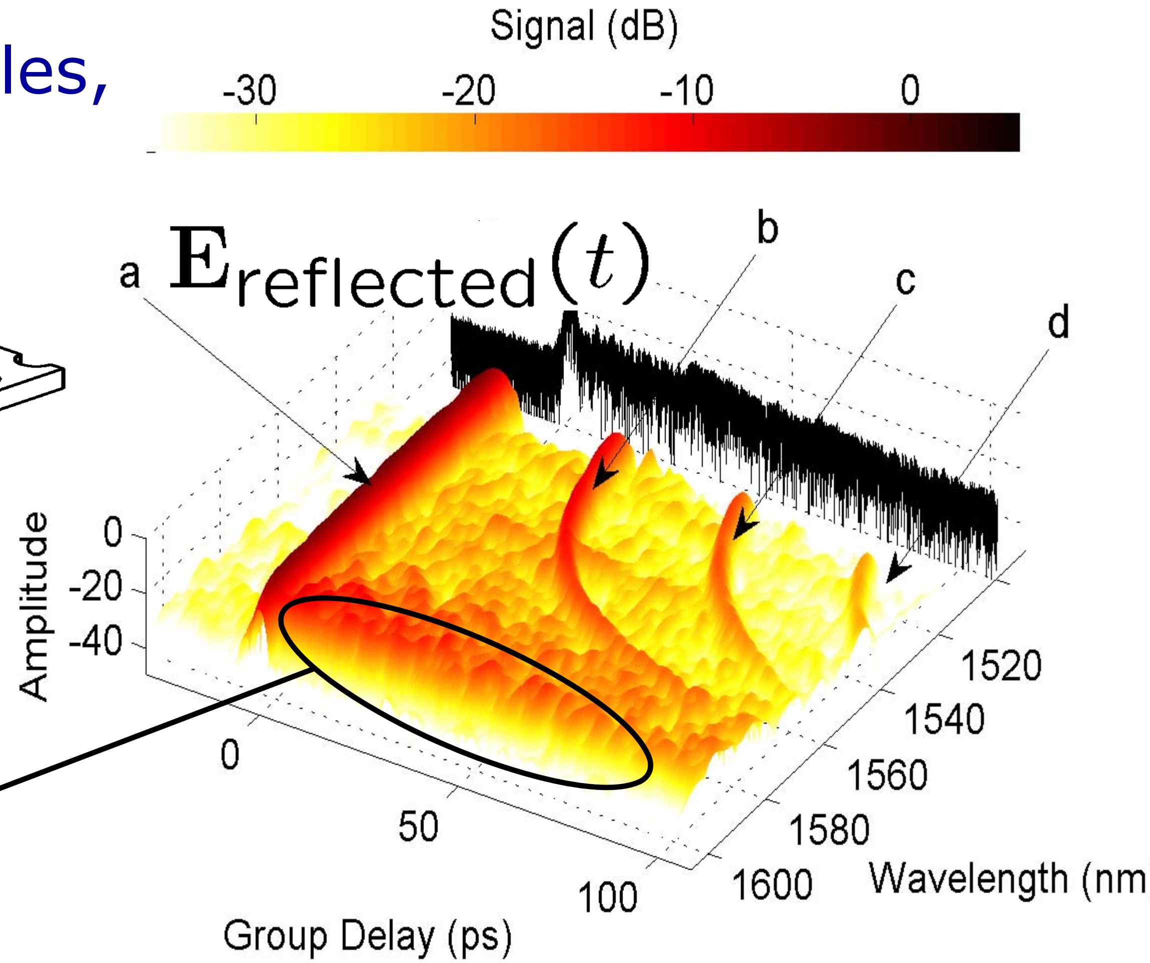


Optical Low-Coherence Reflectometry (OLCR)

De Rossi et al., Thales, **OLCR:**



Interferometric pulse reflection technique: uses broadband source and translation mirror

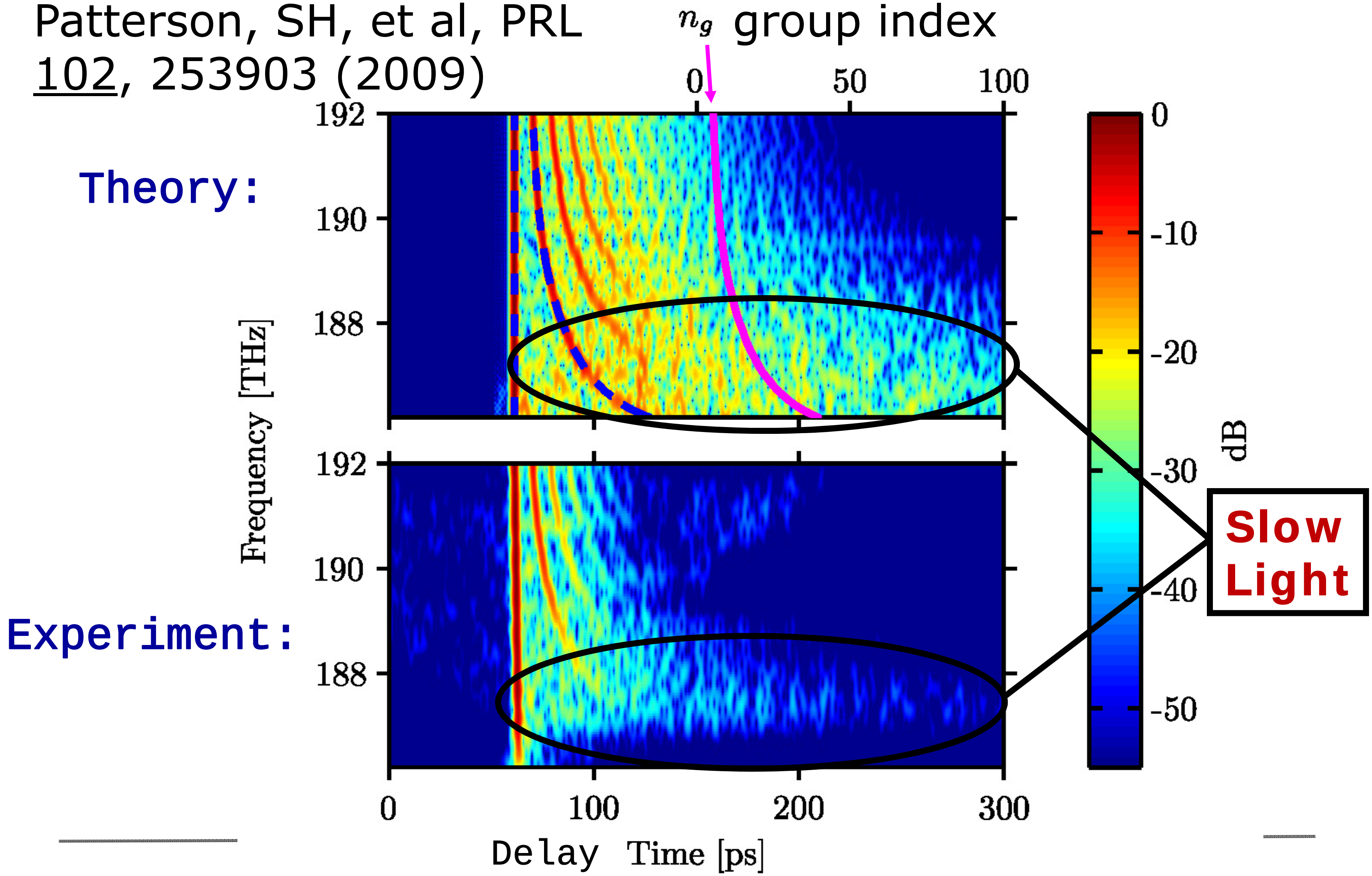


Backscatter

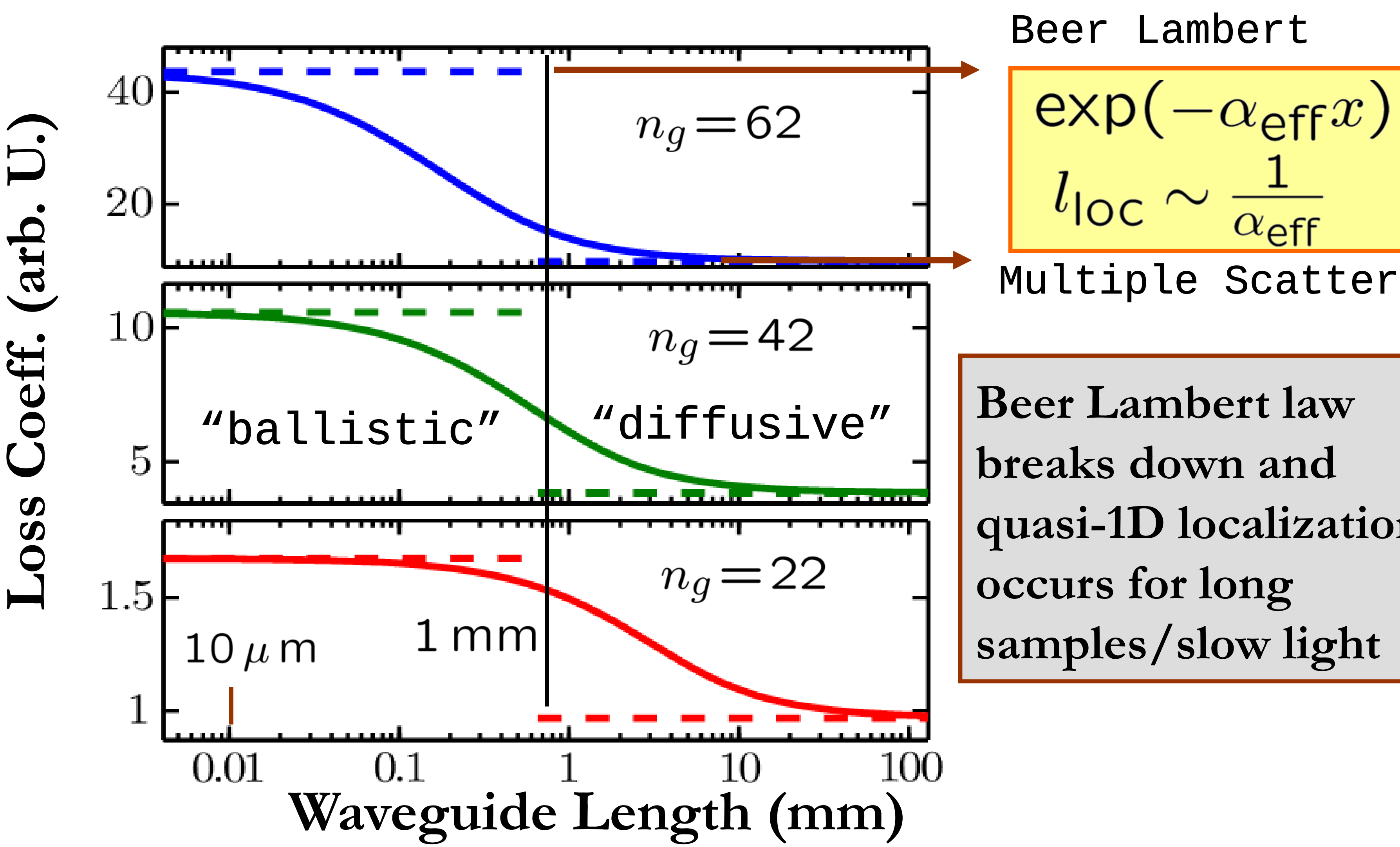
a: reflection at the input facet, b- d: signal after 1,2,3rd round

Theory & Experiment

Patterson, SH, et al, PRL 102, 253903 (2009)



Impact on ensemble averaged transmission



Beer Lambert law breaks down and quasi-1D localization occurs for long samples/slow light

Patterson et al, PRB 80, 195305, 2009

Summary

- Semiconductor PC waveguides can trap and slow down light over thousands of unit cells.
- However, natural disorder in slow-light PC waveguides results in non-perturbative scattering, localization, and fascinating optical physics.
- Physical models of disorder scattering can be constructed using Bloch mode theories and Green functions, allowing one to carry our efficient simulations and gain essential physical insight.
- Disorder is part of the photonic structures – and must be included in a realistic light-matter theory.