Introduction to squeezed states of light

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Based on what we (Alain Aspect, Philippe Grangier, Michel Brune, Antoine Browaeys, and me) teach to master students in Ecole Polytechnique

Outline

- Single-mode quantized field
- Quasi-classical state
- Quadrature operators
- Homodyne detection
- Generation of a squeezed vacuum
- More general squeezed state
- Experimental illustration
- Reduction of quantum noise in a measurement

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Quantized field

- Classical monochromatic e-m field:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos\left(\omega t - \mathbf{k}\cdot\mathbf{r} + \varphi\right)$$

- Quantized free field (Schrödinger pic.):

$$\hat{\mathbf{E}}(\mathbf{r}) = i\sum_{\ell} \mathcal{E}_\ell^{(1)} \vec{\varepsilon}_\ell \left(\hat{a}_\ell e^{i\mathbf{k}_\ell\cdot\mathbf{r}} - \hat{a}_\ell^\dagger e^{-i\mathbf{k}_\ell\cdot\mathbf{r}} \right)$$

with $\left[\hat{a}_\ell, \hat{a}_{\ell'}^\dagger\right] = \delta_{\ell\ell'}$

and single-photon field $\mathcal{E}_\ell^{(1)} = \sqrt{\dfrac{\hbar\omega_\ell}{2\varepsilon_0 L^3}}$

The field states belong to a Hilbert space and evolve with Hamiltonian $\hat{H} = \sum_{\ell} \hbar\omega_\ell \left(\hat{a}_\ell^\dagger \hat{a}_\ell + \frac{1}{2} \right)$

Single-mode quantized field

- Consider only one mode:

$$\hat{\mathbf{E}}(\mathbf{r}) = i\,\mathcal{E}^{(1)}\vec{\varepsilon}\left(\hat{a}e^{i\mathbf{k}\cdot\mathbf{r}} - \hat{a}^{\dagger}e^{-i\mathbf{k}\cdot\mathbf{r}}\right)$$

- Introduce quadrature operators:

$$\hat{E}_P = \mathcal{E}^{(1)}\left(\hat{a}e^{i\omega t} + \hat{a}^{\dagger}e^{-i\omega t}\right)$$

$$\hat{E}_Q = -i\mathcal{E}^{(1)}\left(\hat{a}e^{i\omega t} - \hat{a}^{\dagger}e^{-i\omega t}\right)$$

Hermitian ➔ Observables

- From $\left[\hat{a},\hat{a}^{\dagger}\right] = 1$, we get:

$$\Delta E_P\,\Delta E_Q \geq \left[\mathcal{E}^{(1)}\right]^2$$

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Quasi-classical state (1)

- We have: $\Delta E_P \, \Delta E_Q \geq \left[\mathcal{E}^{(1)} \right]^2$
- Minimum uncertainty state: $\Delta E_P \, \Delta E_Q = \left[\mathcal{E}^{(1)} \right]^2$
- State with symmetric minimum uncertainty:

$$\Delta E_P = \Delta E_Q = \mathcal{E}^{(1)}$$

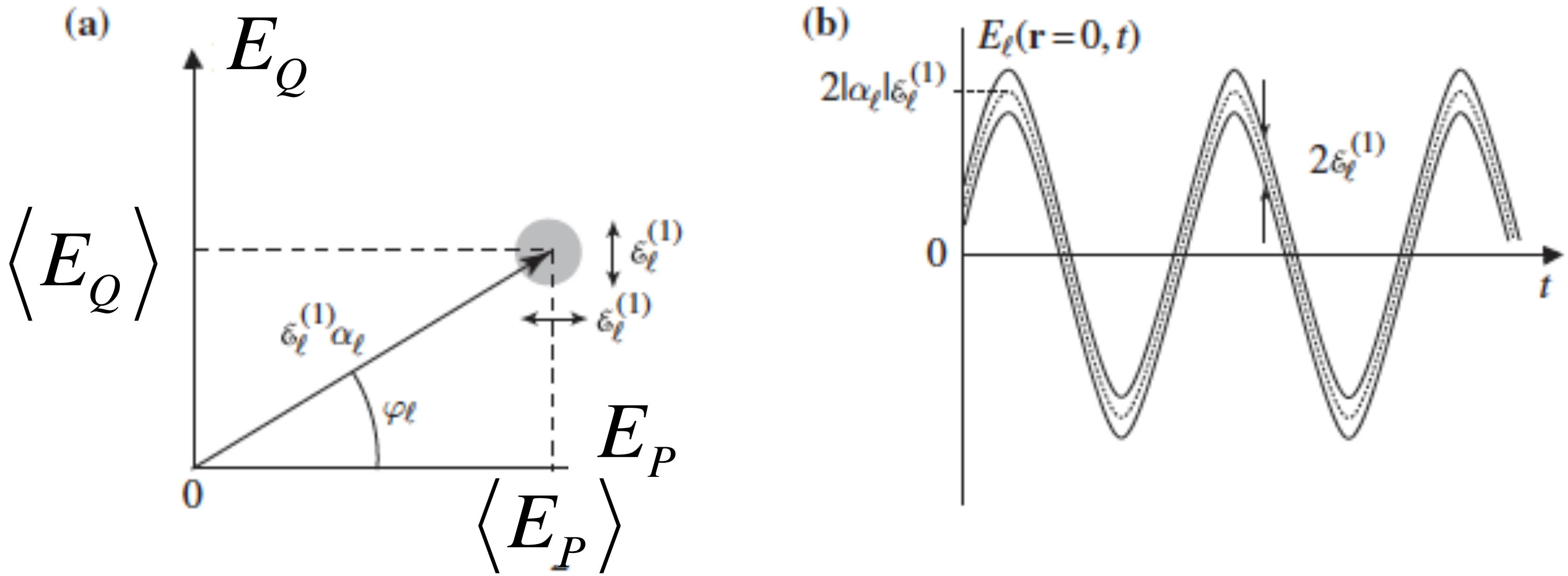
- Such state is called a quasi-classical, or coherent, or Glauber state:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad |\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\left(\hat{a}^\dagger\right)^n}{\sqrt{n!}} |0\rangle$$

Quasi-classical state (2)

- Coherent state:

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$



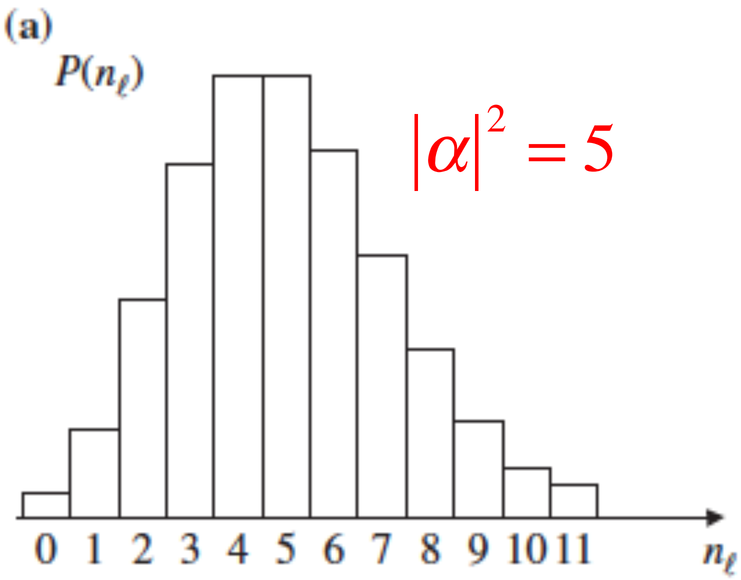
- The "most classical" quantum state of light

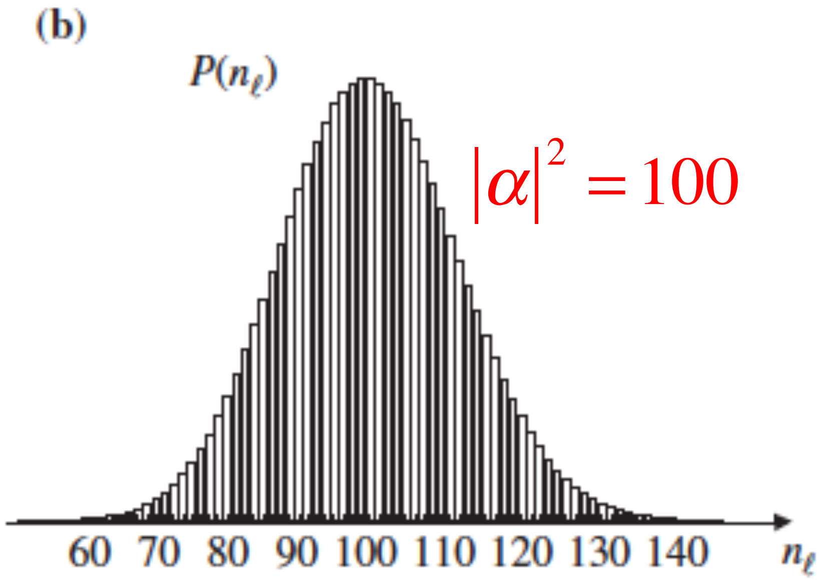
Quasi-classical state (3)

- Coherent state:

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

- Poisson statistics for photons:

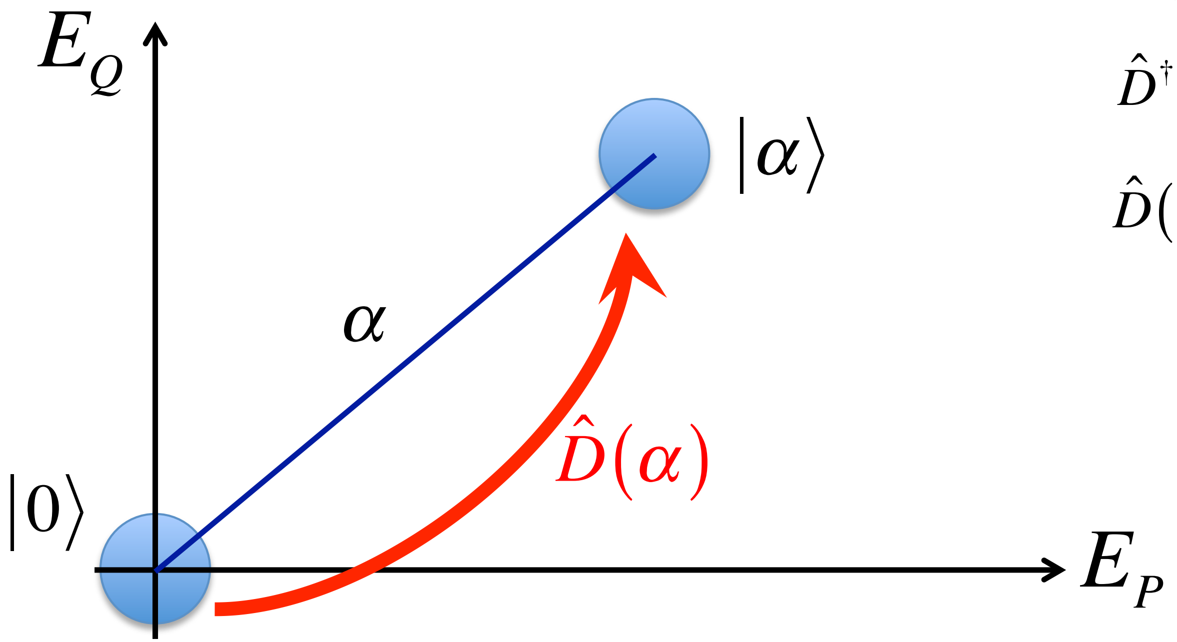




Quasi-classical state (4)

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle$$

With $\hat{D}(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)$ displacement operator



$$\hat{D}(\alpha)\hat{D}^\dagger(\alpha) = \hat{D}^\dagger(\alpha)\hat{D}(\alpha) = \hat{1}$$

$$\hat{D}^\dagger(\alpha) = \hat{D}(-\alpha)$$

$$\hat{D}(\alpha)\hat{D}(\beta) = e^{(\alpha\beta^* - \alpha^*\beta)/2}\hat{D}(\alpha+\beta)$$

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More on quadrature operators

- Generalized quadrature operators:

$$\hat{P}_\beta = \mathcal{E}^{(1)}\left(\hat{a}e^{i\beta} + \hat{a}^\dagger e^{-i\beta}\right)$$

$$\hat{Q}_\beta = -i\mathcal{E}^{(1)}\left(\hat{a}e^{i\beta} - \hat{a}^\dagger e^{-i\beta}\right)$$

$$\Delta\hat{P}_\beta \Delta\hat{Q}_\beta \geq \left[\mathcal{E}^{(1)}\right]^2$$

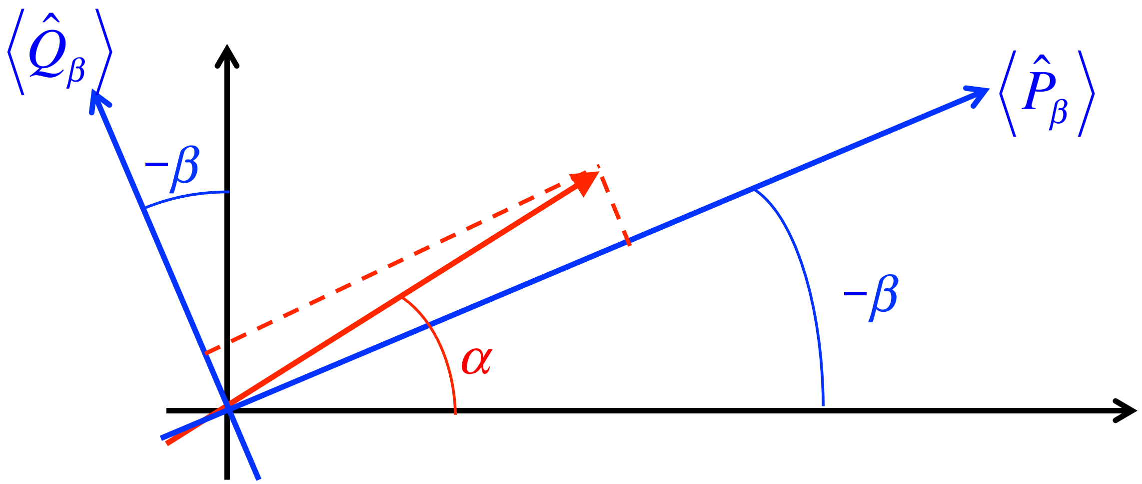
- Electric field:

$$\hat{\mathbf{E}}(\mathbf{r}) = -\vec{\varepsilon}\left\{\hat{P}_\beta \sin(\mathbf{k}\cdot\mathbf{r} - \beta) + \hat{Q}_\beta \cos(\mathbf{k}\cdot\mathbf{r} - \beta)\right\}$$

- Average values for a coherent state:

$$\langle\alpha|\hat{P}_\beta|\alpha\rangle = 2|\alpha|\mathcal{E}^{(1)}\cos(\beta + \varphi)$$

$$\langle\alpha|\hat{Q}_\beta|\alpha\rangle = 2|\alpha|\mathcal{E}^{(1)}\sin(\beta + \varphi)$$



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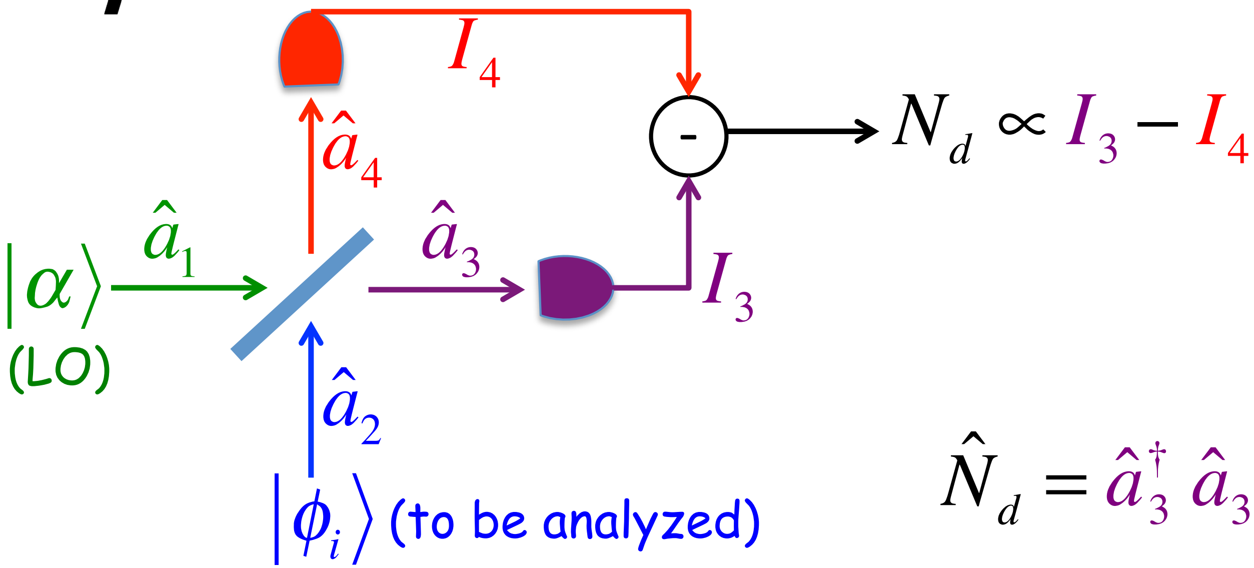
Homodyne detection



$$\hat{a}_3 = \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{a}_2)$$

$$\hat{a}_4 = \frac{1}{\sqrt{2}}(\hat{a}_2 - \hat{a}_1)$$

$$\hat{N}_d = \hat{a}_3^\dagger \hat{a}_3 - \hat{a}_4^\dagger \hat{a}_4 = \hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1$$

LO: $\alpha = \rho e^{i\theta}$ →

$$\langle \hat{N}_d \rangle = \frac{\rho}{\mathcal{E}^{(1)}} \langle \phi_i | \hat{P}_{\beta=-\theta} | \phi_i \rangle$$

→ One can measure the average value of any quadrature

$$\Delta \hat{N}_d^2 = \langle \phi_i | \hat{a}_2^\dagger \hat{a}_2 | \phi_i \rangle + \frac{\rho^2}{\left[\mathcal{E}^{(1)}\right]^2} \Delta \hat{P}_{\beta=-\theta}^2$$

Photon noise in state $|\phi_i\rangle$
Always increases with N_2

Fluctuations of quadrature $\hat{P}$
Can be decreased by squeezing

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Generation of a squeezed vacuum

$$\Omega = \omega + \omega'$$

Degenerate case: $\omega = \omega'$

Effective Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \hbar\Omega\,\hat{A}^\dagger\hat{A} + \hbar\omega\,\hat{a}^\dagger\hat{a}$$

$$\hat{V} = g\left\{\hat{A}\left(\hat{a}^\dagger\right)^2 + \hat{A}^\dagger\left(\hat{a}\right)^2\right\}$$

Incident mode $\mathcal{L}$: coherent state $\alpha \in \mathbb{R}^+$

Neglect pump depletion: $\left|\alpha(t)\right\rangle$ with $\alpha(t) = \alpha e^{-i\Omega t}$

$$\hat{V} \rightarrow \hat{V}_0 = g\alpha\left\{e^{-i\Omega t}\hat{a}^{\dagger 2} + e^{i\Omega t +}\hat{a}^2\right\}$$

Generation of a squeezed vacuum

$$\hat{V}_0 = g\alpha\left\{e^{-i\Omega t}\hat{a}^{\dagger 2} + e^{i\Omega t}\hat{a}^2\right\}$$

$t=0 : |\omega\rangle = |0\rangle$ (vacuum). We apply $\hat{V}_0$ during time τ.

Heisenberg picture: $i\hbar \frac{d}{dt}\hat{a}(t) = \left[\hat{a}(t), \hat{H}_H(t)\right]$

$$\frac{d\hat{a}}{dt} = -i\omega\hat{a} - i\lambda\hat{a}^{\dagger}e^{-i\Omega t} \qquad \lambda = \frac{2g\alpha}{\hbar}$$

Injected into $\hat{P}_\beta$ and $\hat{Q}_\beta$ with $\beta = \omega t - \pi/4$:

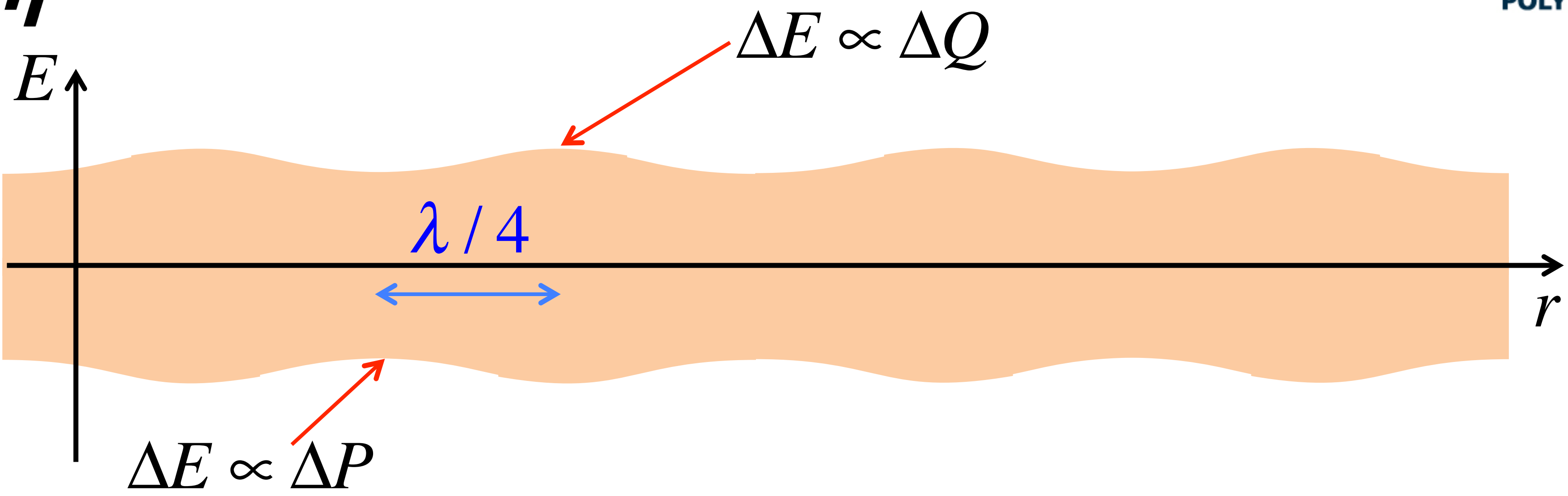
$$\begin{cases} \frac{d\hat{P}_\beta}{dt} = -\lambda\hat{P}_\beta \\ \frac{d\hat{Q}_\beta}{dt} = \lambda\hat{Q}_\beta \end{cases}$$

$$\hat{P}_\beta(\tau) = \hat{P}_\beta(0)e^{-\lambda\tau} \qquad \hat{Q}_\beta(\tau) = \hat{Q}_\beta(0)e^{\lambda\tau}$$

$$\Delta\hat{P}_\beta^{\,2}(\tau) = \left[\mathcal{E}^{(1)}\right]^2 e^{-2\lambda\tau} \qquad \Delta\hat{Q}_\beta^{\,2}(\tau) = \left[\mathcal{E}^{(1)}\right]^2 e^{2\lambda\tau}$$

One quadrature is amplified, while the other one is "deamplified ».

Generation of a squeezed vacuum



Number of Photons:

$$\hat{N} = \frac{1}{2}\left\{ \frac{\hat{P}_\beta{}^2 + \hat{Q}_\beta{}^2}{2\left[\mathcal{E}^{(1)}\right]^2} - 1 \right\} \Rightarrow \boxed{\left\langle \hat{N}(\tau) \right\rangle = \sinh^2 \lambda\tau}$$

The average number of photons diverges exponentially with $\lambda\tau$. The squeezed "vacuum" contains many photons !

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More general squeezed states

$$|\alpha,R\rangle = \hat{S}(R)|\alpha\rangle \qquad (R \in \mathbb{R})$$

With $\hat{S}(R) = \exp\left[R\left(\hat{a}^2 - \hat{a}^{\dagger 2}\right)/2\right]$ squeeze operator

$$\hat{A}_R|\alpha,R\rangle = \alpha|\alpha,R\rangle$$

$$\hat{A}_R = \hat{a}\cosh R + \hat{a}^\dagger \sinh R$$

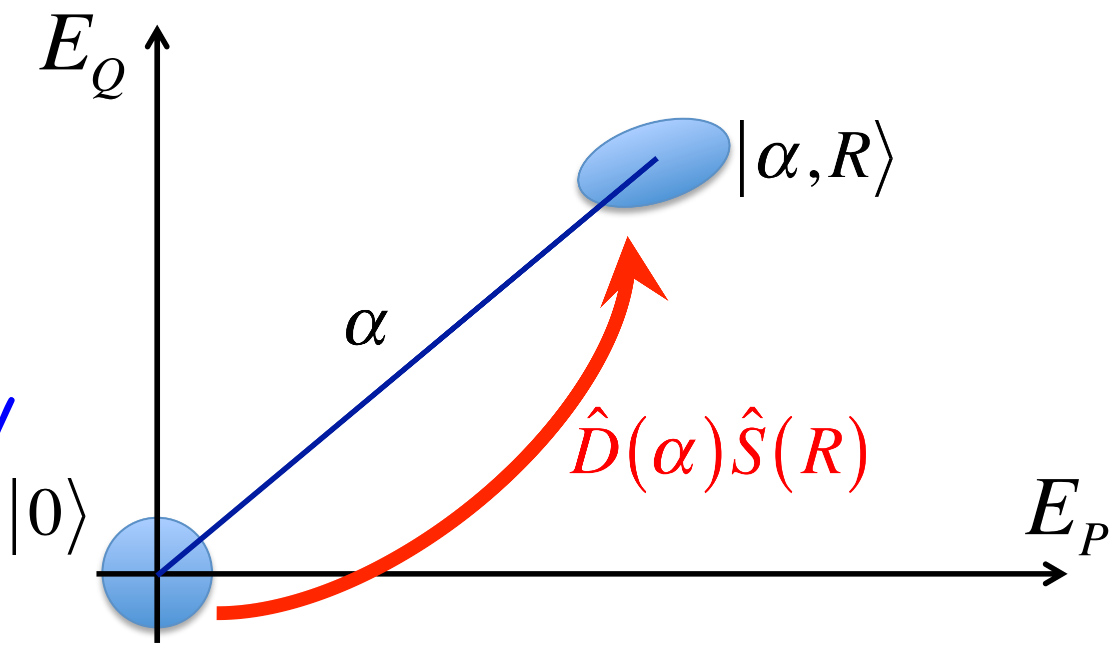
$$\left[\hat{A}_R, \hat{A}_R^\dagger\right] = \hat{1}$$

$$\hat{A}_R = \hat{S}(R)\hat{a}\hat{S}^{-1}(R)$$

$$\left(\Delta E_P\right)^2 = \left[\mathcal{E}^{(1)}\right]^2 e^{2R}$$

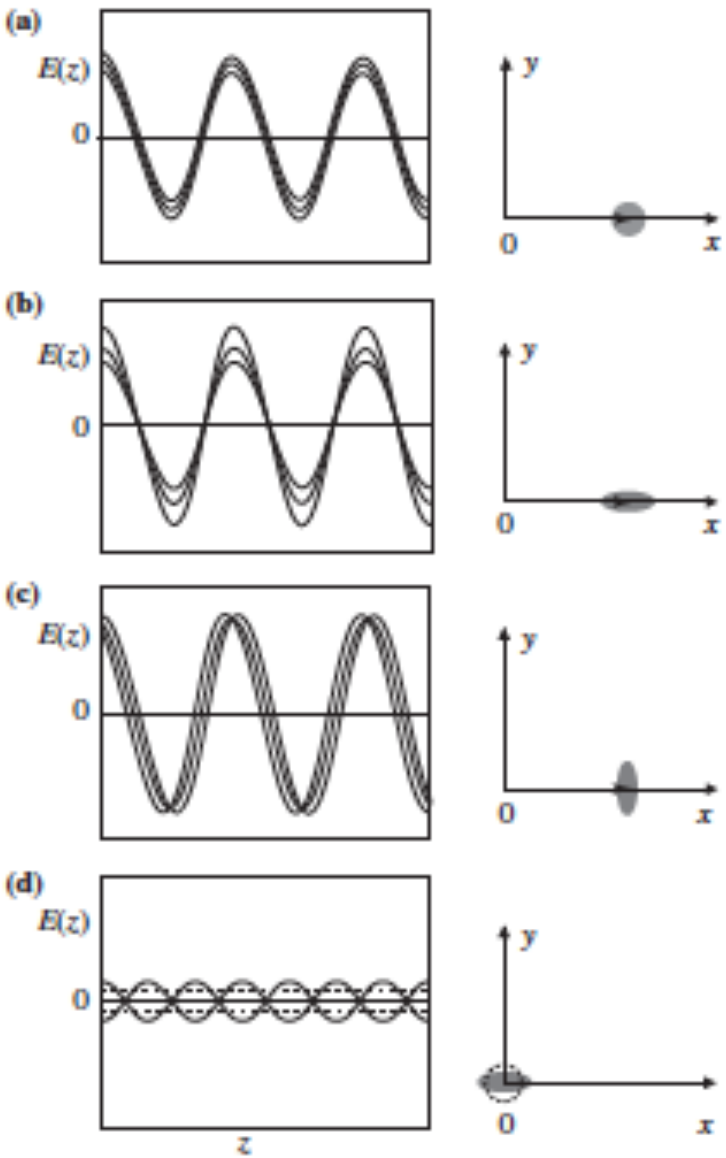
$$\left(\Delta E_Q\right)^2 = \left[\mathcal{E}^{(1)}\right]^2 e^{-2R}$$

$$\Delta E_P \, \Delta E_Q = \left[\mathcal{E}^{(1)}\right]^2$$



In general R is complex and the squeezed quadrature is rotated by arg(R)/2

More general squeezed states



Coherent state

Phase squeezed state

Amplitude squeezed state

Squeezed vacuum

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Generation of Squeezed States by Parametric Down Conversion

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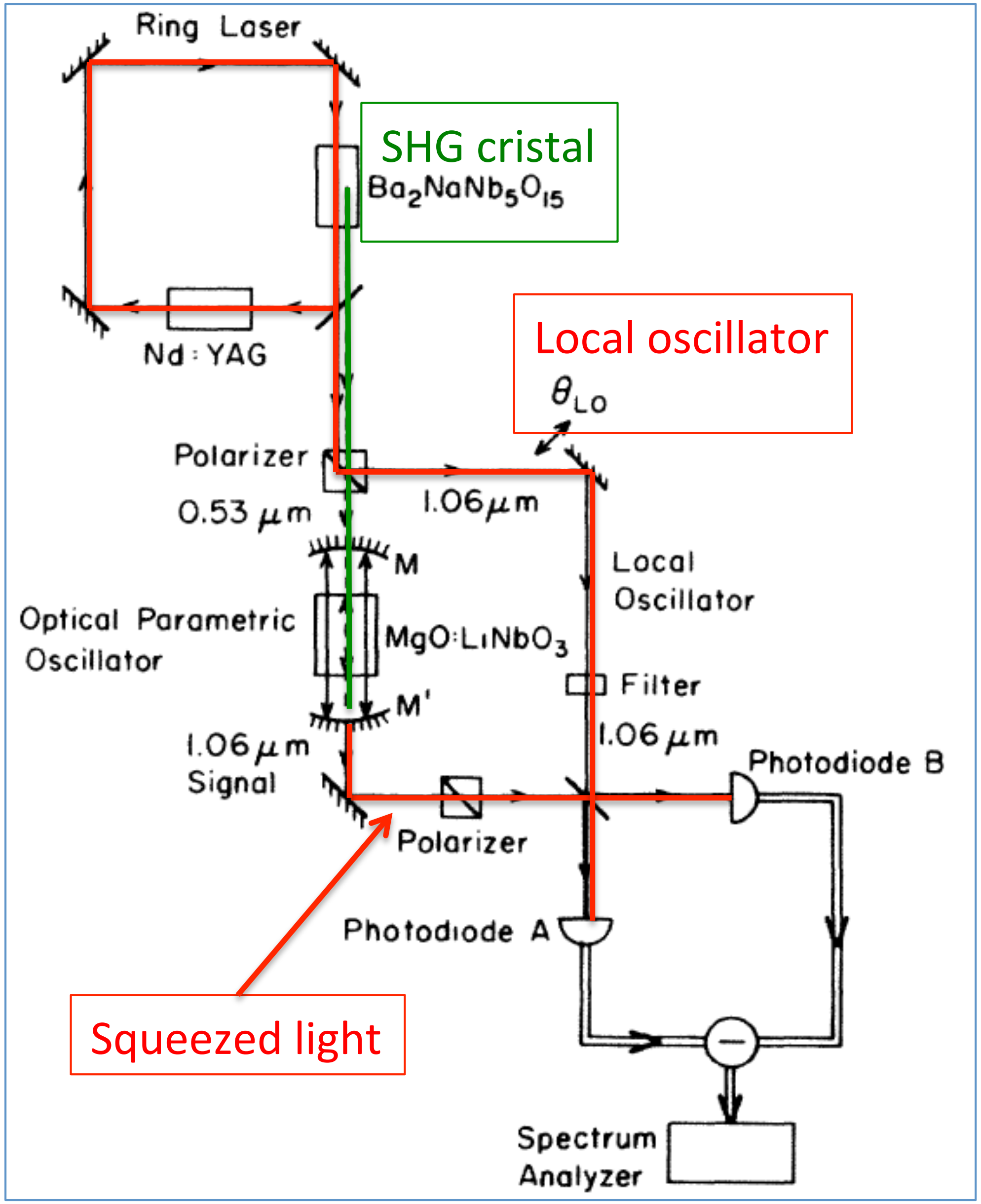

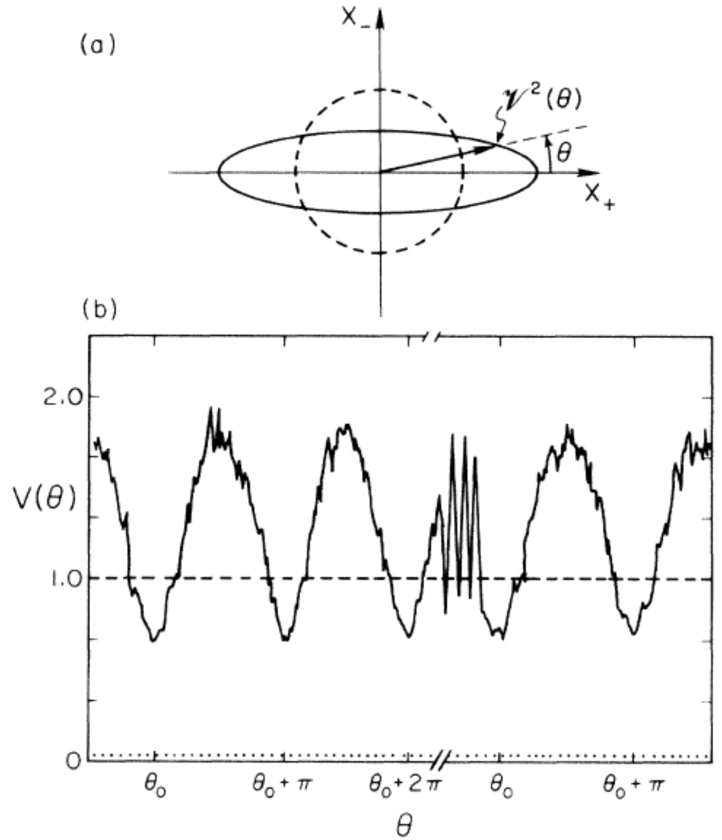

FIG. 1. (a) Phase plot of the uncertainties in the quadrature amplitudes of the electric field. The solid line represents the variance $\nu^2(\theta)$ of the field $\hat{X}(\theta)=\hat{X}_+\cos\theta+\hat{X}_-\sin\theta$ as a function of θ for a squeezed state; the dashed line is for the vacuum state. (b) Measurement of the phase dependence of the quantum fluctuations in a squeezed state produced by degenerate parametric down conversion. The plot corresponds roughly to the quantity $\nu(\theta)$ as in (a). More precisely, the phase depen-

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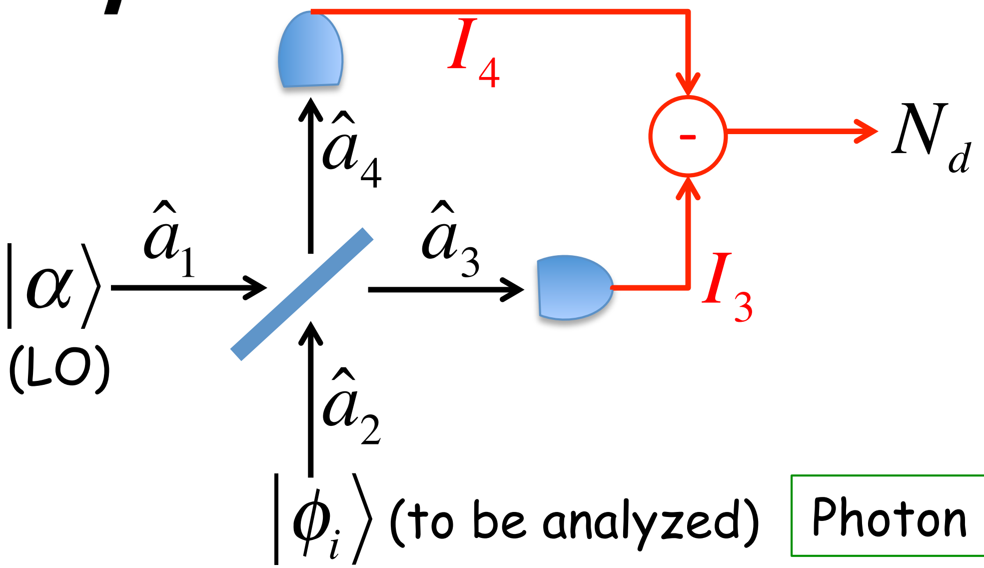
Reduction of quantum noise in a measurement



$$\langle \hat{N}_d \rangle = \frac{\rho}{\mathcal{E}^{(1)}} \langle \phi_i | \hat{P}_{\beta=-\theta} | \phi_i \rangle$$

$$\Delta \hat{N}_d^2 = \langle \phi_i | \hat{a}_2^\dagger \, \hat{a}_2 | \phi_i \rangle + \frac{\rho^2}{\left[\mathcal{E}^{(1)}\right]^2} \Delta \hat{P}_{\beta=-\theta}^{\,2}$$

Photon noise in state $|\phi_i\rangle$

Quadrature noise

First example: vacuum $|\phi_i\rangle = |0\rangle$

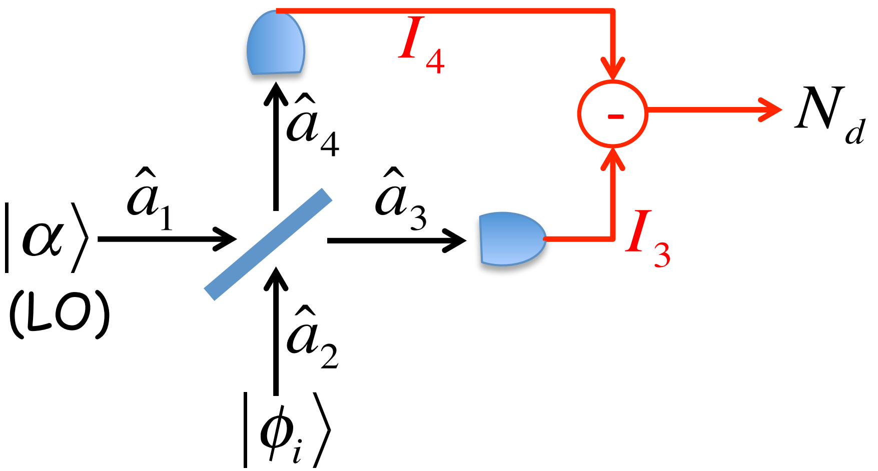
$$\langle \hat{N}_d \rangle = \frac{\rho}{\mathcal{E}^{(1)}} \langle 0 | \hat{P}_{\beta=-\theta} | 0 \rangle = 0$$

$$\Delta \hat{N}_d^2 = \langle 0 | \hat{a}_2^\dagger \, \hat{a}_2 | 0 \rangle + \frac{\rho^2}{\left[\mathcal{E}^{(1)}\right]^2} \langle 0 | \hat{P}_{\beta=-\theta}^{\,2} | 0 \rangle = \rho^2$$

Usual photon noise

Reduction of quantum noise in a measurement

Second example: squeezed vaccum $|\phi_i\rangle$



$$\langle \hat{N}_d \rangle = \frac{\rho}{\mathcal{E}^{(1)}} \langle \phi_i | \hat{P}_{\beta=-\theta} | \phi_i \rangle = 0$$

$$\Delta \hat{N}_d^2 = \langle \phi_i | \hat{a}_2^\dagger \, \hat{a}_2 | \phi_i \rangle + \frac{\rho^2}{\left[\mathcal{E}^{(1)}\right]^2} \Delta \hat{P}^2_{\beta=-\theta}$$

One chooses $-\theta = \omega t - \pi/4$

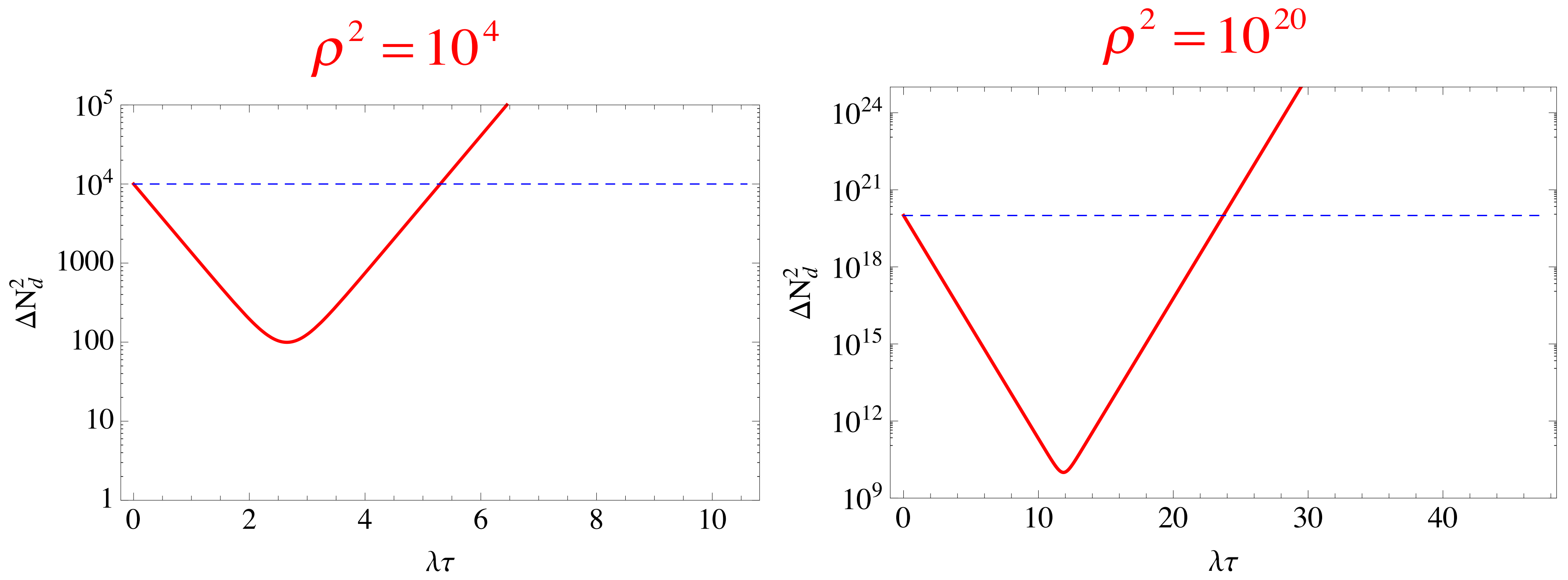
$$\Delta \hat{N}_d^2 = \sinh^2 \lambda\tau + \rho^2 e^{-2\lambda\tau} \approx \frac{e^{2\lambda\tau}}{4} + \rho^2 e^{-2\lambda\tau} \text{ for } \lambda\tau \gg 1$$

Optimum noise compression for $e^{-2\lambda\tau} = 1/2\rho$

Leading to $\Delta \hat{N}_d^2 = \rho$ for a squeezed vacuum

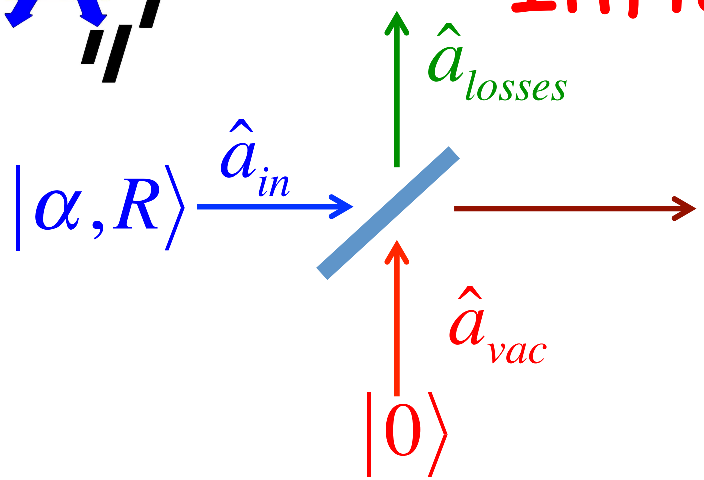
To be compared with $\Delta \hat{N}_d^2 = \rho^2$ for "regular" vacuum

Reduction of quantum noise in a measurement



In practice: reduction limited to a factor of the order of 100, because of losses

Influence of losses



Losses $1 - t^2 = r^2$

$$\hat{a}_{out} = t\hat{a}_{in} - r\hat{a}_{vac}$$

$$\hat{E}_{Qout} = t\hat{E}_{Qin} - r\hat{E}_{Qvac}$$

Incident state: $|\psi\rangle = |\alpha, R\rangle \otimes |0\rangle$ Vacuum field enters through the fourth port -> Fluctuations

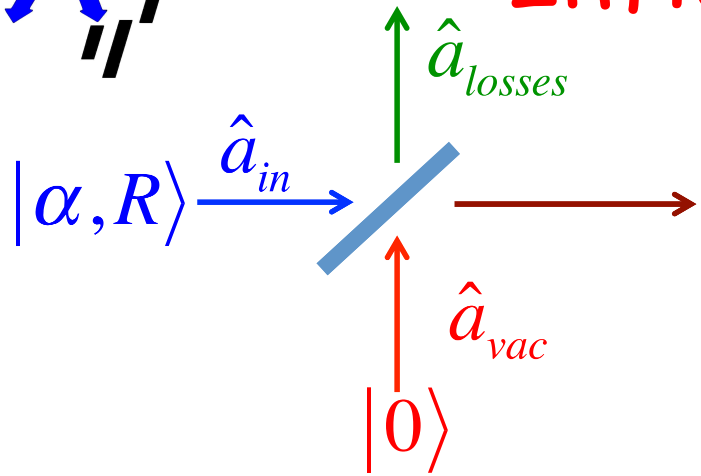
Average: $\langle\psi|\hat{E}_{Qout}|\psi\rangle = t\langle\alpha, R|\hat{E}_{Qin}|\alpha, R\rangle$ Classical transmission t^2

Variance:

$$\left(\Delta\hat{E}_{Qout}\right)^2 = t^2\left(\Delta\hat{E}_{Qin}\right)^2 + r^2\left[\mathcal{E}^{(1)}\right]^2 = \left[\mathcal{E}^{(1)}\right]^2\left(t^2 e^{-2R} + r^2\right)$$

Vacuum fluctuations destroy squeezing !

Influence of losses



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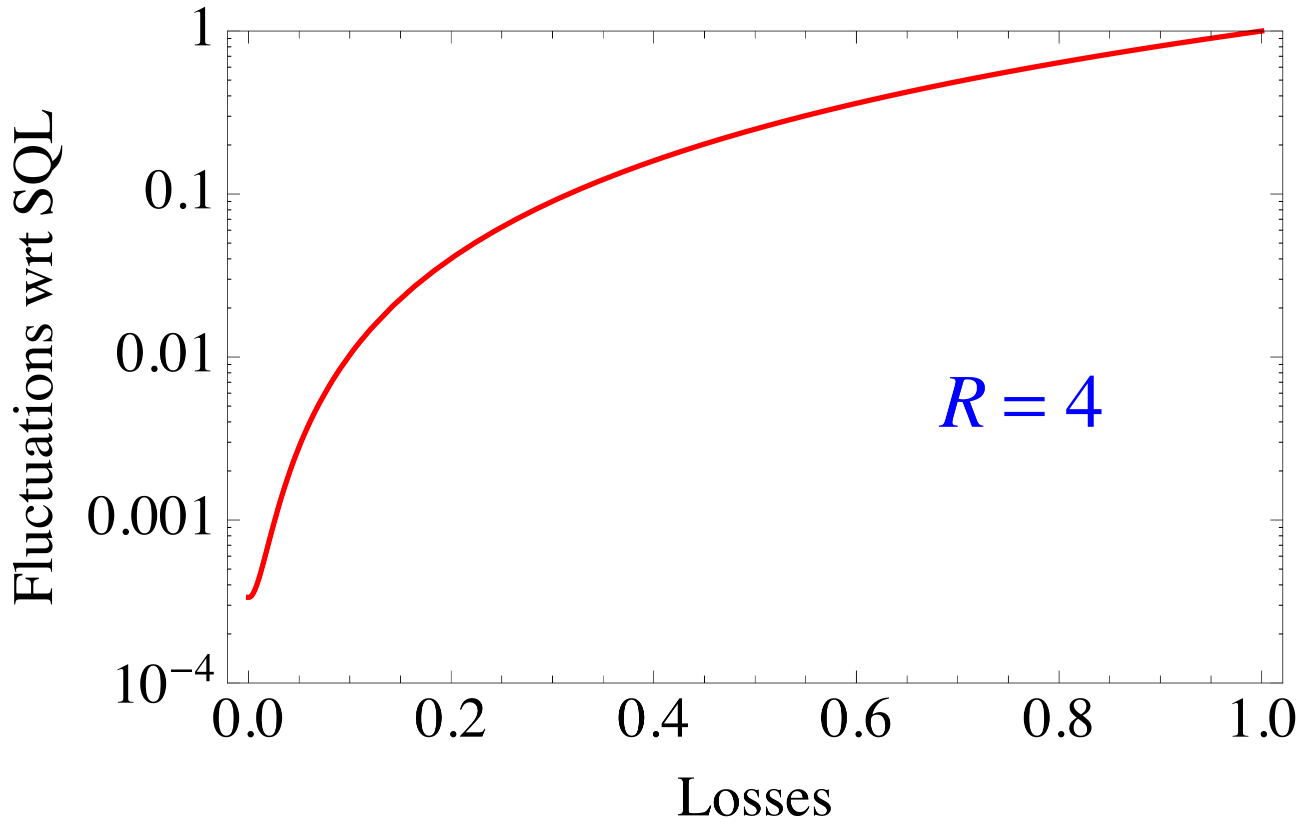
$$\hat{a}_{out} = t\hat{a}_{in} - r\hat{a}_{vac}$$



Vacuum fluctuations destroy squeezing !

Conclusion

- Squeezing is a nice illustration of non-classical states of light
- Squeezing can be produced by second-order nonlinear interaction
- One can imagine use squeezing to reduce quantum fluctuations in measurements
- However squeezing is extremely fragile and sensitive to losses