Amplifier Noise
&
Phase Sensitive Amplification

Fabien Bretenaker

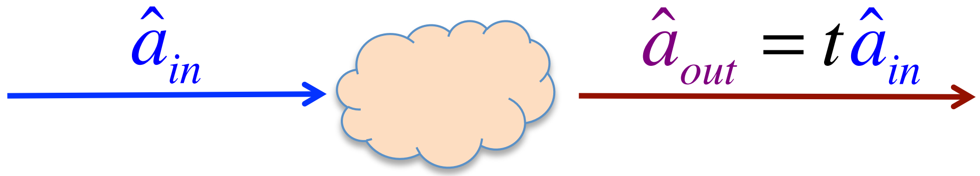
Laboratoire Aimé Cotton (Orsay, France)
&
Ecole Polytechnique (Palaiseau, France)

Outline

- Why do vacuum fluctuations enter?
  - Vacuum fluctuations and losses
  - Vacuum fluctuations and gain
- Phase sensitive amplification
- Implementation

Vacuum fluctuations: losses

Losses: $1-t^2$ with $t<1$



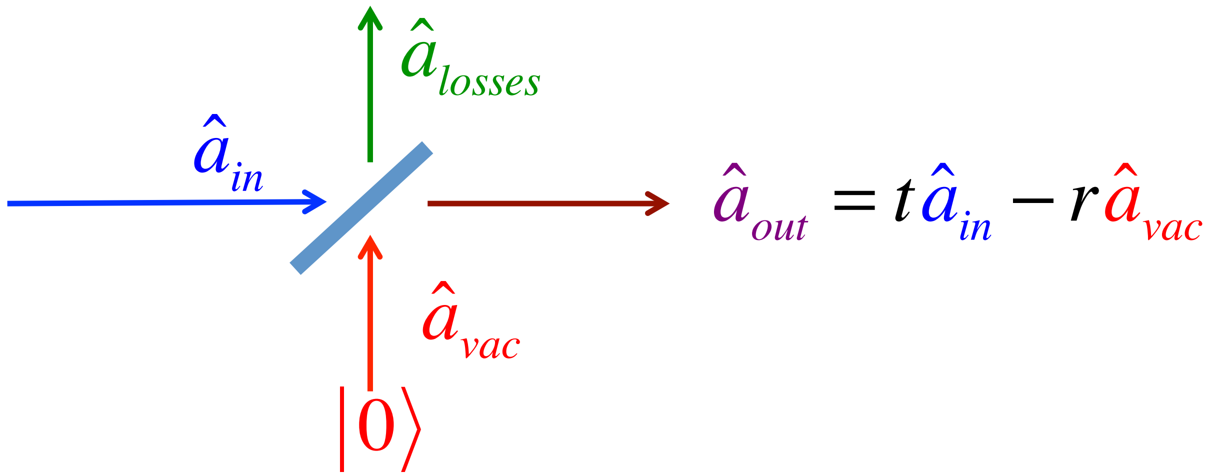
$$\left[\hat{a}_{out},\hat{a}_{out}^{\dagger}\right]=t^2\left[\hat{a}_{in},\hat{a}_{in}^{\dagger}\right]=t^2\neq\hat{1}$$

But all modes should obey $\left[\hat{a}_{\ell},\hat{a}_{\ell}^{\dagger}\right]=\hat{1}$

Something is missing….: coupling to other modes.

Vacuum fluctuations: losses

Losses modeled by a beam splitter: $r^2 = 1 - t^2$ with $t < 1$

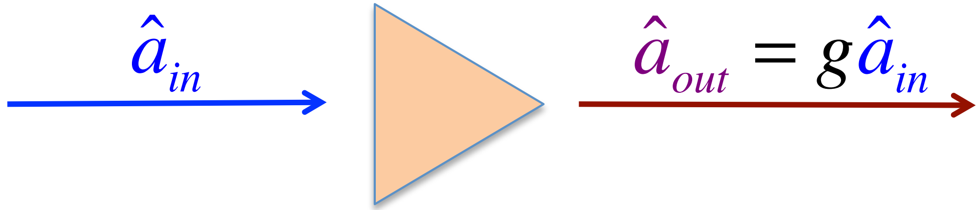


$$\left[\hat{a}_{out}, \hat{a}_{out}^{\dagger}\right] = t^2\left[\hat{a}_{in}, \hat{a}_{in}^{\dagger}\right] + r^2\left[\hat{a}_{vac}, \hat{a}_{vac}^{\dagger}\right] = \hat{1}$$

Extra vacuum fluctuations enter through the empty port, which mimics coupling to other modes due to absorbing or scattering centers.

Vacuum fluctuations: gain

Gain: $G = g^2 > 1$



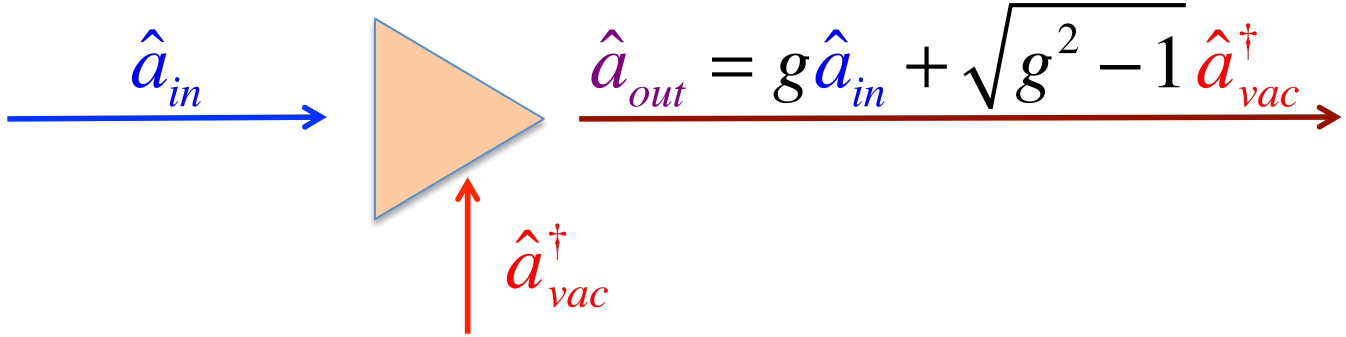
$$\left[\hat{a}_{out}, \hat{a}^{\dagger}_{out}\right] = g^2 \left[\hat{a}_{in}, \hat{a}^{\dagger}_{in}\right] = g^2 \neq \hat{1}$$

But all modes should obey $\left[\hat{a}_{\ell}, \hat{a}^{\dagger}_{\ell}\right] = \hat{1}$

Something is missing again.: coupling to other modes.

Vacuum fluctuations: gain

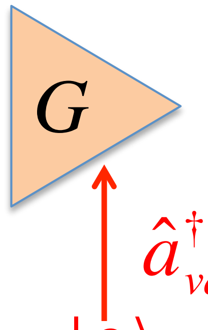
Gain: $G = g^2 > 1$



$$\left[\hat{a}_{out}, \hat{a}^{\dagger}_{out}\right] = g^2\left[\hat{a}_{in}, \hat{a}^{\dagger}_{in}\right] + \left(g^2 - 1\right)\left[\hat{a}^{\dagger}_{vac}, \hat{a}_{vac}\right] = \hat{1}$$

Extra vacuum fluctuations enter through the empty port, which mimics coupling to other modes due to amplifying atoms.

Vacuum fluctuations: gain

$|in\rangle \xrightarrow{\hat{a}_{in}}$ 

$\hat{a}_{out} = g\hat{a}_{in} + \sqrt{g^2 - 1}\,\hat{a}^{\dagger}_{vac}$

$|0\rangle \rightarrow \hat{a}^{\dagger}_{vac}$

$$\hat{E}_P = \mathcal{E}^{(1)}\left(\hat{a} + \hat{a}^{\dagger}\right)$$

$$\hat{E}_Q = -i\mathcal{E}^{(1)}\left(\hat{a} - \hat{a}^{\dagger}\right)$$

$$\hat{E}_{Pout} = g\hat{E}_{Pin} + \sqrt{g^2 - 1}\,\hat{E}_{Pvac}$$

$$\hat{E}_{Qout} = g\hat{E}_{Qin} - \sqrt{g^2 - 1}\,\hat{E}_{Qvac}$$

$$\left(\Delta\hat{E}_{Pout}\right)^2 = G\left(\Delta\hat{E}_{Pin}\right)^2 + (G-1)\left[\mathcal{E}^{(1)}\right]^2$$

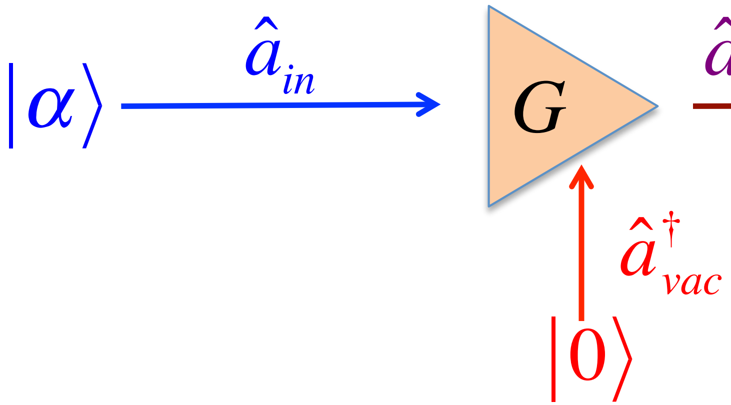
$$\left(\Delta\hat{E}_{Qout}\right)^2 = G\left(\Delta\hat{E}_{Qin}\right)^2 + (G-1)\left[\mathcal{E}^{(1)}\right]^2$$

Input noise amplification

Excess noise

The amplification process necessarily introduces noise.

Vacuum fluctuations: gain



$$\hat{a}_{out} = g\hat{a}_{in} + \sqrt{g^2 - 1}\,\hat{a}^{\dagger}_{vac}$$

$$\left(\Delta\hat{E}_{Pout}\right)^2 = G\left(\Delta\hat{E}_{Pin}\right)^2 + (G-1)\left[\mathcal{E}^{(1)}\right]^2$$

$$\left(\Delta\hat{E}_{Qout}\right)^2 = G\left(\Delta\hat{E}_{Qin}\right)^2 + (G-1)\left[\mathcal{E}^{(1)}\right]^2$$

If $|in\rangle = |\alpha\rangle$ (coherent state), then: $\left(\Delta\hat{E}_{Pin}\right)^2 = \left(\Delta\hat{E}_{Qin}\right)^2 = \left[\mathcal{E}^{(1)}\right]^2$

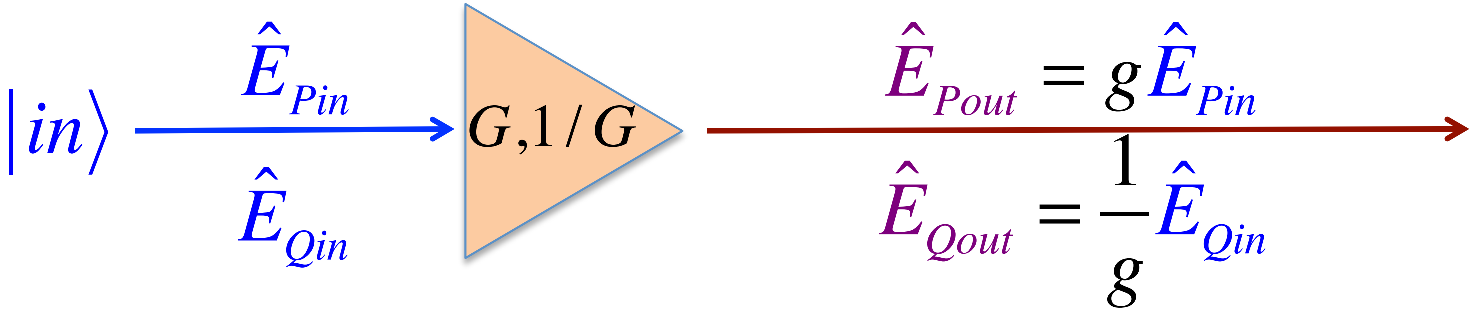
leading to: $$\left(\Delta\hat{E}_{Pout}\right)^2 = \left(\Delta\hat{E}_{Qout}\right)^2 = (2G-1)\left[\mathcal{E}^{(1)}\right]^2$$

The intensity is amplified by a gain G, while the noise is amplified by a factor (2G-1). Thus the signal-to-noise ratio is degraded by a factor:

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{2G-1}{G} > 1$$

$$F \simeq 2 = 3\text{dB when } G \gg 1$$

Phase sensitive amplification



One quadrature is amplified while the other is "deamplified"

Then: $\left[\hat{E}_{Pout}, \hat{E}_{Qout}\right] = \left[\hat{E}_{Pin}, \hat{E}_{Qin}\right]$ and $\left[\hat{a}_{Pout}, \hat{a}^{\dagger}_{Qout}\right] = \hat{1}$

$$\left(\Delta\hat{E}_{Pout}\right)^2 = G\left(\Delta\hat{E}_{Pin}\right)^2$$

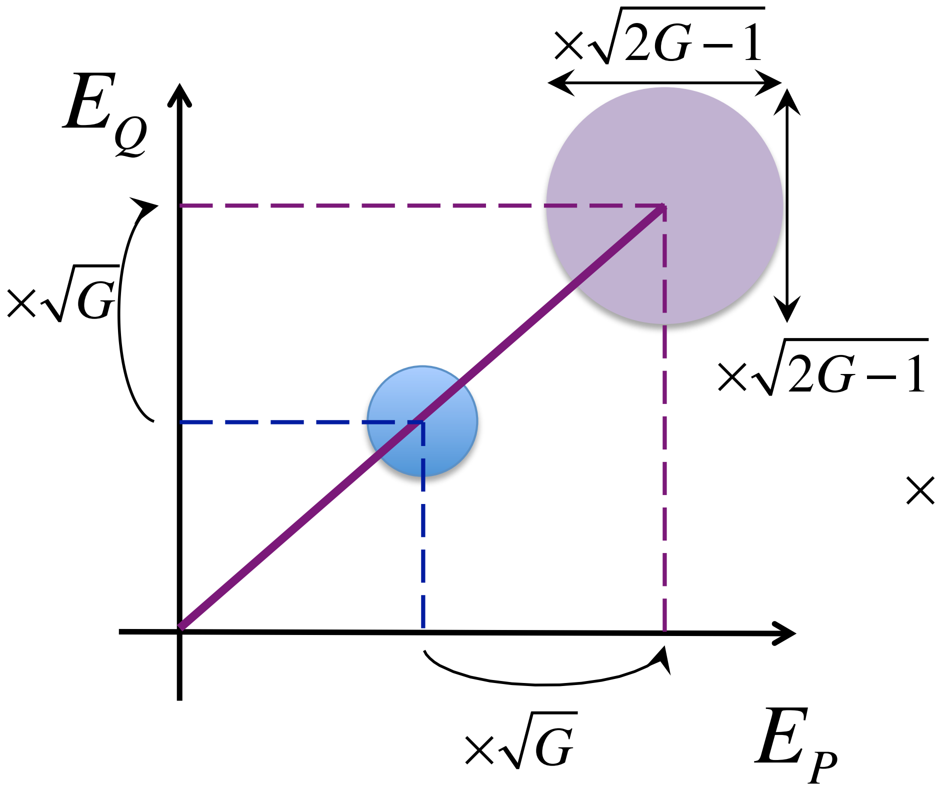
$$\left(\Delta\hat{E}_{Qout}\right)^2 = \frac{1}{G}\left(\Delta\hat{E}_{Qin}\right)^2$$

Intensity and noise are (de-) amplified by the same factor.

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{G}{G} = \frac{1/G}{1/G} = 1$$

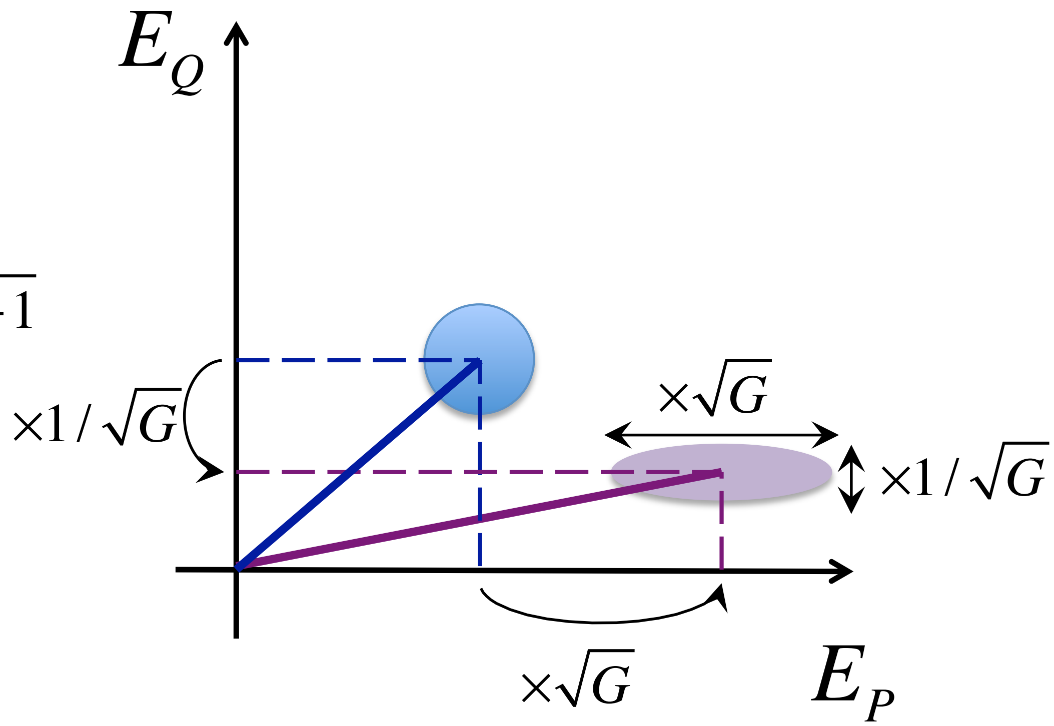
PIA vs. PAS

Phase insensitive amplification



Coupling to other modes through amplifying atoms (open system)

Phase sensitive amplification



No coupling to other modes than the pump (Hamiltonian system)

Implementation

$$\Omega = \omega + \omega'$$

Degenerate case: $\omega = \omega'$

Effective Hamiltonian: $\hat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \hbar\Omega\,\hat{A}^\dagger\hat{A} + \hbar\omega\,\hat{a}^\dagger\hat{a}$$
$$\hat{V} = g\left\{\hat{A}\left(\hat{a}^\dagger\right)^2 + \hat{A}^\dagger\left(\hat{a}\right)^2\right\}$$

$$\begin{cases} \dfrac{d\hat{P}_\beta}{dt} = -\lambda\hat{P}_\beta \\ \dfrac{d\hat{Q}_\beta}{dt} = \lambda\hat{Q}_\beta \end{cases}$$

$$\hat{P}_\beta(\tau) = \hat{P}_\beta(0)e^{-\lambda\tau}$$
$$\hat{Q}_\beta(\tau) = \hat{Q}_\beta(0)e^{\lambda\tau}$$

One quadrature is amplified, while the other one is "deamplified ».

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Reduction of quantum noise in optical parametric amplification

J. A. Levenson, I. Abram, and Th. Rivera

France Telecom, Centre National d'Etudes des Télécommunications, Paris B, BP 107, 92225 Bagneux Cedex, France

Ph. Grangier

Institut d'Optique, BP 147, 91403 Orsay, France

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We experimentally demonstrate that a type-II pulsed optical parametric amplifier operated in a phase-insensitive configuration works as a near-perfect classical optical amplifier whose noise figure approaches 3 dB at high gains. We further demonstrate that, when operated in a phase-sensitive configuration, this amplifier works as a quantum-optical amplifier whose noise figure goes below 3 dB and approaches 0 dB at high gains. The noise figure of 1.45 ± 0.2 dB, measured for a gain of 9 dB, is clearly in the quantum regime.

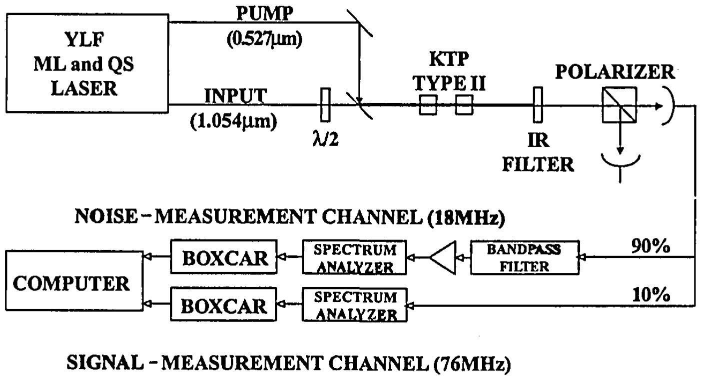

Fig. 2. Experimental setup. A frequency-doubled mode-locked (ML) Q-switched (QS) YLF:Nd laser pumps (at 527 nm) a traveling-wave parametric amplifier composed of two type-II KTP crystals. The polarization of the input signal (at 1054 nm) is rotated by the half-wavelength plate (λ/2) to achieve phase-insensitive or phase-sensitive amplification. The photodiode current is split 90/10, with each partial current measured by use of a spectrum analyzer and a boxcar combination.

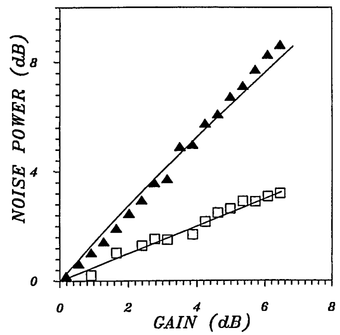

Fig. 3. Noise power of the parametric output in the phase-insensitive configuration as a function of gain (triangles), fitted to Eq. (28). The squares represent the measured noise power of a coherent beam.

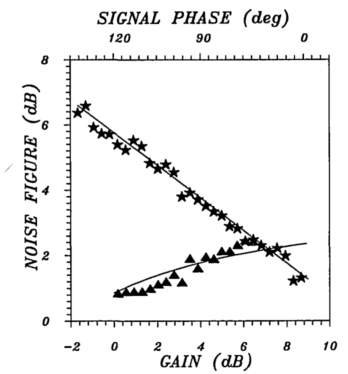

Fig. 4. NF of the phase-insensitive (triangles) and phase-sensitive (stars) parametric amplifiers, fitted to Eqs. (29) and (33), respectively.

Conclusion

- Phase insensitive amplifiers degrade the signal-to-noise ratio, due to coupling to vacuum modes
- Phase sensitive amplifiers can amplify without degrading the SNR
- Practical implementation is based on parametric processes, based on $\chi^{(2)}$ or $\chi^{(3)}$ processes.
- See Tarek Labidi's poster for implementation in nonlinear optical fibers