Translational symmetry broken Mott states of ultracold bosons

K. Sengupta

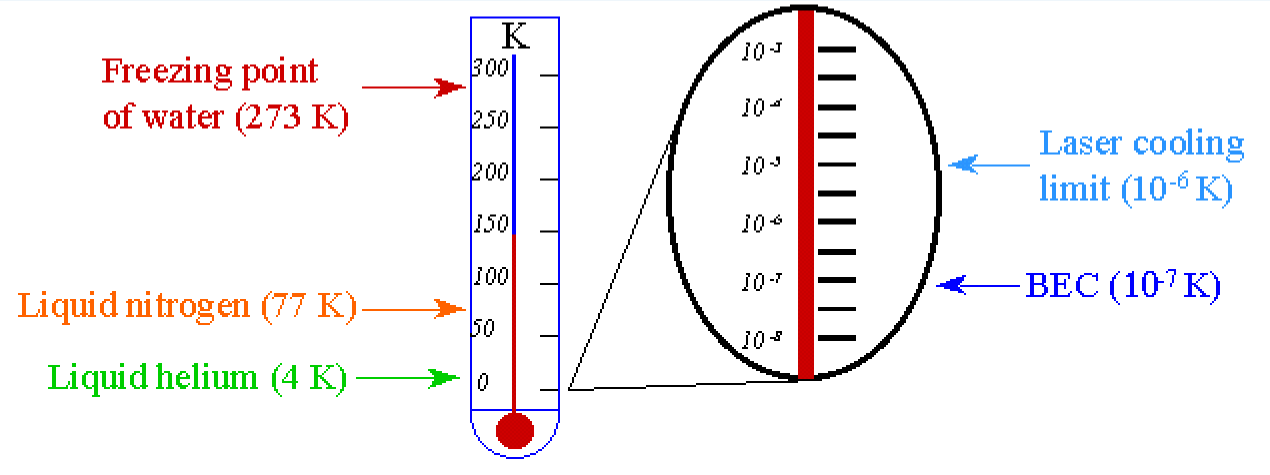
Theoretical Physics Department, IACS, Kolkata

Outline

1. *Emulating correlated systems: Bose Hubbard model*
2. *Translational symmetry breaking via "electric field" application*
3. *Realizing antiferromagnetic spin chains*
4. *Observing quantum dynamics: Kibble Zureck scaling in a nutshell*
5. *Tuning the dipole model*
6. *Breaking symmetries with Rydberg atoms*
7. *Conclusion*

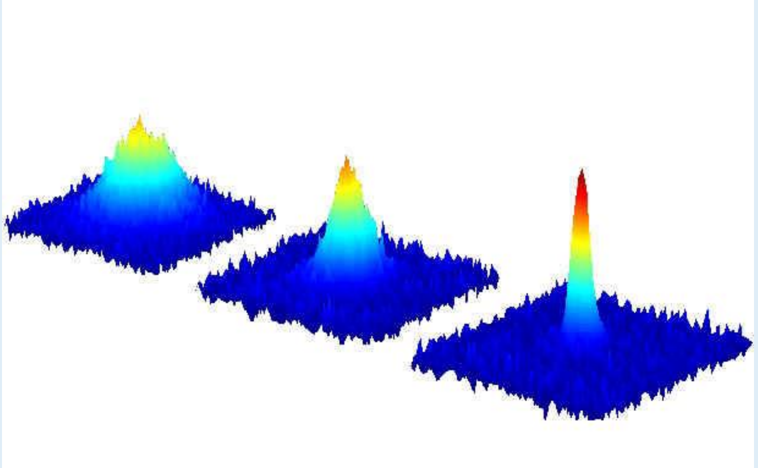
Ultracold atoms: Emulating strong correlation using light

How cold is ultracold



Perspective about how cold these atoms really are: coldest place in the universe

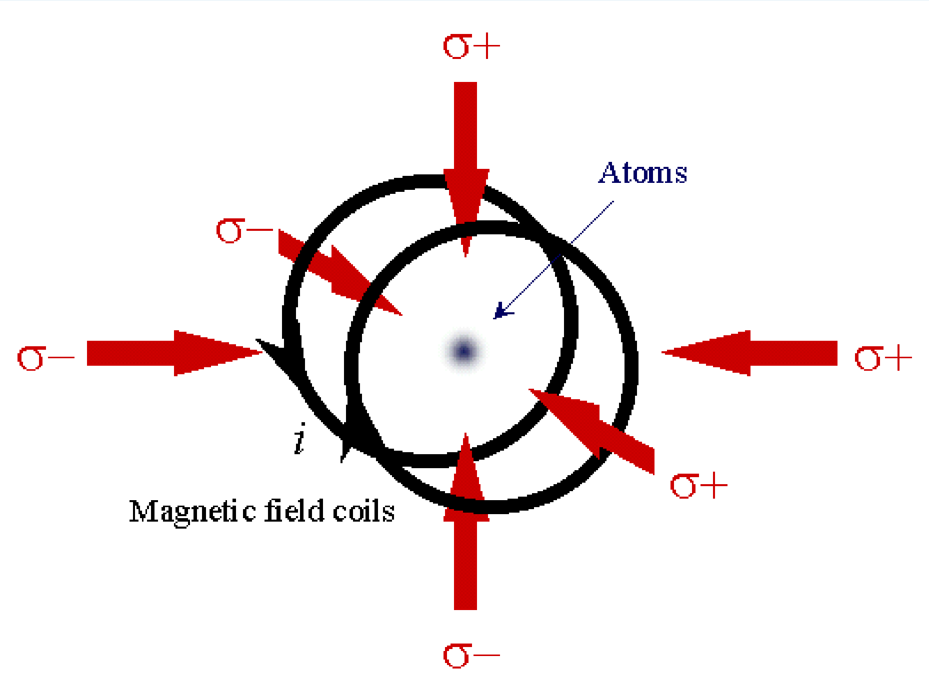
How do you measure such temperatures?



Hot → Cold

Form a BEC and measure the width of the central peak in its momentum distribution.

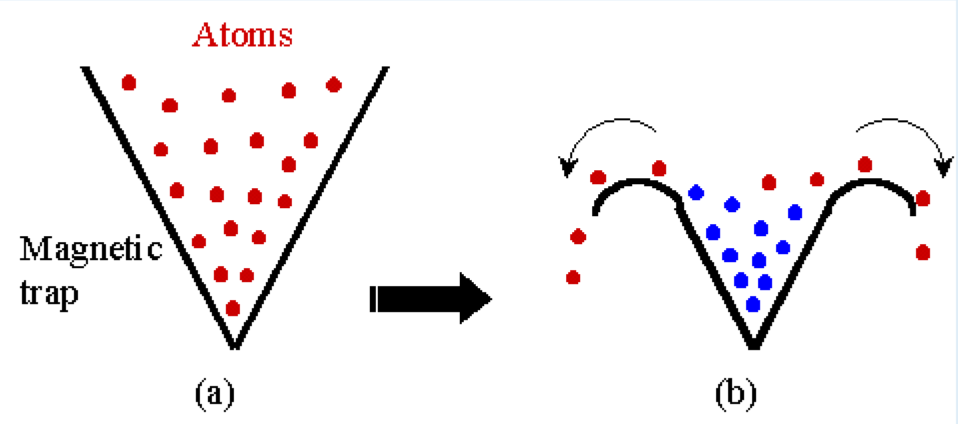
Cooling techniques: two methods



Doppler cooling of atoms

⇩

Creation of optical molasses leading to microkelvin temperatures



Further evaporative cooling of the atoms leading to temperature ~ 10 nK

This is well below critical Temperature of a BEC

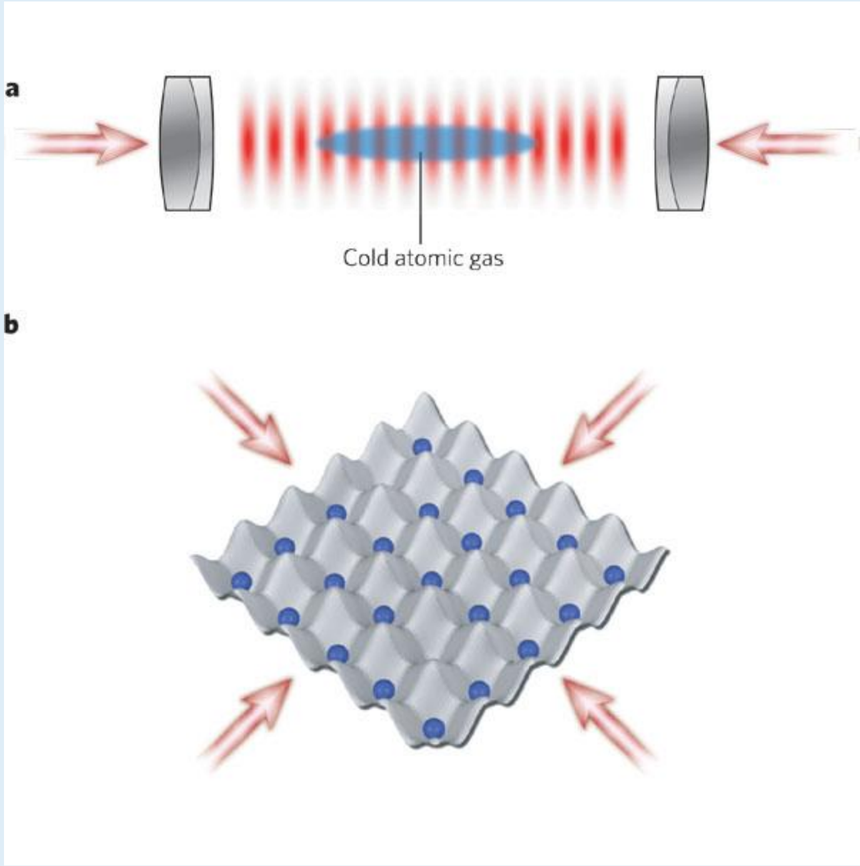
Emulating lattices for bosons with light

Apply counter propagating laser: standing wave of light.

The atoms feel a potential $V = -a\,|E|^2$

For positive α, the atoms sits at the bottom of the potential generated by the lasers:

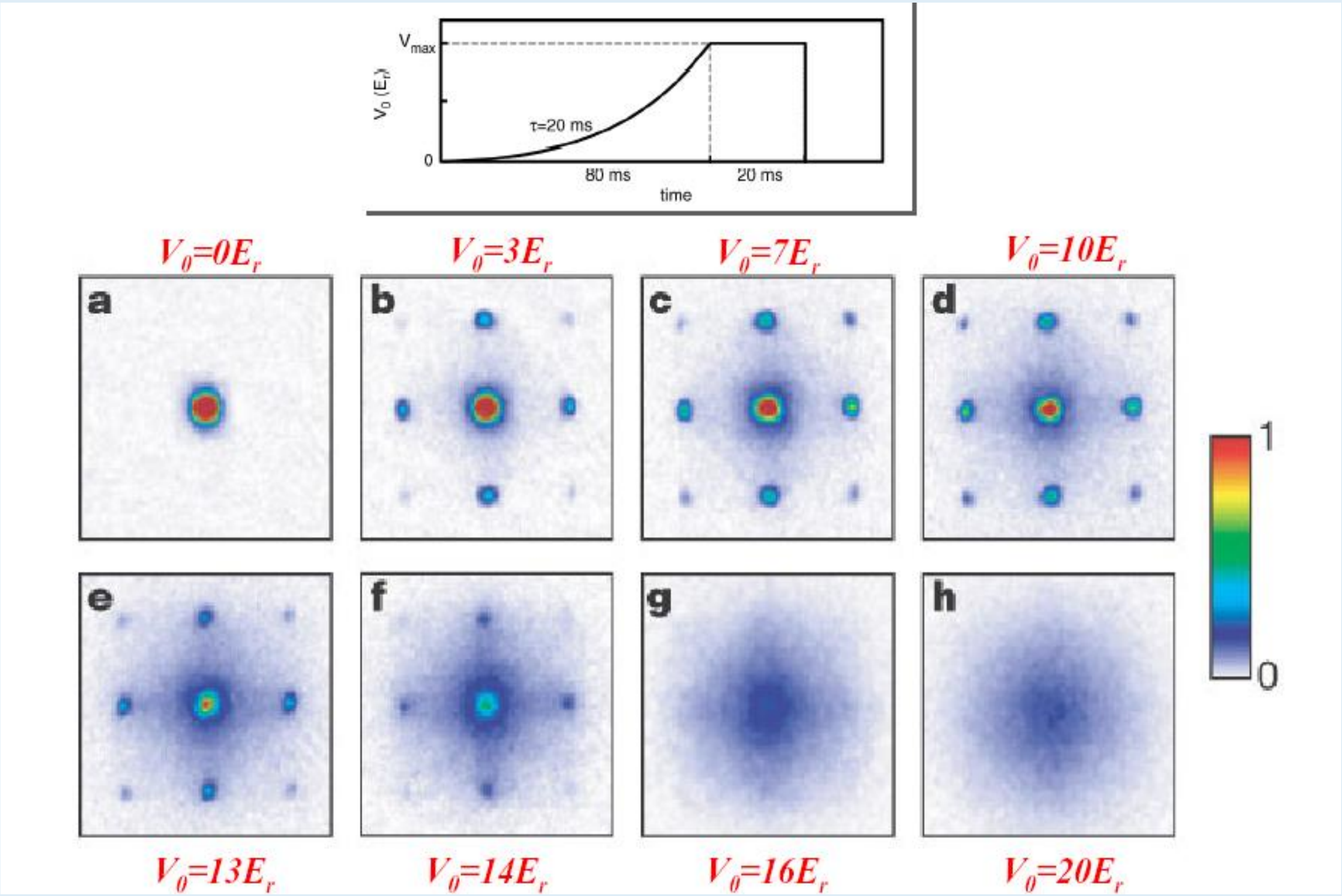
Bloch theory for bosons



Optical Lattices	Solid state crystals
• Fully controllable, no defects, no vibrations	• Very complex condensed matter environment
• Lattice spacing micrometers	• Lattice spacing Angstroms
• Trapped atom mass ~ 10-100 amu	• Electron mass 1/1900 amu
• Temperature : T~1 nK	• Temperature : T~ 100 K

How does one realize physics of interacting quantum matter using such bosons

Emulating strong correlation

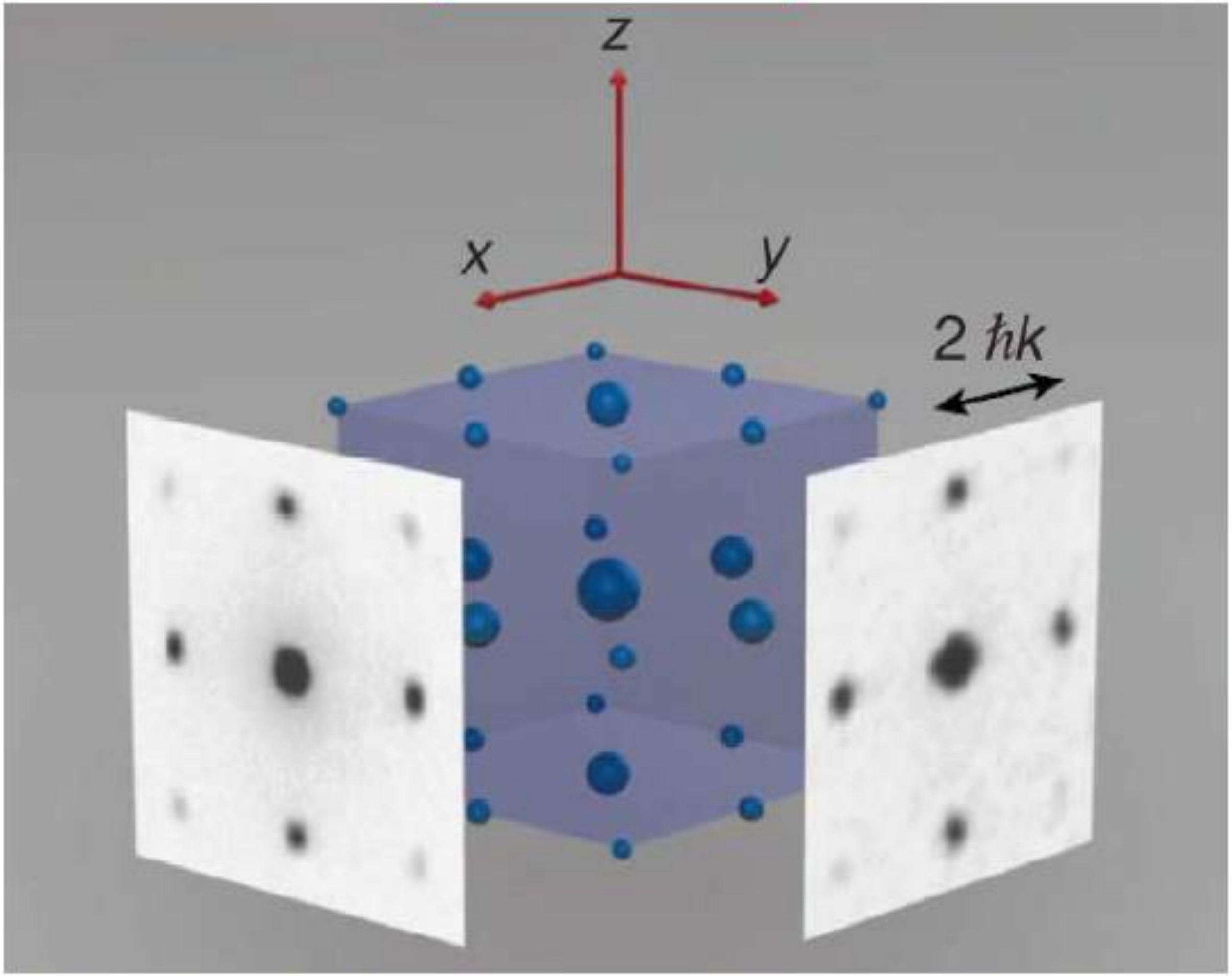


Experiments with cold atoms in an optical lattice (Greiner et al. 2001)

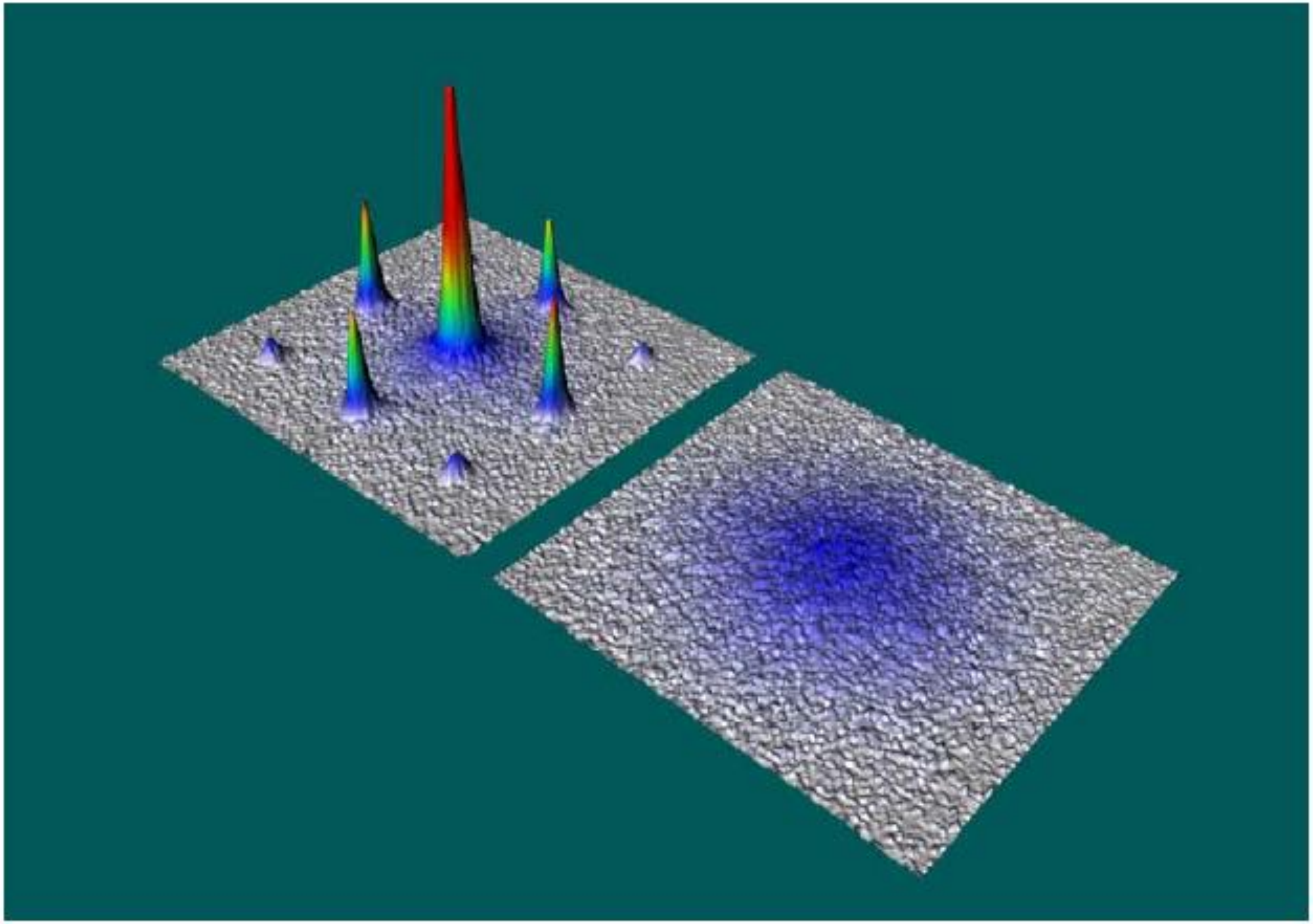
Unit of energy

$$E_r = 2\hbar^2\pi^2/m\lambda^2$$

Superfluid state



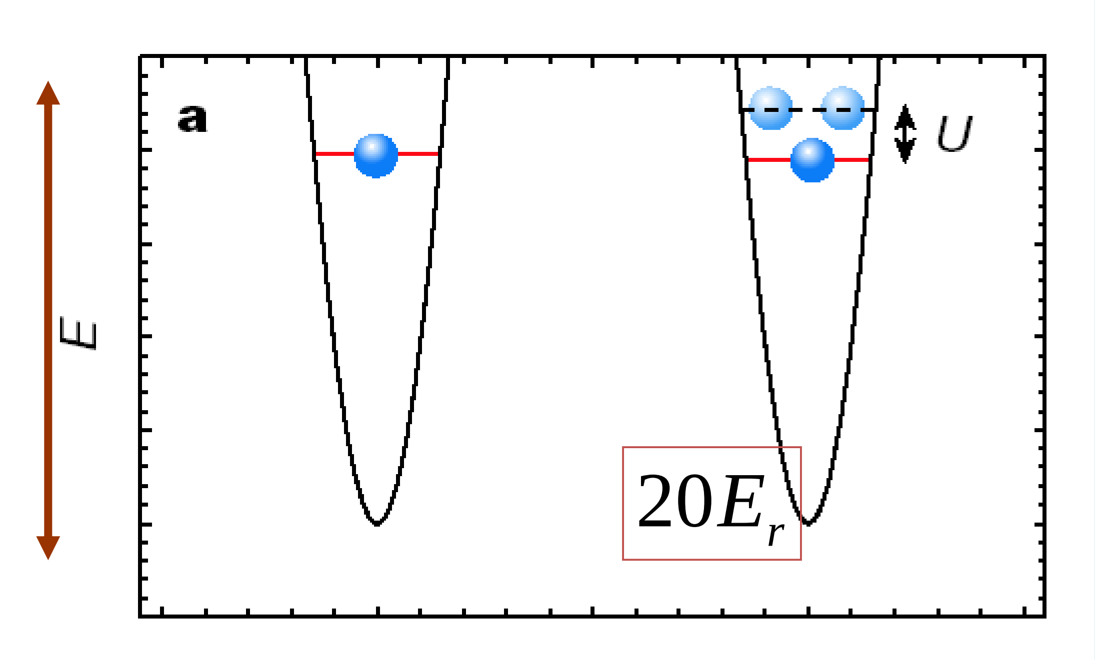
Schematic three-dimensional interference pattern with measured absorption images taken along two orthogonal directions. The absorption images were obtained after ballistic expansion from a lattice with a potential depth of $V_0 = 10\ E_r$ and a time of flight of 15 ms.



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Theoretical Modeling

For a deep enough potential, the atoms are localized : Mott insulator described by single band Bose-Hubbard model.



Energy Scales

$\delta E_n = 5E_r \sim 20$ U
U ~10-300 t

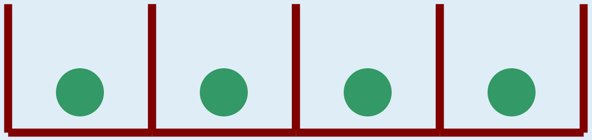
Ignore higher bands

Model Hamiltonian

$$\mathcal{H} = -t \sum_{<i,j>} \left(b_i^\dagger b_j + \text{h.c.}\right) + \frac{U}{2}\sum_i n_i(n_i-1) - \mu \sum_i n_i$$

Mott-Superfluid transition: preliminary analysis

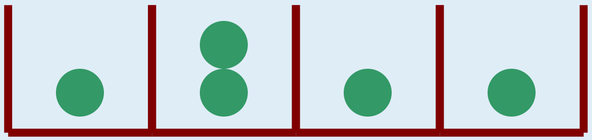
Mott state with 1 boson per site



$$\mathcal{H}_{\text{on-site}} = \frac{U}{2}\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

Stable ground state for 0 < m < U

Adding a particle to the Mott state

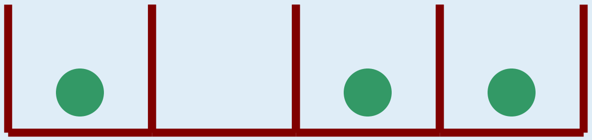


Mott state is destabilized when the excitation energy touches 0.

$$\delta E_p = (-\mu + U) - 2zt$$

$$t_p^c = (-\mu + U)/2z$$

Removing a particle from the Mott state

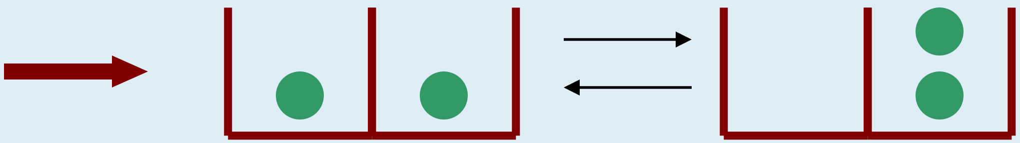


$$\begin{aligned} \delta E_h &= \mu - zt \\ t_c^h &= \mu/z \end{aligned}$$

Destabilization of the Mott state via addition of particles/hole: onset of superfluidity

Beyond this simple picture

Higher order energy calculation by Freericks and Monien: Inclusion of up to $O(t^3/U^3)$ virtual processes.

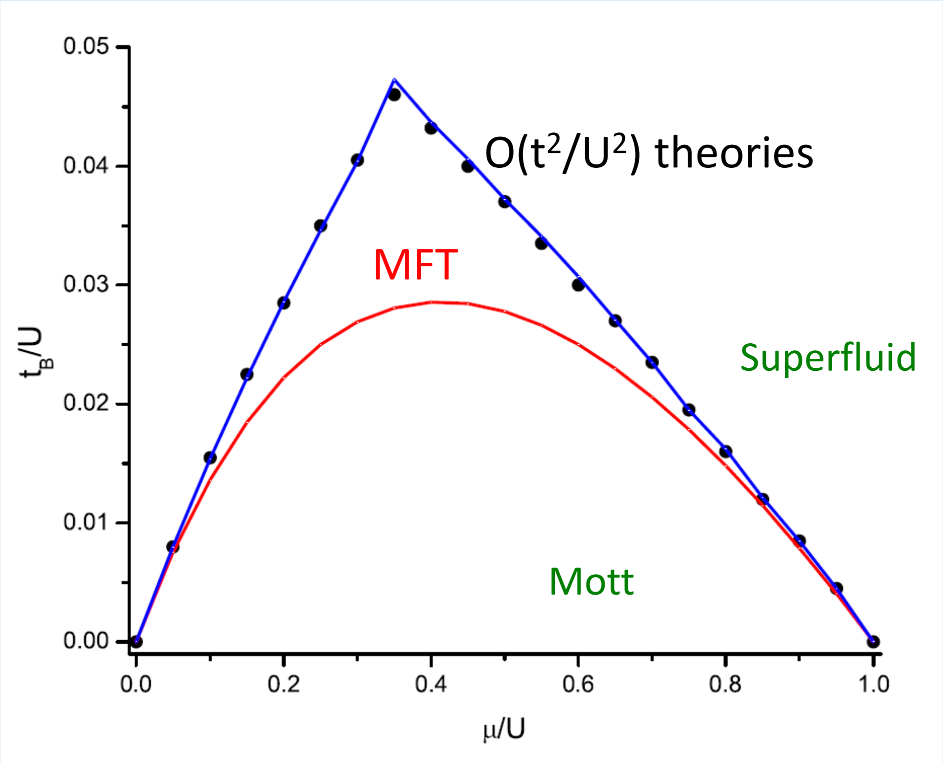


Mean-field theory (Fisher 89)

Quantum Monte Carlo studies for 2D systems: Trivedi and Krauth.

Predicts a quantum phase transition with z=2 (except at the tip of the Mott lobe where z=1).

Phase diagram for n=1 and d=3

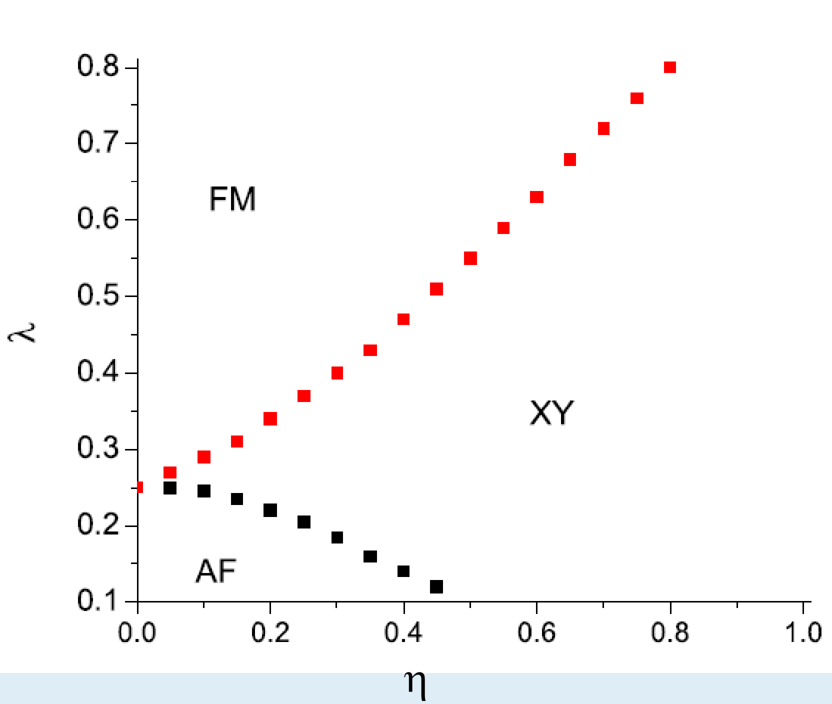


Breaking Translational invariance

Theoretical routes for translational symmetry breaking: Mott states of bosons

Realize two or more species of ultracold bosons on a lattice and rely on order-by-disorder.

$$\begin{aligned}\mathcal{H} &= \sum_{\alpha=1,2}\left[\sum_{<ij>}\left(-t_\alpha b^\dagger_{i,\alpha} b_{j\alpha} + \text{h.c.}\right) - \mu \sum_i n_{i\alpha}\right] \\ &\quad + \frac{U}{2}\left[\sum_{i,\alpha} n_{i,\alpha}\left(n_{i\alpha}-1\right) + 2\lambda \sum_i n_{i1} n_{i2}\right] \\ &= T + H_0\end{aligned}$$



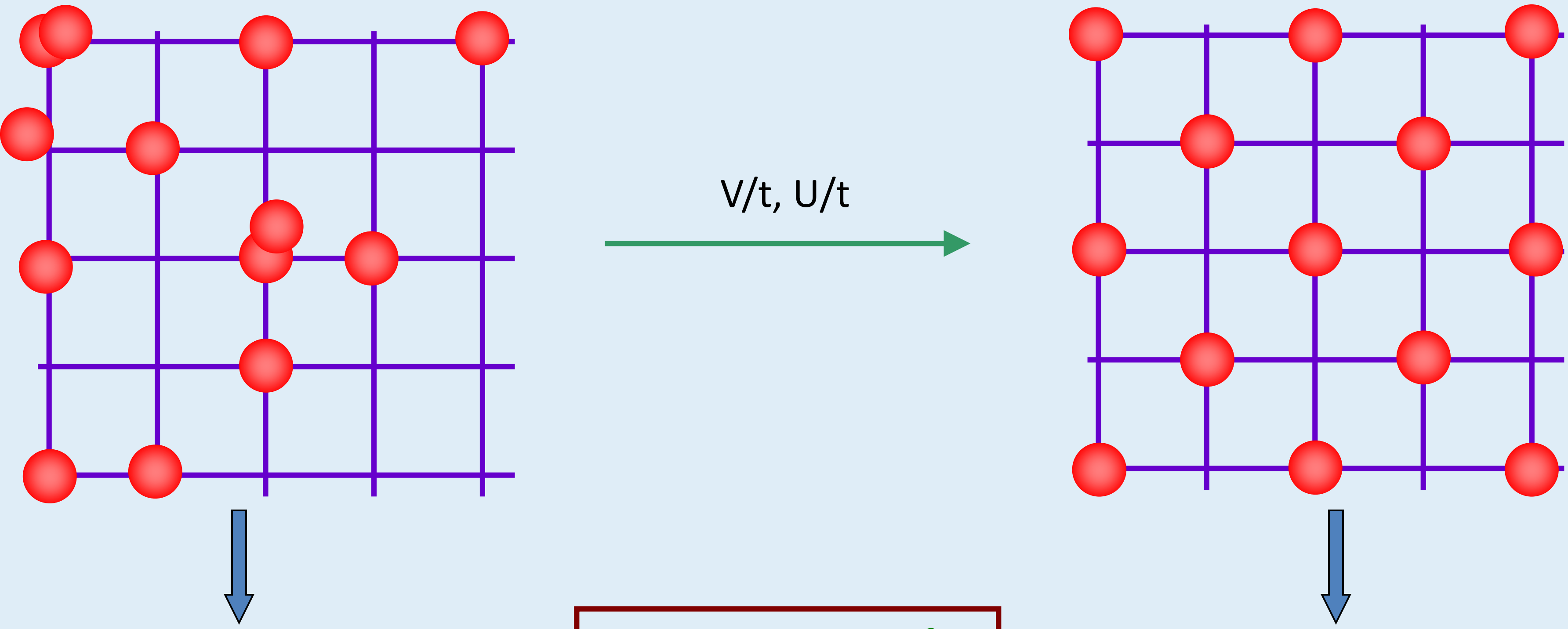
$\text{FM} \equiv |1,0\rangle \qquad \theta_A = \theta_B = 0$

$\text{AF} \equiv |1,0\rangle_A |0,1\rangle_B \qquad \theta_A = 0, \theta_B = \pi$

$\text{XY} \equiv \cos(\theta/2)|1,0\rangle + \sin(\theta/2)|0,1\rangle$
$\theta_A = \theta_B \neq 0$

Add a nearest neighbor interaction between the bosons

$$H_{\text{Bose}} = -t\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + \text{h.c}\right) + \frac{U}{2}\sum_i \hat{n}_i\left(\hat{n}_i - 1\right) + V\sum_{\langle ij\rangle}\hat{n}_i\hat{n}_j - \mu\sum_i \hat{n}_i$$



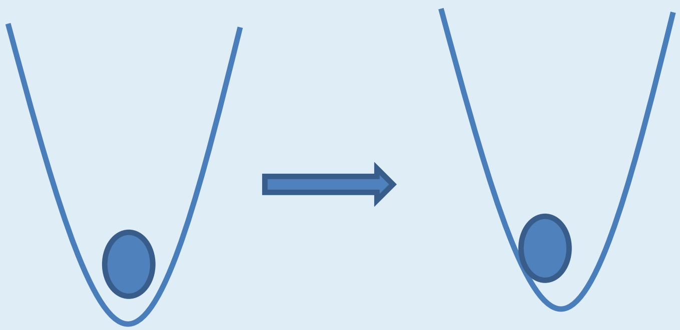
Broken U(1) symmetry

Question: Nature of quantum phase transition?

Broken translation symmetry

Apply an "electric field" to neutral bosons

Shift the center of the trap holding the bosons



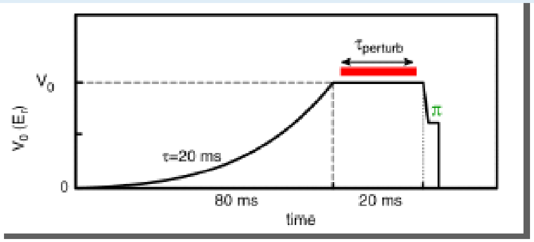
Apply a linearly (spatial) varying magnetic field to spin polarized bosons.

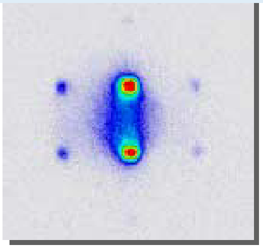
H → H = g μ_B s_z B(x)

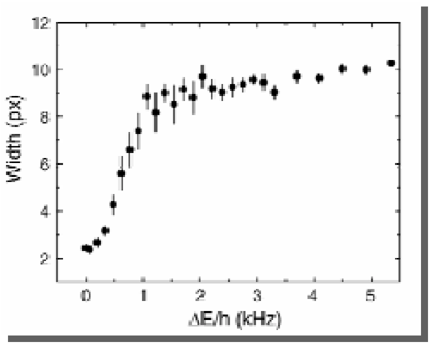
Linear term in the boson Hamiltonian

$$H_{\text{bosons}} = -J\sum_{\langle ij\rangle}(b_i^\dagger b_j + \text{h.c.}) - \sum_i (\mu + \mathcal{E} i)\hat{n}_i + U/2\sum_i \hat{n}_i(\hat{n}_i - 1)$$

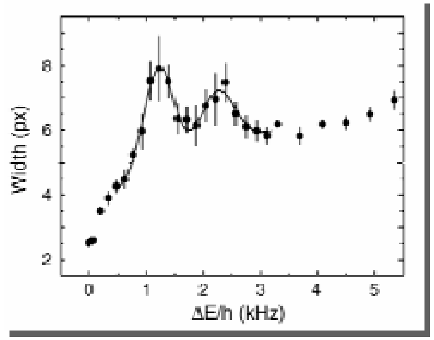
Generation of electric field for neutral bosons





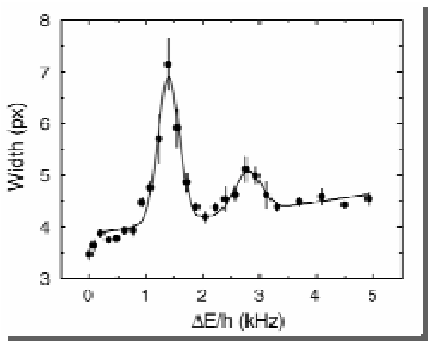


V_0=10 E_{recoil} $\tau_{perturb}$ = 2 ms

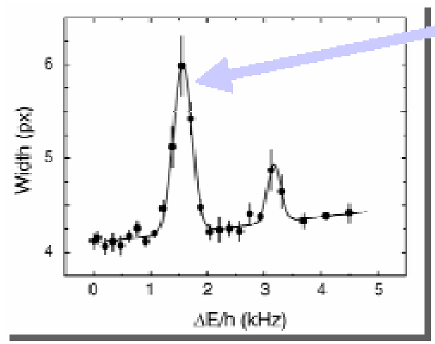


V_0= 13 E_{recoil} $\tau_{perturb}$ = 4 ms

What is the quantum state here ?



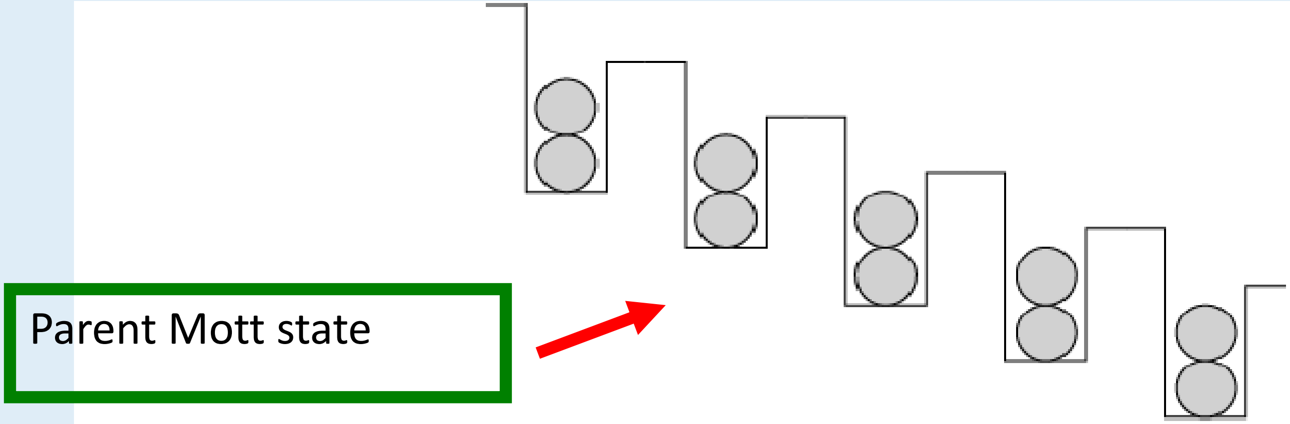
V_0= 16 E_{recoil} $\tau_{perturb}$ = 9 ms



V_0= 20 E_{recoil} $\tau_{perturb}$ = 20 ms

There seem to be sharp resonant absorption at fixed “electric field” amplitude indicating a definite quantum state in the Mott regime

Construction of an effective model: 1D



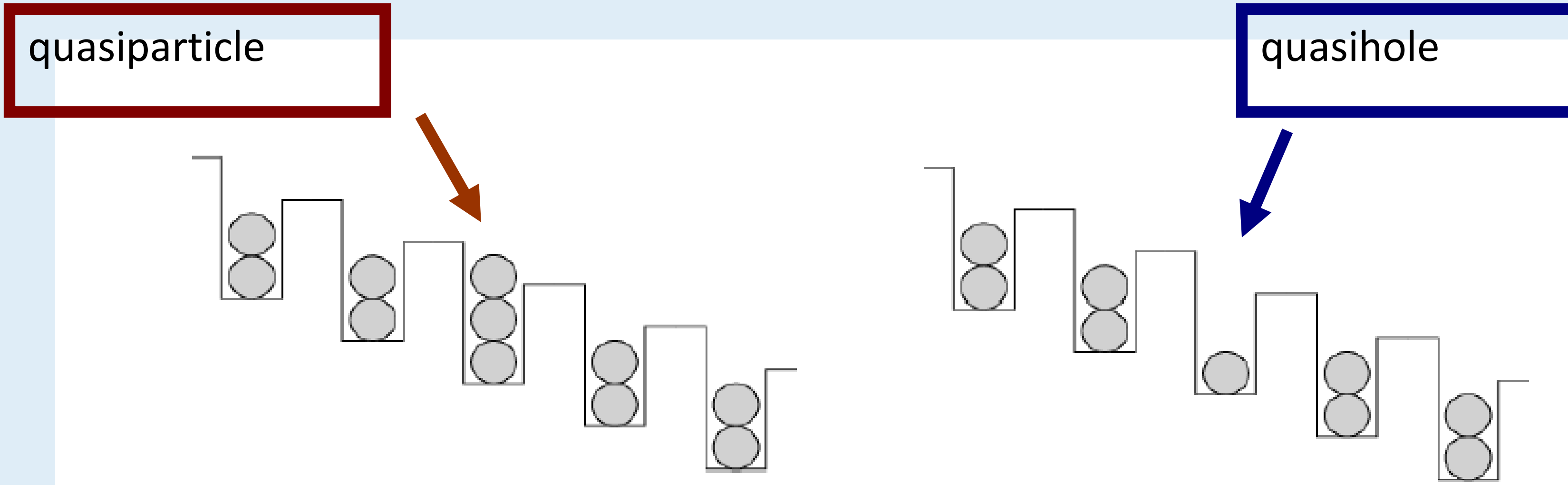
$$H = -w\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

$$n_i = b_i^\dagger b_i$$

$$|U - E|,\, w \ll E, U$$

Describe spectrum in subspace of states resonantly coupled to the Mott insulator

Charged excitations



Effective Hamiltonian for a quasiparticle in one dimension (similar for a quasihole):

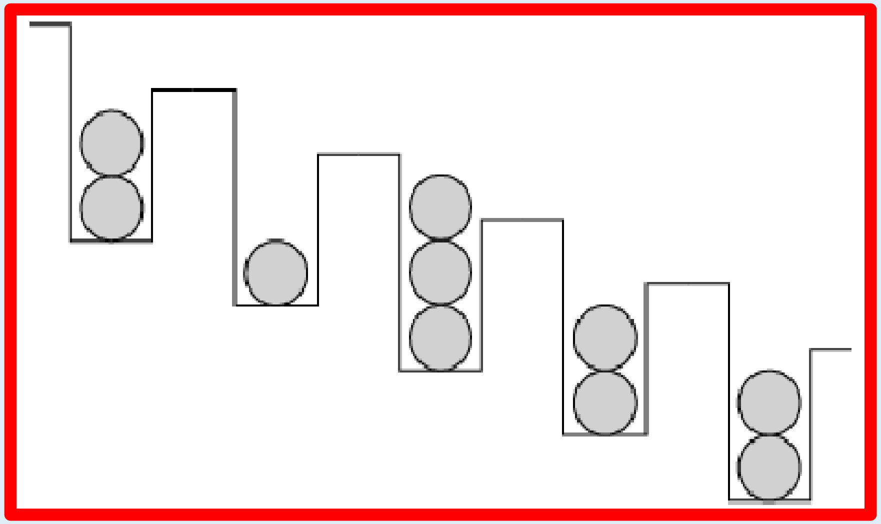
$$H_{\text{eff}} = -\sum_j \left[3w\left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) + Ejb_j^\dagger b_j \right]$$

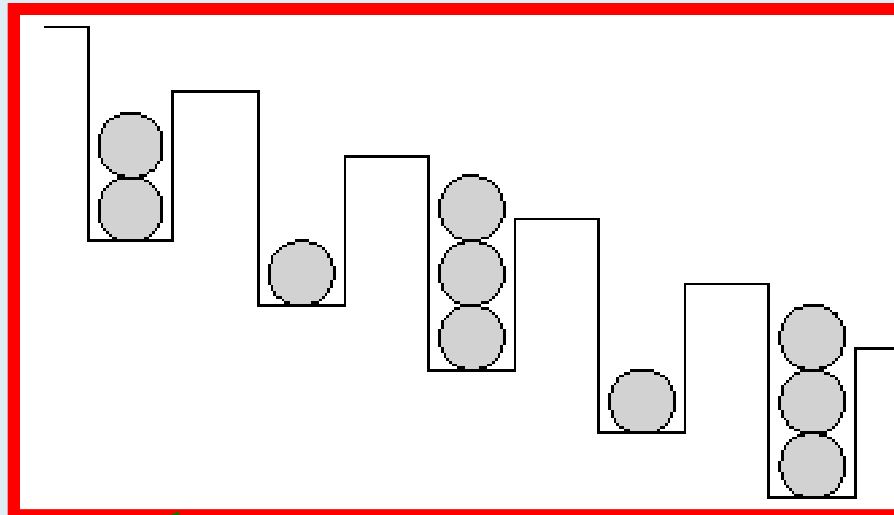
Exact eigenvalues $\varepsilon_m = Em \quad ; \quad m = -\infty \cdots \infty$

Exact eigenvectors $\psi_m(j) = J_{j-m}(6w/E)$

All charged excitations are strongly localized in the plane perpendicular electric field.
Wavefunction is periodic in time, with period h/E (Bloch oscillations)
Quasiparticles and quasiholes are not accelerated out to infinity

Neutral dipoles





Resonantly coupled to the parent Mott state when U=E.

Neutral dipole state with energy U-E.

Two dipoles which are not nearest neighbors with energy 2(U-E).

Effective dipole Hamiltonian: 1D

$d_\ell^\dagger \Rightarrow$ Creates dipole on link ℓ

$$H_d = -\sqrt{6}w\sum_\ell \left(d_\ell^\dagger + d_\ell\right) + (U-E)\sum_\ell d_\ell^\dagger d_\ell$$

Constraints: $d_\ell^\dagger d_\ell \le 1 \quad ; \quad d_{\ell+1}^\dagger d_{\ell+1} d_\ell^\dagger d_\ell = 0$

Determine phase diagram of H_d as a function of $(U\text{-}E)/w$

Note: there is no explicit dipole hopping term.

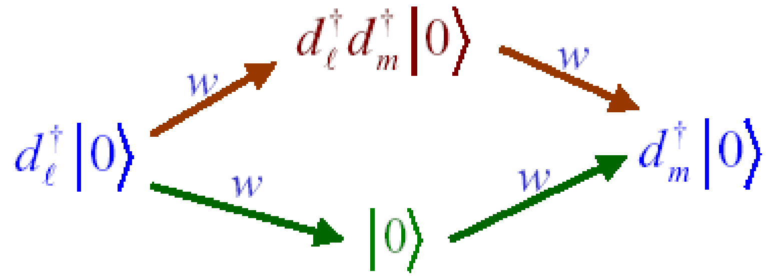
However, dipole hopping is generated by the interplay of terms in H_d and the constraints.

Weak Electric Field

Ground state is dipole vacuum (Mott insulator) $|0\rangle$

First excited levels: single dipole states $d_\ell^\dagger |0\rangle$

Effective hopping between dipole states



If both processes are permitted, they exactly cancel each other.
The top processes is blocked when ℓ, m are nearest neighbors

$\Rightarrow$ A nearest-neighbor dipole hopping term $\sim \dfrac{w^2}{U-E}$ is generated

- The effective Hamiltonian for the dipoles for weak E:

$$\mathcal{H}_{d,\text{eff}} = (U-E)\sum_l \Big[|l><l| + \frac{w^2 n_0(n_0+1)}{(U-E)^2}\Big(|l><l| + |l+1><l| + |l><l+1|\Big)\Big]$$

- Lowest energy excitations: Single band of dipole excitations.

- These excitations soften as E approaches U. This is a precursor of the appearance of Ising density wave with period 2.

- Higher excited states consists of multiparticle continuum.

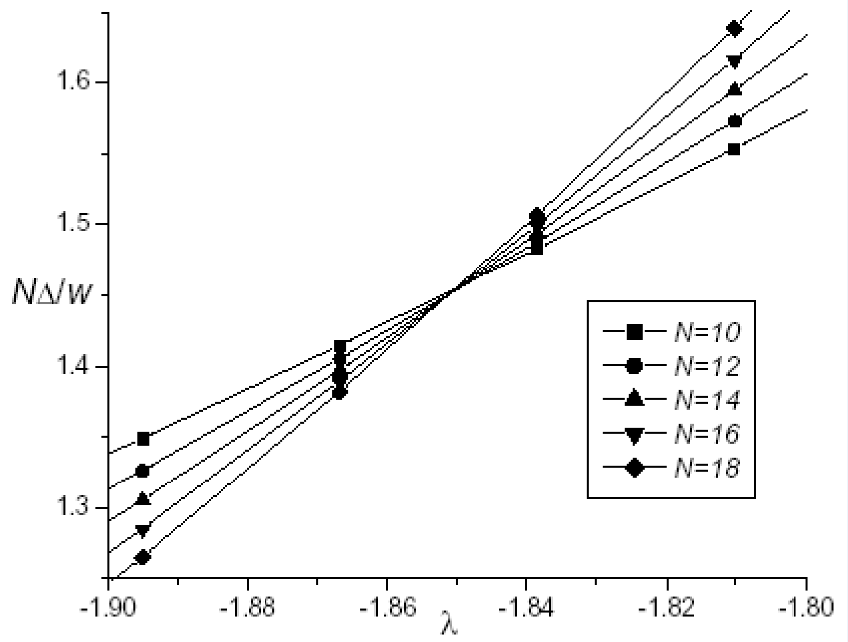
Strong Electric field

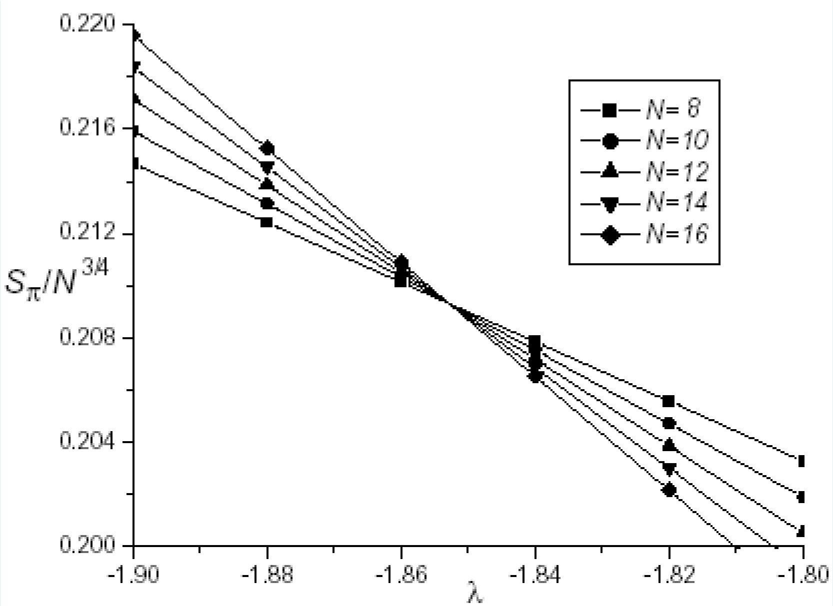
- The ground state is a state of maximum dipoles.
- Because of the constraint of not having two dipoles on consecutive sites, we have two degenerate ground states

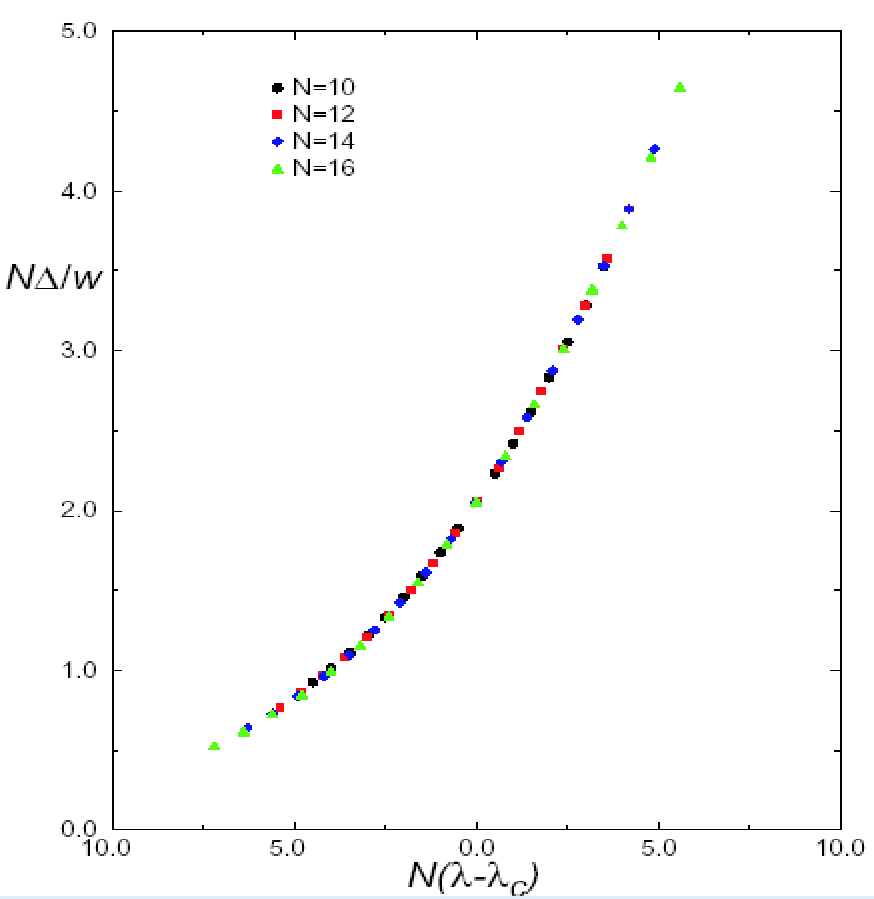
$$\cdots d_1^\dagger d_3^\dagger d_5^\dagger d_7^\dagger d_9^\dagger d_{11}^\dagger \cdots |0\rangle \qquad or \qquad \cdots d_2^\dagger d_4^\dagger d_6^\dagger d_8^\dagger d_{10}^\dagger d_{12}^\dagger \cdots |0\rangle$$

- The ground state breaks Z2 symmetry.
- The first excited state consists of band of domain walls between the two filled dipole states.
- Similar to the behavior of Ising model in a transverse field.

Intermediate electric field: QPT







Quantum phase transition at E-U=1.853w. Ising universality.

Experimental Realization: Antiferromagnetic Spin Chains

Measurements appropriate for the Mott state

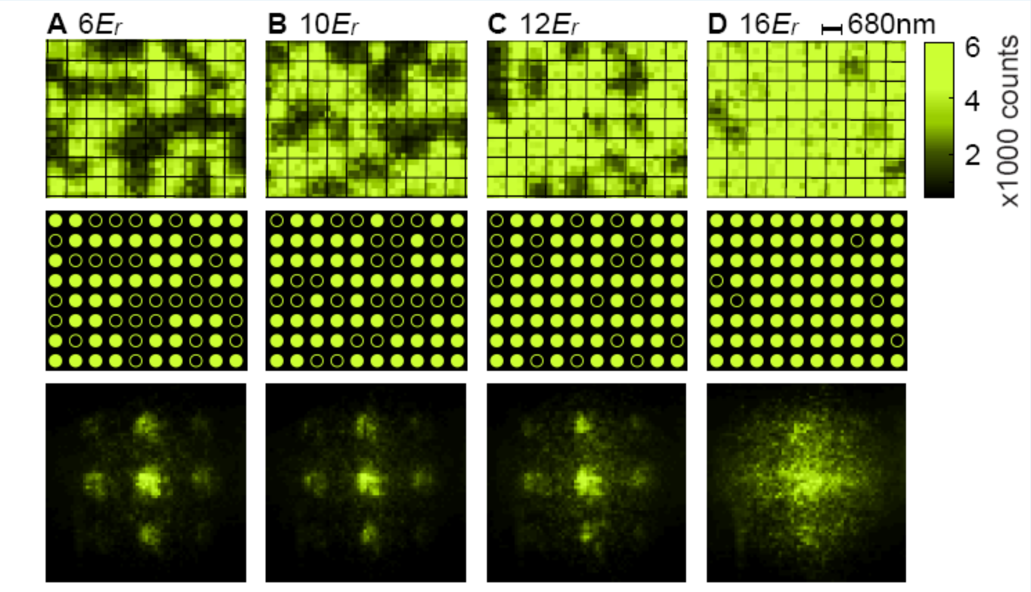


FIG. 1. Single-site imaging of atom number fluctuations across the superfluid-Mott insulator transition. (**A** – **D**) Images within each column are taken at the same final 2D lattice depth of (**A**) $6E_r$, (**B**) $10E_r$, (**C**) $12E_r$ and (**D**) $16E_r$. Top row: in-situ fluorescence images from a region of 10×8 lattice sites within the $n = 1$ Mott shell that forms in a deep lattice. In the superfluid regime (**A**, **B**), sites can be occupied with odd or even atom numbers, which appear as full or empty sites respectively in the images. In the Mott insulator, occupancies other than 1 are highly suppressed (**D**). Middle row: results of the atom detection algorithm [16] for images in the top row. A full (empty) circle indicates the presence (absence) of an atom on a site. Bottom row: time of flight fluorescence images after 8ms expansion of the cloud in the 2D plane as a result of non-adiabatically turning off the lattice and the transverse confinement (averaged over 5 shots and binned over 5×5 lattice sites).

1. *Prepare the state with 2D ^{87}Rb BEC of 10^5 atoms ($F=m_f=1$): use magnetic trap potential.*

2. Project a square optical lattice on it with λ=680nm and ramp the lattice depth.

3. After the state is prepared, increase the lattice depth to freeze the atoms.

4. Apply light so that light-assisted collision eject pair of atoms from each site.

5. Image the remaining atoms to detect parity of occupation.

6. For SF, $p_{odd} = 0.5(1-e^{-\langle n \rangle}) < 0.5$ while for Mott state p_{odd}=0,1.

Realizing the tilted Bose-Hubbard model

Experimental generation of a linearly varying Zeman field along x to generate the tilt.

One can describe the system using dipoles or spins via the following transformation

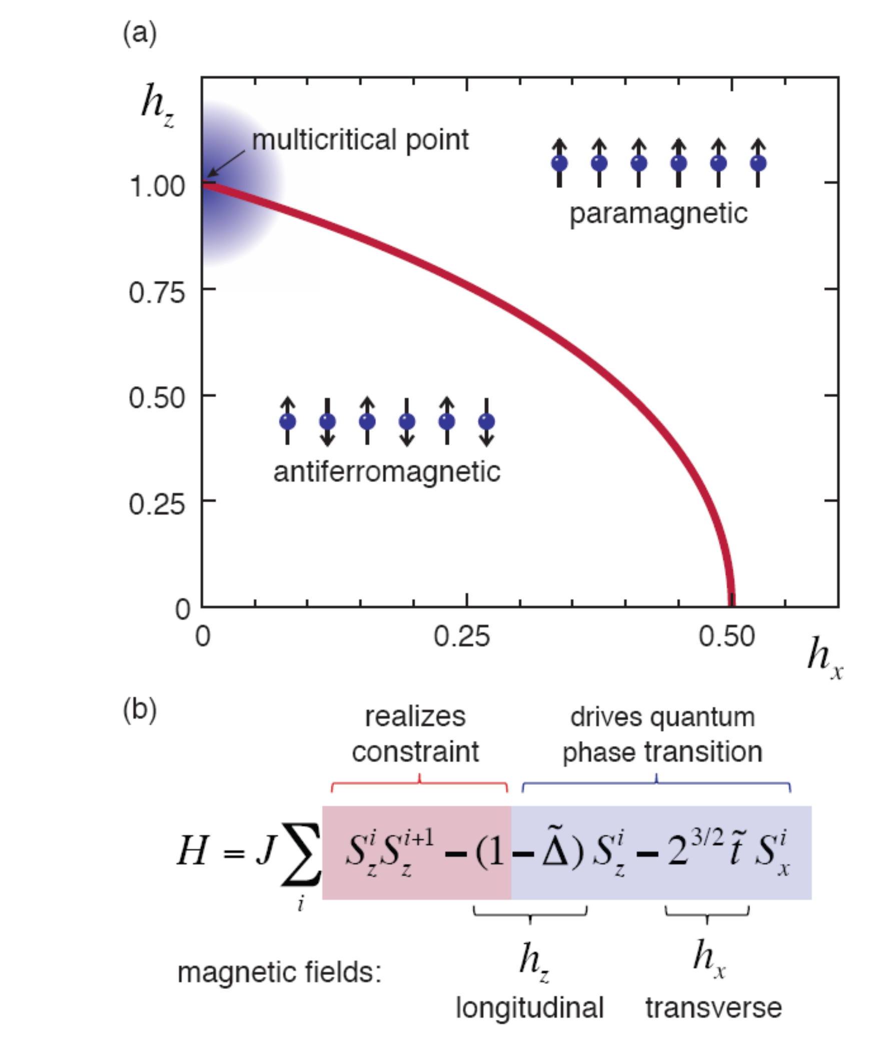
$$S_z^j = \frac{1}{2} - d_j^\dagger d_j, S_x^j = \frac{1}{2}\left(d_j^\dagger + d_j\right), \text{and } S_y^j = \frac{i}{2}\left(d_j^\dagger - d_j\right)$$

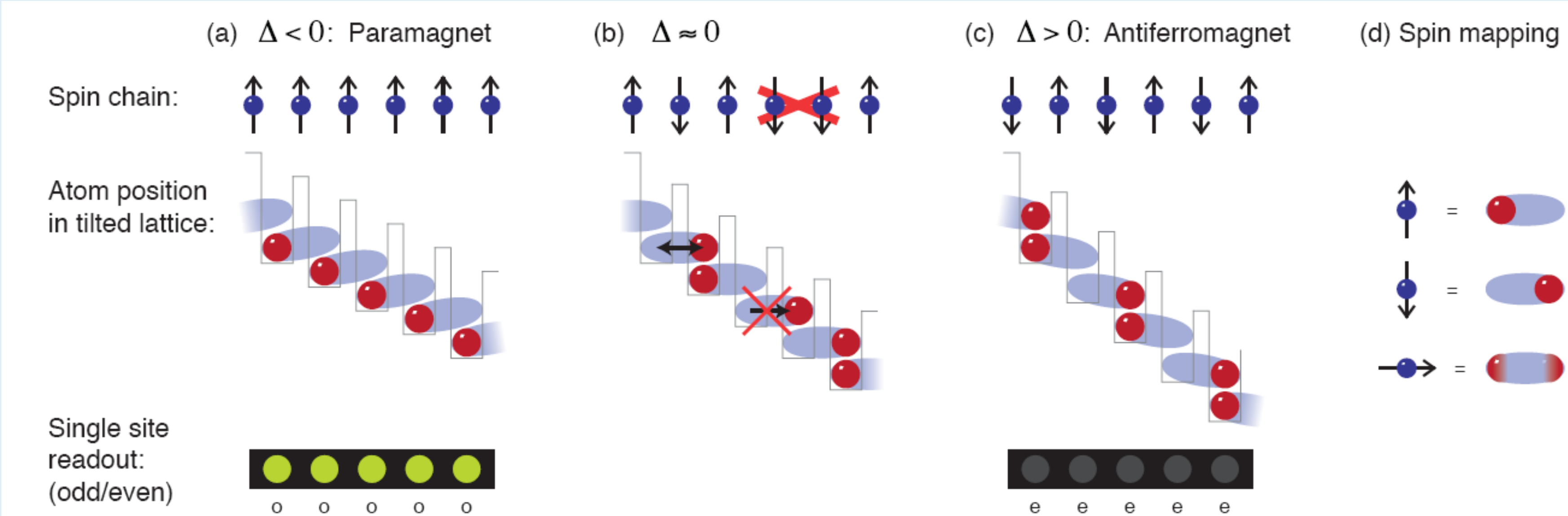
The dipole Hamiltonian can then be mapped to the spin Hamiltonian where the constraint is realized by the J term.

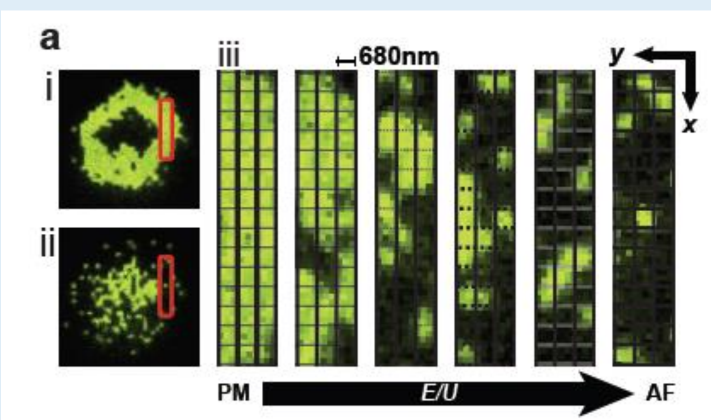
$$H = -\sqrt{M(M+1)}t\sum_j\left(d_j^\dagger + d_j\right) + (U-E)\sum_j d_j^\dagger d_j$$

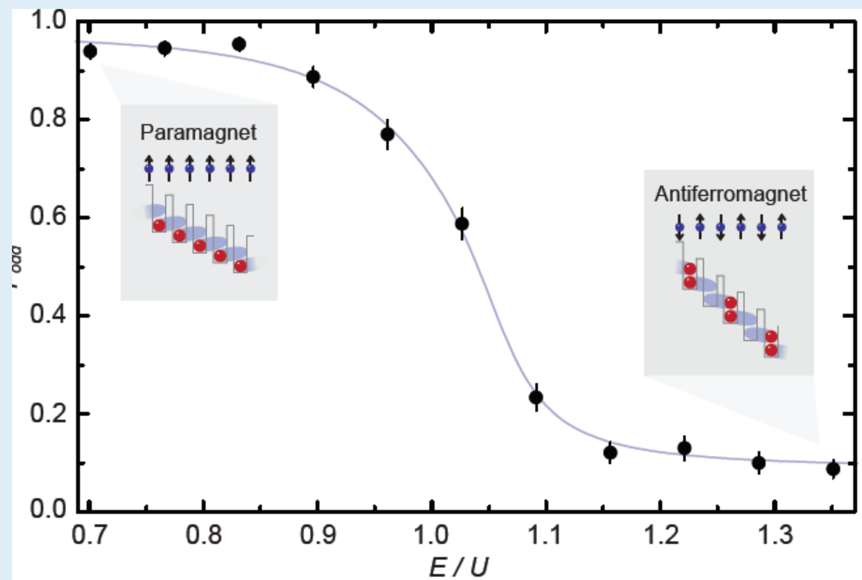
$$\Downarrow$$

$$\begin{aligned} H = & J\sum_j S_z^j S_z^{j+1} - 2\sqrt{M(M+1)}t\sum_j S_x^j \\ & -(J-\Delta)\sum_j S_z^j \\ = & J\sum_j\left(S_z^j S_z^{j+1} - h_x S_x^j - h_z S_z^j\right) \end{aligned}$$




(a) Δ < 0: Paramagnet
(b) Δ ≈ 0
(c) Δ > 0: Antiferromagnet
(d) Spin mapping
Spin chain:
Atom position in tilted lattice:
Single site readout: (odd/even)
o o o o o
e e e e e
=
=
=

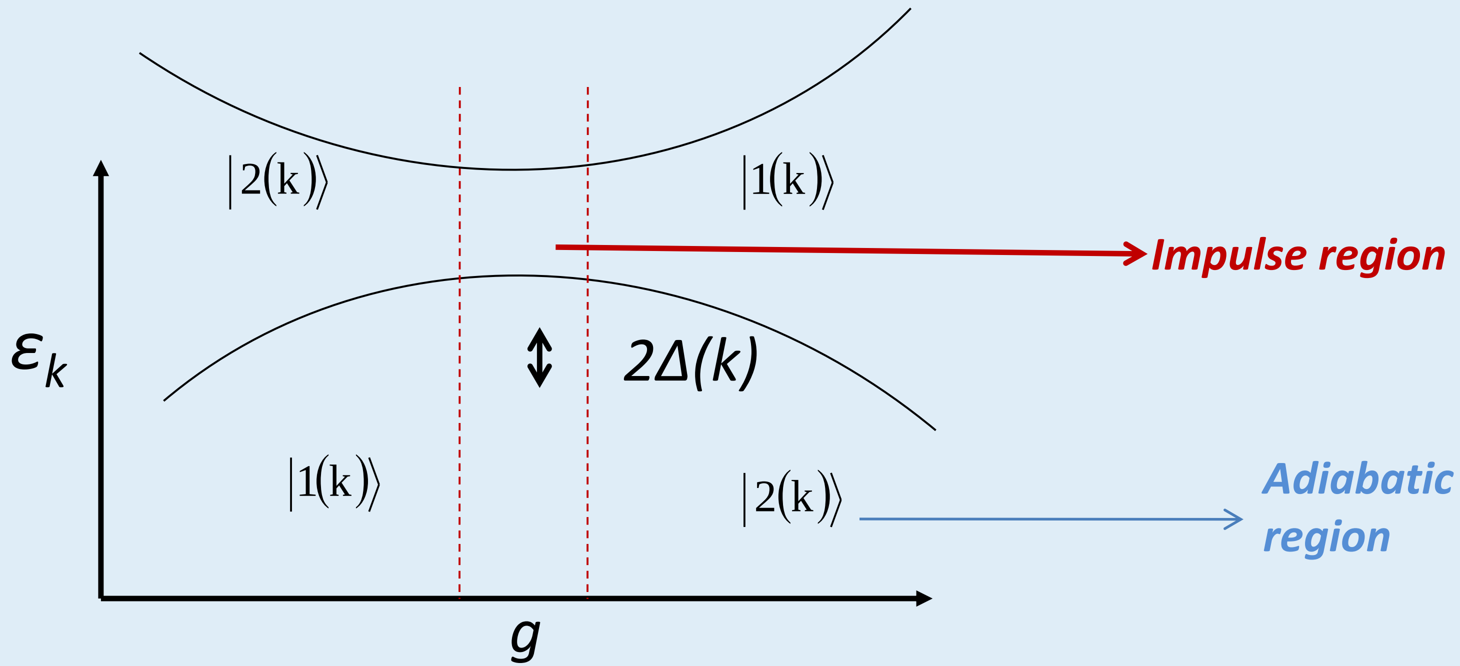

a
i
ii
iii
680nm
y
x
PM
E/U
AF


1.0
0.8
0.6
0.4
0.2
0.0
0.7
0.8
0.9
1.0
1.1
1.2
1.3
E / U
Paramagnet
Antiferromagnet

Quantum dynamics for the tilted Bose-Hubbard model

Introduction to Kibble Zureck Scaling

What is the rate at which excitations are produced when a system is taken "slowly" through a critical point via a ramp?



Away from the avoided crossing, for slow ramp rate, there are no excitations.

Probability of the system to be in the excited state: **$p = \exp[-\pi\, \Delta(t)^2 / d\, \Delta(t)/dt\,]$**

At and near the avoided crossing, the system produces excitations. This happens if

$$d\, \Delta(t)/dt > \Delta(t)^2$$

Landau criteria for excitation production

Generic critical points: A phase space argument

The system enters the impulse region when rate of change of the gap is the same order as the square of the gap.

$$d\ln(\Delta_{\vec{k}})/dt \;\geq\; \Delta_{\vec{k}}$$

For slow dynamics, the impulse region is a small region near the critical point where scaling works

$$\Delta_{\vec{k}} \;\sim\; \lambda^{z\nu}|t/\tau|^{z\nu}$$

The system thus spends a time T in the impulse region which depends on the quench time

$$T \sim \tau^{z\nu/(z\nu+1)}$$

In this region, the energy gap scales as

$$\Delta_{\mathbf{k}} \sim \tau^{-z\nu/(z\nu+1)}$$

Thus the scaling law for the defect density turns out to be

$$\Omega_n \sim |\mathbf{k}|^d \sim \Delta_{\mathbf{k}}^{d/z} \sim \tau^{-\nu d/(z\nu+1)}$$

Since the defects are primarily dipoles, one expects dipole density to have same scaling

Computation for the dipole chain

Solve the Schrodinger equation

$$i\hbar\partial_t|\psi(t)\rangle = H_d(t)|\psi(t)\rangle. \quad \mathcal{E}(t) = \mathcal{E}_0 + (\mathcal{E}_f - \mathcal{E}_0)t/\tau$$

$$H_d(t) = [U - \mathcal{E}(t)]\sum_\ell d_\ell^\dagger d_\ell - J\sum_\ell (d_\ell^\dagger + d_\ell),$$

Numerically, it is easiest to express the wavefunction in terms of the basis of $H_d(t=\tau)$

$$|\psi(t)\rangle = \sum_m c_m(t)|m\rangle_f .$$

The Schrodinger equation can then be written as

$$\begin{aligned} i\hbar\partial_t|\psi(t)\rangle &= H_0(t)|\psi(t)\rangle = [H_0[\mathcal{E}_f] + \Delta H_0(t)]|\psi(t)\rangle \\ \Delta H_0(t) &= (\mathcal{E}_f - \mathcal{E}_0)(1 - t/\tau)\sum_\ell \hat{n}_\ell^d \qquad (\end{aligned}$$

This leads to equations for the coefficients c_m (t)

$$\begin{aligned} (i\hbar\partial_t - E_m^f)c_m(t) &= (\mathcal{E}_f - \mathcal{E}_0)(1 - t/\tau)\Lambda_{nm}(t) \\ \Lambda_{nm} &= \sum_n c_n(t)\, {}_f\langle m|\sum_\ell \hat{n}_\ell^d|n\rangle_f \end{aligned}$$

Finally, one can express any expectation value in terms of $c_m(t)$

$$\begin{aligned} n_d &= \frac{1}{L}\langle\psi(\tau)|\sum_l \hat{n}_\ell^d|\psi(\tau)\rangle = \frac{1}{L}\sum_{m,n} c_m^*(\tau)c_n(\tau)\Lambda_{mn} \\ D &= n_d - \Lambda_{00}, \quad |F|^2 = |\langle 0|\psi(\tau)\rangle|^2 = |c_0(\tau)|^2 \\ Q &= \langle\psi(\tau)|H(\tau)|\psi(\tau)\rangle = \sum_{m\neq 0} E_m^f|c_m(\tau)|^2. \qquad (\end{aligned}$$

Effect of finite size

For slow enough rate, the dynamics is controlled by the presence of system-size induced gap: Landau-Zenner regime

The Kibble-Zureck regime showing critical scaling now shifts to higher ramp rates

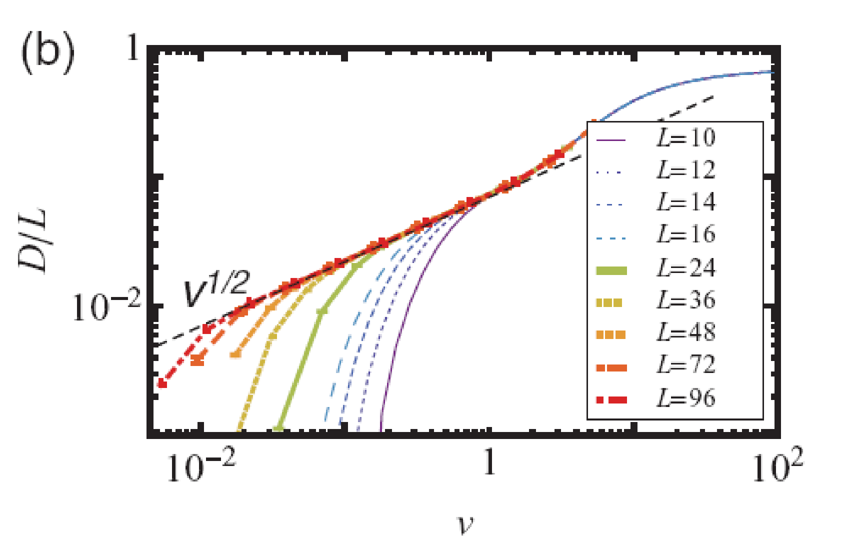
One can encode this fact using a finite-size scaling theory

$$F \sim L^d \tau^{-d\nu/(z\nu+1)} s_1(L^{1/\nu+z}/\tau)$$
$$Q \sim L^d \tau^{-(d+z)\nu/(z\nu+1)} s_2(L^{1/\nu+z}/\tau)$$

$$s_{1,2}(y >> 1) \sim 1$$
$$s_1(y \ll 1) \sim y^{2-d\nu/(z\nu+1)}$$
$$s_2(y \ll 1) \sim y^{2-(d+z)\nu/(z\nu+1)}.$$

Thus the KZ regime occurs for $\tau << L^{(1/\nu+z)}$

Plot of excitation (dipole) density as a function of ramp rate $v=1/\tau$ showing the extent of the Kibble Zureck regime.



Experimental route to verification of Kibble-Zureck scaling in isolated Quantum systems.

Other symmetry broken states

Theoretical route using tilted Bose-Hubbard

Add a repulsive interaction between dipoles on different sites

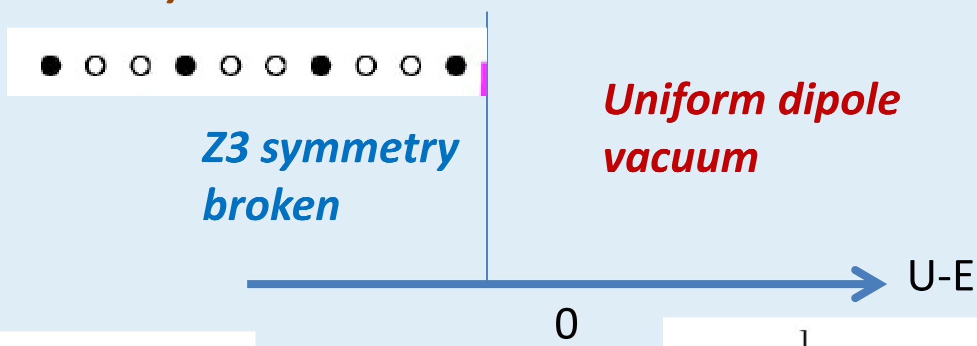
$$H_0 \;=\; H_d + H_1,$$

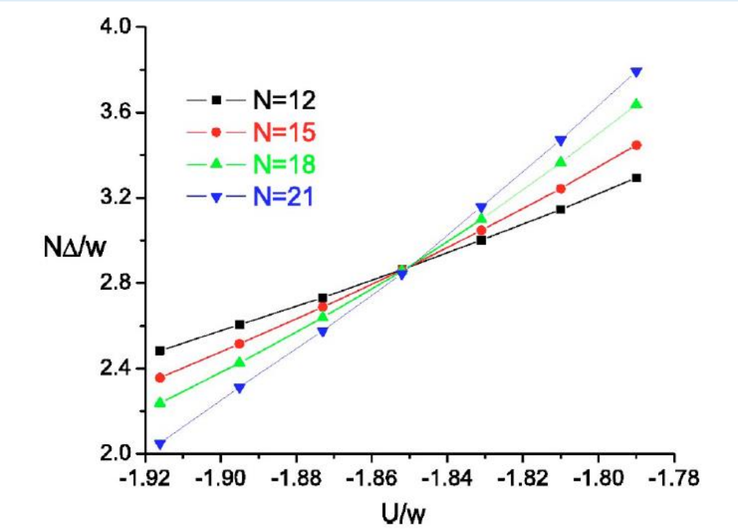
$$H_1 = \sum_{\ell_1 \ell_2} V_{\ell_1 \ell_2} \hat{n}^d_{\ell_1} \hat{n}^d_{\ell_2}$$

$$H_d \;=\; = -w \sum_{\ell} (d_\ell + d_\ell^\dagger) + (U - \mathcal{E}) \sum_i \hat{n}^d_\ell$$

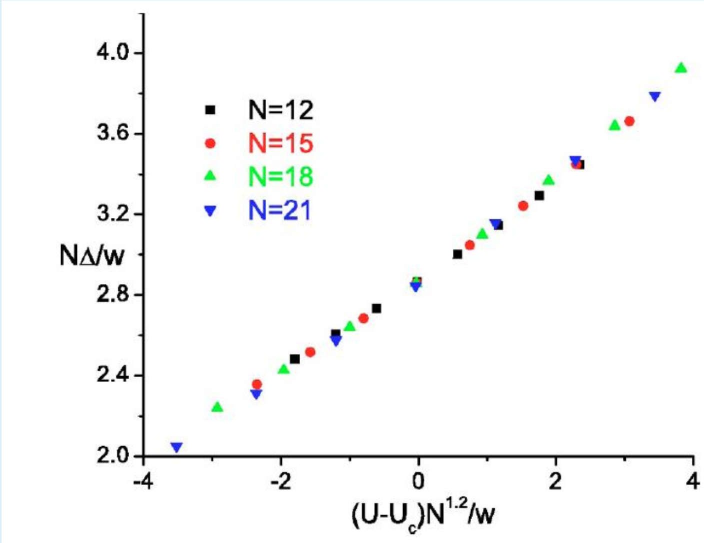
Putting $V_{l,l+2}$ -> infinity ensures a Z_3 symmetry broken state for large negative U-E.

The phase for the model has been worked out. The intermediate critical point belongs to 2D classical 3 state Potts with ν=5/6 and z=1.





Finite size scaling for dipole chains leads to ν=0.833 and z=1 consistent With 3-state Potts Universality.



Ramp dynamics

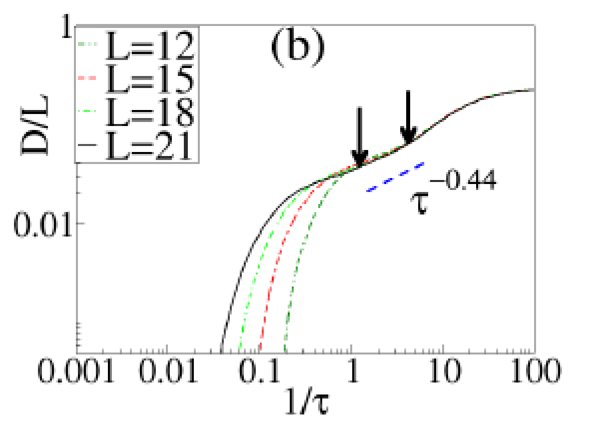
Linear ramp to the critical point from the dipole vacuum state

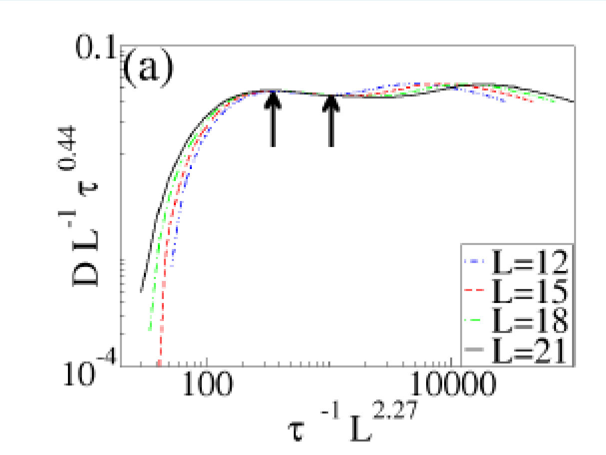
$$\mathcal{E}(t) = \mathcal{E}_0 + (\mathcal{E}_f - \mathcal{E}_0)t/\tau$$

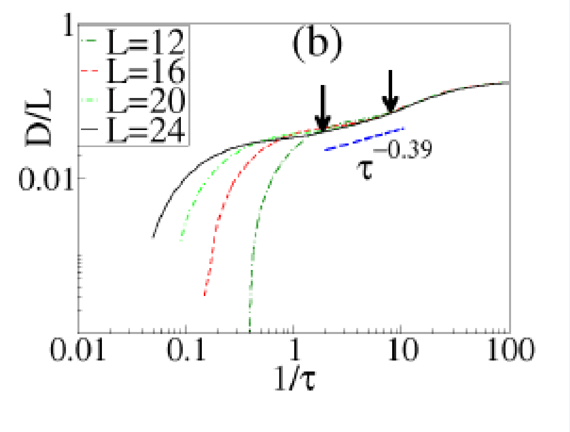
Theoretically expected Kibble-Zureck exponent depends on universality class through z and ν

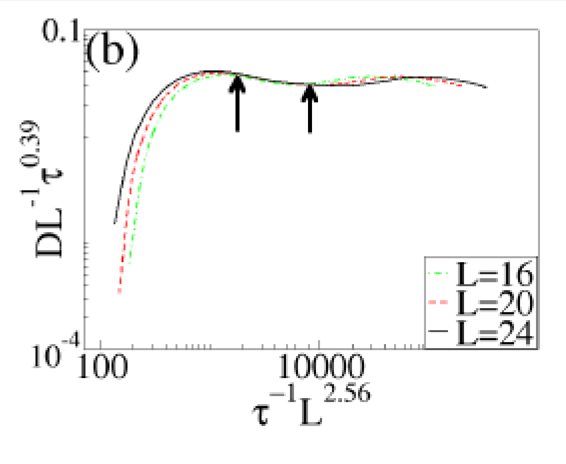
For the 3-state Potts z=1 and ν=5/6 KZ exponent is expected to be 5/11

For the 4-state Potts z=1 and ν=2/3 KZ exponent is expected to be 2/5





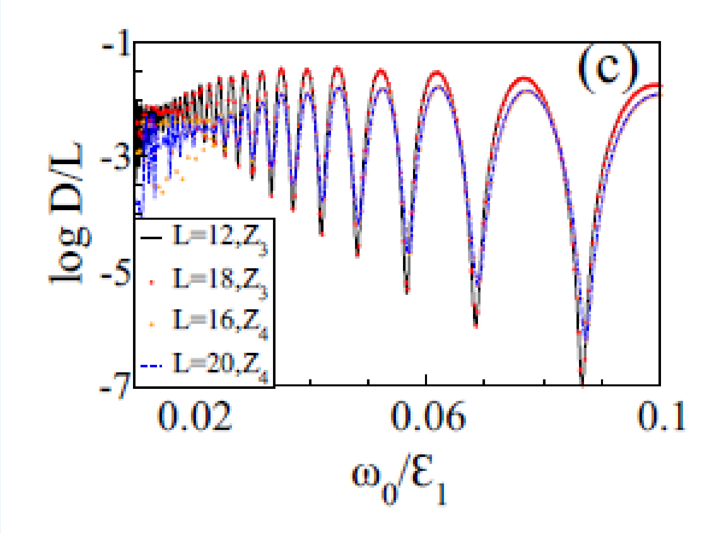




Periodic dynamics

Drive the system periodically through the critical point → ***Measure excitation density D at the end of a single drive cycle***

$$\mathcal{E}(t) = \mathcal{E}_0 - \mathcal{E}_1 \cos(\omega_0 t)$$



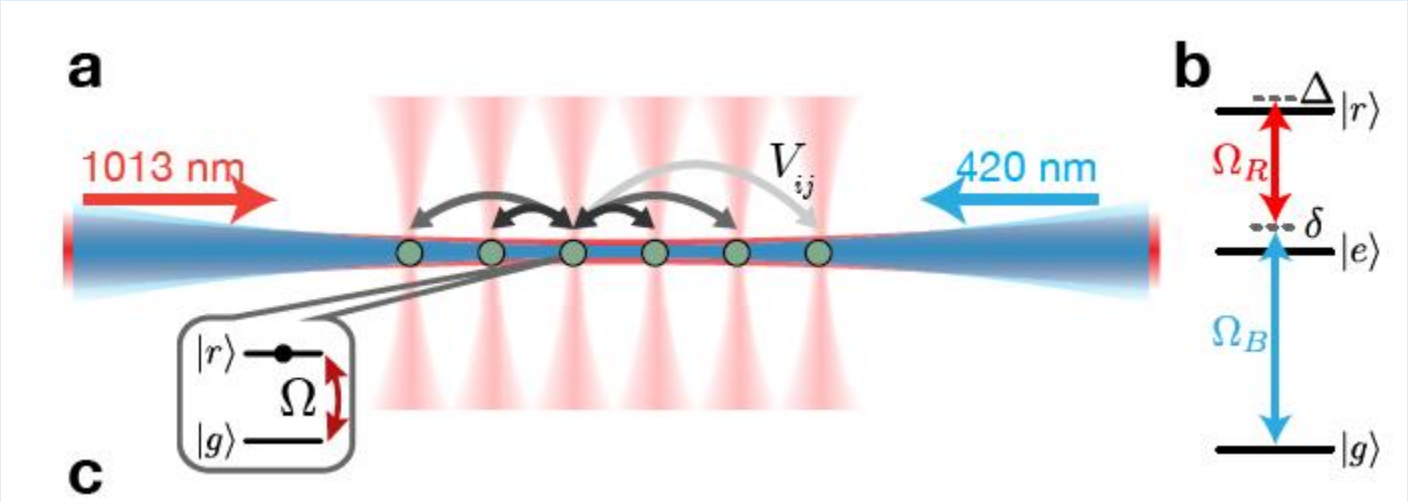
D is a non-monotonic function of the drive frequency.

For special values of the drive frequency D almost vanishes. The system return to its starting (ground) state with unit probability.

This phenomenon occurs due to near-perfect destructive quantum interference on successive Passage through the impulse region around the Quantum critical point. → ***Dynamics induced freezing***

Many-body realization of Stuckelberg interference phenomenon

Experimental realization using Rydberg atoms



$|g\rangle = |5S_{1/2}, F=2, m_F=-2\rangle$ $|r\rangle = |71S_{1/2}, J=1/2, m_J=-1/2\rangle$

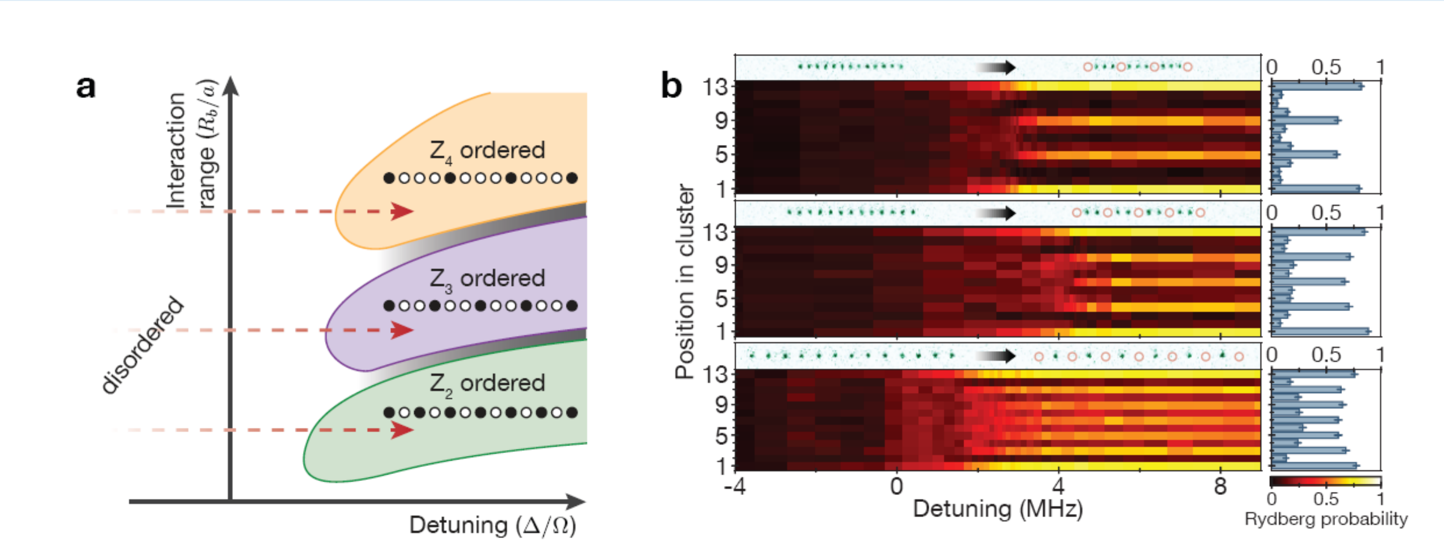
System of ^{87}Rb atoms controllably coupled to their Rydberg excited state.

The interaction between two atoms in their excited (Rydberg) states is denoted by V and is a tunable parameter.

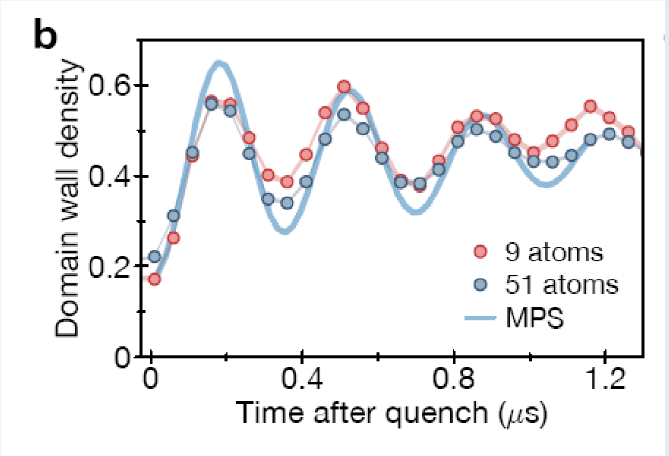
One can vary the detuning parameter Δ which allows one to preferentially put the atom in a Rydberg or ground state

$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

The ground state of the Hamiltonian is analogous to the dipole model with the identification of the Rydberg atom density with the dipole density



Realization of states with broken Z_n symmetries (n=2,3,4) with Rydberg atoms



Evolutions following quench dynamics:
Robust long-lived quantum oscillations

Conclusion