Stochastic and Deterministic Models for Tropical Convection

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Outline

- Introduction
- The deterministic multicloud model for organized convection
- Stochastic interaction system for cloud area fractions
- Effect of congestus detrainment
- Column model and (x,t) simulations for flows over the equator.

Why a stochastic model for convection?

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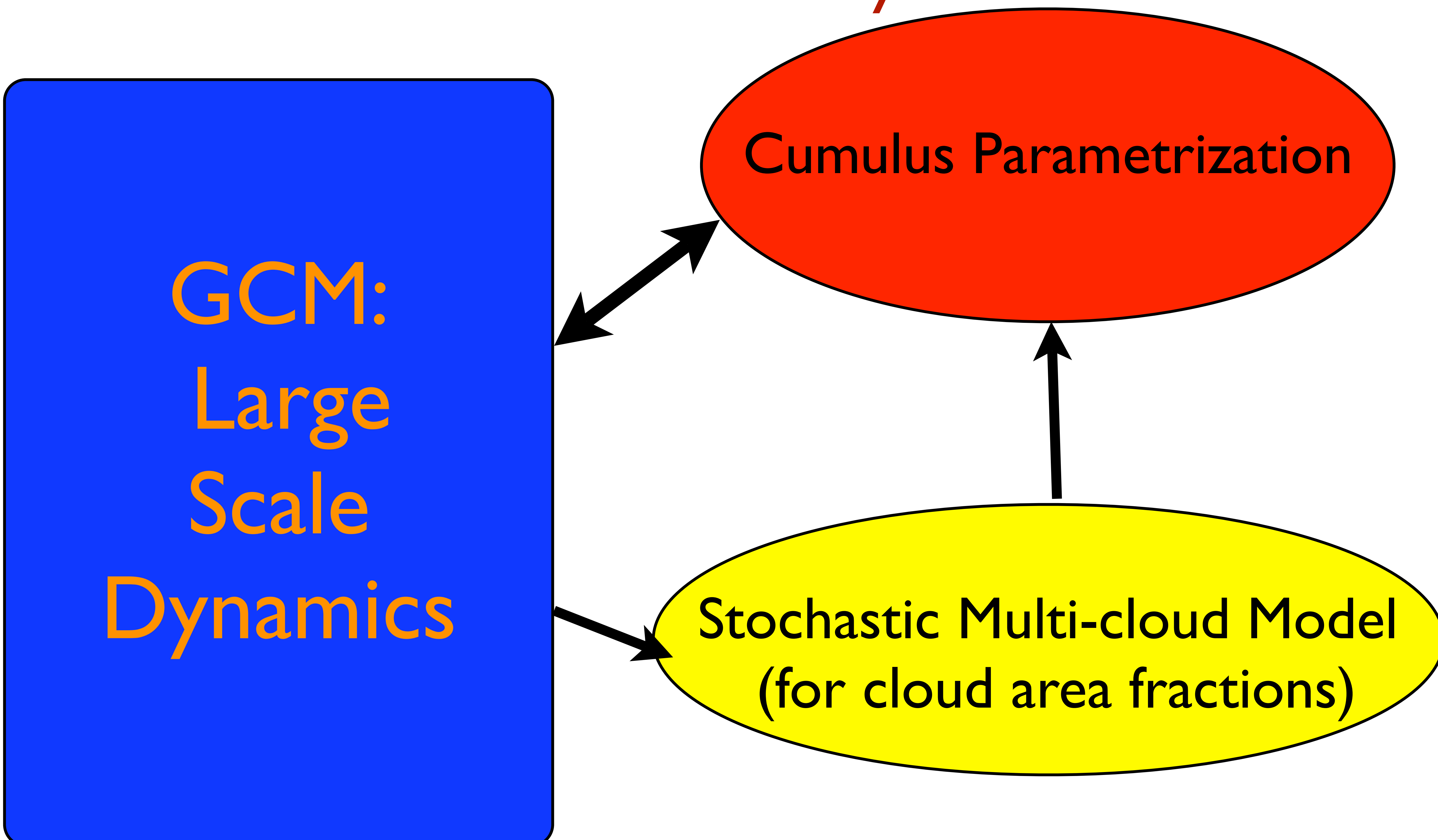
- How cloud systems interact with each other and with the environment?
- Adequate representation of sub-grid dynamics based on Statistical-self similarity across-scales of tropical convective systems
- Capture deviations from quasi-equilibrium paradigm
- Improve tropical variability in climate models ---> reduce model error
- We propose a stochastic model for cloud area-fractions ...

Stochastic Multi-cloud Model to inform cumulus parametrization: represent the missing sub-grid scale variability



Main cloud types of tropical weather

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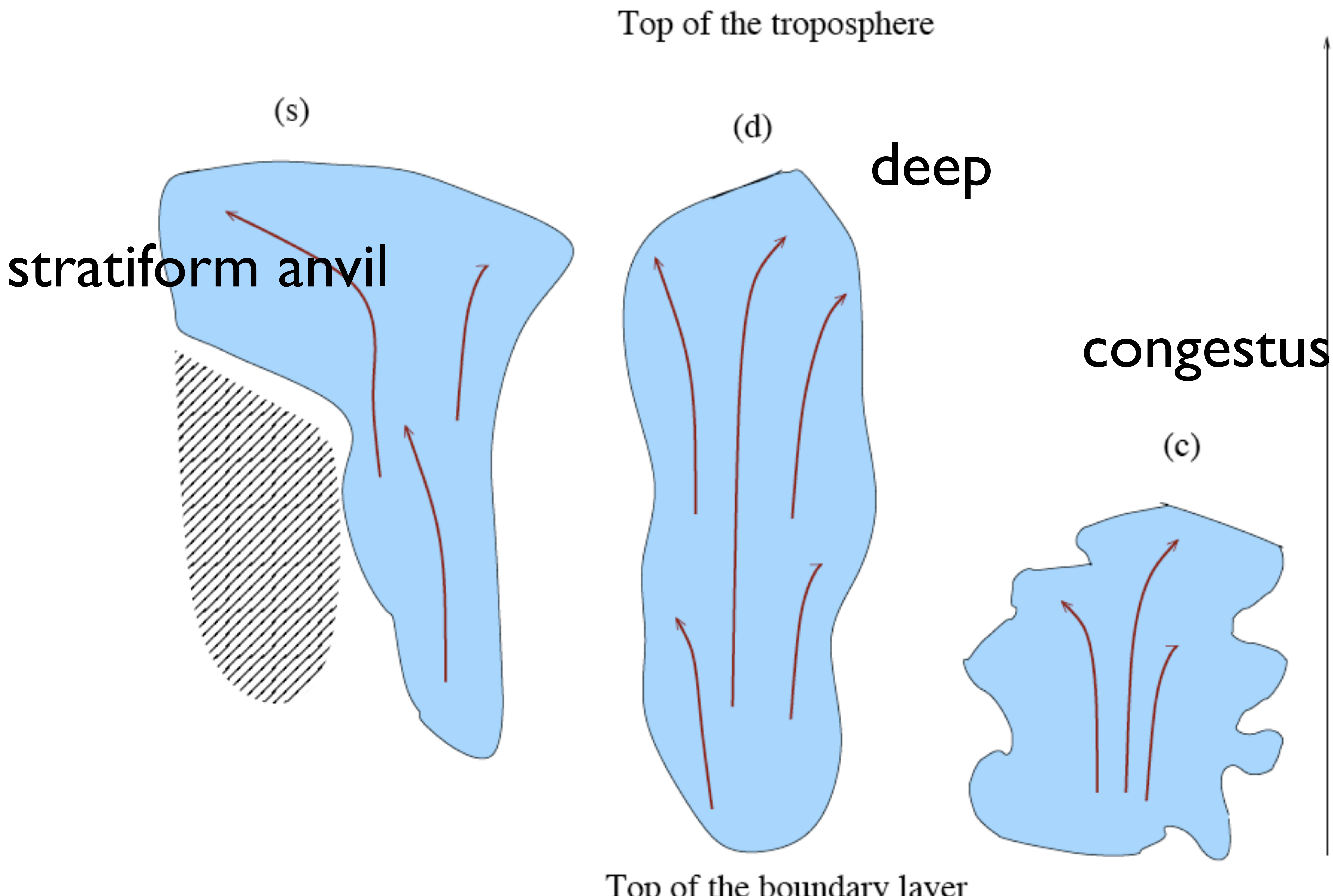
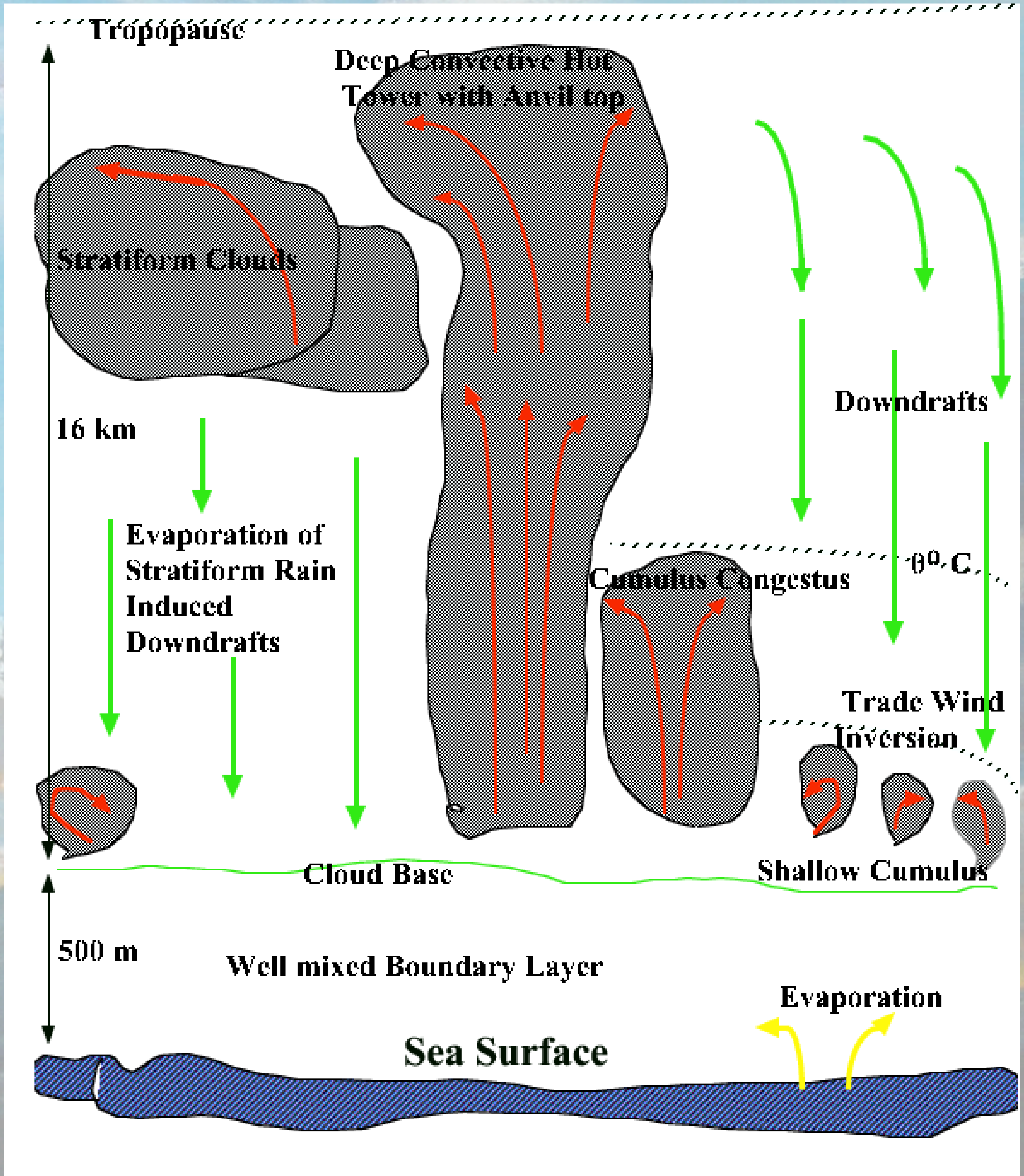
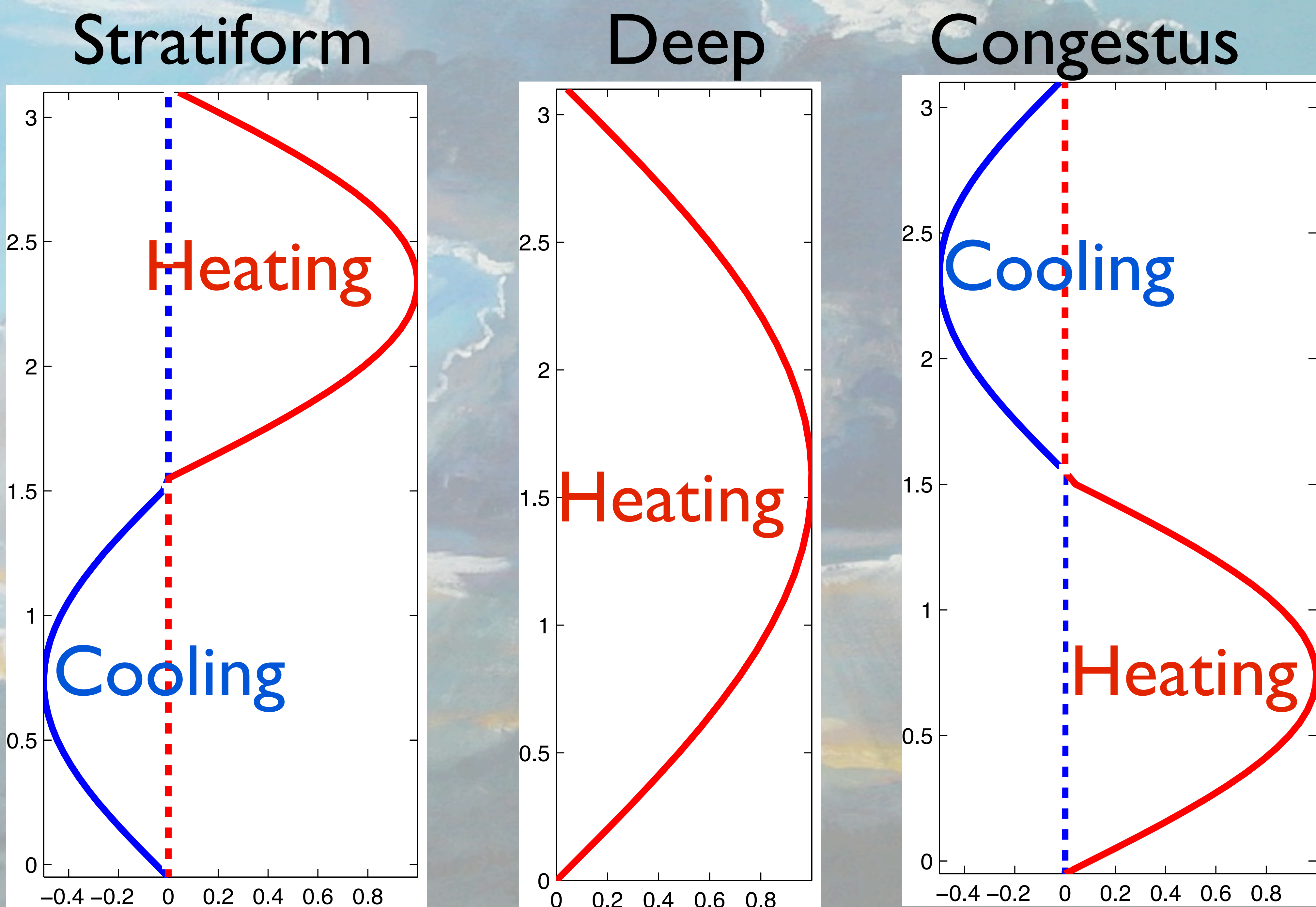


DETERMINISTIC MULTICLOUD MODEL

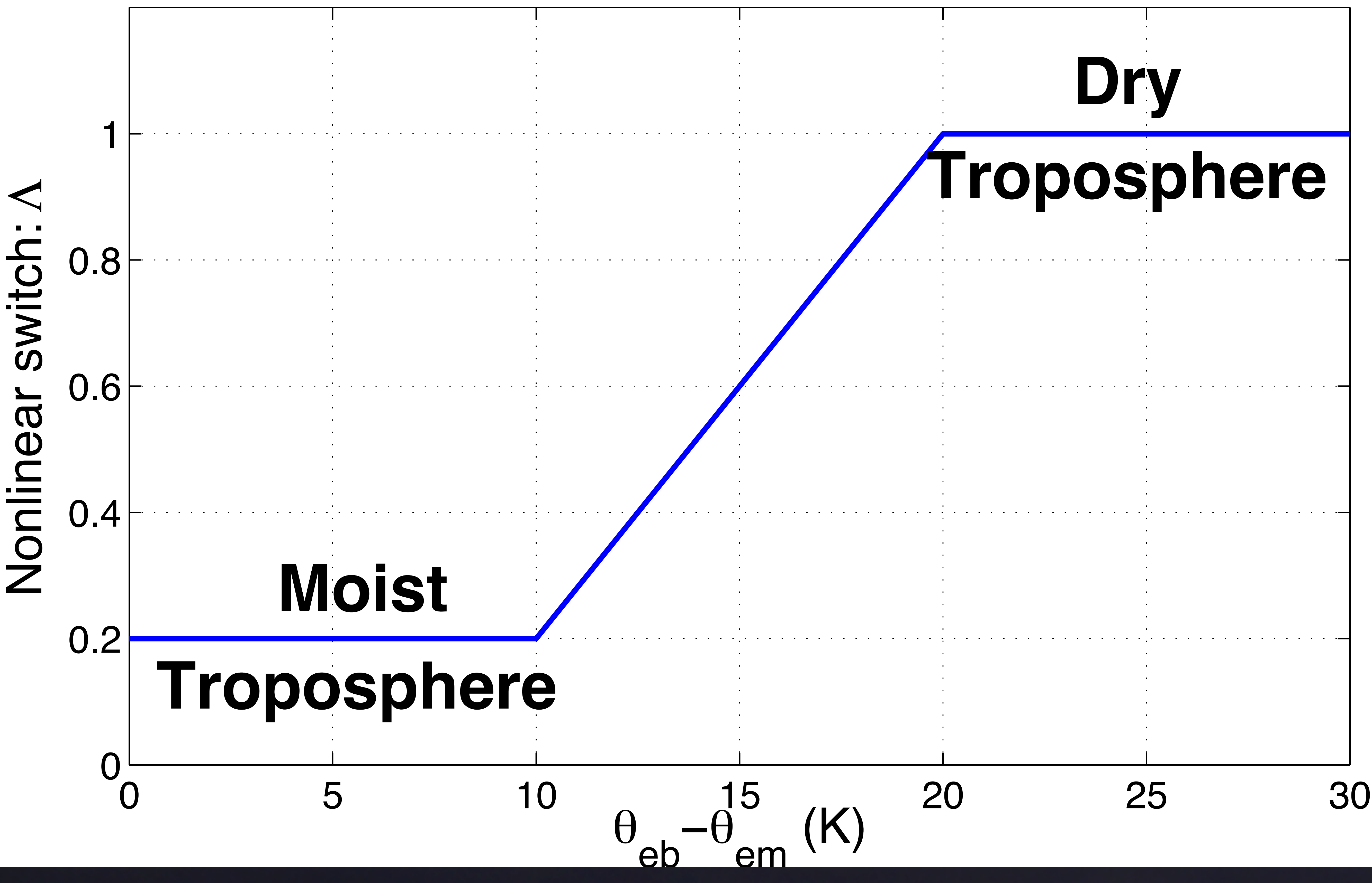


K. and Majda (JAS, 2006, 2008, etc.)

Imposed heating profiles



Moisture Switch to make transition from one type of convection to another



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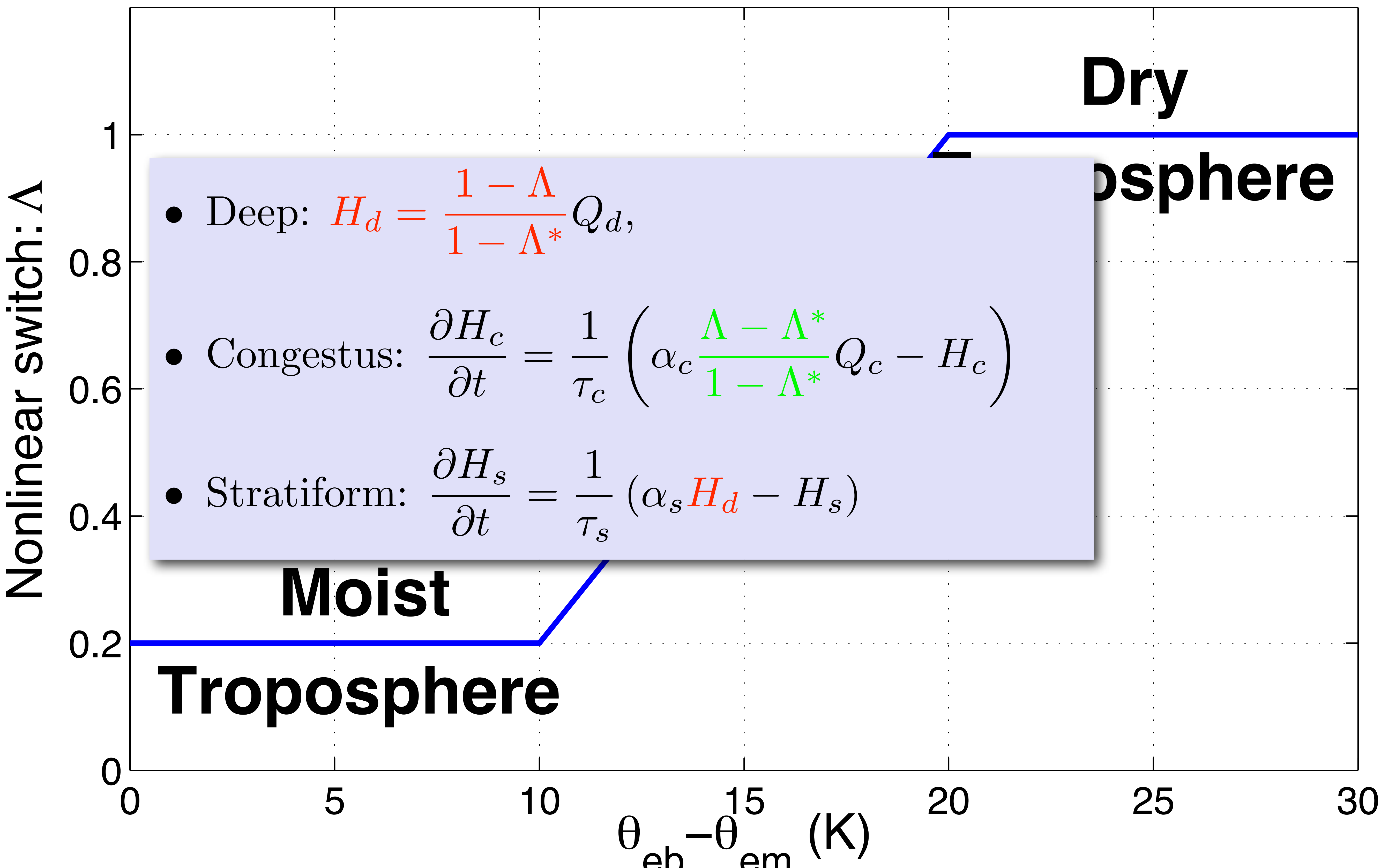


- Deep: $H_d = \dfrac{1-\Lambda}{1-\Lambda^*} Q_d,$
- Congestus: $\dfrac{\partial H_c}{\partial t} = \dfrac{1}{\tau_c}\left(\alpha_c \dfrac{\Lambda-\Lambda^*}{1-\Lambda^*} Q_c - H_c\right)$
- Stratiform: $\dfrac{\partial H_s}{\partial t} = \dfrac{1}{\tau_s}\left(\alpha_s H_d - H_s\right)$

- Tropospheric dynamics

$$
\text{1st Mode}\begin{cases}
\dfrac{\bar{d}\mathbf{v}_1}{dt} + \beta y \mathbf{v}_1^{\perp} - \nabla\theta_1 = -C_d(u_0)\mathbf{v}_1 - \dfrac{1}{\tau_R}\mathbf{v}_1 \\
\dfrac{\bar{d}\theta_1}{dt} - \operatorname{div}\mathbf{v}_1 = H_d + \xi_s H_s + \xi_c H_c + S_1
\end{cases}
$$

$$
\text{2nd Mode}\begin{cases}
\dfrac{\bar{d}\mathbf{v}_2}{dt} + \beta y \mathbf{v}_2^{\perp} - \nabla\theta_2 = -C_d(u_0)\mathbf{v}_2 - \dfrac{1}{\tau_R}\mathbf{v}_2 \\
\dfrac{\bar{d}\theta_2}{dt} - \dfrac{1}{4}\operatorname{div}\mathbf{v}_2 = (-H_s + H_c) + S_2
\end{cases}
$$

- Moisture Eqn: $P = \dfrac{2\sqrt{2}}{\pi}(H_d + \xi_s H_s + \xi_c H_c)$

$$
\frac{\bar{d}q}{dt} + \operatorname{div}\left[(\mathbf{v}_1 + \tilde{\alpha}\mathbf{v}_2)q + \tilde{Q}(\mathbf{v}_1 + \tilde{\lambda}\mathbf{v}_2)\right] = -P + \frac{D}{H_T}
$$

- Boundary layer: $\dfrac{\partial\theta_{eb}}{\partial t} = \dfrac{1}{h_b}(E - D)$

Closures

- Congestus heating prop. to low level CAPE

$$Q_c \propto \sigma_c \int_0^{z_m} (\theta_{eb} - \theta_e^*(z))dz \approx$$

$$Q_c = Q_c^0 + \frac{1}{\tau_{conv}}(\theta_{eb} - a_0'(\theta_1 + \gamma_2'\theta_2)$$

- Deep convection: CAPE & Betts-Miller

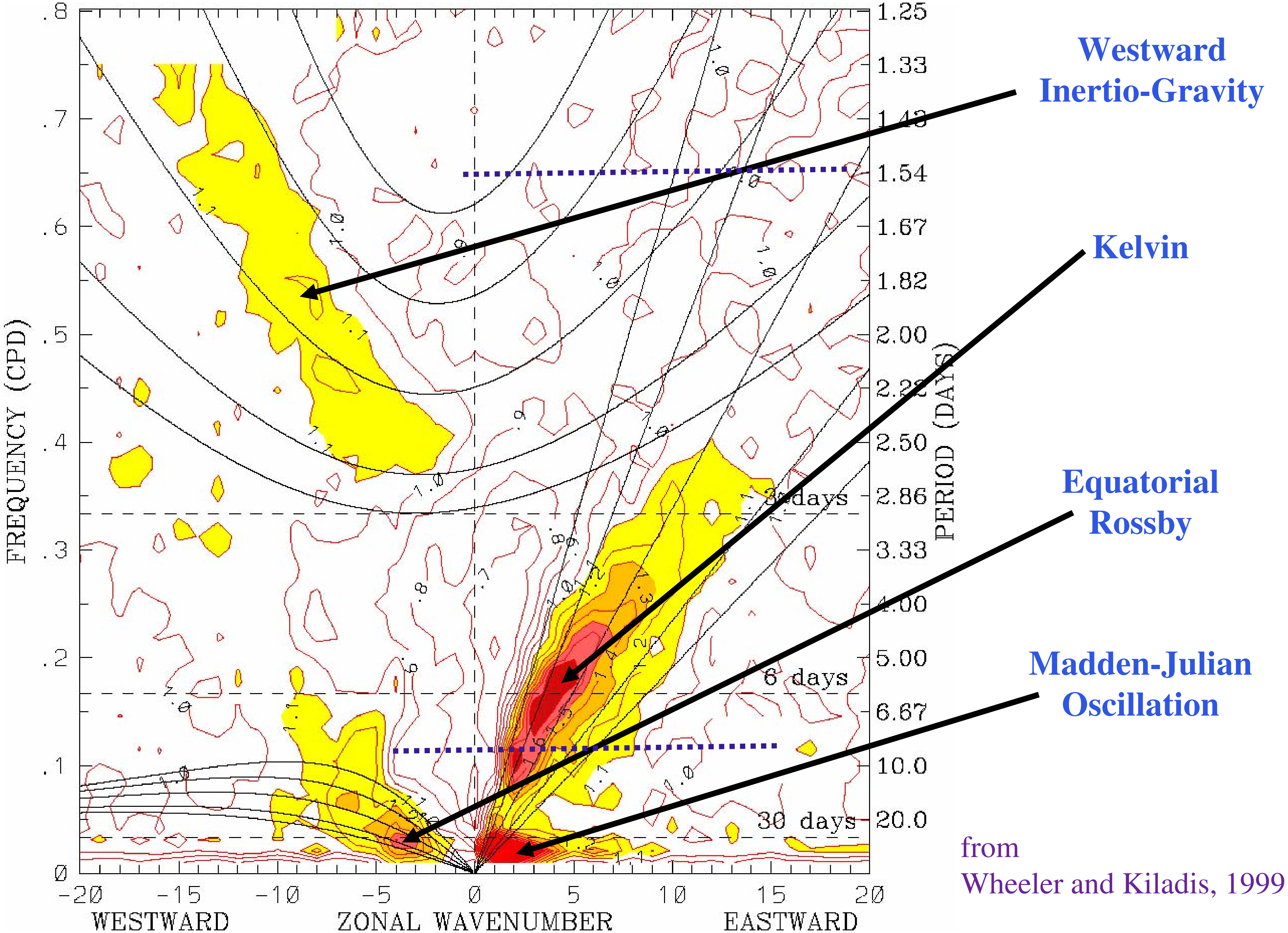
$$Q_d = Q_c^0 + \frac{1}{\tau_{conv}}(a_1\theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2\theta_2))$$

- Downdrafts

$$D = m_0(1 + \mu(H_s - H_c))^+(\theta_{eb} - \theta_{em})$$

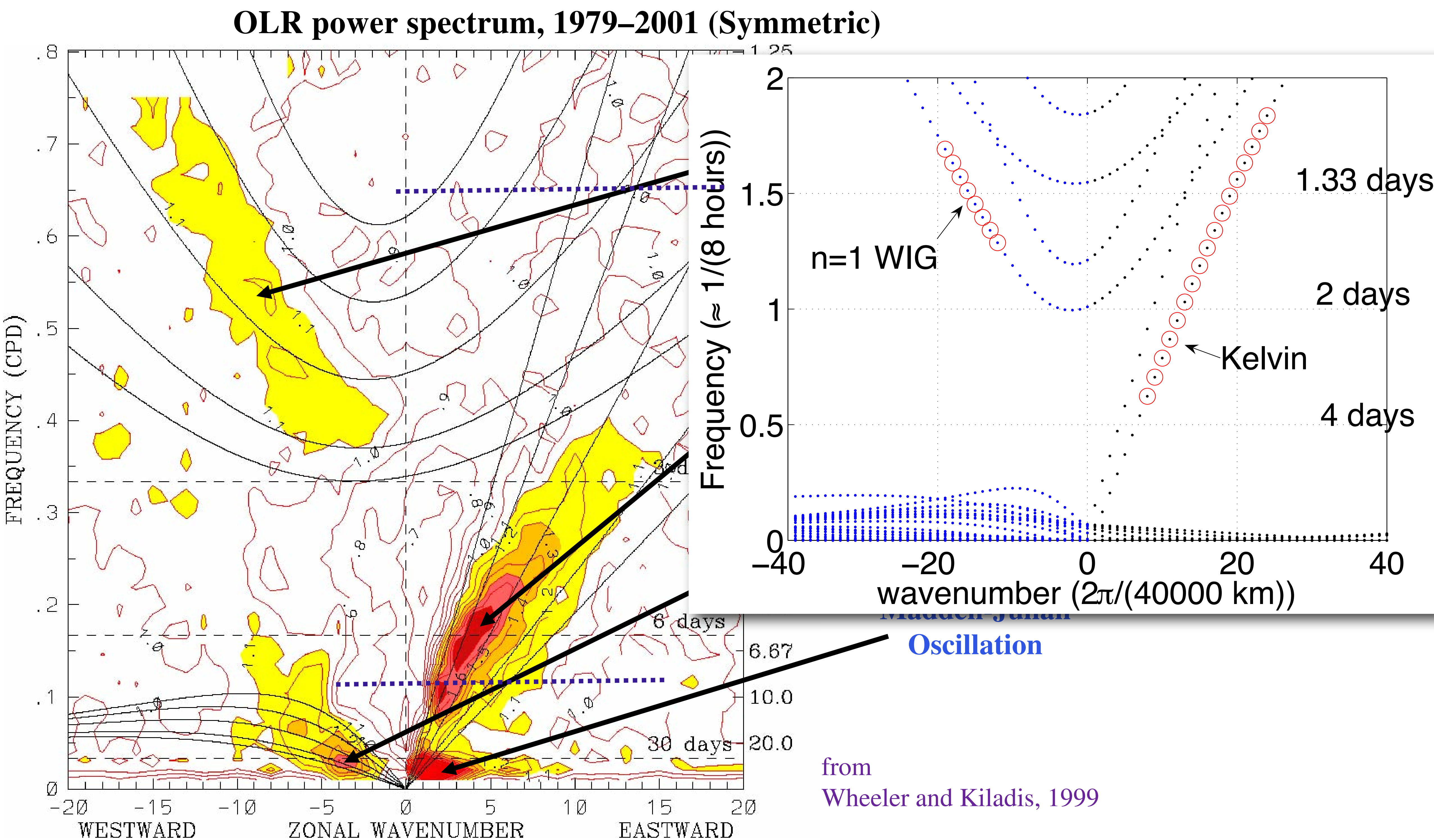
Linear Instabilities and Waves

OLR power spectrum, 1979–2001 (Symmetric)



from
Wheeler and Kiladis, 1999

Linear Instabilities and Waves



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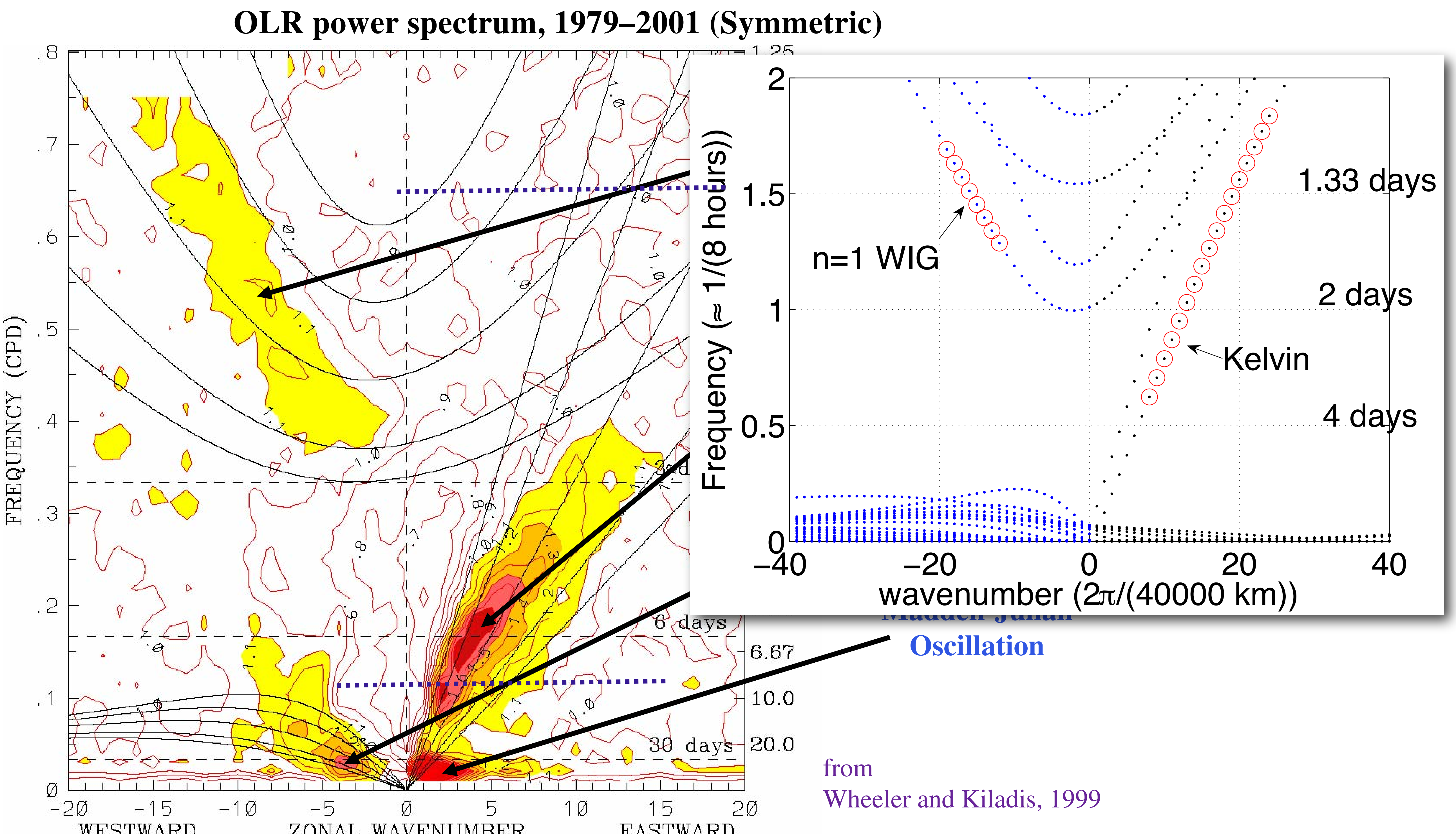
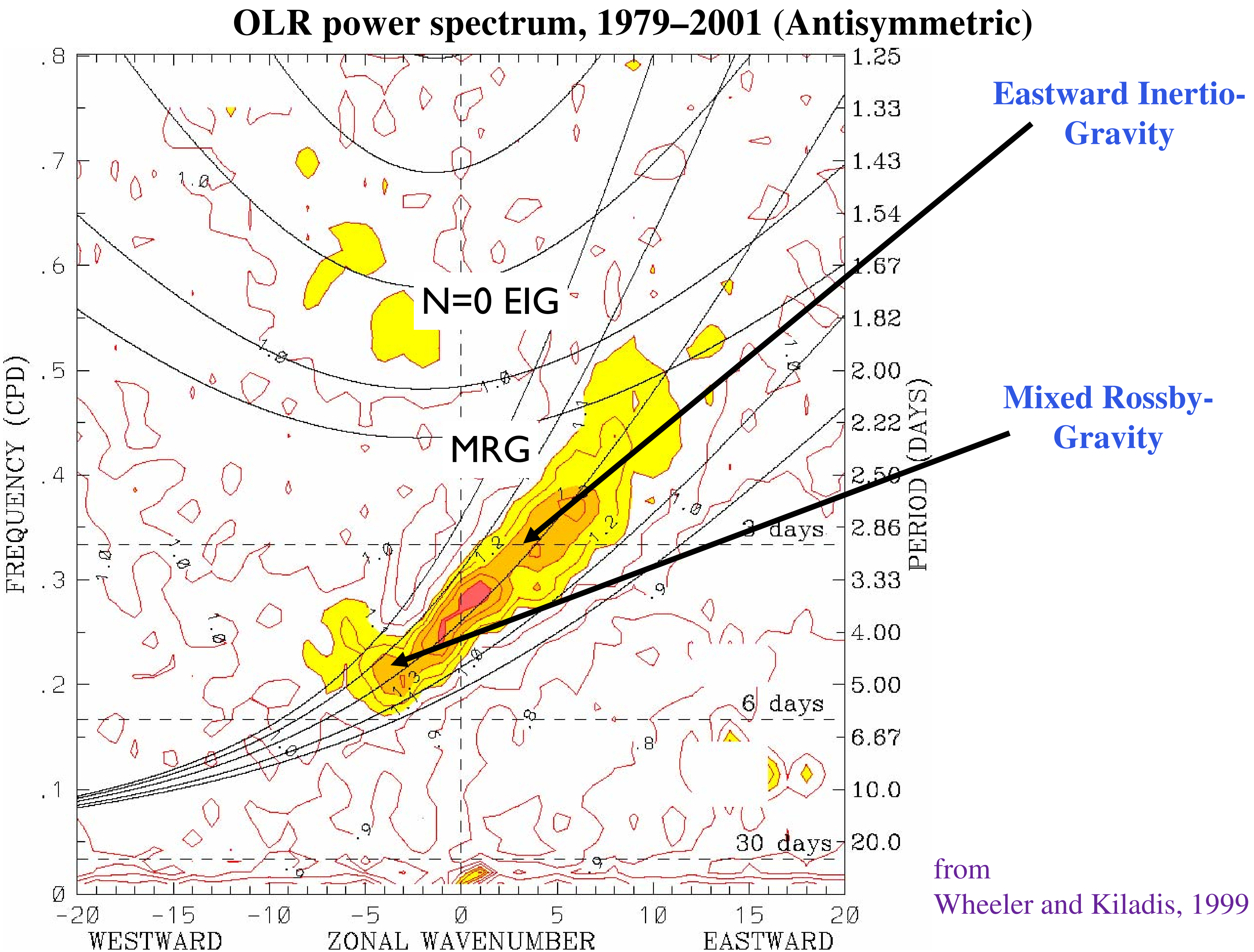


from
Wheeler and Kiladis, 1999

Shorter periods b/c Nonlinear effects ignored.
Model has no Rossby wave instability due to Lack of barotropic flow (Han and K. 2010), GCM results presented next.

Waves and Instabilities: Anti-symmetric



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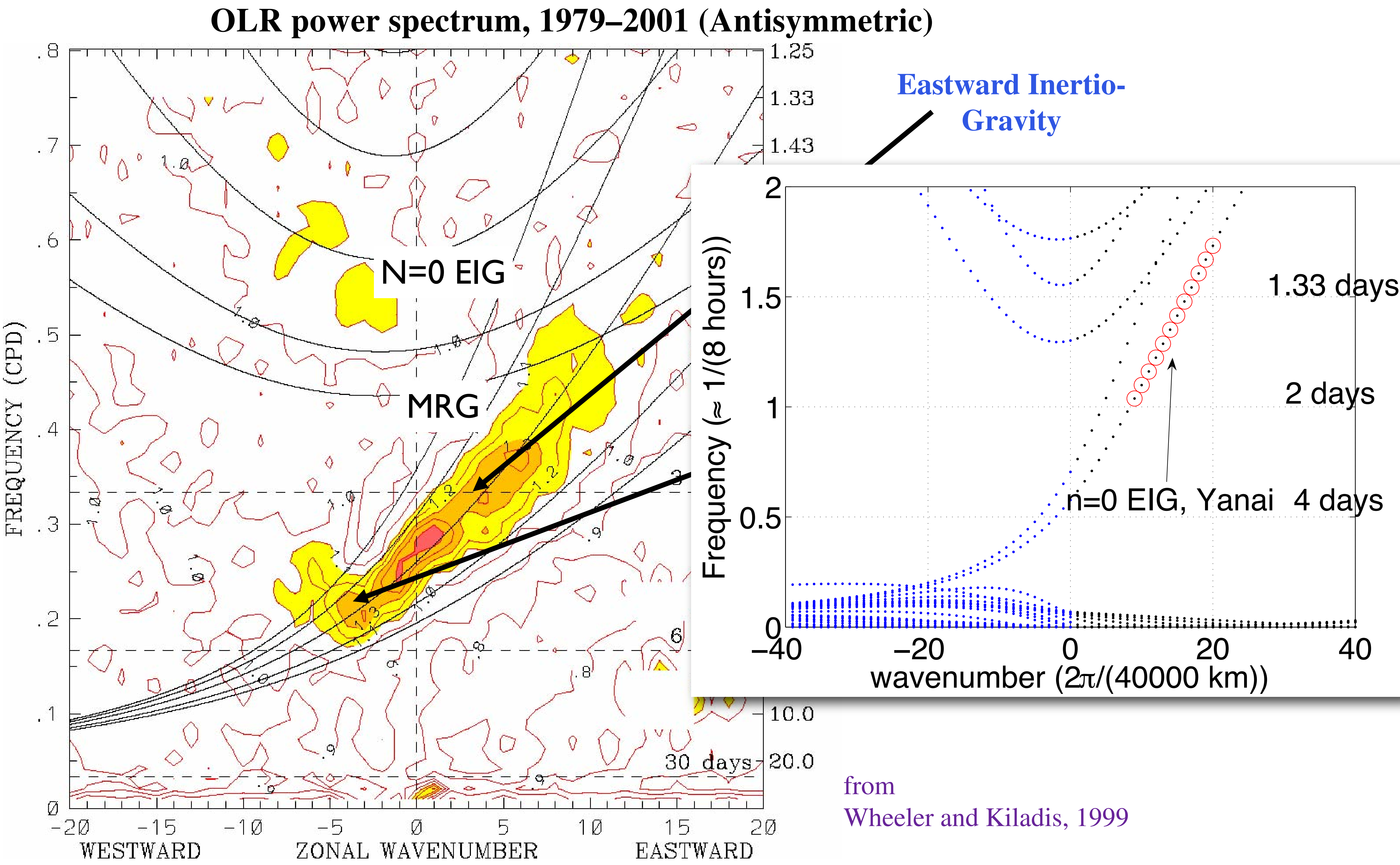
Waves and Instabilities: Anti-symmetric



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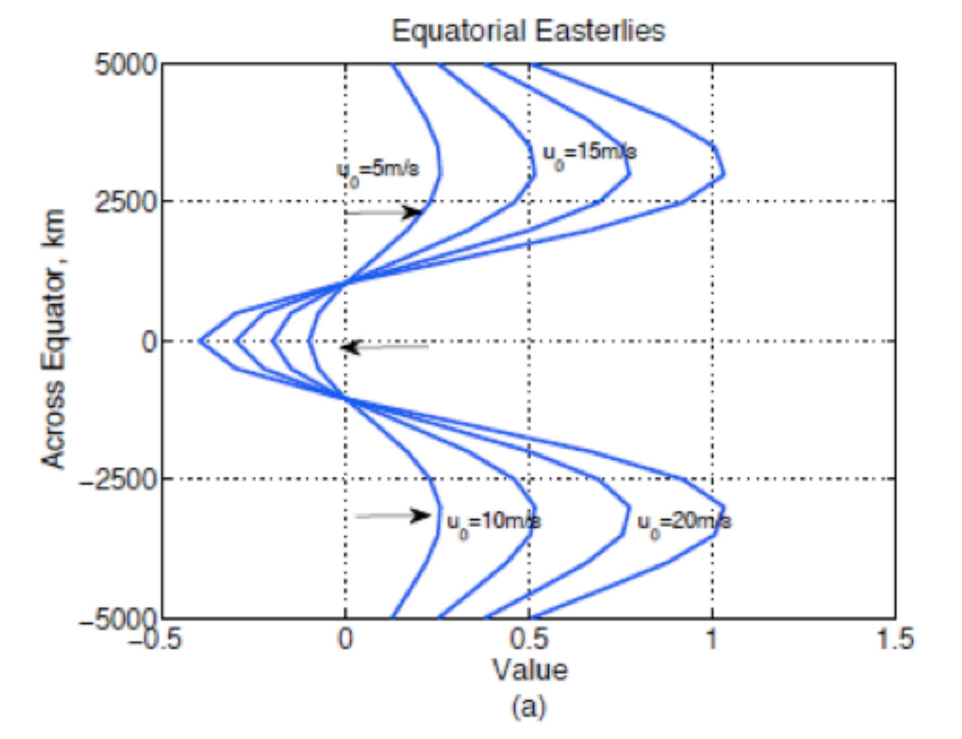
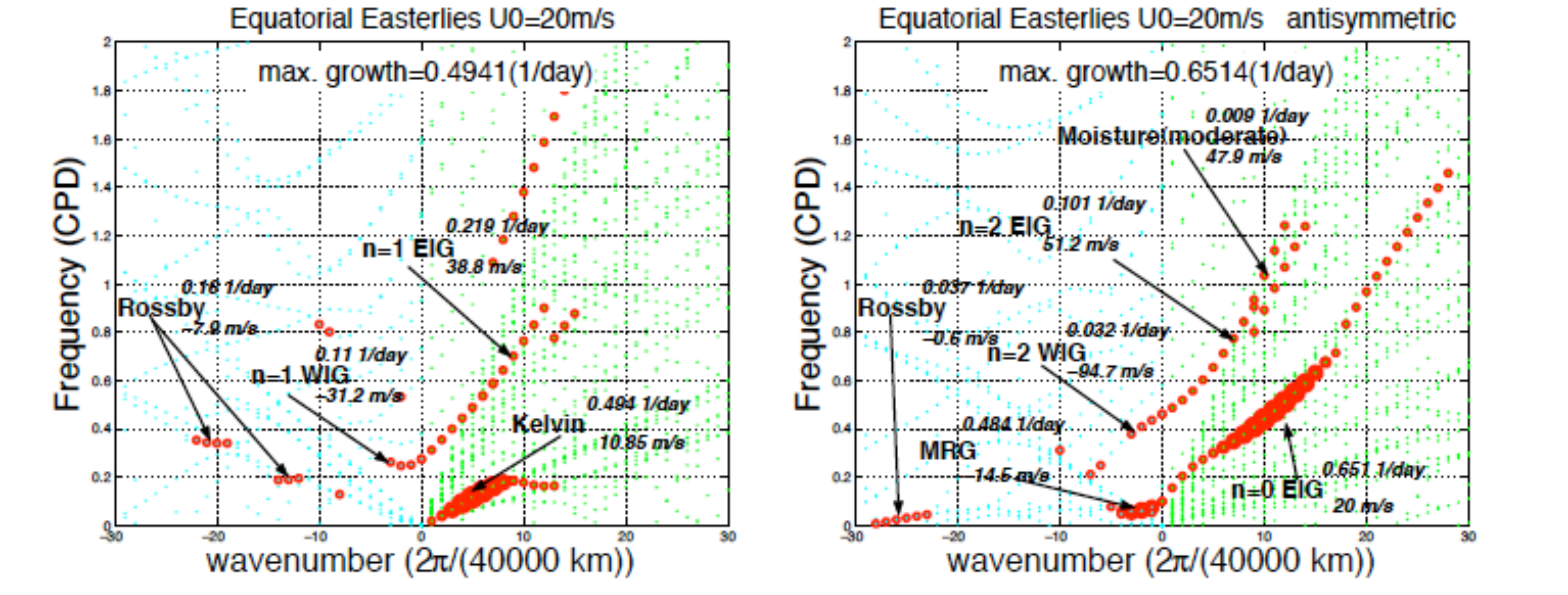
Climatological Shear Background



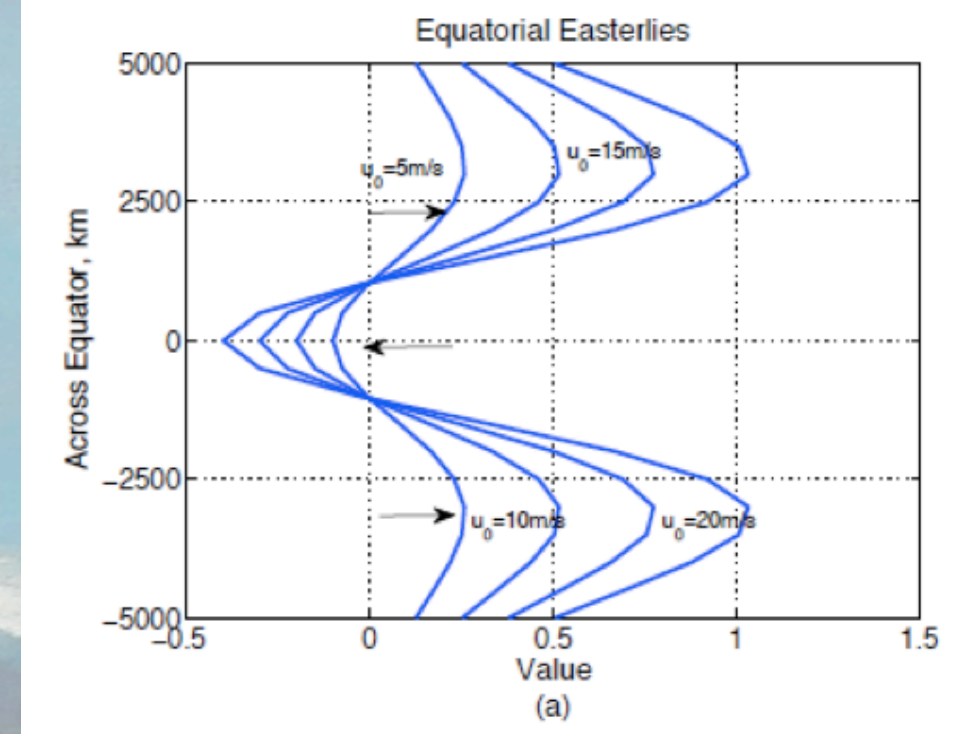


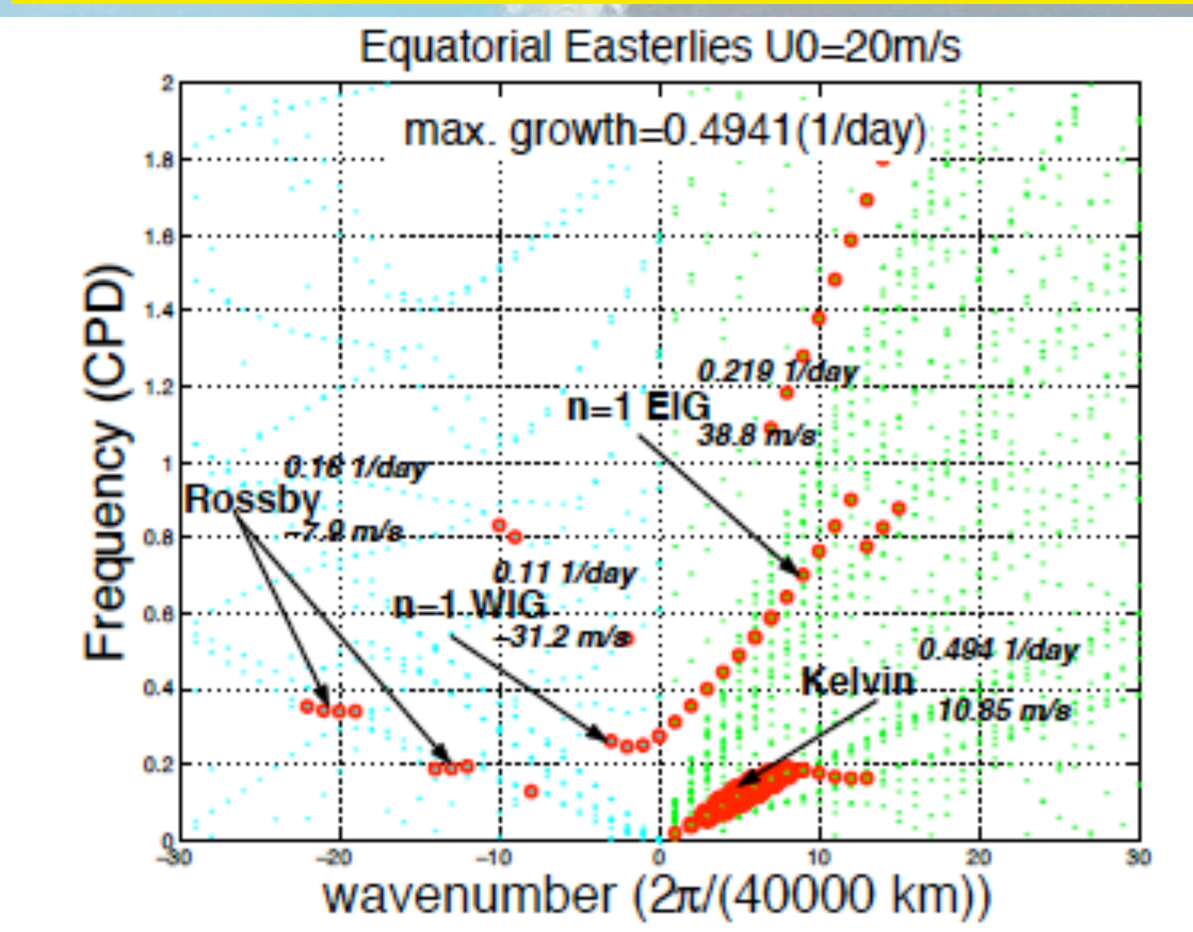
Han and Khouider (2010)

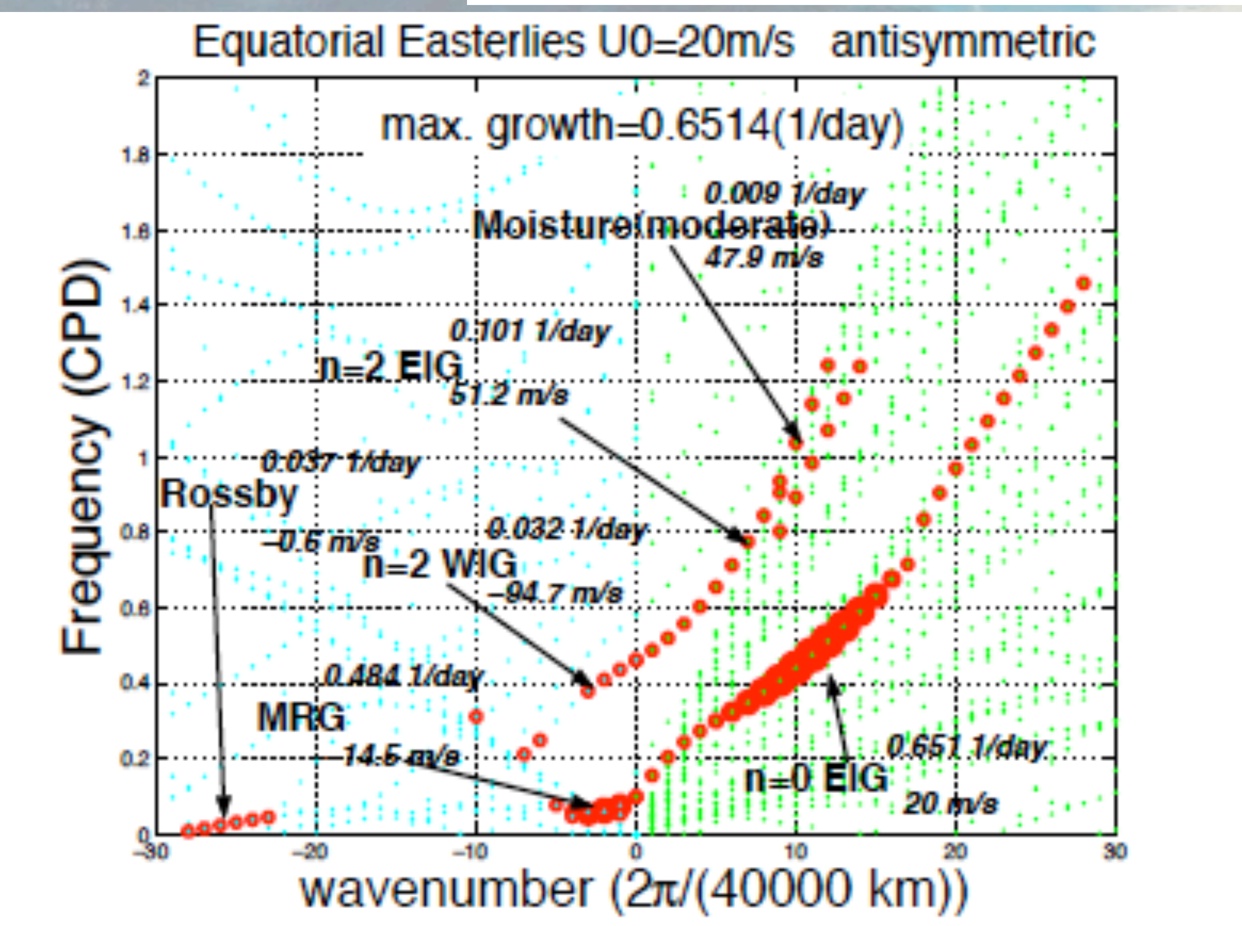
Unlike Kelvin and gravity waves, Rossby waves growth is due to shear instability--->dry instability!

Climatological Shear Background

See Stechmann's talk for effect of vertical shear, CMT and multiscale waves







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Multicloud in Aquaplanet GCM (HOMME)

$$\mathcal{H}^{tot} = \mathcal{H}_d \tilde{\psi}_1(p) + (\mathcal{H}_c - \mathcal{H}_s)\tilde{\psi}_2(p).$$

$$\psi_j(p) = -\int_{p_T}^{p} \phi_j(p')dp'$$

Imposed Heating and Moisture background profiles: Based on GATE sounding (Grabowski et al. 2001)

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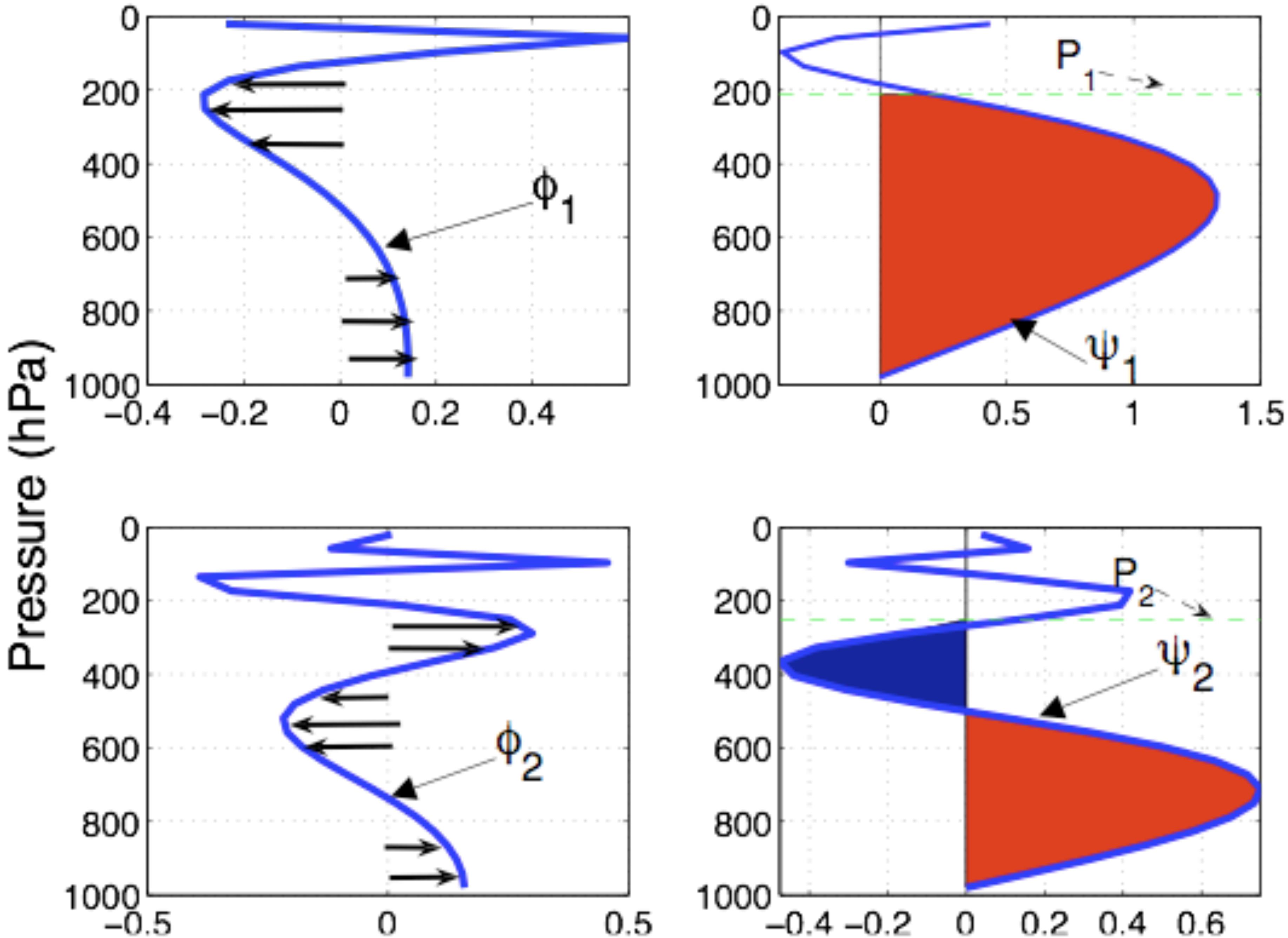
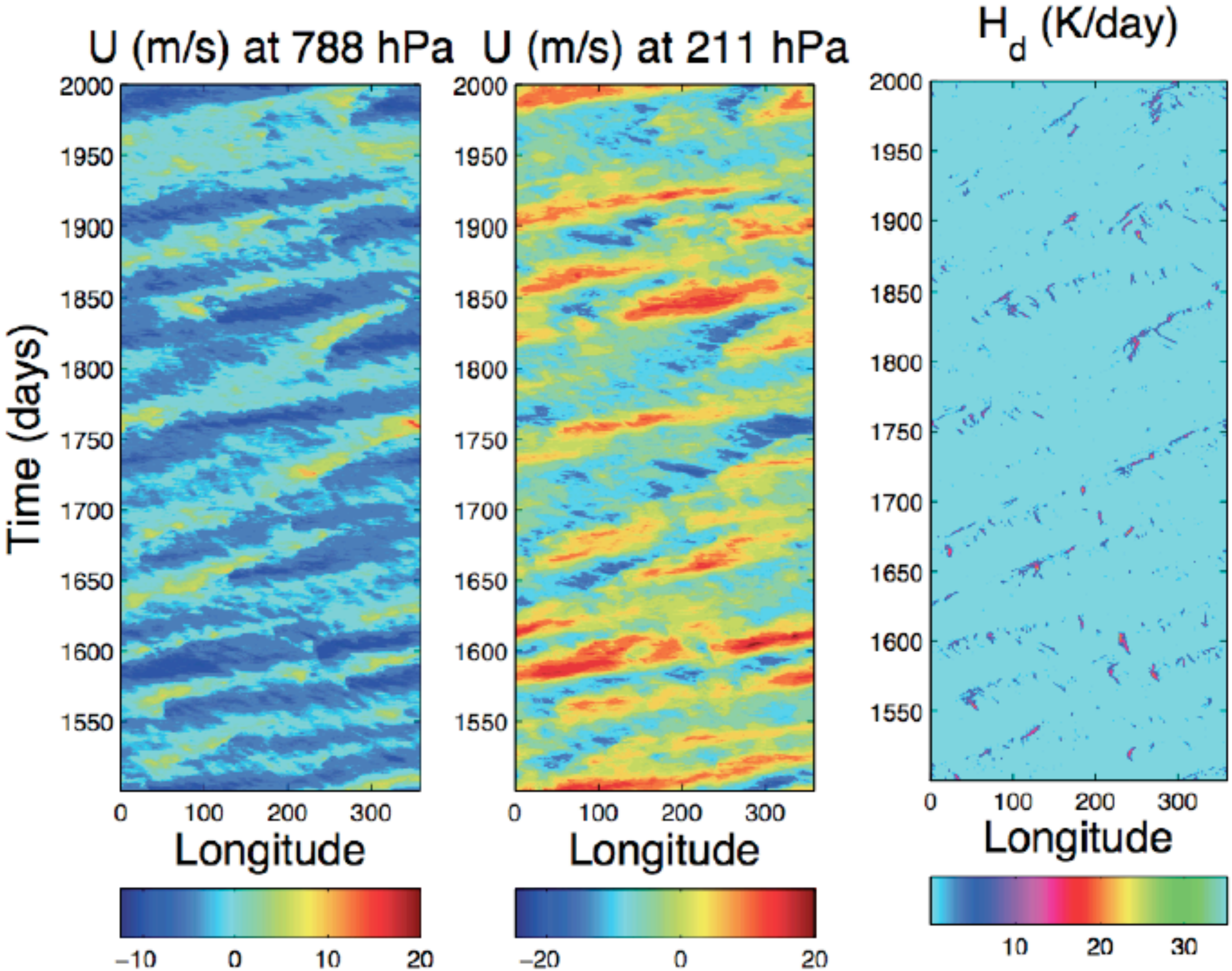


Imposed Heating and Moisture background profiles: Based on GATE sounding (Grabowski et al. 2001)

MJO in MC-GCM (HOMME)



Khouider, St-Cyr, Majda, Tribbia (2010)

Stochastic Multicloud Model

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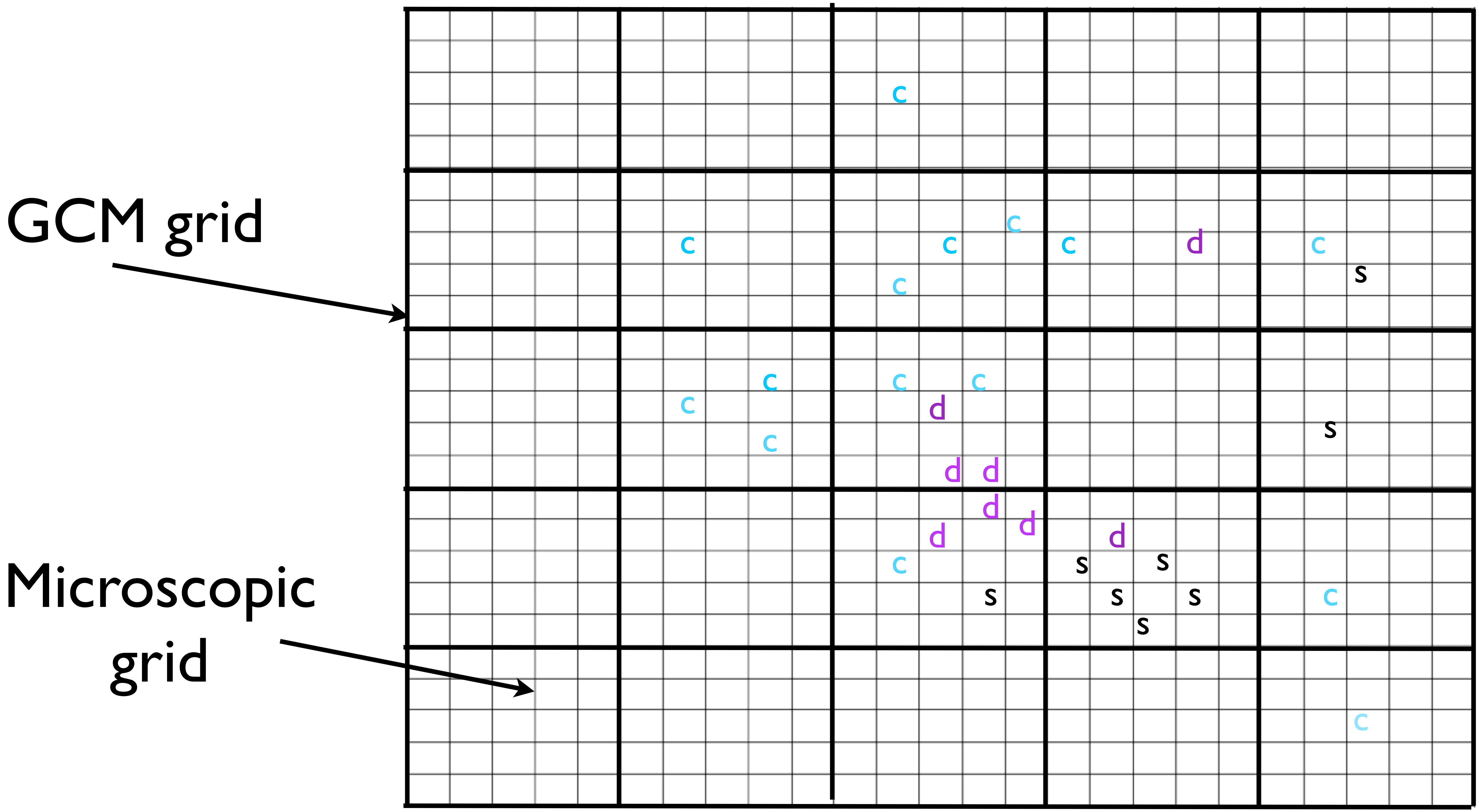

- Lattice points 1-10 km apart.
- Lattice site is occupied by a certain cloud type or is empty site

Intuitive transition rules

- A clear sky site turns into a congestus site with high probability if CAPE>0 and middle troposphere is dry.
- A congestus or clear sky site turns into a deep site with high probability if CAPE>0 and middle troposphere is moist.
- A deep site turns into a stratiform site with high probability.
- All three cloud types decay naturally according to prescribed decay rates.

Particle Interacting System

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- Four state Markov process (at given site):

$$\boxed{X_t = \begin{cases} 0 & \text{at clear sky site} \\ 1 & \text{at congestus site} \\ 2 & \text{at deep site} \\ 3 & \text{at statiform site} \end{cases}}$$

- State Space:

$$\Sigma = \{0,1,2,3\}^N, \;\; N = \text{ total number of lattice sites}$$

- $X \in \Sigma$ is called a configuration

Spin flips or infinitesimal transitions

- Configuration waits an "exponential" time before it makes a transition at a random site
- A transition occurs at site $j \text{ in } (t, t+\Delta t)$, if

$$X^i_{t+\Delta t} = \begin{cases} X^i_t & \text{if } i \neq j \\ X^j_t + \eta, & \text{if } i = j; \end{cases} \qquad \eta \in \{-3, -2, -1, 1, 2\}.$$

$$\eta = 1 : \text{clear} \longrightarrow \text{congestus or}$$

$$\text{congestus} \longrightarrow \text{deep or deep} \longrightarrow \text{stratiform}$$

$$\eta = -1 : \text{congestus} \longrightarrow \text{clear}$$

Special Case: No Local interactions

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- C = CAPE/low-level CAPE,
- D = mid-tropospheric dryness
- tau_kl transition time scales (parameters)
- Transition rates depend only on large-scale/external factors

$$R_{01} = \frac{1}{\tau_{01}}\Gamma(C)\Gamma(D) \qquad R_{02} = \frac{1}{\tau_{02}}\Gamma(C)(1-\Gamma(D))$$

$$R_{10} = \frac{1}{\tau_{10}}\Gamma(D) \qquad R_{12} = \frac{1}{\tau_{12}}\Gamma(C)(1-\Gamma(D))$$

$$R_{20} = \frac{1}{\tau_{20}}[1-\Gamma(C)] \qquad R_{23} = \frac{1}{\tau_{23}} \text{ or } \frac{\Gamma(C)}{\tau_{23}}$$

$$\Gamma(x) = 1 - e^{-x} \text{ if } x > 0$$

$$\Gamma(x) = 0 \text{ if } x \leq 0$$

$$R_{30} = 1/\tau_{30}$$

Cloud area fraction and Equilibrium measure

- When local interactions are ignored, X_t^i, are N independent four state Markov chains with the common equilibrium measure

$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1, \pi_1 = \frac{R_{01}}{R_{10} + R_{12}} \pi_0,$$

$$\pi_2 = \frac{R_{02}\pi_0 + \pi_1 R_{12}}{R_{20} + R_{23}}, \pi_3 = \frac{R_{23}}{R_{30}} \pi_2$$

- Cloud area fractions on coarse mesh (e.g. congestus)

$$N_c^j(t) = \sum_{j \in D_i} \mathbb{I}_{\{X_t^i = 1\}}, \quad \sigma_c^j(t) = \frac{1}{Q} N_c^j(t)$$

$$0 \le N_c \le Q$$

$$E\sigma_c^j(t) = \pi_1(U_j) \text{ at equilibrium}$$

Time evolution and statistics of filling fraction

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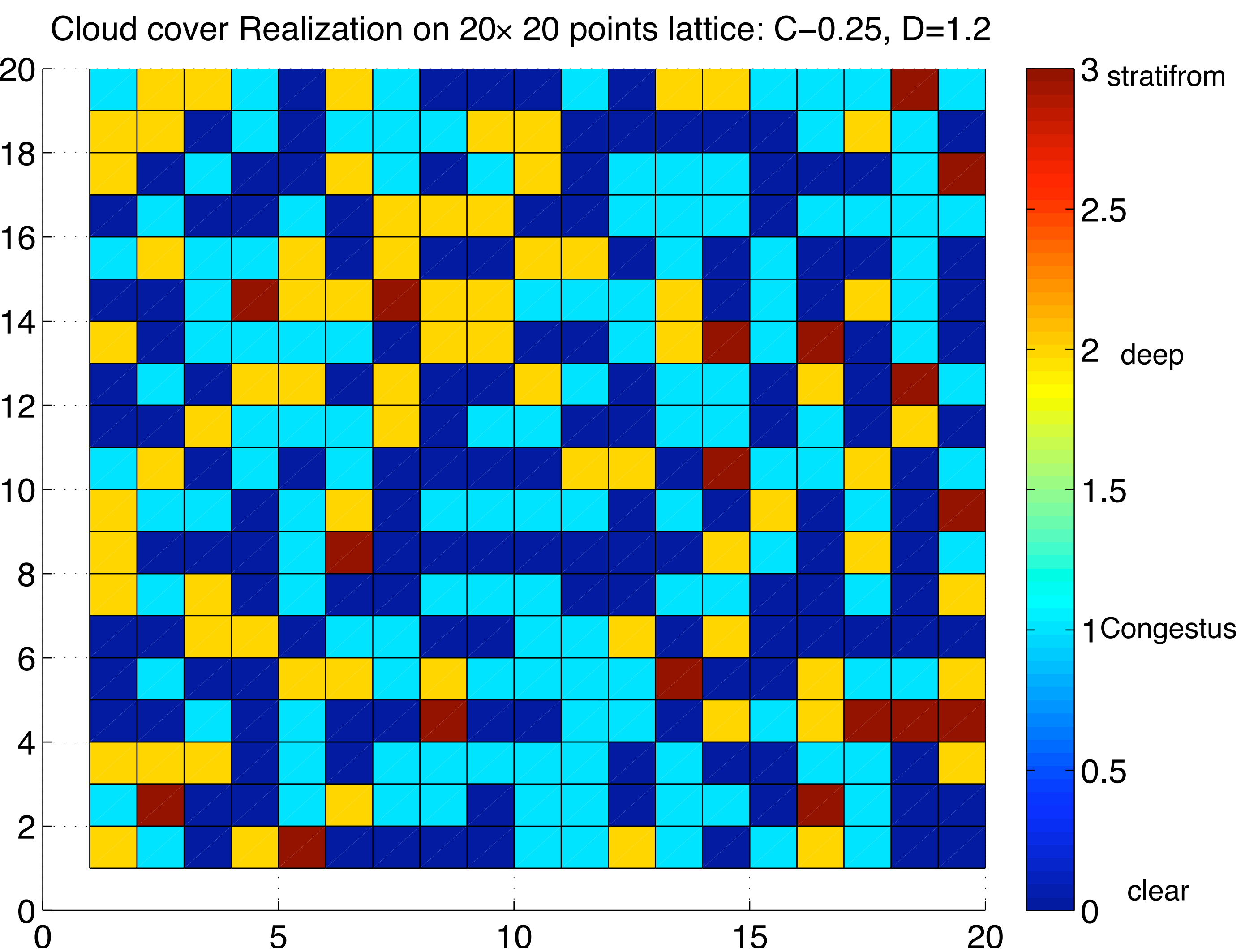

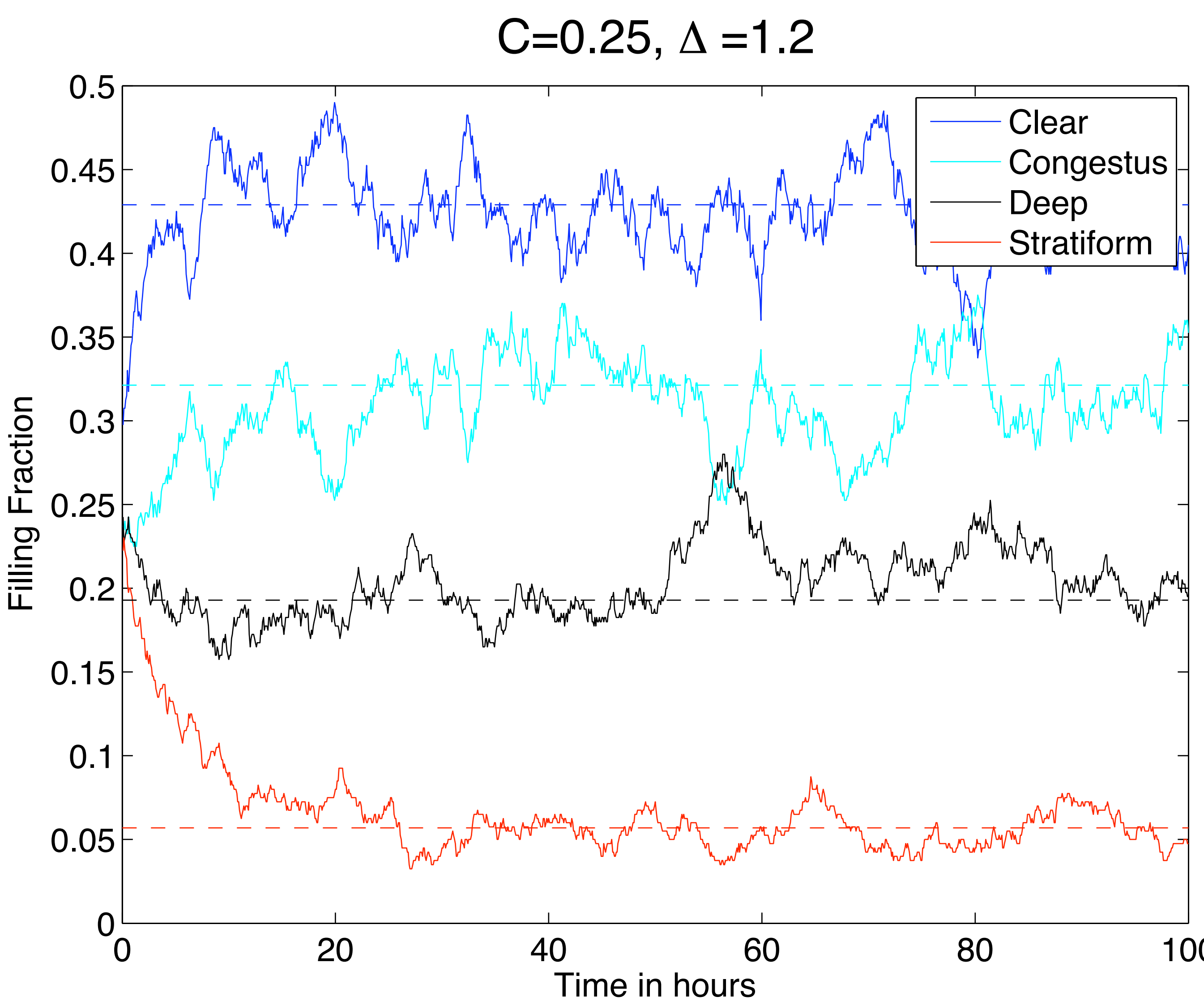

Case When Local Interactions are Ignored

- Coarse grained Process: Nx = number of sites filled with cloud type x within GCM grid box. e.g.
- NG talk (Thursday) will discuss local interactions
- Transition rates depend only on large scale variables
- $X_t^i, i = 1, 2, \cdots, N$ are i.i.d random variables
- Exact Dynamics for Coarse Grained process: Birth-death process with immigration:

$$Prob\{N_c^{t+\Delta t} = k+1 / N_c^t = k\} = N_{cs} R_{01} \Delta t + o(\Delta t)$$

$$Prob\{N_c^{t+\Delta t} = k-1 / N_c^t = k\} = N_c (R_{10} + R_{12}) \Delta t + o(\Delta t)$$

$$Prob\{N_d^{t+\Delta t} = k+1 / N_d^t = k\} = (N_{cs} R_{01} + N_c R_{12}) \Delta t + o(\Delta t)$$

clear sky $\quad N_{cs} = N - (N_c + N_d + N_s)$

Linking the stochastic model to the cumulus parameterization

$$H_c = \sigma_c \frac{\alpha_c \bar{\alpha}}{H_m} \sqrt{CAPE_l^+},$$

$$H_d = \sigma_d [\bar{Q} + \frac{1}{\tau_c^0 \bar{\sigma}_d} (a_1 \theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2 \theta_2))]^+,$$

$$H_s = \sigma_s \alpha_s [\bar{Q} + \frac{1}{\tau_c^0 \bar{\sigma}_s} (a_1 \theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2 \theta_2))]^+$$

Diagnostic

or

$$\partial_t H_s = \frac{1}{\tau_s} (\alpha_s \sigma_s H_d / \bar{\sigma}_d - H_s)$$

Prognostic (more consistent with deterministic MC)

Warm Pool Simulation using the stochastic MC Model

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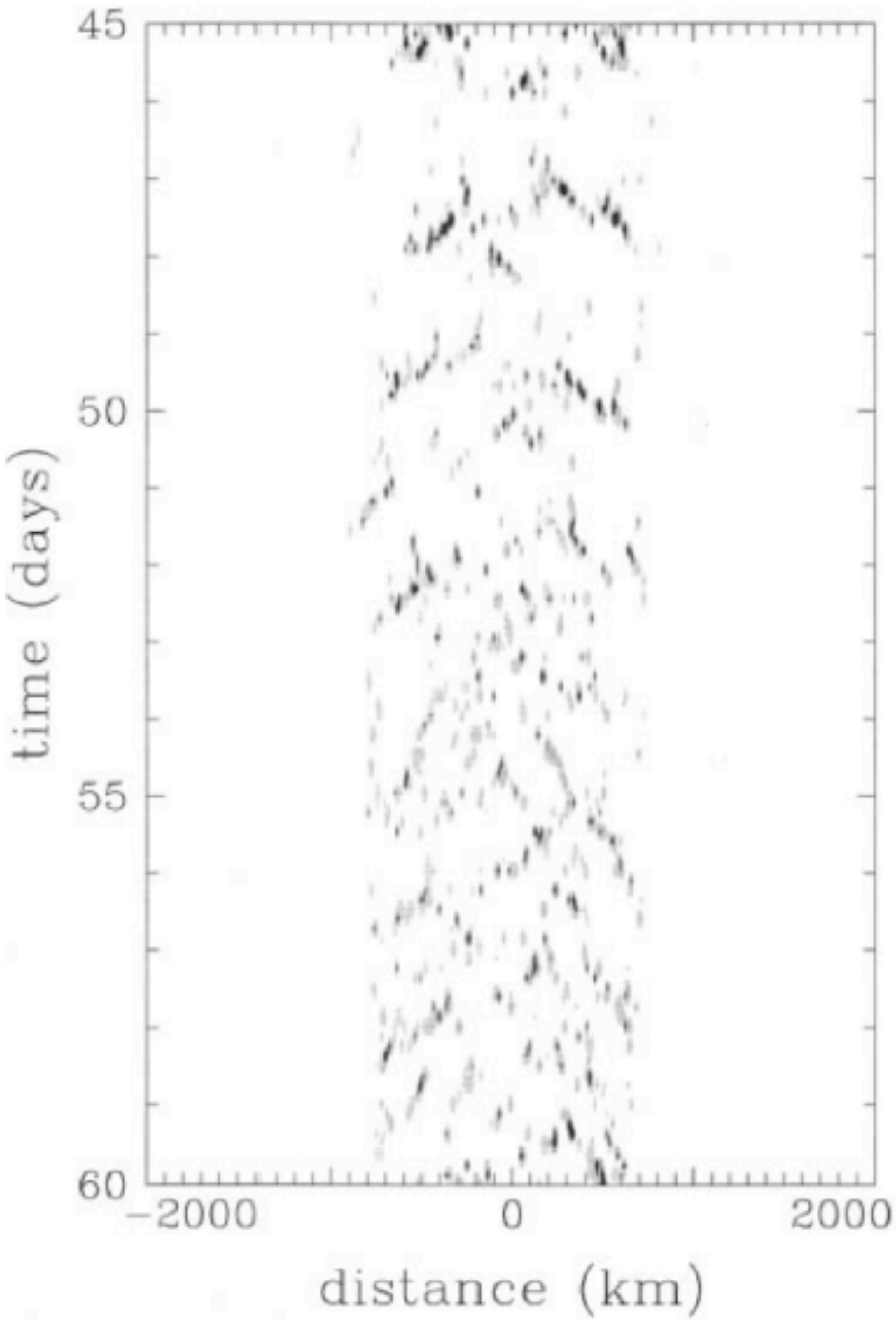
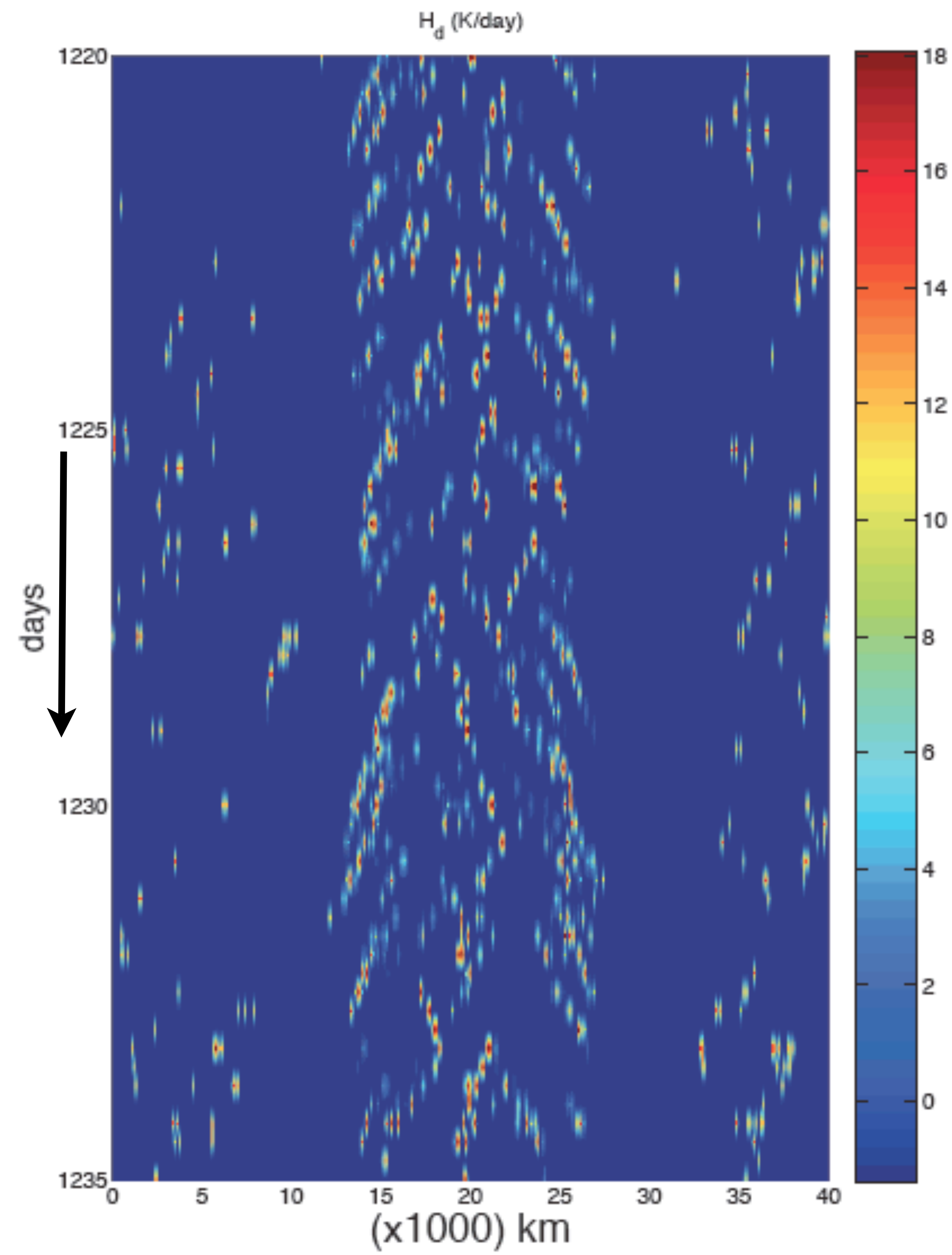
CRM (Grabowski et al. 2000)



Stochastic MC



Congestus Moistening

- Believed to be main driver of deepening of convection: transition from shallow to deep convection
- Occurs via two main mechanisms
  - ✦ Large-scale low-level moisture convergence due to congestus heating of second baroclinic mode. Instability of convectively coupled waves in multicloud model disappears when low-level moisture convergence is ignored (K. and Majda, 2006) .
  - ✦ Detrainment of non-precipitating congestus clouds of up to 2g/kg/day occurs prior the deepening of convection in a small domain CRM simulation (Waite and K. 2010).

What is next?

1. Compare diagnostic and prognostic Stratiform Closures
2. Add effect of congestus detrainment
3. Systematic Link and comparison between stochastic and deterministic multicloud models

Detrainment of congestus clouds

- Introduce evaporation due to detrainment of congestus

$$E_c = \frac{\sqrt{2}}{\pi}\frac{H_c}{Q_{R,1}^0}(\theta_{eb}-\theta_{el}); \quad \theta_{el} = 2q + \frac{2\sqrt{2}}{\pi}(\theta_1 + 2\theta_2)$$

- The new moisture budget equations ...

$$\frac{\partial q}{\partial t} + ... = -\frac{2\sqrt{2}}{\pi}P + (D + E_c)/H_T$$

$$\partial_t \theta_{eb} = \frac{1}{h_b}(E - E_c - D)$$

The Deterministic Mean Field Limit

- Mean field equations of cloud area fraction

$$\dot{\sigma}_c = (1 - \sigma_c\sigma_d - \sigma_s)R_{01} - \sigma_c(R_{10} + R_{12})$$
$$\dot{\sigma}_d = (1 - \sigma_c\sigma_d - \sigma_s)R_{02} + \sigma_c R_{12} - \sigma_d(R_{20} + R_{23})$$
$$\dot{\sigma}_s = \sigma_d R_{23} - \sigma_s R_{30}$$

- Analogy with (original) Deterministic MC

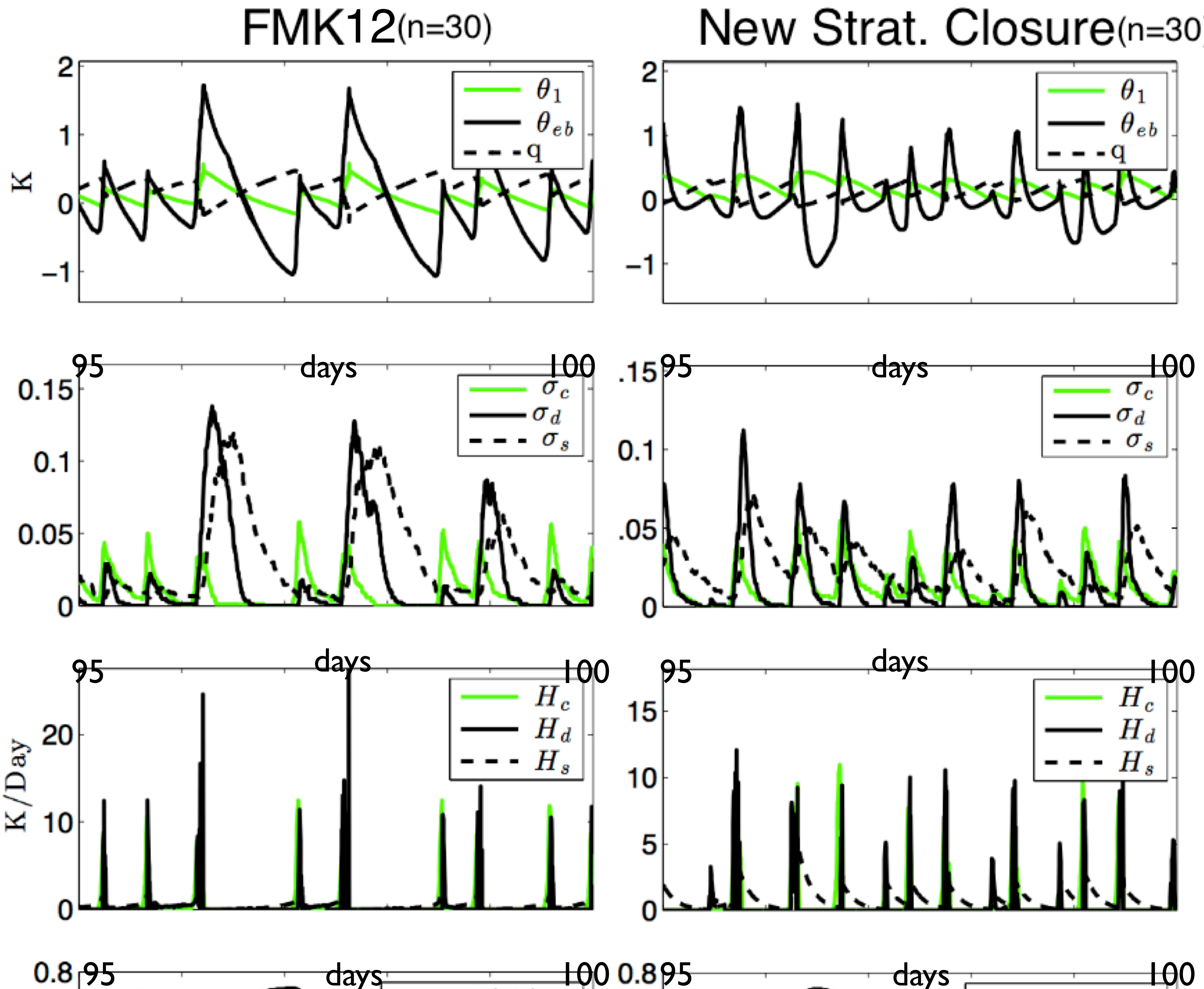
$$\partial_t H_c \approx \frac{1}{\tau_c^{MFL}}(\alpha_c^{MFL}\Gamma(D)\sqrt{CAPE_l^+} - H_c), \quad \tau_c^{MFL} = \frac{\tau_{12}}{\Gamma(\bar{C})}$$

$$H_d \approx \left[\frac{\tau_{23}\tau_{20}\Gamma(\bar{C})}{\tau_{02}(\tau_{20} + \tau_{23}(1 - \Gamma(\bar{C}))}\right]\Bigg(1 - \Gamma(D)\Bigg)\times$$
$$\left[\bar{Q} + \frac{1}{\tau_{conv}^0 \bar{\sigma_d}}(a_1\theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2\theta_2))\right]^+$$

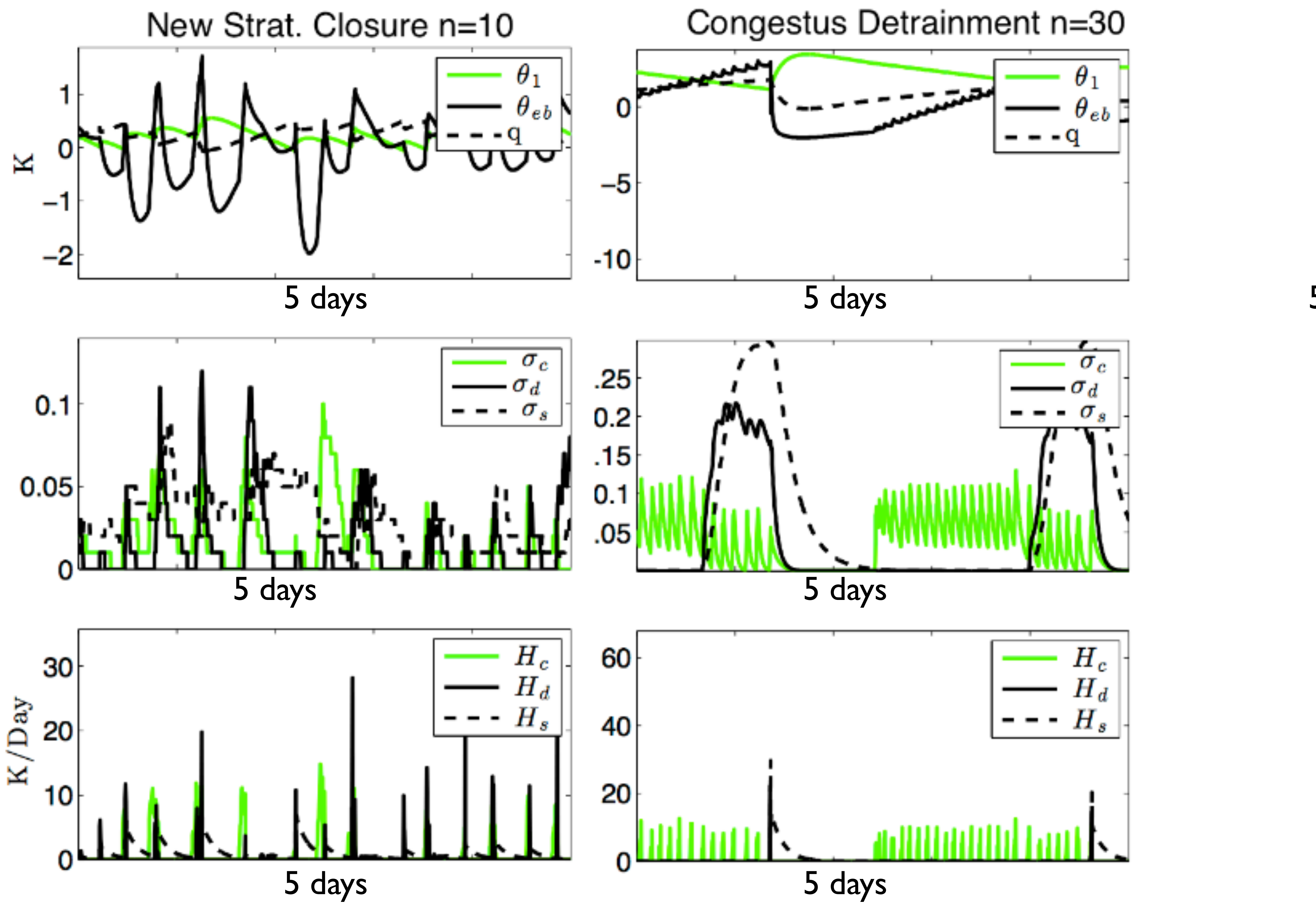
- $\Gamma(D)$ is an increasing function of $\theta_{eb} - \theta_{em}$

  like Λ but highly non-linear

Single Column Simulations



Congestus detrainment

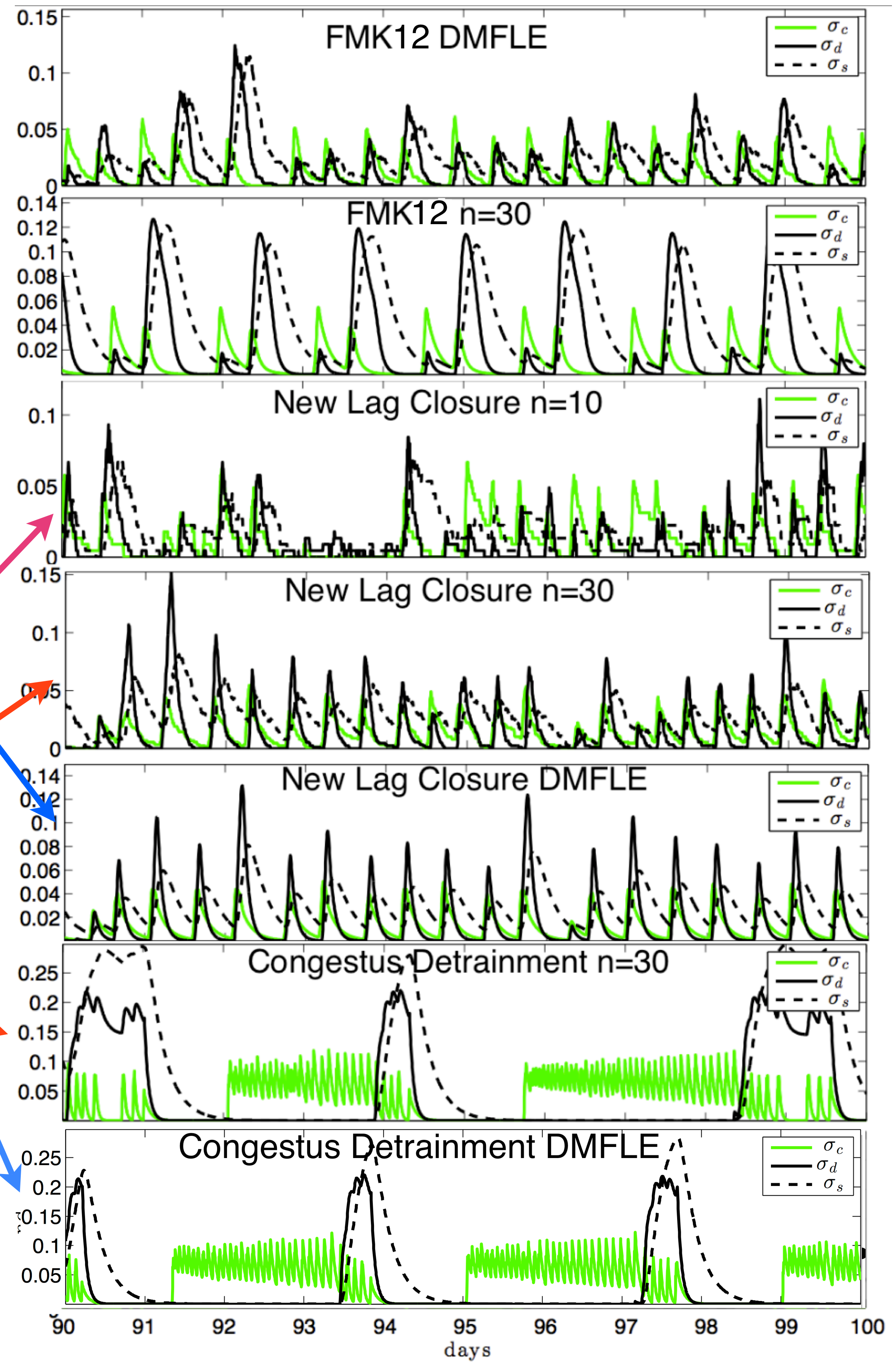


Deterministic Mean Field Limit

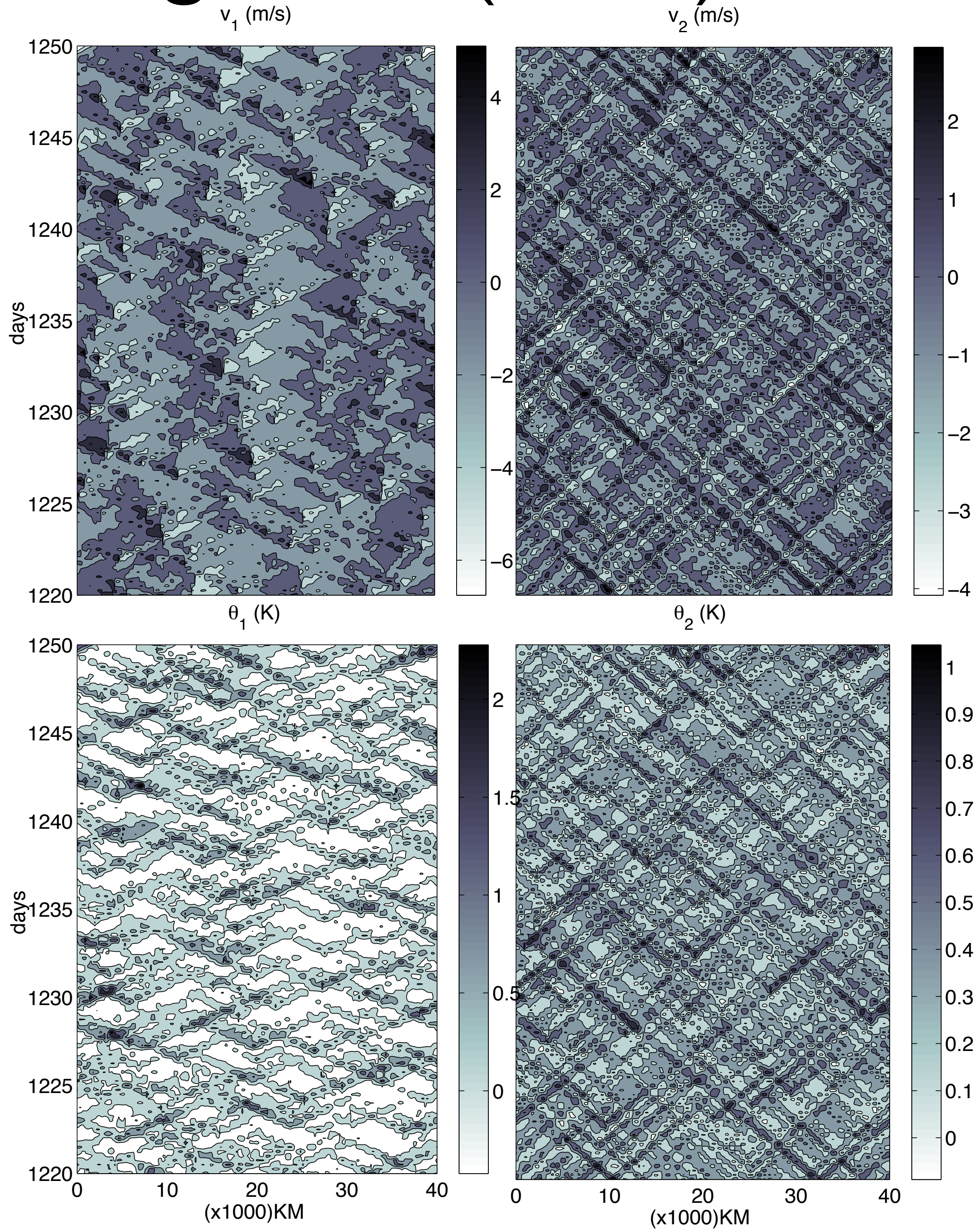
Deterministic Nonlinearity

V.S.

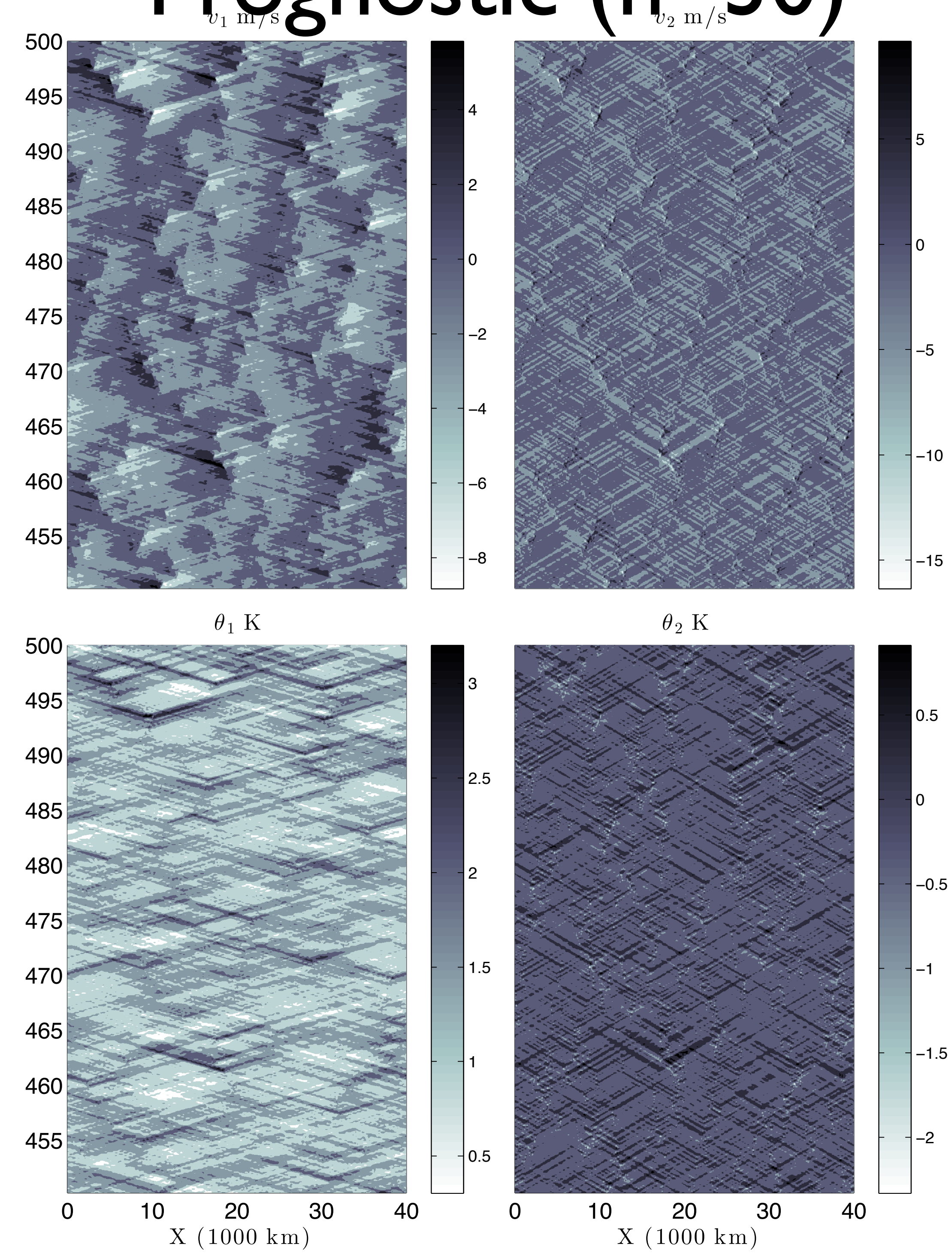
Stochasticity



Diagnostic (n=30)

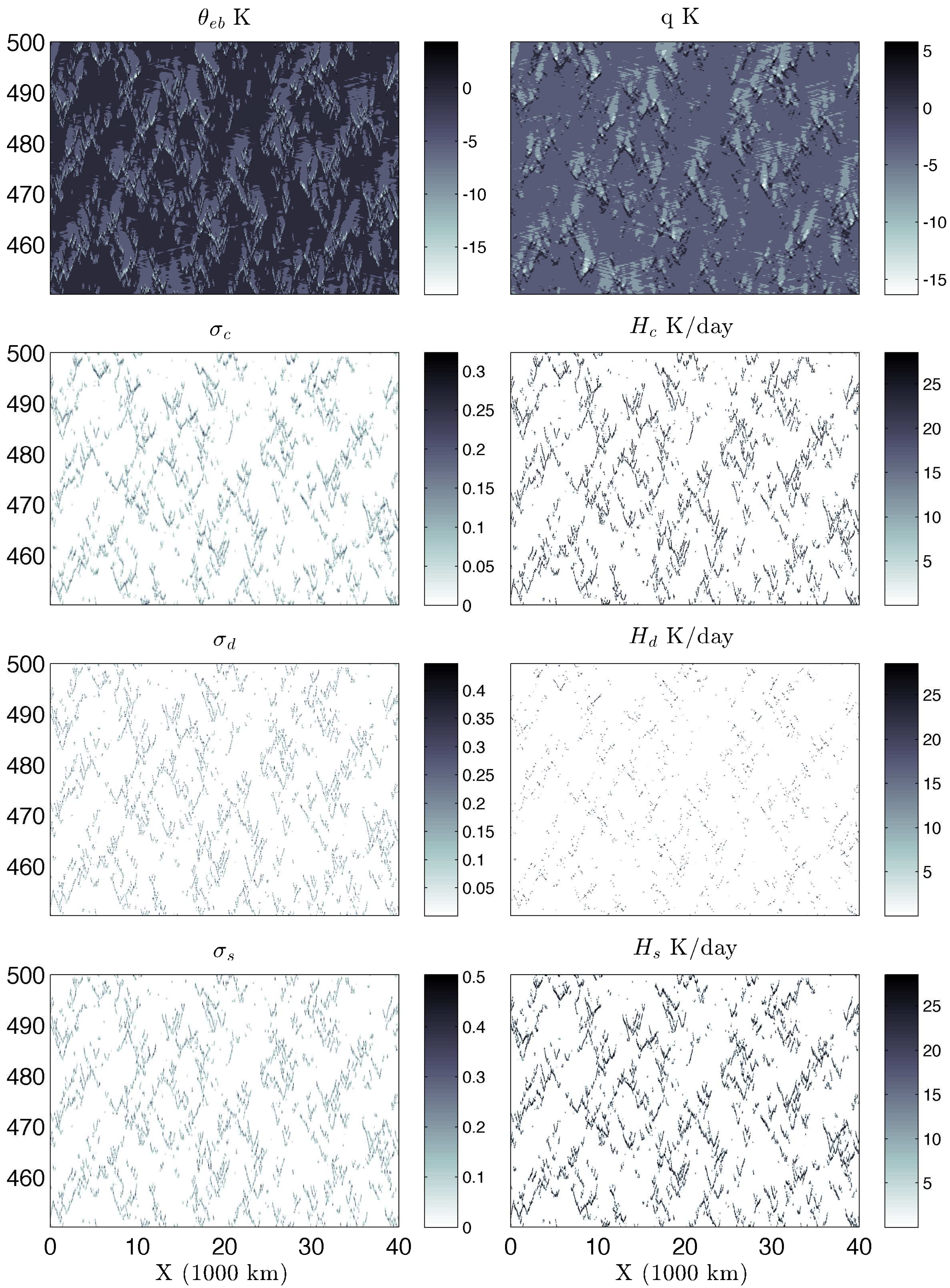


Prognostic (n=30)



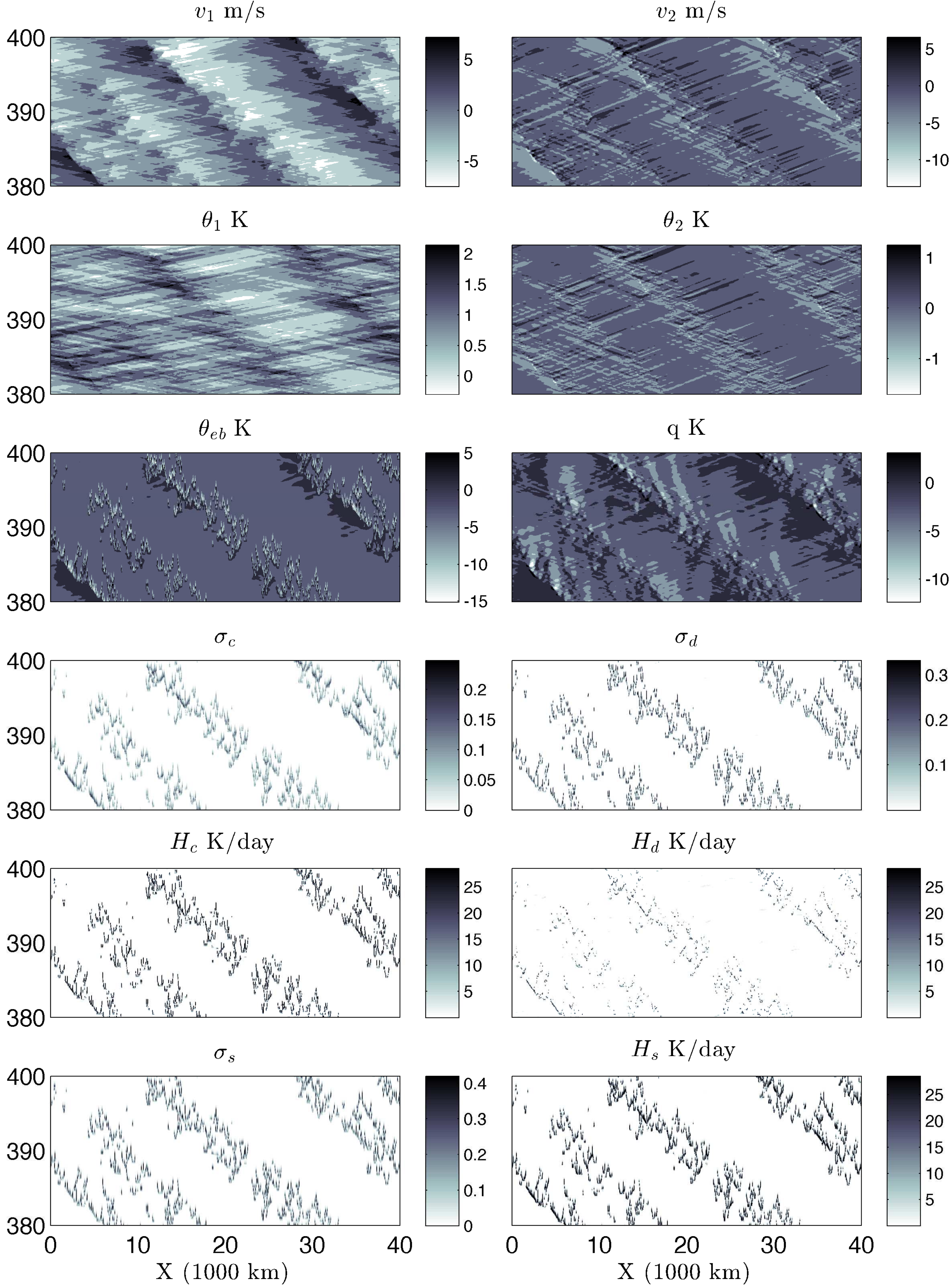
(x,t) Simulations Uniform SST background:

- Slow moving envelopes of convection
- Fast moving dry gravity waves
- Convective envelopes are more coherent in Prognostic case



Convective Variables for Lag-Prognostic Stratiform Closure

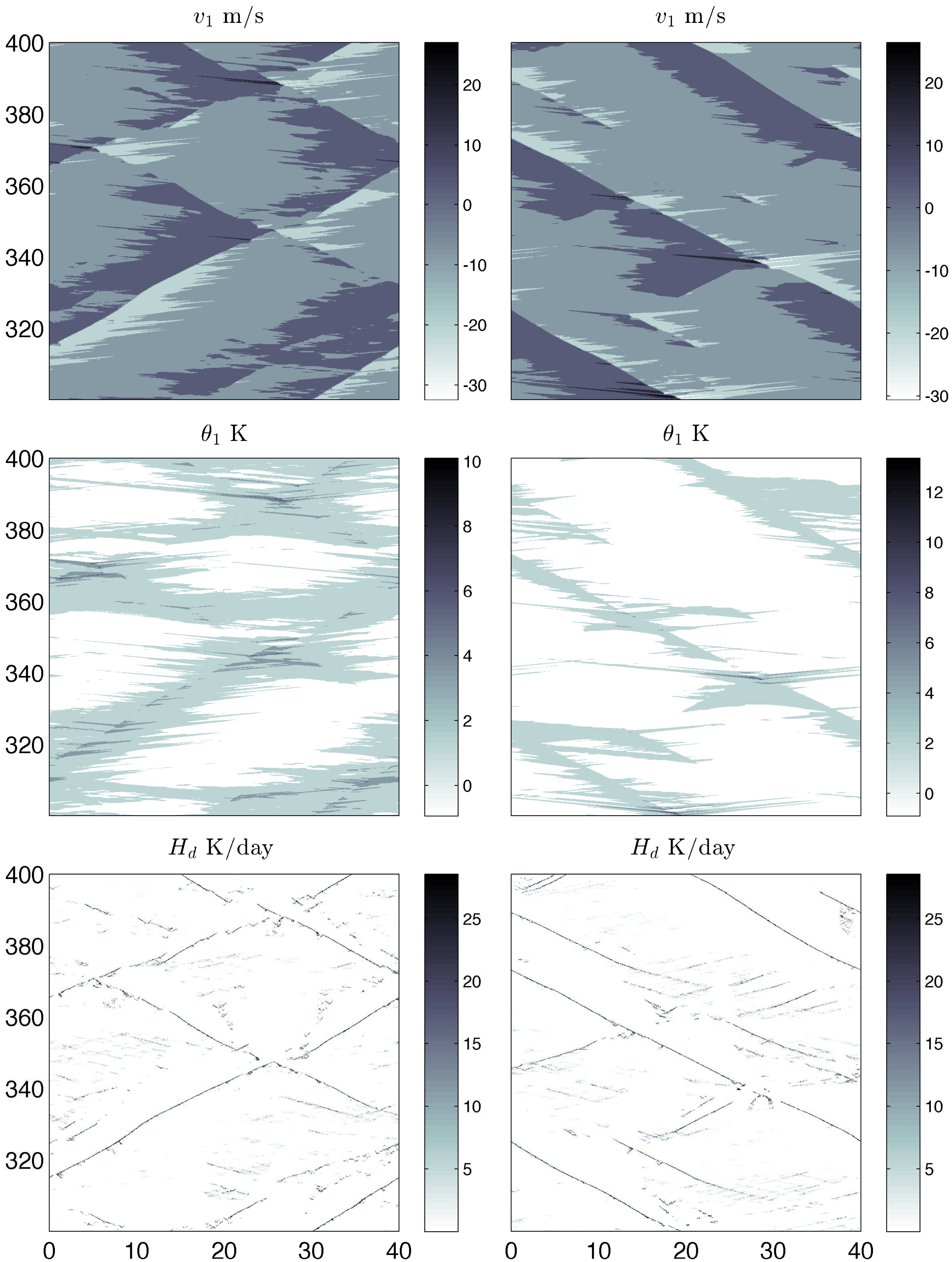
Moist Gravity waves move in both directions and interact with each other



Diagnostic (Lag) Stratifrom Closure n=100:

Waves are more organized, less chaotic, and locked. Close to mean field limit-deterministic behaviour...forced convection "regime"

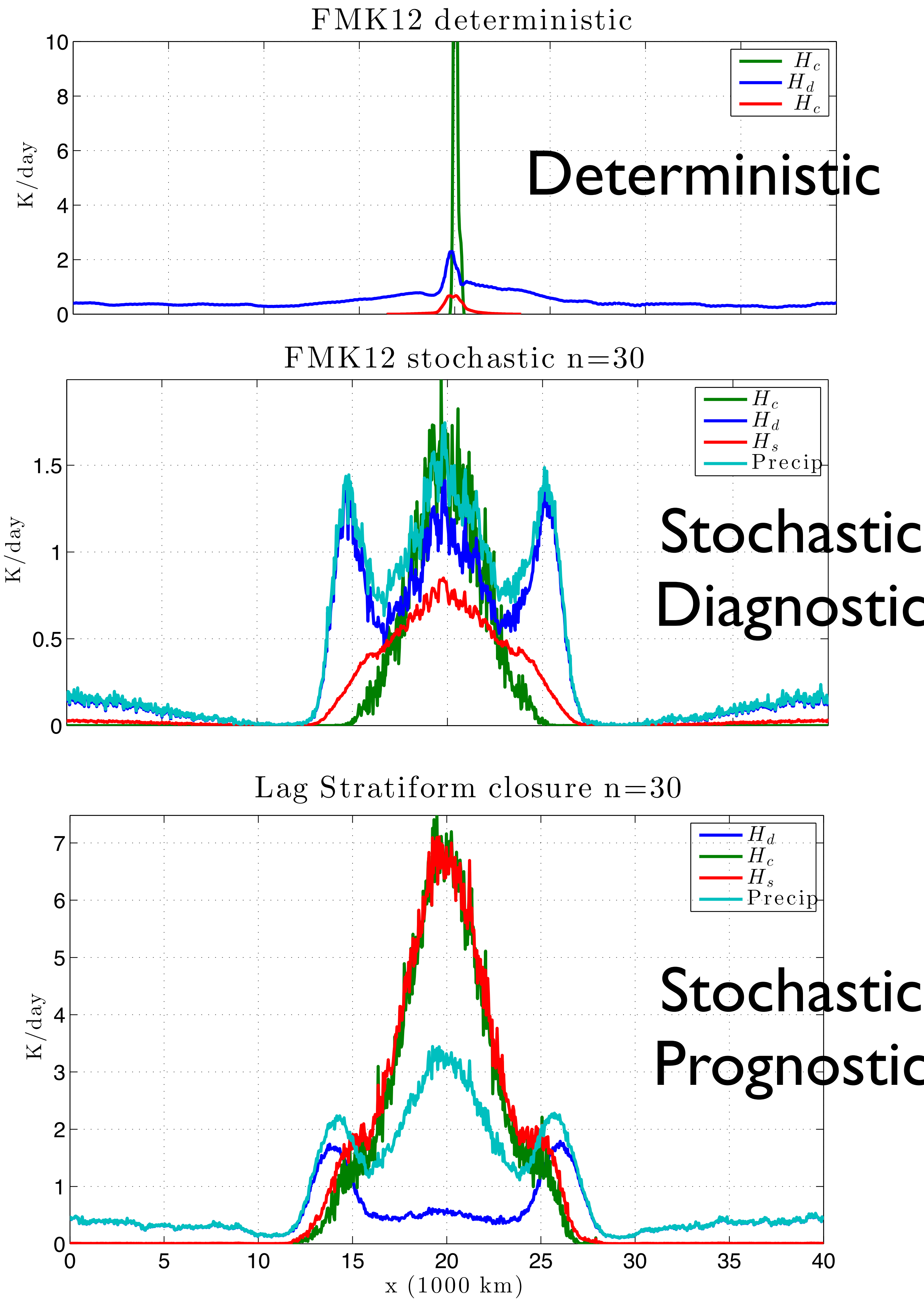
Weak / detrainment / Strong

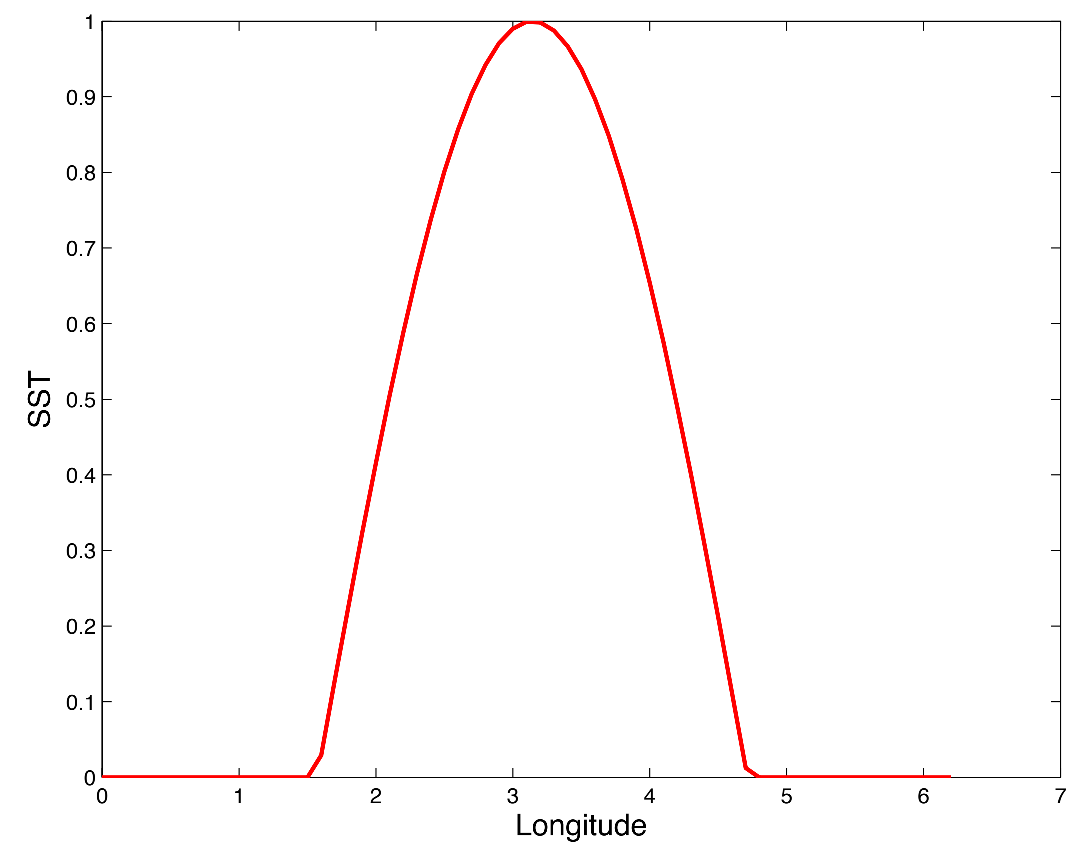


Congestus detrainment

- More organized wave envelopes
- Chaotic convection in subsiding regions
- Dry gravity waves emanate from convective envelope may shut convection

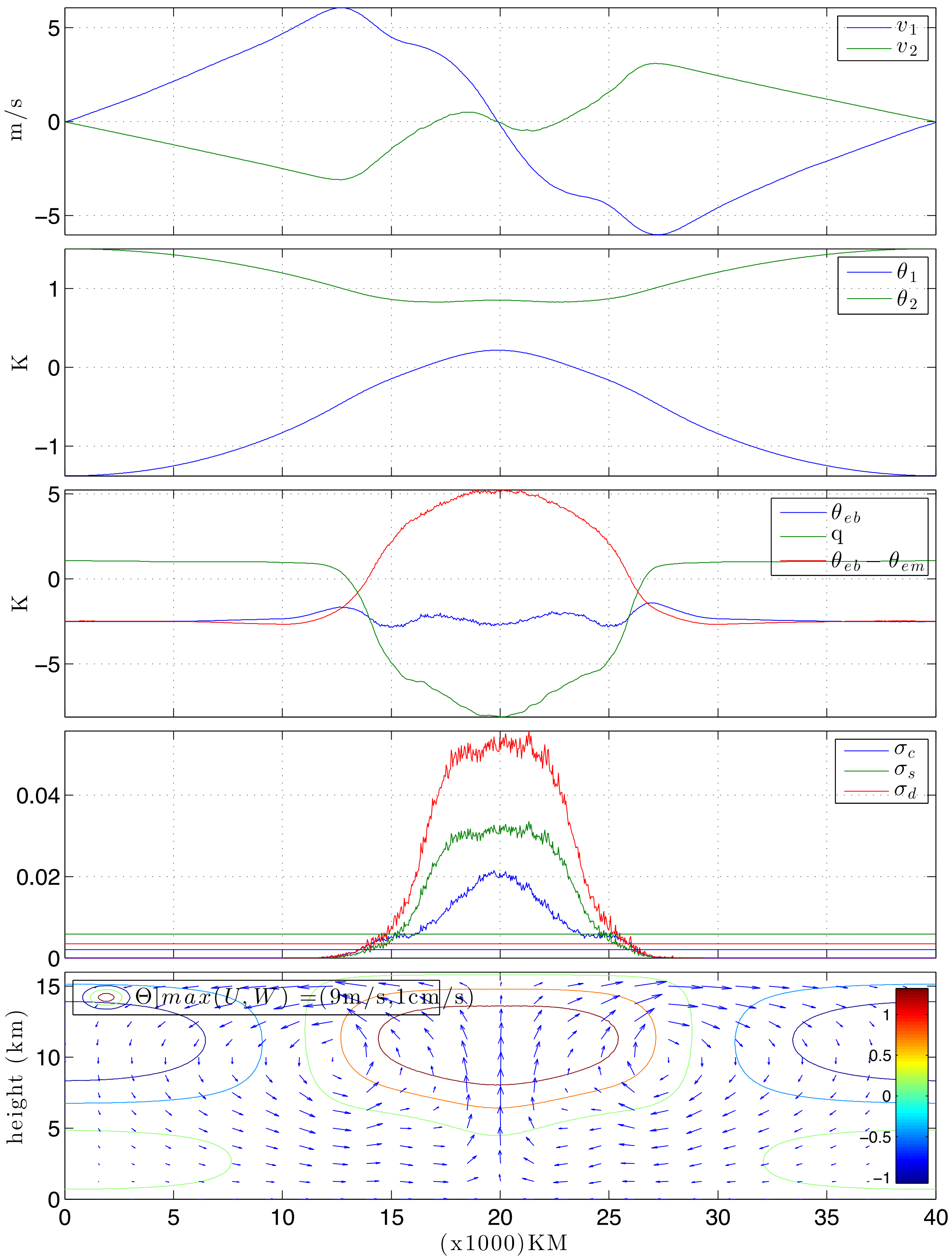
Warm Pool Background: Stochastic MC improves climatology:

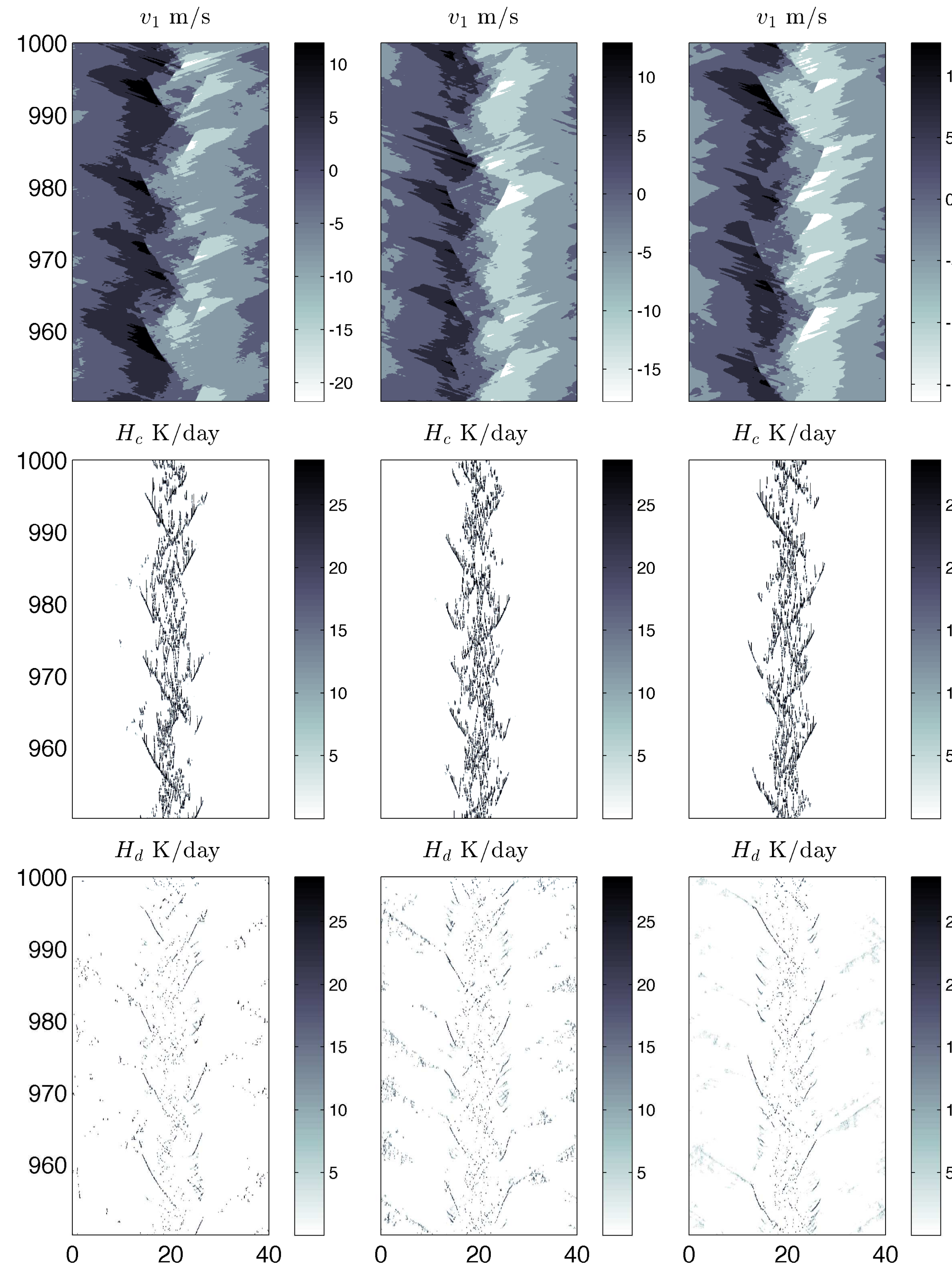




Precipitation triple Peak: Amplification of waves on edges of warm pool.

Walker Circulation and Waves





Some Statistics

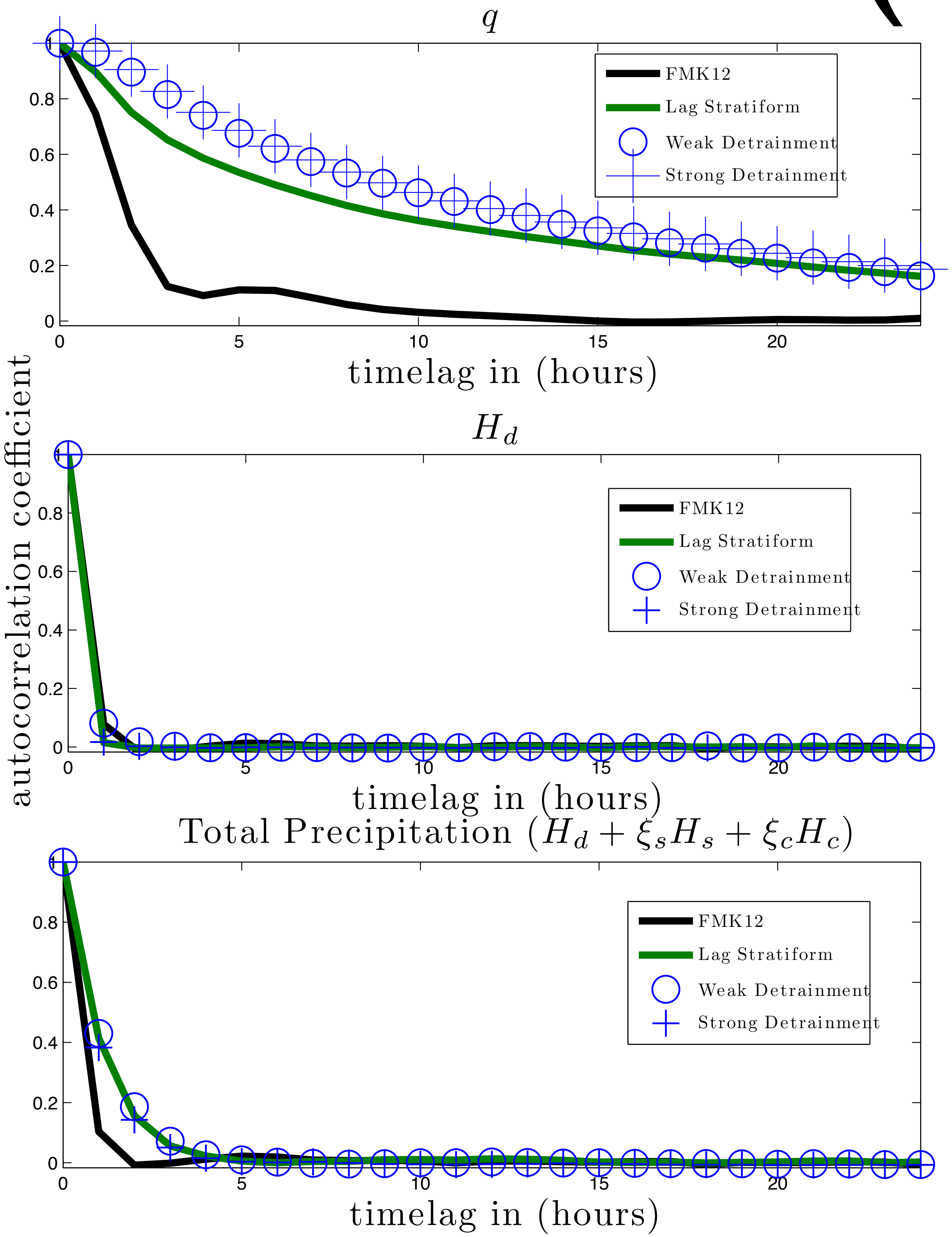
- Stochastic model boosts both variability of convection and strength of zonal winds
- Variability is insensitive to number of lattice sites n when in the range of 10 to 30
- Variability of deep convection decreases but congestus variability increases, consistent with stiffness of mean field limit--->deterministic chaos--->formation of persistent congestus decks.

Some Statistics

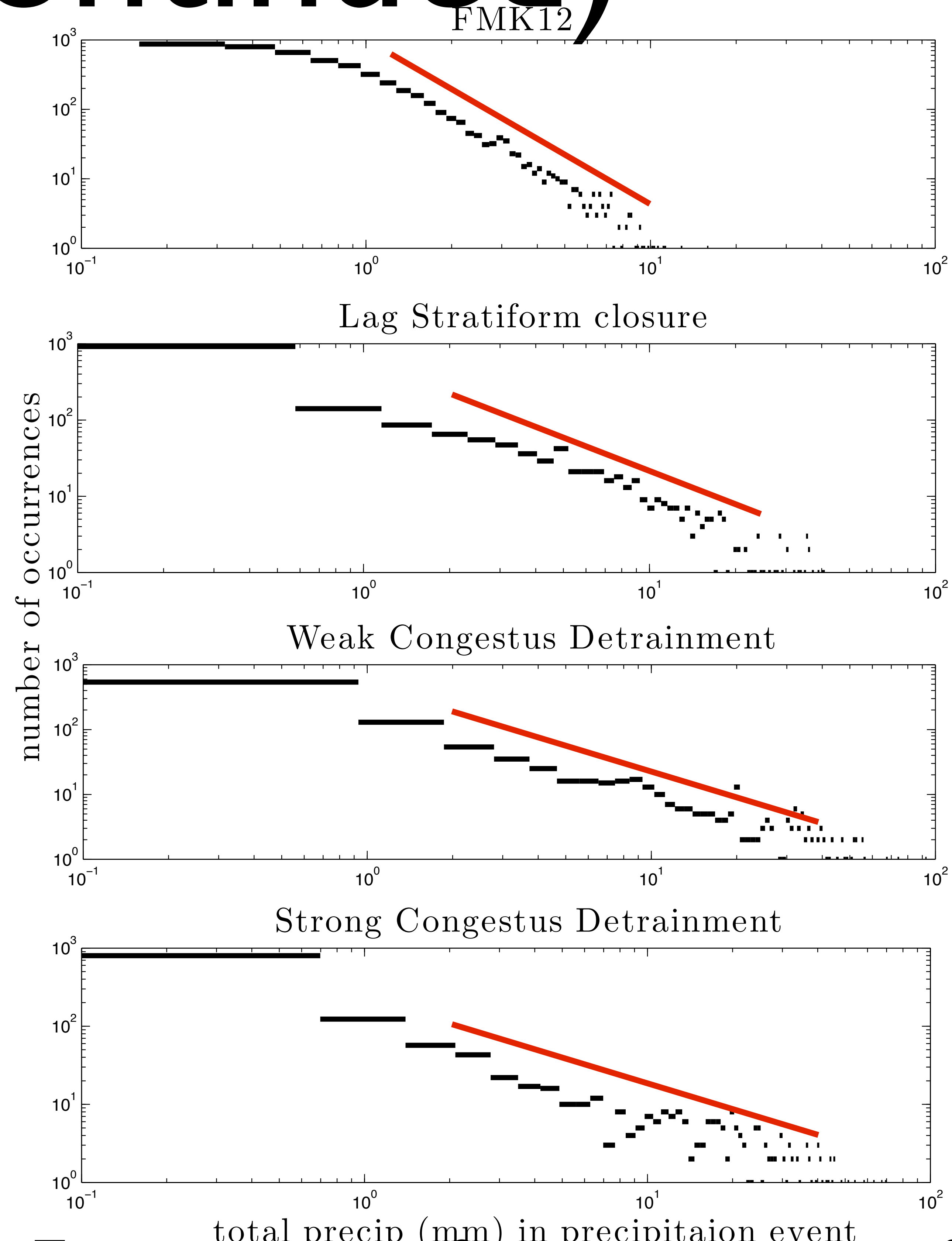
Regime	model	n	warm pool max (U, W)	std(H_d)	std(H_c)
FMK12	deterministic	-	$(4m/s, 4cm/s)$	0.97 K/Day	0.14 K/Day
FMK12	stochastic	$n = 30$	$(10m/s, 2cm/s)$	2.14 K/Day	2.83 K/Day
New heating closure	stochastic	$n = 10$	$(10m/s, 1cm/s)$	3.21 K/Day	4.21 K/Day
New heating closure	stochastic	$n = 30$	$(11m/s, 1cm/s)$	2.34 K/Day	4.26 K/Day
New heating closure	stochastic	$n = 100$	$(11m/s, 1cm/s)$	1.96K/Day	5.02 K/Day

- Stochastic model boosts both variability of convection and strength of zonal winds
- Variability is insensitive to number of lattice sites n when in the range of 10 to 30
- Variability of deep convection decreases but congestus variability increases, consistent with stiffness of mean field limit--->deterministic chaos--->formation of persistent congestus decks.

Statistics (continued)



Lag Correlation



Frequency of occurrence of precipitation events

Conclusions

- Multicloud model captures key features of organized tropical convection including MJO
- Here we showed how to build physically constrained stochastic model to account for sub-grid scale variability
- More chaotic behaviour for small values of n
- Dry gravity waves may play non-trivial roles in shutting convection in subsidence regions
- Congestus detrainment elucidate this clearly
- Variability increases as more physics are added