Course Title: Introduction to Knots, Knotoids and Virtual Knots.

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Summary: This is an introductory course on combinatorial knot theory with emphasis on virtual knot theory and knotoids. Virtual knot theory is a generalization of classical knot theory that includes the study of knots in thickened surfaces, and a diagrammatic approach to this structure. Knotoids are generalizations of classical knots to knotted intervals whose endpoints can be in any regions of the diagram. Knotoids have applications to the study of open-ended interval embeddings in space, and so to the embeddings of long molecules. One of the key topics in this study is cobordism of knots, and we shall discuss the relationship of knot and link cobordism with vortex reconnection for knotted vortices.

Course Outline:
1. Introductory Lecture
(a) Formulation of knot theory in terms of embeddings of circles in three dimensional space. Isotopy generated by piecewise linear triangle moves.
(b) Formulation of knot theory in terms of Reidemeister moves on knot and link diagrams. Proof of this combinatorial formulation by projection and
the use of the triangle moves.
(c) Basic invariants of knots and links: linking number, fundamental group, Fox coloring, Quandle.
(d) Spanning surfaces for knots and Seifert's algorithm.
(e) Rational Tangles and their fractions. Hints of the applications of rational tangles and rational knots to DNA recombination.
(f) Combinatorial properties of diagrams, including moves on arc diagrams and description of the Dynnikov unknotting algorithm.

2. The Jones Polynomial and the Kauffman Bracket
(a) Definition and Kauffman Bracket State summation. Properties of the bracket. Examples. Relation with the Jones polynomial.
(b) Alternating and Adequate knots and links and their bracket polynomials.
(c) Temperley Lieb algebra and representation of Artin Braid Group to Temperley Lieb algebra.

3. Introduction to Virtual Knot Theory and to Knotoids.
(a) Definitions and surface interpretations for virtual knots and links. Different categories of virtuals and related structures such as welded knots and links.
Basic invariants of virtuals including Odd Writhe, Jones polynomial and Manturov Parity Bracket.
(b) Knotoids and their virtual closures. Applications to polymer chains and proteins.
(c) Affine Index polynomial and cobordism of virtual knots.

4. Introduction to Khovanov Homology for Classical and Virtual Knots.
(a) Definitions and description of gradings, Frobenius algebra and methods of calculation.
(b) Results about cobordism of virtual knots via Khovanov homology.
(c) Cobordism and Vortex Reconnection (this part is self-contained with movies).

5. Applications and Relations of Knot theory and Quantum Computing
(a) Review of basic quantum theory.
(b) Description of simple quantum algorithms.
(c) Constructions of unitary representations of the Braid group — including Majorana Fermions and the Fibonacci Model via the bracket polynomial and
Temperley Lieb Recoupling Theory.
(d) Topological Quantum Computing.

Problem Classes
Each lecture has many examples associated with it. The problem classes will explore these examples with the students doing much of the
work. Since this course will develop as it goes along, I shall not formulate problems at this time, but each lecture will have a number of suggested problems and explorations associated with it.