Abstract:

We study the Zariski topology of the ind-groups of polynomial and free associative algebras $\Aut(K[x_1,...,x_n])$ (which is equivalent to the automorphism group of the affine space $\Aut(K^n))$) and $\Aut(K< x_1,..., x_n>$ via $\Ind$-schemes, toric varieties, approximations and singularities.

We obtain some nice properties of $\Aut(\Aut(A))$, where A is polynomial or free associative algebra over a field K. We prove that all $\Ind$-scheme automorphisms of $\Aut(K[x_1,...,x_n])$ are inner for $n\ge 3$, and all $\Ind$-scheme automorphisms of $\Aut(K< x_1,..., x_n>)$ are semi-inner. We also establish that any effective action of torus T_n on $\Aut(K< x_1,..., x_n>)$ is linearizable provided $|K|$ is infinity. That is, it is conjugated to a standard one. As an application, we prove that $\Aut(K[x_1,...,x_n])$ cannot be embedded into $\Aut(K<x_1,...,x_n>)$ induced by the natural abelianization. In other words, the {\it Automorphism Group Lifting Problem} has a negative solution.
We explore the close connection between the above results and the Jacobian conjecture, and Kontsevich-Belov conjecture, and formulate the Jacobian conjecture for fields of any characteristic.